Supplier Selection for Supply Chains in the Processed Food Industry

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Abstract

This work addresses an integrated framework for deciding about the supplier selection for supply chains in the processed food industry. The relevance of including tactical production and distribution planning in this procurement decision is assessed. The contribution of this paper is three-fold. Firstly, we propose a new two-stage stochastic mixed-integer programming model for the supplier selection in the process food industry that maximizes profit and minimizes risk of low customer service. Secondly, we reiterate the importance of considering main complexities of food supply chain management, such as: perishability of both raw materials and final products; uncertainty at both downstream and upstream parameters; and age dependent demand. Thirdly, we develop a solution method based on a multi-cut Benders decomposition and generalized disjunctive programming. Results indicate that sourcing and branding actions vary significantly between using an integrated and a decoupled approach. The integrated framework grasps the advantages of branding a given product as local through the acknowledgement of relevant supply chain costs. The proposed solution method proved to be suitable for solving large instances of this problem.

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1. Introduction

The importance of food supply chain management has been growing both at the industrial and scientific levels. The challenges faced in food supply chains are at the intersection of several disciplines and go beyond the traditional cost minimization concern. Particularly, in the process food industry, companies have to deal with higher uncertainties both upstream and downstream of the supply chain. These uncertainties are related to an ever increasing product variety, more demanding customers and a highly interconnected distribution network. This implies that companies operating in the process food industry need to manage the risk/cost trade-off without disregarding freshness, sustainability and corporate social responsibility issues (Maloni and Brown, 2006).

Effective and efficient decision support models and methods for supply chain planning are critical for this sector that is the largest manufacturing sector in Europe with a turnover of 1,048 billion euros, employing over 4.2 million people (FoodDrink Europe, 2014). It is widely acknowledged that the standard tools for supply chain management perform poorly when applied to process food industries (Rajurkar and Jain, 2011). The characteristics of food supply chains are significantly different from other supply chains. The main difference is the continuous change in the quality of raw materials - from the time they are shipped from the grower to the time they are processed at the plant, and in the quality of final products - from the time they are shipped from the plant to the time they are consumed. Ahumada and Villalobos (2009) state that food supply chains are more complex and harder to manage than other supply chains. The shelf-lives of raw, intermediate and final goods together with the strong uncertainties in the whole chain endanger a good supply chain management and planning. Despite the relevant specificities of process food industries, the consideration of perishability, customers willingness to pay and risk management at the strategic and planning levels has been seldom addressed in the literature.

The present work addresses the joint decision of choosing which suppliers to select, and the planning of procurement, production and distribution in a medium-term planning horizon. We focus on companies that process a main perishable raw material and convert it into perishable final food products. These conditions happen for instance in the dairy, fresh juices

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and tomato sauce industries. Within this scope we integrate strategic and tactical decisions in a common framework. We consider a setting in which companies have their plants and distribution channels well established and, therefore, the supply chain strategic decisions address the supplier selection and the related product branding. We classify the suppliers and the product branding as local or mainstream. This differentiation has already been made for the agri-business (Ata et al, 2012), but never for the food processing industry. However, there are several practical examples of the leverage than can be achieved in the customers willingness to pay by branding a product as local, and correspondingly sourcing raw-materials from local suppliers (Martinez, 2010; Oberholtzer et al, 2014; Frash et al, 2014). Therefore, the demand and the list price is assumed higher for fresh products that are branded and produced with local raw materials. In contrast, a similar product with a low remaining shelf-life and produced with mainstream raw materials has a lower demand and a lower list price.

Within this context, we propose a two-stage stochastic mixed-integer programming model to tackle this supplier selection problem. In the first-stage we decide the branding of products and the quantities to be procured in advance from each supplier. In the second-stage, we decide on the produced and transported quantities as well as on the quantities procured in the spot market. Uncertainties relate to the suppliers' raw material availability, suppliers' lead time, suppliers' spot market prices and demand for final products.

The different sources of uncertainty in this supplier selection problem render the corresponding stochastic programming model hard to solve as a considerable number of scenarios have to be considered. Therefore, to solve this problem we propose a multi-cut Benders decomposition approach. Moreover, to improve its convergence we test several acceleration techniques.

The remainder of this paper is as follows. Section 2 reviews relevant contributions in the supplier selection problem. Section 3 describes formally the problem and the proposed mathematical formulations. Section 4 is devoted to an illustrative example of a solution and to discuss uncertainty issues as well as the integration of the supplier selection with tactical planning decisions. Section 5 presents the implementation of a multi-cut Benders decomposition algorithm for this problem. Section 6 reports computational results for larger instances. Finally, Section 7 draws the main conclusions and indicates future lines of research.

2. Literature Review

The research on supplier selection problems has been traditionally divided between the operations management community that seeks an intuitive understanding of this problem, and the operations research community that explores the advantages of structuring this decision process and unveils hidden trade-offs through the use of techniques, such as mathematical programming (De Boer et al, 2001). For a thorough review of quantitative methods for the supplier selection problem the readers are referred to Ho et al (2010).

Most of the approaches to tackle the supplier selection problem are based on the Analytic Hierarchy Process (AHP) method to help decision makers in dealing with both uncertainty and subjectivity (Deng et al, 2014). There are also examples of works that combine AHP with other techniques, such as fuzzy linear programming (Sevkli et al, 2008). Data Envelopment Analysis (DEA) is also another widely used technique for supplier selection problems. For example, Kumar et al (2014) propose a methodology for the supplier selection taking into consideration the carbon footprints of suppliers as an attribute of the DEA model.

The most straightforward extension to the supplier selection problem is to couple it with decisions about inventory management (Aissaoui et al, 2007). Guo and Li (2014) integrate supplier selection and inventory management for multi-echelon systems. Other works incorporate other decisions, such as the carrier selection, besides determining the ordering quantities (Choudhary and Shankar, 2014).

More recently, researchers start to address other relevant aspects that can be studied under this general problem. Chen and Guo (2013) study the importance of supplier selection in competitive markets, and indicated that besides the more evident conclusion that dual sourcing can help to mitigate supply chain risks, strategic sourcing can also be an effective tool in approaching retail competition. Qian (2014) develops an analytic approach that incorporates extensive market data when determining the supplier selection in a make-to-order production strategy. With a more practical emphasis, Hong and Lee (2013) lay the foundations of a decision support system for effective risk-management when selecting suppliers in a spot market using measures similar to the Conditional Value-at-Risk (Rockafellar and Uryasev, 2000, 2002), such as the Expected Profit-Supply at Risk. Another relevant aspect is disruption management, especially regarding the suppliers' availability. Silbermayr and Minner (2014) develop an analytic model based on Markov decision processes in which suppliers may be completely

unavailable at a given (stochastic) interval of time. Due to the complexity of the optimal ordering policies, they also derive a heuristic approach.

Uncertainty has been incorporated in supplier selection problems either through stochastic programming or simulation. Moreover, distinct sources of uncertainty and different distributions for these uncertainties have been considered. Using a hybrid simulation optimization methodology, Ding et al (2005) are able to estimate the impact of the supplier selection on the tactical processes of the supply chain, and use this information back in the decision about which suppliers to select. Stochastic programming has proved to be a suitable methodology to address complex issues involved in supplier selection. Sawik (2013) proposes a similar model to the one presented in this paper as it is able to deal with multiple periods and it accounts for uncertainty through stochastic programming. Hammami et al (2014) propose a model for supplier selection considering uncertainty on the currency fluctuation. Through a case-study, the authors were able to show the value of the stochastic solution when compared to the deterministic model.

In light of this discussion, the main contributions of this paper to the supplier selection literature relate to accounting for uncertainty in the lead time, the consideration of distribution decisions and the emphasis on the characteristics of processed food industries. This last point is in line with an ongoing discussion about the sustainability and profitability of local sourcing for processed food industries (Schönhart et al, 2009).

In terms of solution methods, as the complexity of our problem required the use of a more sophisticated approach rather than solving the monolithic model, we show the applicability of a multi-cut Benders decomposition approach to this supplier selection problem. Moreover, we show that the valid inequalities that can be obtained from a generalized disjunctive programming formulation (Raman and Grossmann, 1994) can be used in order to tighten the Benders master problem.

3. Problem Statement and Mathematical Formulations

This section describes the supplier selection problem for supply chains in the processed food industry and the mathematical models that have been developed. Let $k = 1, ..., K$ be the products that are produced in the different factories ($f \in \mathsf{F}$). To produce these products the factories have to procure raw materials from the different available suppliers ($s \in \mathsf{S}$). These raw materials are classified either as mainstream ($u = 0$) or local ($u = 1$) depending on the distance between the supplier and the customers. Notice that the focus is on a divergent production structure in which a main

raw material (milk, oranges or tomatoes, for example) is transformed into several final products that vary only in the packaging or in the incorporation of small amounts of other ingredients. The planning horizon is divided into periods $t = 1, ..., T$. These periods correspond to months as we are dealing with tactical planning. After production, which can take place in regular schedules or overtime, products are transferred to retailers ($r \in \mathsf{R}$), which face an uncertain demand (D_{ktr}^{av}) that also depends on the products' age $a \in \mathsf{A_k} = \{0, ..., (SL_k - 1)\}$, where SL_k corresponds to the shelf-life of product k. Notice that raw materials also have limited age, $a \in \mathsf{A_u} = \{0, ..., (\hat{SL}_u - 1)\}$, where $\hat{SL}_u$ corresponds to the shelf-life of raw material u.

The stochastic data is initially given by continuous distributions and it is then modeled on some probability space, where V is a set of discrete scenarios with corresponding probabilities of occurrence ϕ_v, such that $\phi_v > 0$ and $\sum_v \phi_v = 1$. This discretization relates to the sampling strategy used in the computational experiments. In our two-stage stochastic program, we define the quantities to procure in advance from each supplier (s_{tsf}) and the branding strategy for each product: local ($\chi_k = 1$) or mainstream ($\chi_k = 0$) as first-stage decisions. Notice that when a product is branded as local it has to be produced only using local raw materials, whereas when a product is branded as mainstream, it is possible to use a dual sourcing strategy, and therefore procure raw materials from local and mainstream suppliers. Procured quantities in the spot market ($\bar{s}_{tsf}^v$), production quantities in regular schedules and in overtime (p_{kuft}^{av} and $\bar{p}_{kuft}^{av}$), transportation flows (x_{ktfr}^v), inventory levels of both raw materials ($\hat{w}_{utf}^{av}$) and final products (w_{ktr}^{av}), and demand satisfaction (ψ_{kutr}^{av}) are the second-stage decisions.

When making the first-stage decisions there are four sources of uncertainty to be considered. The first regards the demand for final products that is not known with certainty when negotiating contracts with suppliers and while deciding the branding strategy for each product. This reflects the real-world setting in which demand for fast moving consumer food goods is highly variable. The other three sources of uncertainty are related to the suppliers of raw materials: availability, lead time and spot price. While local suppliers have more uncertainty in the available quantities to be delivered through the considered planning horizon, mainstream suppliers have their lead times more volatile. Generally, local suppliers have less structured operations as well as smaller possibilities of engaging in risk mitigating strategies, such as producing in several locations. Mainstream suppliers are by definition located in farther locations, and therefore their lead times usually have more variability. Regarding the spot price, it usually depends more on the

negotiation undertaken and on the yields of that period than on the type of supplier. Negotiating in the spot market has a critical component of price uncertainty that is reflected in our model. Figure 1 illustrates the general problem framework and clarifies the connection between the formulation stages and the supply chain processes.
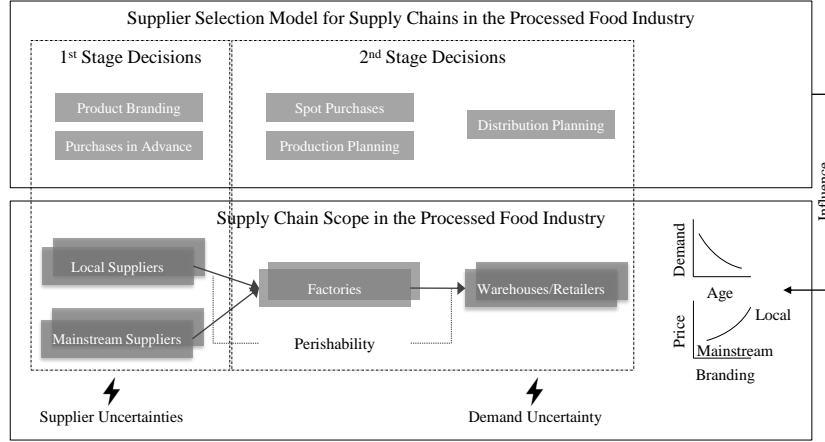
Figure 1: Overview of the scope of this research.

Consider the following indices, parameters, and decision variables that are used in the stochastic formulation.

Indices and Sets

$k \in \mathsf{K}$ final products

$u \in \mathsf{U}$ supplier / raw material classification: 0 for mainstream, 1 for local

$s \in \mathsf{S}$ suppliers

$f \in \mathsf{F}$ factories

$r \in \mathsf{R}$ retailers

$t \in \mathsf{T}$ periods

$a \in \mathsf{A}$ ages (in periods)

$v \in \mathsf{V}$ scenarios

$\mathsf{S_u}$ set of suppliers that supply raw material of type u

$\mathsf{A_k}$ set of ages that product k may have

$\mathsf{A_u}$ set of ages that raw-material u may have

Deterministic Parameters

SP_s	unit purchasing cost of raw material when bought in advance at supplier s
$TC_{sf}(\hat{TC}_{fr})$	transportation cost from supplier s (factory f) to factory f (retailer r)
$SL_k(\hat{SL}_u)$	shelf-life duration of product k (raw material u) right after being produced (time)
$HC_k(\hat{HC}_u)$	holding cost for product k (raw material u)
$PC_{kf}(\bar{PC}_{kf})$	normal (extra) production cost for product k when produced in factory f
LP_{ku}	list price for product k when branded as product of type u
E_{kf}	capacity consumption (time) needed to produce one unit of product k in factory f
$CP_{tf}(\bar{CP}_{tf})$	normal (extra) capacity of factory f available in period t

Stochastic Parameters

D_{ktr}^{av}	demand at retailer r for product k with age a in period t in scenario v
LT_{ts}^{v}	lead time offset of a shipment due to arrive in period t from supplier s in scenario v
$\bar{SP}_s^{v}$	unit purchasing cost of raw material when bought in a spot deal from supplier s in scenario v
AQ_{ts}^{v}	availability of raw material at supplier s for supplying in period t in scenario v
ϕ_v	probability of occurrence of scenario v

First-Stage Decision Variables

s_{tsf}	quantity of raw material procured in advance from supplier s in period t for supplying factory f
χ_k	equals 1, if product k is produced using only local raw materials (0 otherwise)
η	value-at-risk of the customer service

Second-Stage Decision Variables

τ_{tsf}^{av}	auxiliary variable that quantifies the amount of raw material procured in advance from supplier s that arrives in period t with age a for supplying factory f in scenario v
$\bar{s}_{tsf}^{v}$	quantity of raw material procured with a spot deal from supplier s in period t for supplying factory f in scenario v
$p_{kutf}^{av}(\bar{p}_{kutf}^{av})$	regular (overtime) produced quantity of product k in factory f using raw-materials of type u with age a in period t and scenario v
x_{ktfr}^{v}	transported quantity of product k from factory f to retailer r in period t and scenario v
$w_{ktr}^{av}(\hat{w}_{utf}^{av})$	initial inventory of product k (raw material u) with age a in period t in scenario v at retailer r (factory f), $a = 0,...,\min\{SL_k, t-1\}(a = 0,...,\min\{\hat{SL}_u, t-1\})$
ψ_{kutr}^{av}	fraction of the demand for product k produced with suppliers of type u delivered with age a in period t in scenario v from retailer r, $a = 0,...,\min\{SL_k, t-1\}$
δ_v	auxiliary variable for calculating the conditional value-at-risk of the customer service

3.1. Mixed-integer Linear Programming Formulation

The mixed-integer linear programming formulation of the problem is described next. The constraints that this problem is subject to are organized around the respective supply chain echelon.

3.1.1. Objective Function

The first part of objective function (1) maximizes the profit of the producer over the tactical planning horizon. Expected revenue, which depends on the products' branding is subtracted by supply chain related costs: purchasing costs of raw materials, both when bought in advance and or in the spot market, holding costs for raw materials and final products, transportation costs between the supply chain nodes, and normal and extra production costs. The second part the objective function maximizes the $(1 - \alpha) \cdot 100$ scenarios that yield the lowest customer service. This is an adaptation of the Conditional Value-at-Risk (Rockafellar and Uryasev, 2000, 2002) focusing on the customer service. Similar concepts, such as the supply-at-risk (SaR) were developed in a similar context Hong and Lee (2013). The main advantages of these type of metrics is that they are less influenced by very unlikely uncertain scenarios that could steer the whole solution structure to

a very conservative behavior. The risk-aversion of the objective function is controlled by weight parameter γ.

$$\max \sum_v \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D_{ktr}^{0v} \cdot \psi_{kutr}^{av} - \sum_{k,t,r,a<SL_k} HC_k \cdot w_{ktr}^{av} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x_{ktfr}^{v}$$

$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p_{kutf}^{av} + \bar{PC}_{kf} \cdot \bar{p}_{kutf}^{av}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}_{utf}^{av}$$

$$- \sum_{s,f} (\bar{SP}_s^v + TC_{sf}) \cdot \bar{s}_{tsf}^v - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau_{tsf}^{av}] + \gamma \cdot (\eta - \frac{1}{1-\alpha} \sum_v \phi_v \cdot \delta_v) \quad (1)$$

Decision variable η retrieves an approximation of the customer service value-at-risk and the auxiliary variable δ_v is defined using Eq.(2). The equation defines variable δ_v making it zero when the customer service in a given scenario is higher that the customer service value-at-risk. Otherwise, variable δ_v determines the difference between the customer service value-at-risk and the corresponding mean customer service of the scenario. Figure 2 shows a graphical representation of the customer service conditional value-at-risk. The graph represents the distribution of the random customer service (ω). The customer service conditional value-at-risk is given by $\mathbb{E}[\omega | \omega \leq VaR_\alpha(\omega)]$, where $VaR_\alpha(\omega)$ is the value-at-risk of the customer service with a confidence of α.

$$\delta_v \geq \eta - \sum_{k,u,t,r,a \in A_k} \psi_{kutr}^{av} \quad \forall v \in \mathsf{V} \quad (2)$$

3.1.2. Procurement Constraints

Eq.(3) translates the first stage decision that defines the quantity and the arrival time of raw material from each supplier (s_{tsf}) to a second stage decision variable (τ_{tsf}^{av}) that is affected by the uncertainty on the lead time (LT_{ts}^v) and by the availability of the supplier (AQ_{ts}^v). Lead time uncertainty offsets both the arrival time and the age of the product. Therefore, if $LT_{tc}^v = 2$ then a product that was supposed to arrive on $t = 2$ with age 0, will arrive on $t = 4$ with age 2 in scenario v. The availability AQ_{ts}^v is defined a fraction between 0 and 1.

$$\tau_{t+LT_{ts}^v,sf}^{LT_{ts}^v,v} = AQ_{ts}^v \cdot s_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, v \in \mathsf{V} \quad (3)$$

Over the entire planning horizon it is also important to enforce that the amount of product arriving with different ages is equal to the quantities that
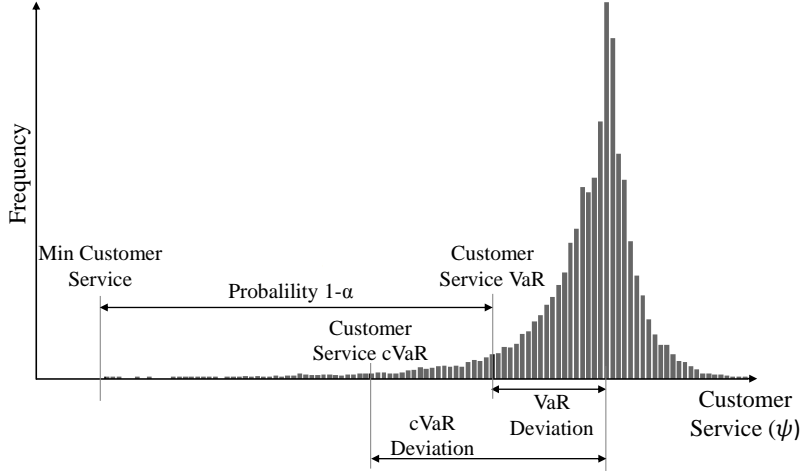
Figure 2: Graphical representation of the customer service conditional value-at-risk (adapted from Sarykalin et al (2008)).

the producer has ordered discounted by the availability and arrivals outside the planning horizon (4).

$$\sum_{t,a\in A_u} \tau_{tsf}^{av} = \sum_t AQ_{ts}^v \cdot s_{tsf} \quad \forall s \in S, f \in F, v \in V \tag{4}$$

Eq.(5) indicates that the inventory amount of raw material available to process at factory f with age 0 is equivalent to the amount bought in the spot market and bought in advance when there were no delivery delays.

$$\sum_{s\in S_u} (\tau_{tsf}^{0v} + \bar{s}_{tsf}^v) = \hat{w}_{utf}^{0v} \quad \forall u \in U, t \in T, f \in F, v \in V \tag{5}$$

3.1.3. Production Constraints

Eq.(6) acts as an inventory balance constraint for the stock of the raw-materials. It also updates the age of the raw material stock and takes into account the raw materials arriving with older ages (larger than 0). Notice that the domain of the inventory variables is constrained in its definition in the beginning of Section 3.

$$\hat{w}_{utf}^{av} = \hat{w}_{u,t-1,f}^{a-1,v} + \sum_{s\in S_u} \tau_{tsf}^{av} - \sum_k (p_{ku,t-1,f}^{a-1,v} + \bar{p}_{ku,t-1,f}^{a-1,v})$$
$$\forall u \in U, t \in \{2,...,T+1\}, f \in F, a \in A_u : a \geq 1, v \in V \tag{6}$$

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Eq.(7) forces the utilization of local raw material in case the product is branded as local.

$$p^{av}_{k0tf} + \bar{p}^{av}_{k0tf} \leq M(1 - \chi_k) \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, a \in \mathsf{A_u} : u = 0, v \in \mathsf{V} \quad (7)$$

Eqs.(8)-(9) limit both normal and extra production to the available factory capacity, respectively.

$$\sum_{k,u,a \in \mathsf{A_u}} E_{kf} p^{av}_{kutf} \leq CP_{ft} \quad \forall t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \quad (8)$$

$$\sum_{k,u,a \in \mathsf{A_u}} E_{kf} \bar{p}^{av}_{kutf} \leq \bar{CP}_{ft} \quad \forall t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \quad (9)$$

3.1.4. Distribution Constraints

Eq.(10) forces all production made in the different factories to flow to retailers within the same planning period.

$$\sum_{u,a \in \mathsf{A_u}} p^{av}_{kutf} = \sum_{r} x^{v}_{ktfr} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \quad (10)$$

The amount of final products entering each retailer corresponds to the inventory available to satisfy demand with age 0 (11). Therefore, notice that after processing the raw-materials, the age of the final products is always set to 0.

$$\sum_{f} x^{v}_{ktfr} = w^{0v}_{ktr} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, v \in \mathsf{V} \quad (11)$$

3.1.5. Demand Fulfillment Constraints

Eqs.(12)-(13) link the choice on the product branding as local ($\chi_k = 1, u = 1$) or mainstream ($\chi_k = 0, u = 0$) to the type of demand fulfilled that will determine the list price that the customer pays. These constraints define the revenue of the solution with the first term of the objective function (1).

$$\psi^{av}_{k0tr} \leq 1 - \chi_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \quad (12)$$

$$\psi^{av}_{k1tr} \leq \chi_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \quad (13)$$

Eq.(14) is another inventory balance constraint, but this time in the retailers' premises for final products. This constraint updates the age of final products' inventory throughout the planning periods.

$$w_{ktr}^{av} = w_{k,t-1,r}^{a-1,v} - \sum_{u} D_{k,t-1,r}^{0v} \psi_{ku,t-1,r}^{a-1,v} \qquad (14)$$

$$\forall k \in \mathsf{K}, t \in \{2, ..., (T+1)\}, r \in \mathsf{R}, a \in \mathsf{A_k} : a \geq 1, v \in \mathsf{V}$$

Eq.(15) keeps the demand fulfilled at different inventory ages below the respective demand profile (Amorim et al, 2013b).

$$\sum_{u} D_{ktr}^{0v} \psi_{kutr}^{av} \leq D_{ktr}^{av} \quad \forall k \in \mathsf{P}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \qquad (15)$$

Eq.(16) ensures that the demand fulfilled with different ages is always below the demand that the customer would be willing to pay for the product in the fresher state.

$$\sum_{u,a \in \mathsf{A_k}} \psi_{kutr}^{av} \leq 1 \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, v \in \mathsf{V} \qquad (16)$$

$$s_{tsf}, \tau_{tsf}^{v}, \bar{s}_{tsf}^{v}, w_{ktr}^{av}, \hat{w}_{utf}^{av}, p_{kutf}^{av}, \bar{p}_{kutf}^{av}, x_{ktfr}^{v}, \psi_{kutr}^{av}, \delta_v \geq 0; \chi_k \in \{0,1\};$$
$$\eta \in \Re \quad \forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \quad (17)$$

Property 3.1. *The supplier selection problem for supply chains in the processed food industry (1)-(17) has complete recourse, i.e., there exists a feasible second-stage decision for every first-stage decision and independently of the uncertainties (Wets, 1983).*

3.2. Generalized Disjunctive Programming Formulations

The supplier selection for supply chains in the processed food industry may be formulated with generalized disjunctive programming (GDP) (Raman and Grossmann, 1994). With GDP the different boolean decisions are represented through disjunctions. These disjunctions are then related through propositions. After having a problem formulated using GDP it is possible to derive other formulations, such as big-M (Nemhauser and Wolsey, 1988) or convex hull reformulations (Balas, 1985). For an overview of the fundamentals of GDP please refer to Grossmann and Trespalacios (2013); Castro and Grossmann (2012).

3.2.1. Initial GDP Formulation

One possible GDP formulation of the problem addressed in this paper is formalized next. Boolean variables Y_k indicate if product k is branded as local.

$$\max \sum_v \phi_v \Big[\sum_{k,u,t,r,a} LP_{ku} \cdot D_{ktr}^{0v} \cdot \psi_{kutr}^{av} - \sum_{k,t,r,a<SL_k} HC_k \cdot w_{ktr}^{av} - \sum_{k,t,f,r} \hat{T}C_{fr} \cdot x_{ktfr}^{v}$$

$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p_{kutf}^{av} + \bar{P}C_{kf} \cdot \bar{p}_{kutf}^{av}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{H}C_u \cdot \hat{w}_{utf}^{av}$$

$$- \sum_{s,f} (\bar{SP}_s^v + TC_{sf}) \cdot \bar{s}_{tsf}^v - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau_{tsf}^{av} \Big] + \gamma \cdot \Big(\eta - \frac{1}{1-\alpha} \sum_v \phi_v \cdot \delta_v \Big)$$

subject to:

$$\begin{bmatrix} Y_k \\ p_{k0tf}^v = 0 \\ \bar{p}_{k0tf}^v = 0 \\ \psi_{k0tr}^{av} = 0 \\ w_{ktr}^{av} = w_{k,t-1,r}^{a-1,v} - D_{k,t-1,r}^{0v}\psi_{k1,t-1,r}^{a-1,v} \\ 0 \le D_{ktr}^{0v}\psi_{k1tr}^{av} \le D_{ktr}^{av} \\ \sum_a \psi_{k1tr}^{av} \le 1 \end{bmatrix} \vee \begin{bmatrix} \neg Y_k \\ \psi_{k1tr}^{av} = 0 \\ w_{ktr}^{av} = w_{k,t-1,r}^{a-1,v} - D_{k,t-1,r}^{0v}\psi_{k0,t-1,r}^{a-1,v} \\ 0 \le D_{ktr}^{0v}\psi_{k0tr}^{av} \le D_{ktr}^{av} \\ \sum_a \psi_{k0tr}^{av} \le 1 \end{bmatrix} \quad \forall k \in \mathsf{K} \quad (18)$$

$$(2)\text{-}(6), \ (8)\text{-}(11)$$

$$s_{tsf}, \tau_{tsf}^v, \bar{s}_{tsf}^v, w_{ktr}^{av}, \hat{w}_{utf}^{av}, p_{kutf}^{av}, \bar{p}_{kutf}^{av}, x_{ktfr}^v, \psi_{kutr}^{av}, \delta_v \ge 0; \eta \in \Re$$
$$\forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \quad (19)$$

$$Y_k \in \{True, False\} \quad \forall k \in \mathsf{K} \quad (20)$$

Disjunctions (18) use the information about the branding choice to set to zero part of the linear decision variables. In particular, when branding a product as local (Y_k) it is possible to set to zero both the decisions variables related with production and demand fulfillment of products using mainstream raw-materials ($p_{k0tf}^v, \bar{p}_{k0tf}^v$ and ψ_{k0tr}^{av}).

3.2.2. Improved GDP Formulation

One of the advantages of formulating a problem using GDP is the potential of deriving alternative models. The supplier selection for supply chains in the processed food industry can be addressed from a strategic level in which it is necessary to first choose between three different options: (1) use local single sourcing to produce all products, (2) use mainstream single sourcing to produce all products, (3) use dual sourcing and choose individually which product to brand as local and mainstream. The formulations that are presented next make no use of global constraints as it is possible to fit all constraints inside of the mentioned disjunctions. Boolean variables Z_i indicate if option (i) is chosen.

$$
\max \sum_v \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^v_{ktfr}
$$

$$
- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}
$$

$$
- \sum_{s,f} (\bar{SP}^v_s + TC_{sf}) \cdot \bar{s}^v_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf}] + \gamma \cdot (\eta - \frac{1}{1-\alpha} \sum_v \phi_v \cdot \delta_v)
$$

subject to:

$$
\begin{bmatrix}
Z_1 \\
s_{tsf}, \bar{s}^v_{tsf}, \tau^v_{tsf} = 0 \,\forall s \in \mathsf{S_0} \\
\hat{w}^{av}_{0fd} = 0 \\
p^v_{k0tf}, \bar{p}^v_{k0tf} = 0 \\
\psi^{av}_{k0tc} = 0 \\
\delta_v \geq \eta - \sum_{k,t,r,a \in \mathsf{A_k}} \psi^{av}_{k1tr} \\
\tau^{LT^v_{ts},v}_{t+LT^v_{ts},sf} = AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S_1} \\
\sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_t AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S_1} \\
\sum_{s \in \mathsf{S_1}} (\tau^{0v}_{tsf} + \bar{s}^v_{tsf}) = \hat{w}^{0v}_{1tf} \\
\hat{w}^{av}_{1tf} = \hat{w}^{a-1,v}_{1,t-1,f} + \sum_{s \in \mathsf{S_1}} \tau^{av}_{tsf} - \sum_k p^{a-1,v}_{k1,t-1,f} \\
\sum_{k,a \in \mathsf{A_u}:u=1} E_{kf} p^{av}_{k1tf} \leq CP_{ft} \\
\sum_{k,a \in \mathsf{A_u}:u=1} E_{kf} \bar{p}^{av}_{k1tf} \leq \bar{C}P_{ft} \\
\sum_{a \in \mathsf{A_u}:u=1} p^{av}_{k1tf} = \sum_r x^v_{ktfr} \\
\sum_f x^v_{ktfr} = w^{0v}_{ktr} \\
w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - \sum_u D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k1,t-1,r} \\
D^{0v}_{ktr} \psi^{av}_{k1tr} \leq D^{av}_{ktr} \\
\sum_{a \in \mathsf{A_k}} \psi^{av}_{k1tr} \leq 1
\end{bmatrix}
\vee
\begin{bmatrix}
Z_2 \\
s_{tsf}, \bar{s}^v_{tsf}, \tau^v_{tsf} = 0 \,\forall s \in \mathsf{S_1} \\
\hat{w}^{av}_{1fd} = 0 \\
p^v_{k1tf}, \bar{p}^v_{k1tf} = 0 \\
\psi^{av}_{k1tc} = 0 \\
\delta_v \geq \eta - \sum_{k,t,r,a \in \mathsf{A_k}} \psi^{av}_{k0tr} \\
\tau^{LT^v_{ts},v}_{t+LT^v_{ts},sf} = AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S_0} \\
\sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_t AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S_0} \\
\sum_{s \in \mathsf{S_0}} (\tau^{0v}_{tsf} + \bar{s}^v_{tsf}) = \hat{w}^{0v}_{0tf} \\
\hat{w}^{av}_{0tf} = \hat{w}^{a-1,v}_{0,t-1,f} + \sum_{s \in \mathsf{S_0}} \tau^{av}_{tsf} - \sum_k p^{a-1,v}_{k0,t-1,f} \\
\sum_{k,a \in \mathsf{A_u}:u=0} E_{kf} p^{av}_{k0tf} \leq CP_{ft} \\
\sum_{k,a \in \mathsf{A_u}:u=0} E_{kf} \bar{p}^{av}_{k0tf} \leq \bar{C}P_{ft} \\
\sum_{a \in \mathsf{A_u}:u=0} p^{av}_{k0tf} = \sum_r x^v_{ktfr} \\
\sum_f x^v_{ktfr} = w^{0v}_{ktr} \\
w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - \sum_u D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k0,t-1,r} \\
D^{0v}_{ktr} \psi^{av}_{k0tr} \leq D^{av}_{ktr} \\
\sum_{a \in \mathsf{A_k}} \psi^{av}_{k01tr} \leq 1
\end{bmatrix}
$$

$$
\vee
\begin{bmatrix}
Z_3 \\
(2) - (11) \\
(18)
\end{bmatrix} (21)
$$

$$
Z_1 \,\underline{\vee}\, Z_2 \,\underline{\vee}\, Z_3 \tag{22}
$$

$$
Z_3 \iff Y_k \,\underline{\vee}\, \neg Y_k \tag{23}
$$

$$
Z_3 \implies \vee_k [\neg Y_k] \tag{24}
$$

$$
s_{tsf}, \tau^v_{tsf}, \bar{s}^v_{tsf}, w^{av}_{ktr}, \hat{w}^{av}_{utf}, p^{av}_{kutf}, \bar{p}^{av}_{kutf}, x^v_{ktfr}, \psi^{av}_{kutr}, \delta_v \geq 0; \eta \in \Re
$$
$$
\forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \tag{25}
$$

$$
Z_1, Z_2, Z_3 \in \{True, False\} \tag{26}
$$

$$
Y_k \in \{True, False\} \quad \forall k \in \mathsf{K} \tag{27}
$$

Disjunctions (21) use the information about the sourcing strategy choice to narrow the search space. The first and the second disjunctions (Z_1 and Z_2) set to zero all variables related to mainstream sourcing / branding and to local sourcing / branding, respectively. The third disjunction (Z_3) has a embedded disjunction similar to the one presented in the previous section (cf. Section 3.2.1). Logic proposition (22) forces the choice of one of the sourcing strategies. Logic proposition (23) states that if a dual sourcing strategy is chosen then it is necessary to decide for each product the branding (mainstream or local). Finally, logic proposition (24) ensures that when choosing a dual sourcing strategy there exists at least one product that is not branded as local.

The use of GDP modeling in this context will be clearer in Section 5.1 where the related convex hull reformulation is used.

4. Illustrative Example

In this section we explore with an illustrative example the importance of considering uncertainty and the impact of the integrated approach in the supplier selection for supply chains in the processed food industry.

4.1. Instances Generation

We consider a mainstream and a local supplier ($S = 2$) that supply raw-material to a factory ($F = 1$). This factory converts the raw-material into six products ($K = 6$) and fulfills demand for 3 retailers over a horizon of 1 year, discretized in $T = 12$ time periods. Purchasing raw-materials in advance from the mainstream supplier costs 0.3 monetary units and from the local supplier it costs 0.5. Both raw-materials have a shelf-life of 3 periods. All transportation costs are given in Table 1. Holding costs of both raw and final products are 0.05. All final products spend one unit of time of capacity to be produced ($E_{kf} = 1$). There is constant available normal capacity throughout the planning horizon that is equal to the expected demand across all scenarios for products in its fresher state. Therefore, the capacity per period CP_{tf} is determined as

$$CP_{tf} = \sum_{k,r} \mathbb{E}(D_{ktr}^{0v}), \quad \forall t, f.$$

Extra capacity ($\bar{CP}_{tf}$) is 25% of the normal one. Producing within the normal capacity (PC_{kf}) costs 0.1, while using the extra capacity costs 10% more ($\bar{PC}_{kf} = 0.11$).

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Table 1: Transportation costs.

Origin	Destination	TC_{sf} and TC_{fr}
Mainstream Supplier	Factory	0.06
Local Supplier	Factory	0.02
Factory	Retailer 1	0.01
Factory	Retailer 2	0.02
Factory	Retailer 3	0.03

The remaining deterministic parameters for products k (SL_k and LP_{ku}) and the statistics used to generate the demand for final products (D_{ktr}^{av}) are given in Table 2. Note that the list price for a product branded as local is 10% higher than for a product branded as mainstream. Demand for final products follows a gamma distribution (Van Donselaar et al, 2006). For more information about these values please refer to Amorim et al (2013a). We consider that final products have a medium product quality risk, and therefore, a linear decay of demand over the age of the product (Amorim et al, 2013b).

Table 2: Demand related product parameters.

Product	SL_k	LP_{k0}	LP_{k1}	$\mathbb{E}(D_{ktr}^{0v})$	$\sigma(D_{ktr}^{0v})$
1	3	2.49	2.74	52.80	11.09
2	2	2.7	2.97	76.80	22.27
3	2	2.99	3.29	135.20	25.69
4	3	1.69	1.86	52.80	11.09
5	3	0.62	0.68	76.80	22.27
6	2	2.68	2.95	135.20	25.69

As already mentioned, the supply uncertainties are related to three stochastic parameters: LT_{ts}^v, $\bar{SP}_s^v$ and AQ_{ts}^v. The local supplier has no uncertainty in the delivery dates and the mainstream is characterized by an exponential negative offset (Qian, 2014) with an expected value of one period. The raw-material spot cost for both suppliers is on average 10% more expensive than the corresponding cost when buying in advance (SP_s). This cost surplus follows a normal distribution (Fu et al, 2010) and has a coefficient of variation of one. Finally, the mainstream supplier has no availability issues and the local supplier has a randomly distributed availability (Federgruen and Yang, 2008) in the interval $[0.7, 1]$. The value for α is set to 0.95 in all experiments.

To measure the importance of uncertainty we use the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS). These two metrics are often used to evaluate the importance of using stochastic solutions over deterministic approximations.

Let RP be the optimal value of solving the two-stage stochastic programming problem (1)-(17), and consider WS_v the optimal value of solving the same problem only for scenario $v \in V$. Then, the wait-and-see (WS) solution is determined as the expected value of WS_v over all scenarios. EVPI is obtained with the difference between WS and the RP:

$$EVPI = WS - RP. \tag{28}$$

The EVPI may be seen as the cost of uncertainty or the maximum amount the decision maker is willing to pay in order to make a decision without uncertainty. Higher EVPIs mean that uncertainty is important to the problem (Wallace and Ziemba, 2005).

Now, let EV be the solution obtained by solving the problem in which stochastic parameters are replaced by their expected values. The expected value of using the first-stage decisions of EV over all scenarios is denoted as EEV (expected value of using the EV solution). VSS is obtained as follows:

$$VSS = RP - EEV. \tag{29}$$

VSS estimates the profit that may be obtained by adopting the stochastic model rather than using the approximated mean-value one. Therefore, VSS shows the cost of ignoring the uncertainty in choosing a first-stage decision (Birge and Louveaux, 1997).

In general, there may be cases in which fixing first-stage decision variables may result in unfeasible EEV problems. However, as the supplier selection for supply chains in the processed food industry has complete recourse that is not the case (cf. Property 3.1).

To approximate both EVPI and VSS for the supplier selection problem we have sampled 1296 scenarios with equal probability from the stochastic parameters and solved the supplier selection problem with parameter γ set to 0 and 100. Table 3 presents the results for these two metrics.

Both metrics are far from zero and they seem to increase with the risk-aversion of the solution. The importance of uncertainty grows along with the concerns about customer service. Acquiring more precise information about uncertain parameters seems not to be as critical as acknowledging the stochastic nature of this problem. The values of the EVPI show that the

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Table 3: EVPI and VSS values for the supplier selection problem.

	WS	RP	EEV	EVPI	VSS	EVPI/RP	VSS/RP
$\gamma = 0$	36910	36594	20657	316	15937	0.9%	43.6%
$\gamma = 100$	36910	36502	18373	408	18129	1.1%	49.7%

recourse decisions are able to correct substantially previous actions. The relative VSS values are higher than 40%, which denotes the importance of incorporating the variability of the possible outcomes instead of using expected values to make supplier selection decisions in the processed food industry context.

4.3. Integrated vs. Decoupled Approach

In order to assess the impact of considering an integrated approach to the supplier selection and production-distribution planning, we have performed sensitivity analysis on the key parameters that may influence the advantages of this approach. Trough preliminary computational tests, weight γ is changed such that customer service conditional value at risk (cscVaR) is either 90% or 95%, the shelf-life of the raw materials ($\hat{SL}_u$) is varied between 3 and 9 periods and the list price of a product branded as local (LP_{k0}) is 0% or 10% higher than the mainstream list price.

Solutions are obtained with a sample average approximation scheme (Shapiro and Homem-de Mello, 1998). We sampled 81 scenarios and solved 50 instances of the approximating stochastic programming. We then evaluated the objective function by solving 1296 independently sampled scenarios. In the Decoupled approach, first problem (1)-(17) is solved without production-distribution planning constraints (6)-(11). Afterwards, having the procurement and demand fulfillment variables fixed the overall problem is solved. In the Integrated approach, problem (1)-(17) is solved simultaneously.

In Tables 4 and 5 we report for the Decoupled and Integrated approach, respectively, several indicators:

- profit - first part of objective function (1)

- % local - quantity of local procured raw material over the total procured raw material, $\sum(\tau_{ts'f}^{av} + \bar{s}_{ts'f}^{v})/\sum(\tau_{tsf}^{av} + \bar{s}_{tsf}^{v}) : s' \in \mathsf{S}_1$.

- % spoiled - amount of spoiled raw material over the total procured raw material, $\sum w_{utf}^{\hat{SL}_u v}/\sum(\tau_{tsf}^{av} + \bar{s}_{tsf}^{v})$.

- % raw - total procured raw material over the total demand $\sum(\tau_{tsf}^{av} + \bar{s}_{tsf}^{v})/\sum D_{ktr}^{0v}$.

- # local - number of products that the model chose to be branded as local, $\sum \chi_k$

Table 4: Indicators for the decoupled approach.

LP_{k1}	+0%				+10%			
$\hat{SL}_u$	9		3		9		3	
cscVaR	90%	95%	90%	95%	90%	95%	90%	95%
Profit	32801	32496	33064	32866	32966	32734	33151	32920
%local	3.2%	4.2%	2.8%	3.6%	3.1%	3.5%	3.2%	6.3%
%spoiled	0.2%	0.0%	6.0%	7.4%	0.0%	0.0%	5.9%	7.4%
%raw	104.4%	107.9%	105.3%	108.5%	105.2%	107.7%	105.0%	108.8%
#local	0	0	0	0	0	0	0	0

Table 5: Indicators for the integrated approach.

LP_{k1}	+0%				+10%			
$\hat{SL}_u$	9		3		9		3	
cscVaR	90%	95%	90%	95%	90%	95%	90%	95%
Profit	35232	34834	35345	35114	35206	34863	35368	34960
%local	3.4%	3.8%	2.5%	2.9%	56.3%	66.8%	64.1%	78.5%
%spoiled	0.1%	0.1%	6.9%	6.5%	0.0%	0.0%	3.8%	3.9%
%raw	103.6%	106.0%	106.0%	106.7%	112.4%	117.3%	116.1%	121.6%
#local	0	0	0	0	3	4	4	5

Comparing the results of both approaches it is clear that the integrated approach is relevant as it is able to grasp the advantages of having a product branded as local in order to dilute key supply chain costs. These costs may arise, for example, from producing in overtime. With the 10% increase in the list price, the decoupled approach does not lead to any product being branded as local, whereas the integrated approach suggests to brand several products this way. These decisions force the amount of local raw material to rise considerably above 50%. This indicates that the sourcing/branding decisions in the processed food industry may need to take a wider view over the supply chain than just focusing on the procurement processes. Results also show that the logistics characteristics of local suppliers (for instance

smaller and less variable lead time) may not constitute a significant attribute to rise considerably the amount of raw-materials bought from such suppliers.

Taking advantage of the higher customer willingness to pay for local products increases profit and lowers the amount of spoiled raw-material. The lower levels of raw-material reaching their shelf-lives is related with the difference in the lead time uncertainty between mainstream and local suppliers. On the other hand, both higher service levels and lower shelf-lives of raw materials lead to an increase in the amount of spoiled material and an increase of the quantities purchased from suppliers in relation to the actual demand. The quantity of raw-materials procured is also related to the amount of local supplies due to the availability uncertainties that corresponding suppliers are subject to.

Across all solutions a dual sourcing strategy is used. Regarding the trade-off between profit and cscVaR, it seems that small losses in the average profit may lead to substantial shift in cscVaR.

5. Multi-Cut Benders Decomposition Algorithm

Even while using a sample average approximation scheme, it is necessary to solve a large number of two-stage stochastic programs that are not trivial to solve using monolithic models as the ones described in Section 3. In this Section we discuss a multi-cut Benders decomposition approach that can be embedded in the sample average approximation scheme in a hybrid solution approach to solve this supplier selection problem (Santoso et al, 2005).

Benders decomposition (Benders, 1962) is a solution approach that is more commonly named as L-Shaped method when applied to stochastic programming (Van Slyke and Wets, 1969). This solution approach partitions the complete formulation into two models. The Benders master problem approximates the cost of the scenarios in the space of first-stage decision variables and the Benders subproblems are obtained from the original one by fixing the first stage variables to the values obtained in the master problem. This solution approach iterates between these two models improving the lower bounds obtained in the master problem (LB) with information coming from the upper bounds of the subproblems (UB).

The resulting Benders subproblems (BSP) that may be decomposed for each scenario $v \in \mathsf{V}$ are formulated for each iteration i as follows:

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$$\max \sum_v \phi_v \big[\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^v_{ktfr}$$

$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}$$

$$- \sum_{s,f} (\bar{SP}^v_s + TC_{sf}) \cdot \bar{s}^v_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf} \big] - \gamma \cdot \big(\frac{1}{1-\alpha} \sum_v \phi_v \cdot \delta_v \big)$$

subject to:

$$\delta_v \geq \eta^i - \sum_{k,u,t,r,a \in \mathsf{A_k}} \psi^{av}_{kutr} \qquad (30)$$

$$\tau^{LT^v_{ts},v}_{t+LT^v_{ts},sf} = AQ^v_{ts} s^i_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F} \qquad (31)$$

$$\sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_t AQ^v_{ts} s^i_{tsf} \quad \forall s \in \mathsf{S}, f \in \mathsf{F} \qquad (32)$$

$$(5)\text{-}(6)$$

$$p^{av}_{k0tf} + \bar{p}^{av}_{k0tf} \leq M(1-\chi^i_k) \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, a \in \mathsf{A_u} : u = 0 \qquad (33)$$

$$(8)\text{-}(11)$$

$$\psi^{av}_{k0tr} \leq 1 - \chi^i_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k} \qquad (34)$$

$$\psi^{av}_{k1tr} \leq \chi^i_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k} \qquad (35)$$

$$(14) \text{ - } (16)$$

$$\tau^v_{tsf}, \bar{s}^v_{tsf}, w^{av}_{ktr}, \hat{w}^{av}_{utf}, p^{av}_{kutf}, \bar{p}^{av}_{kutf}, x^v_{ktfr}, \psi^{av}_{kutr}, \delta_v \geq 0;$$
$$\forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R} \qquad (36)$$

In the Benders subproblems we use optimal first-stage solution of variables s_{tsf}, χ_k and η coming from the solution of the master problem in the previous iteration i that are denoted as s^i_{tsf}, χ^i_k and η^i.

The Benders master problem (BMP) is formulated as follows:

$$\max \gamma \cdot \eta - \sum_v \phi_v \cdot \theta_v \qquad (37)$$

$$
\begin{aligned}
\theta_v \geq -\eta \Gamma^{vi} &+ \sum_{t,s,f} AQ^v_{ts} s_{tsf} \Delta^{vi}_{tsf} + \sum_{t,s,f} AQ^v_{ts} s_{tsf} \Theta^{vi}_{sf} + \sum_{k,t,f,a} M(1-\chi_k)\Lambda^{avi}_{ktf} \\
&+ \sum_{t,f} CP_{ft} \Xi^{vi}_{tf} + \sum_{t,f} \bar{C}P_{ft} \Pi^{vi}_{tf} + \sum_{k,t,r,a} (\Omega^{avi}_{ktr} - \chi_k \Omega^{avi}_{ktr}) + \sum_{k,t,r,a} \chi_k \Upsilon^{avi}_{ktr} \\
&+ \sum_{k,t,r,a} D^{av}_{ktr} \Phi^{avi}_{ktr} + \sum_{k,t,r} \Psi^{vi}_{ktr} \quad \forall v \in \mathsf{V}(38)
\end{aligned}
$$

$$s_{tsf} \geq 0; \chi_k \in \{0,1\}; \eta \in \Re \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}$$

In the Benders master problem we use the dual values Γ^{vi}, Δ^{vi}_{tsf}, Θ^{vi}_{sf}, Λ^{avi}_{ktf}, Ξ^{vi}_{tf}, Π^{vi}_{tf}, Ω^{avi}_{ktr}, Υ^{avi}_{ktr}, Φ^{avi}_{ktr} and Ψ^{vi}_{ktr} of constraints (30), (31), (32), (33), (8), (9), (34), (35), (15) and (16), respectively, in iteration i. Note that after preliminary experiments we chose to add an optimality cut per scenario (38) in the master problem instead of a single global cut (Birge and Louveaux, 1988; You and Grossmann, 2013). As mentioned before this problem has complete recourse, therefore, no feasibility cuts are necessary (cf. Property 3.1).

Remark 5.1. *It is possible to use a more intensive multi-cutting strategy by introducing cuts per time period t. However, it is necessary to distinguish the customer service in each period and, therefore, to rewrite Eq.(2) as $\delta_v \geq \eta - \sum_{k,u,r,a \in \mathsf{A}_k} \psi^{av}_{kutr} \quad \forall t \in \mathsf{T}, v \in \mathsf{V}$. Consequently dual values Γ^{vi} have to be extended to incorporate the time dimension.*

Algorithm 1 outlines the main steps of the solution approach, where ε is a very small threshold value.

Algorithm 1: Outline of Benders solution approach.

---

initialize s_{tsf}^0, χ_k^0 and η^0;
set $UB = \infty$ and $LB = -\infty$;
while $UB - LB > \varepsilon$ **do**
  Solve BSP;
  Get Second-Stage Variables;
  Update UB;
  Get Duals;
  Add Optimality Cuts to BMP;
  Get s_{tsf}, χ_k, η;
  Update LB;
  Update s_{tsf}^i, χ_k^i and η^i on the BSP;
end

The Benders decomposition algorithm is known to have some convergence issues that can be mitigated through acceleration techniques. In the remainder of this section we discuss approaches that can be used to this end.

5.1. Tightening the Benders Master Problem

The resulting BMP from the original formulation (cf. Section 3.1) has no constraints besides the on-the-fly optimality cuts and the decision variables domain constraints. Therefore, before "enough" cuts are added into the BMP the convergence of the solution approach is expected to be rather slow. The lack of first-stage constraints is related to two characteristics of this problem. Firstly, the uncertainty of suppliers both in the available quantity and on the lead time forces a translation of the purchased quantities in advance s_{tsf} into a second-stage decision variable τ_{tsf}^{av}. Secondly, the two main first-stage decision variables (s_{tsf} and χ_k) are not tightly related due to the multi-echelon scope of the supplier selection for supply chains in the processed food industry, which separates the acquisition of raw-materials u from the transformation and selling of final products k.

With the GDP formulation presented in Section 3.2.2 we are able to tighten the first-stage decisions by introducing the three disjunctions Z_i related with the sourcing strategy. Transforming the GDP formulation into a mixed-integer programming model by applying classical Boolean algebra rules to convert the logic propositions (Williams, 1999) and reformulating the disjunctions using a hull reformulation (Lee and Grossmann, 2000) results in the following first-stage constraints that are added to the BMP.

$$z_1 \leq \chi_k \quad \forall k \in \mathsf{K} \tag{39}$$

$$1 - z_2 \geq \chi_k \quad \forall k \in \mathsf{K} \tag{40}$$

$$K - z_3 \geq \sum_k \chi_k \tag{41}$$

$$z_1 + z_2 + z_3 = 1 \tag{42}$$

$$s_{tsf} = s_{tsf}^2 + s_{tsf}^3 \quad \forall t \in \mathsf{T}, s \in \mathsf{S_0}, f \in \mathsf{F} \tag{43}$$

$$s_{tsf} = s_{tsf}^1 + s_{tsf}^3 \quad \forall t \in \mathsf{T}, s \in \mathsf{S_1}, f \in \mathsf{F} \tag{44}$$

$$0 \leq s_{tsf}^1 \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft})z_1 \quad \forall t \in \mathsf{T}, s \in \mathsf{S_1}, f \in \mathsf{F} \tag{45}$$

$$0 \leq s_{tsf}^2 \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft})z_2 \quad \forall t \in \mathsf{T}, s \in \mathsf{S_0}, f \in \mathsf{F} \tag{46}$$

$$0 \leq s_{tsf}^3 \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft})z_3 \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F} \tag{47}$$

$$z_1, z_2, z_3 \in \{0, 1\} \tag{48}$$

Note that χ_k are the binary variables resulting from transforming Boolean variables Y_k and z_1, z_2, z_3 are the binary variables resulting from transforming Boolean variables Z_1, Z_2, Z_3, respectively. Moreover, $s_{tsf}^1, s_{tsf}^2, s_{tsf}^3$ are the disaggregated variables of s_{tsf} for each disjunctive term.

5.2. Convex Combinations

The key idea in this acceleration scheme is to consider prior solutions to the BMP, and then to modify the evaluation of the objective function to also optimize over best convex combination multipliers (Smith, 2004).

Let s_{tsf}^i, χ_k^i and η^i for $i = 1, ..., I$ be the solutions found after solving the BMP over the last I iterations. Parameter I controls the frequency for which the modified BSP (mBSP) is solved. In this problem the solution of the first-stage decision variables s_{tsf}, χ_k and η that was found in the previous iteration is replaced by the convex combination of these variables

over the past I iterations $\sum_i \lambda_i s_{tsf}^i$, $\sum_i \lambda_i \chi_k^i$ and $\sum_i \lambda_i \eta^i$, respectively. The objective function (30) is modified by adding the following term

$$\sum_i \lambda_i \cdot \gamma \cdot \eta^i.$$

Moreover, it is necessary to add the following constraints:

$$0 \leq \lambda \leq 1 \tag{49}$$

$$\sum_i \lambda_i = 1 \tag{50}$$

After solving mBSP in a given iteration, Algorithm 1 proceeds by getting the dual values of the subproblem constraints and adding the associated cuts to the BMP. Once mBSP is solved in one iteration, BSP is solved for the next I iterations.

5.3. Solving a Single Benders Master Problem

In the classical Benders solution approach, outlined in Algorithm 1, we alternate between solving a master problem and the subproblems. In this acceleration scheme we solve a single master problem and generate Benders cuts on the fly as we find feasible master solutions. This general approach is named branch-and-check (Thorsteinsson, 2001). This approach can be also seen as a branch-and-cut algorithm with the Benders subproblems sourcing the cuts. In the reminder of the paper we use modern Benders to refer to this approach.

This approach takes advantage of callback functions in the solver of the master problem. Its main advantage is that it avoids considerable rework in the branch-and-bound because we are keeping the same tree throughout the iterations of the Benders algorithm. Its main drawback is the harder implementation procedure.

6. Computational Experiments

In this section we describe computational experiments using the multi-cut Benders decomposition algorithm presented in Section 5. We sampled 81, 256 and 625 scenarios from the instances described in Section 4.1 and solved for the case in which it is necessary to decide about the supplying/branding strategy of 6, 12 and 24 products. Parameter γ was set to 0 and to 1000. Therefore, in total we report results for 18 instances. All the

programs were implemented in C++ and solved using IBM ILOG CPLEX Optimization Studio 12.4 on an Intel E5-2450 processor under a Scientific Linux 6.5 platform. For instances with 81 scenarios, each run was limited to 2 cores of the processor and 8GB of RAM. For instances with 12 and 24 products under 256 and 625 scenarios the execution was limited to 3 cores and 12GB of RAM.

In order to achieve better computational results, we solved the scenario subproblems with parallel computing. We grouped the scenarios into subproblems, in a way that each subproblem has 9 to 50 scenarios, depending of the number of scenarios in each instance. This method was effective in improving the computational performance and in reducing the amount of RAM required.

Table 6 shows the size of the monolithic model for each instance.

Table 6: Size of monolithic model for each instance.

Products (K)	Scenarios	Constraints	Variables	Binary Variables
6	81	167,265	235,903	6
6	256	922,624	1,038,368	6
6	625	2,252,500	2,535,032	6
12	81	330,561	468,460	12
12	256	1,788,928	1,983,014	12
12	625	4,367,500	4,841,288	12
24	81	657,153	933,574	24
24	256	3,521,536	3,872,306	24
24	625	8,597,500	9,453,800	24

We report in Table 7 and 8 results for each instance using the mixed-integer solver to solve the monolithic model (Monolithic), modern Benders decomposition algorithm (MB) (cf. Section 5.3) and the same algorithm with the hull reformulation (MB+H) (cf. Section 5.1). In Table 7, instances were run with the parameter parameter γ set to 0. The results of Table 8 represent the runs with γ set to 1000. All solution methods were limited to 21600 seconds (6 hours). The complete Table comparing the performance of all the approaches can be seen in the Appendix. There, each row represents the performance of each method, respectively: monolithic model (Monolithic), classical Benders (CB), classical Benders with hull reformulation (CB+H), classical Benders with convex combination (CB+CC) (cf. Section 5.2), classical Benders with hull and convex combination (CB+H+CC), modern Benders (MB), modern Benders with hull reformulation (MB+H), modern Benders with convex combination (MB+CC) and modern Benders

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with hull and convex combination (CB+H+CC).

Table 7: Results for the supplier selection problem with parameter γ set to 0.

Products (K)	Scenarios		Monolithic	MB	MB+H
		Lower Bound	34,625.94	34,625.94	34,625.94
	81	Optimality Gap	0.00%	0.00%	0.00%
		Runtime (s)	4,710	937	1,072
		Lower Bound	33,280.164	34,451.741	34,451.741
6	256	Optimality Gap	9.01%	0.00%	0.00%
		Runtime (s)	21,610*	3,842	6,262
		Lower Bound	-	34,193.966	34,193.966
	625	Optimality Gap	-	0.00%	0.0 0%
		Runtime (s)	21,621*	16,152	17,838
		Lower Bound	55,439.168	55,433.481	55,501.203
	81	Optimality Gap	5.14%	9.15%	9.64%
		Runtime (s)	21,606*	21,609*	21,604*
		Lower Bound	-	45,737.085	45,756.344
12	256	Optimality Gap	-	17.59%	19.47%
		Runtime (s)	21,615*	21,610*	21,622*
		Lower Bound	-	64,207.166	64,129.093
	625	Optimality Gap	-	21.10%	16.33%
		Runtime (s)	21,641*	21,679*	21,677*
		Lower Bound	90,341.772	90,709.587	90,728.562
	81	Optimality Gap	9.21%	19.38%	19.49%
		Runtime (s)	21,613*	21,609*	21,610*
		Lower Bound	-	95,675.549	95,642.661
24	256	Optimality Gap	-	23.76%	20.72%
		Runtime (s)	21,633*	21,663*	21,750*
		Lower Bound	-	97,325.228	97,776.63
	625	Optimality Gap	-	27.90%	23.77%
		Runtime (s)	21,750*	22,157*	21,760*

* Execution time limit reached.
- No feasible solution found.

Results show that for the instances with 6 products and 81 scenarios CPLEX was able to solve to optimality within the given time. However, for the more realistic and larger instances, with more scenarios and products, most of the times the solver was not able to find a feasible solution to the model. The Benders algorithms did not have enough time to converge with 12 or 24 products, but they achieved better overall results in instances with 6 products and in all instances with 256 and 625 scenarios. When comparing the lower bound, modern Benders decomposition approaches (with or without hull reformulation) outperformed all other methods as they were able to find reasonable or good solutions in all instances.

Table 8: Results for the supplier selection problem with parameter γ set to 1000.

Products (K)	Scenarios		Monolithic	MB	MB+H
		Lower Bound	35,295.29	35,295.29	35,295.29
	81	Optimality Gap	0.00%	0.00%	0.00%
		Runtime (s)	5538	971	1,101
		Lower Bound	35,060.015	35,060.015	35,060.015
6	256	Optimality Gap	4.87%	0.00%	0.00%
		Runtime (s)	21608*	4,733	7,202
		Lower Bound	-	34,880.382	34,880.382
	625	Optimality Gap	-	0.00%	0.0 0%
		Runtime (s)	21,622*	16,725	19,798
		Lower Bound	56,169.77	56,169.77	56,249.42
	81	Optimality Gap	5.01%	8.41%	10.45%
		Runtime (s)	21,606*	21,605*	21,604*
		Lower Bound	-	46,273.255	46,293.80
12	256	Optimality Gap	-	22.95%	21.43%
		Runtime (s)	21,618*	21,678*	21,614*
		Lower Bound	-	64,848.425	64,885.77
	625	Optimality Gap	-	19.15%	21.15%
		Runtime (s)	21,645*	21,691*	21,772*
		Lower Bound	89,800.494	91,150.52	90,994.117
	81	Optimality Gap	10.63%	22.57%	21.06%
		Runtime (s)	21,609*	21,619*	21,624*
		Lower Bound	-	96,001.70	95,912.45
24	256	Optimality Gap	-	24.38%	27.28%
		Runtime (s)	21,677*	21,923*	21,900*
		Lower Bound	-	98,215.09	97,459.08
	625	Optimality Gap	-	27.73%	28.35%
		Runtime (s)	21,700*	21,842*	21,931*

Although Benders decomposition achieved better solutions in almost every case, it was not able to obtain good upper bounds in instances with a larger number of products, which resulted in higher GAPs. This may be caused by the structure and the size of the model, and also by the slow convergence of benders decomposition in some cases (You and Grossmann, 2013).

When compared with classical Benders decomposition, modern Benders decomposition achieved better convergence in all instances. This can be explained by the faster solving time of the master problem and also by the usage of only one single exploration tree, in a way that it is not necessary to create a search tree and revisit the same nodes in each iteration.

The other acceleration techniques did not perform as well as expected. Convex combination was not able to improve the convergence and in instances with 256 and 625 scenarios, it reached the total amount of memory allowed in the first iterations. Nonetheless, we can not conclude that these methods are not effective for other models or instances. In instances with more products, the hull reformulation constraints were able to improve the optimality gap of the solutions.

The solution performance of all approaches seems to decrease when the parameter γ changes from 0 to 1000. This is line with the work of Miller and Ruszczynski (2011) which shows that the more traditional decomposition algorithms have a better performance for risk-neutral models ($\gamma = 0$) rather than for risk-averse ones ($\gamma = 1000$).

Figure 3 shows the convergence of the upper and lower bound for the multi-cut Benders decomposition algorithm variants and the monolithic approach when solving the instance with 6 products, 81 scenarios and γ set to 0. In this case, the modern Benders approach had a faster convergence than other variants of Benders and than the monolithic approach.

7. Conclusions and Future Work

This paper proposes a novel formulation to tackle the integrated decision of supplier selection and production-distribution planning for processed food supply chains. Uncertainty is present in the suppliers' processes namely in lead time, availability and spot price, and in customers' demand, which furthermore depends on the age of the sold product. Results show that only by taking such an integrated approach of tactical and strategic levels it is possible to make better decisions regarding sourcing of perishable raw materials to produce processed food products. The advantages of the premium price customers are willing to pay is undervalued by decoupled approaches.
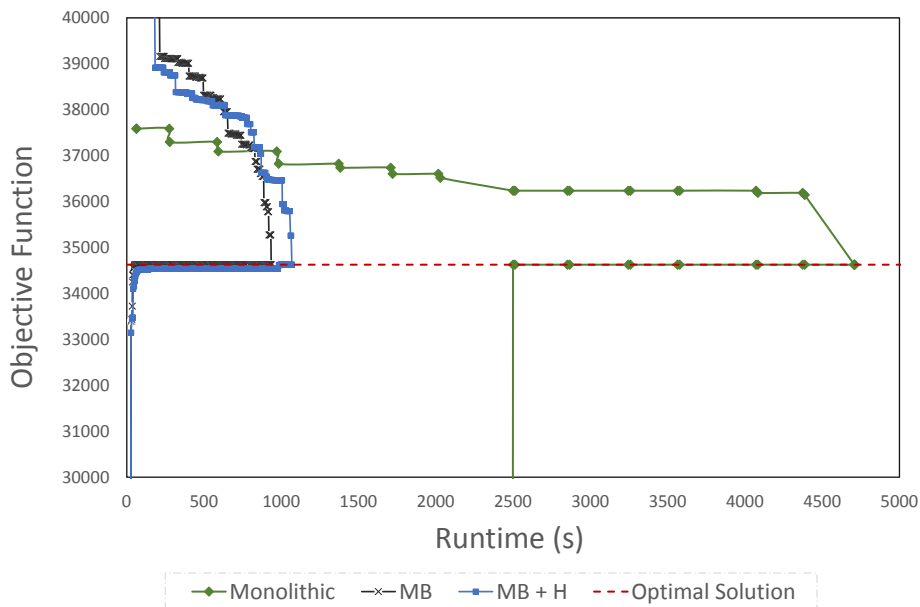
Figure 3: Comparison of the convergence of the multi-cut Benders decomposition algorithm variants and the monolithic approach for the instance with 6 products, 81 scenarios and γ set to 0

Due to the difficulty in solving the problem, we explored a multi-cut Benders decomposition algorithm that leverages the different proposed formulations. This algorithm suits the structure of this supplier selection problem, however, in a short time span it is hard to obtain optimal solutions. Modern Benders decomposition was able to improve significantly the results in comparison with the classical Benders approach and the monolithic model. Although acceleration techniques did not perform effectively, hull reformulation applied to the Benders master problem showed as a promising method to improve the convergence of Benders, particularly in problems where one can take more advantage of disjunctive programming to tighten the master problem.

Future research in terms of modelling should focus on improving the reality of the models, for example by considering setup costs. In terms of solution methods, it would be interesting to explore other possible decomposition algorithms, such as Lagrangian decomposition and Cross decomposition strategies.

Appendix A. Full Computational Results

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Table 9: Instance with 6 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	34,625.94	34,625.94	0.00%	4,710.27	-
			CB	34,296.58	34,790.36	1.42%	21,760.88	431
			CB+H	34,375.16	35,029.88	1.87%	21,640.94	372
			CB+CC	33,610.70	35,395.69	5.04%	21,663.20	185
6	0	81	CB+H+CC	33,535.97	35,392.65	5.25%	22,131.42	210
			MB	34,625.94	34,625.94	0.00%	936.97	297
			MB+H	34,625.94	34,625.94	0.00%	1,071.81	373
			MB+CC	34,625.94	34,625.94	0.00%	8,427.81	296
			MB+H+CC	34,625.94	34,625.94	0.00%	11,223.45	368
			Monolithic	35,295.29	35,295.29	0.00%	5,538.30	-
			CB	35,127.94	35,589.97	1.30%	21,729.64	431
			CB+H	34,891.62	35,803.53	2.55%	21,666.42	369
			CB+CC	34,057.49	36,239.22	6.02%	21,616.17	178
6	1000	81	CB+H+CC	33,562.71	36,241.16	7.39%	21,654.41	197
			MB	35,295.29	35,295.29	0.00%	971.27	341
			MB+H	35,295.29	35,295.29	0.00%	1,100.93	326
			MB+CC	35,295.29	35,295.29	0.00%	9,223.33	319
			MB+H+CC	35,295.29	35,295.29	0.00%	11,142.54	389

Table 10: Instance with 6 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	33,280.16	36,576.38	9.01%	21,609.53	-
			CB	33,115.09	36,105.12	8.28%	21,765.39	190
			CB+H	33,538.78	35,790.16	6.29%	21,647.58	187
			CB+CC	26,682.87	38,022.24	29.82%	24,217.46	35
6	0	256	CB+H+CC	28,083.33	37,736.18	25.58%	21,944.72	40
			MB	34,451.74	34,451.74	0.00%	3,842.05	285
			MB+H	34,451.74	34,451.74	0.00%	6,262.13	410
			MB+CC	34,451.74	39,907.50	13.67%	21,895.63	65
			MB+H+CC	34,429.84	38,287.25	10.07%	21,838.79	70
			Monolithic	35,060.02	36,854.03	4.87%	21,608.35	-
			CB	33,600.90	36,892.35	8.92%	21,698.79	186
			CB+H	32,903.77	37,035.98	11.16%	32,325.77	188
			CB+CC	29,471.37	38,839.82	24.12%	*	30
6	1000	256	CB+H+CC	28,506.75	38,688.55	26.32%	25,079.78	45
			MB	35,060.02	35,060.02	0.00%	4,733.58	347
			MB+H	35,060.02	35,060.02	0.00%	7,202.22	448
			MB+CC	35,060.02	39,913.96	12.16%	22,077.57	80
			MB+H+CC	27,491.61	46,625.42	41.04%	21,600.00	5

* Execution reached the total amount of memory allowed.

Table 11: Instance with 6 products and 625 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,621.15	-
			CB	33,036.91	35,923.02	8.03%	21,876.07	113
			CB+H	32,945.15	35,999.78	8.49%	21,731.70	101
			CB+CC	26,053.69	42,374.80	38.52%	*	5
6	0	625	CB+H+CC	26,053.69	43,160.81	39.64%	*	5
			MB	34,193.97	34,193.97	0.00%	16,152.40	360
			MB+H	34,193.97	34,193.97	0.00%	17,837.94	291
			MB+CC	26,053.69	43,247.82	39.76%	*	5
			MB+H+CC	33,922.85	44,814.07	24.30%	*	13
			Monolithic	-	-	-	21,621.67	-
			CB	32,497.52	37,015.75	12.21%	21,837.17	114
			CB+H	31,908.47	37,270.94	14.39%	21,858.08	98
			CB+CC	24,549.19	43,023.66	42.94%	*	5
6	1000	625	CB+H+CC	24,549.19	44,332.19	44.62%	*	5
			MB	34,880.38	34,880.38	0.00%	16,724.76	356
			MB+H	34,880.38	34,880.38	0.00%	19,797.74	314
			MB+CC	24,549.19	44,380.23	44.68%	*	5
			MB+H+CC	27,491.61	46,625.42	41.04%	*	5

* Execution reached the total amount of memory allowed.

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Table 12: Instance with 12 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
12	0	81	Monolithic	55,439.17	58,442.98	5.14%	21,605.66	-
			CB	50,484.34	64,681.70	21.95%	21,666.29	442
			CB+H	51,102.18	63,831.85	19.94%	21,772.82	507
			CB+CC	46,632.97	63,709.48	26.80%	22,516.73	130
			CB+H+CC	44,675.22	63,571.25	29.72%	22,205.16	150
			MB	55,433.48	61,017.34	9.15%	21,608.65	2,115
			MB+H	55,501.20	61,424.72	9.64%	21,604.37	1,735
			MB+CC	55,419.77	63,297.14	12.45%	21,652.34	315
			MB+H+CC	55,439.17	62,056.83	10.66%	21,636.85	320
12	1000	81	Monolithic	56,169.77	59,169.07	5.07%	21,605.96	-
			CB	52,134.08	65,299.08	20.16%	21,696.03	458
			CB+H	51,734.74	65,187.02	20.64%	21,665.77	447
			CB+CC	47,801.51	64,095.92	25.42%	21,934.02	105
			CB+H+CC	45,910.74	64,088.12	28.36%	21,877.99	145
			MB	56,169.77	61,325.38	8.41%	21,604.89	1,930
			MB+H	56,249.42	62,815.97	10.45%	21,603.88	1,783
			MB+CC	56,169.77	64,297.71	12.64%	21,604.13	305
			MB+H+CC	56,175.02	62,743.29	10.47%	22,286.51	245

Table 13: Instance with 12 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,615.41	-
			CB	36,621.68	59,548.02	38.50%	21,810.67	204
			CB+H	35,477.93	60,708.37	41.56%	21,769.32	171
			CB+CC	34,629.32	63,396.40	45.38%	*	5
12	0	256	CB+H+CC	34,629.32	55,975.29	38.13%	*	15
			MB	45,737.09	55,497.62	17.59%	21,610.27	670
			MB+H	45,756.34	56,820.73	19.47%	21,621.66	496
			MB+CC	34,629.32	63,927.58	45.83%	*	10
			MB+H+CC	44,963.00	63,587.85	29.29%	23,496.69	20
			Monolithic	-	-	-	21,617.57	-
			CB	39,200.11	59,888.91	34.55%	21,613.60	224
			CB+H	36,880.65	61,064.91	39.60%	21,909.67	171
			CB+CC	30,701.42	57,631.58	46.73%	*	5
12	1000	256	CB+H+CC	30,701.42	57,296.91	46.42%	23,881.59	15
			MB	46,273.26	60,055.04	22.95%	21,678.06	527
			MB+H	46,293.80	58,918.95	21.43%	21,614.10	452
			MB+CC	45,706.40	65,412.08	30.13%	*	29
			MB+H+CC	30,701.42	65,371.93	53.04%	*	10

* Execution reached the total amount of memory allowed.

Table 14: Instance with 12 products and 625 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,641.11	-
			CB	44,141.80	80,824.70	45.39%	21,931.61	121
			CB+H	42,989.67	81,343.90	47.15%	21,923.29	127
			CB+CC	42,540.54	83,548.43	49.08%	*	5
12	0	625	CB+H+CC	42,540.54	84,250.43	49.51%	*	5
			MB	64,207.17	81,376.87	21.10%	21,678.92	260
			MB+H	64,129.09	76,647.69	16.33%	21,677.19	272
			MB+CC	42,540.54	84,494.45	49.65%	*	5
			MB+H+CC	42,540.54	84,264.90	49.52%	*	5
			Monolithic	-	-	-	21,645.42	-
			CB	45,328.80	83,039.43	45.41%	21,744.54	115
			CB+H	44,680.32	83,737.90	46.64%	21,895.01	131
			CB+CC	39,760.23	85,350.90	53.42%	*	5
12	1000	625	CB+H+CC	39,668.25	86,634.98	54.21%	*	5
			MB	64,848.43	80,205.05	19.15%	21,690.88	252
			MB+H	64,885.77	82,287.11	21.15%	21,771.56	239
			MB+CC	39,760.23	86,512.37	54.04%	*	5
			MB+H+CC	39,668.25	88,640.02	55.25%	*	5

* Execution reached the total amount of memory allowed.

Table 15: Instance with 24 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	90,341.77	99,507.39	9.21%	21,613.16	-
			CB	67,105.07	128,833.00	47.91%	21,607.23	269
			CB+H	67,105.07	129,728.31	48.27%	21,673.73	255
			CB+CC	67,105.07	114,071.54	41.17%	21,626.39	38
24	0	81	CB+H+CC	67,105.07	111,488.86	39.81%	22,208.15	60
			MB	90,709.59	112,509.59	19.38%	21,608.58	1,080
			MB+H	90,728.56	112,697.81	19.49%	21,609.70	1,132
			MB+CC	88,903.13	113,039.05	21.35%	22,014.27	60
			MB+H+CC	89,059.80	132,040.08	32.55%	24,147.66	61
			Monolithic	89,800.49	100,486.77	10.63%	21,609.45	-
			CB	62,027.09	131,093.64	52.68%	21,677.83	221
			CB+H	62,027.09	131,018.59	52.66%	21,612.24	230
			CB+CC	62,027.09	115,202.26	46.16%	23,289.52	40
24	1000	81	CB+H+CC	62,027.09	111,807.41	44.52%	21,621.70	55
			MB	91,150.52	117,726.59	22.57%	21,619.00	1,153
			MB+H	90,994.12	115,267.05	21.06%	21,624.06	1,067
			MB+CC	89,397.69	113,507.63	21.24%	22,085.62	75
			MB+H+CC	89,553.10	134,107.81	33.22%	*	55

* Execution reached the total amount of memory allowed.

Table 16: Instance with 24 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,632.83	-
			CB	69,149.34	131,738.64	47.51%	21,743.15	135
			CB+H	69,149.34	132,855.58	47.95%	22,268.11	145
			CB+CC	69,149.34	133,987.87	48.39%	*	5
24	0	256	CB+H+CC	69,149.34	134,804.56	48.70%	*	5
			MB	95,675.55	125,488.51	23.76%	21,663.49	219
			MB+H	95,642.66	120,637.20	20.72%	21,750.99	150
			MB+CC	69,149.34	135,562.01	48.99%	*	5
			MB+H+CC	69,149.34	136,239.94	49.24%	*	5
			Monolithic	-	-	-	21,676.96	-
			CB	64,518.52	135,794.92	52.49%	21,870.41	152
			CB+H	64,518.52	138,001.51	53.25%	21,841.07	136
			CB+CC	64,518.52	138,035.11	53.26%	*	5
24	1000	256	CB+H+CC	64,518.52	139,917.62	53.89%	*	5
			MB	96,001.70	126,951.50	24.38%	21,923.17	110
			MB+H	95,912.45	131,891.60	27.28%	21,899.53	115
			MB+CC	64,518.52	139,306.45	53.69%	*	5
			MB+H+CC	64,518.52	139,970.70	53.91%	*	5

* Execution reached the total amount of memory allowed.

Table 17: Instance with 24 products and 625 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,750.91	-
			CB	68,383.36	136,187.58	49.79%	22,219.27	73
			CB+H	68,383.36	136,928.79	50.06%	21,968.92	89
			CB+CC	68,383.36	138,002.49	50.45%	*	5
24	0	625	CB+H+CC	68,383.36	138,534.54	50.64%	*	5
			MB	97,325.23	134,981.14	27.90%	22,157.37	104
			MB+H	97,776.63	128,259.24	23.77%	21,760.02	99
			MB+CC	68,383.36	138,918.40	50.77%	*	5
			MB+H+CC	68,383.36	138,628.79	50.67%	*	5
			Monolithic	-	-	-	21,700.43	-
			CB	63,012.39	137,238.47	54.09%	21,614.62	77
			CB+H	63,012.39	138,335.81	54.45%	21,857.03	66
			CB+CC	63,012.39	139,274.51	54.76%	*	5
24	1000	625	CB+H+CC	63,012.39	139,773.61	54.92%	*	5
			MB	98,215.09	135,906.19	27.73%	21,842.04	94
			MB+H	97,459.075	136,016.63	28.35%	21,930.87	83
			MB+CC	63,012.39	139,974.67	54.98%	*	5
			MB+H+CC	63,012.39	139,949.60	54.97%	*	5

* Execution reached the total amount of memory allowed.

Acknowledgments

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