Continuous-Time Formulations for the Optimal Planning of Multiple Refracture Treatments in a Shale Gas Well

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ABSTRACT. This work presents a continuous-time optimization model for planning multiple refracture treatments over the lifespan of a shale gas well. In contrast to previous continuous-time formulations (Cafaro, Drouven and Grossmann, AIChE J., 62(12), 4297–4307), we deal with multiple restimulations and account for economic criteria including natural gas sales, refracturing costs and discount rates, seeking to maximize the net present value of the well development and refracturing plan. We model the well productivity as a continuous-time, piecewise power function. Gas production also depends on when and how often the well has been refractured. We illustrate the effectiveness of the proposed representation and the impact of the time value of money in the restimulation strategy.

INTRODUCTION

One of the most critical challenges facing shale gas companies is the rapid decline of well production rates. The production rate from a typical shale gas well can decline as much as 85 % after the first year of operation[1]. As a result, most of the revenues from selling methane and heavier hydrocarbons are usually obtained within the first or second year after turning a well in line, leaving only marginal benefits for the rest of the well lifespan. However, recent studies demonstrate that the production from a

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shale gas well can be increased by refracturing; that is, by hydraulically fracturing the well again when the production is too low. In essence, refracturing is an effective strategy to recover large volumes of untapped oil and gas reserves from mature shale wells. According to Kotov and Freitag (2015)[2], roughly 90 % of the wells' potential production has historically been left untapped by following a single drilling/fracturing approach for each well. By restoring connections to original fractures and tapping into zones that were missed in the initial stimulation treatment, refracturing can reinvigorate the overall production profile of a well. Moreover, refracturing treatments are useful to improve completion and stimulation designs, often by increasing the amount proppant used. In this context, even the most productive shale gas wells have proven to be good candidates for refracturing.

Some contributions have recently been published in the literature with the aim of deciding on the potential of a further stimulation, eventually determining the optimal planning of refracture treatments over the lifespan of a shale gas well. Sharma (2013)[3] studies the redistribution of stresses around a fractured well using numerical simulation models. The work suggests that secondary fractures make it possible to access higher pressure regions, thus improving the productivity of the well. Some guidelines and type-curves are proposed to estimate the expected production increase and to choose the timing of the refracture operation. In turn, Eshkalak et al. (2014)[4] address the refracturing planning problem from an economic perspective. By estimating the net present value and the internal rate of return for different assumed gas prices, they demonstrate that refracturing of shale gas wells plays an important role in the economic success of an unconventional asset. The effect of the time to refracture in the production decline after restimulation is also addressed in detail by Tavassoli et al. (2013)[5]. The authors develop a numerical simulation model that predicts the performance of a shale gas well after refracturing based on reservoir parameters, completions design, and the time to refracture the well over its lifespan. More recently, Zeng and Cremaschi (2017)[6] introduce deterministic and stochastic mathematical programming models to solve the artificial lift infrastructure planning problem to help sustain shale gas well performance.

In contrast to the previously cited works, only a few contributions have accounted for the possibility of multiple refracture treatments over the lifespan of the shale gas well. Cafaro et al. (2016)[7] present an optimization framework to plan multiple shale gas well refracture treatments, assuming that the decision to drill and complete a prospective shale well has already been made. In a recent extension, Drouven et al. (2017)[8] propose a moving horizon framework to determine the optimal time to develop a new shale gas well and refracture treatments as recourse actions against uncertainties, such as natural gas price or well performance.

In order to determine if, when and how often a well should be restimulated over its lifespan, two kinds of optimization models can be adopted: continuous-time nonlinear programming (NLP) models and discrete-time (multiperiod) mixed-integer linear programming (MILP) models. To date, NLP models are strictly concerned with the optimal timing to refracture a single well such that its expected ultimate gas recovery (EUR) is maximized. Cafaro et al. (2016)[7] point out that unlike continuous models, multiperiod MILP models are capable of planning multiple refracturing operations over the lifespan of a shale gas well, also allowing for a straightforward evaluation of economic objective functions, such as the net present value (NPV) of a well development and refracturing project. However, this work demonstrates that continuous-time representations can also deal with multiple restimulations over the lifespan of a well, and account for economic criteria including natural gas sales, refracturing costs and the time value of money.

CONTINUOUS-TIME, MULTI-REFRACTURE PLANNING MODELS

Shale Gas Production After Multiple Refracture Treatments

We rely on a generalized production estimate function that predicts how much gas a well is expected to produce over time as a function of when and how often it has been restimulated. As suggested by Cafaro et al. (2016)[7], the gas production of an unconventional well can be represented adequately by a decreasing power function. This power function is defined by an expected initial production peak parameter k and an expected initial production decline parameter a. In essence, the decreasing power function adopted by Cafaro et al. (2016)[7] is an adaptation of the hyperbolic equation for general decline in a well, first proposed by Arps (1945)[9]. However, parameters k and a by themselves can only represent the production of a shale well that has not been refractured. Hence, to account for the production rate of a shale gas well that can be refractured once or more often, we propose Eq. (1).

$$p(t) = \gamma_i \cdot k \cdot t^{-a} + r_i \cdot \left(t - \hat{t}_i - rt_i\right)^{-a_i - b_i \cdot \hat{t}_i} \tag{1}$$

Eq. (1) estimates the shale gas production rate at time t after the i-th refracture. It is based on the expected initial production peak parameter k and the expected initial production decline parameter a. However, this function also accounts for the number of refracture treatments (i) previously performed to the shale gas well up to time t. More precisely, the elements $i \in I$ constitute the ordered set of refractrure treatments, $\hat{t}_i$ is the time at which the i-th refracture treatment is accomplished, and rt_i is the time it takes to recomplete the well.

By focusing on the first term of Eq. (1), it can be observed that every time a shale well is refractured, the contribution of its initial fractures to the overall production changes. Depending on the effectiveness of the initial completion, the contribution of original fractures can either increase or decrease after refracturing. We introduce the parameter γ_i to capture this aspect. When the well has been drilled and fractured for the first time, this parameter equals one ($\gamma_o = 1$). After every restimulation, however, the parameter will be set to a different value (usually, $\gamma_i < 1$ for $i \geq 1$), which is typically forecasted by completions design engineers.

Moreover, we include a second term to capture the characteristic production peak following a well restimulation. We introduce the refracturing production peak parameter r_i, whose value changes depending on how many times the well has been refractured (index i). With every additional refracture treatment, this supplemental production peak becomes less pronounced ($r_i < r_{i+1}$). However, for simplicity, we assume that this value is independent of the time the well is refractured.

In turn, we address the exponent ($-a_i - b_i \cdot \hat{t}_i$) in Eq. (1). Essentially, this exponent is an estimate of the post-refracture production decline after *i* restimulations. The term is made up of three criticial factors that are believed to determine the production decline following a refracture treatment: a) the initially expected production decline of the well after *i* restimulations a_i (which may vary depending on how many stages of the well are actually recompleted), b) the decline increase rate after *i* restimulations b_i, and c) the timing of the *i*-th refracture treatment $\hat{t}_i$. This composite decline exponent is motivated by the work of Tavassoli et al. (2013)[5] who show that the post-refracture production decline increases the longer one waits to refracture a shale well.

Finally, we can express the productivity of a shale gas well before, during and after having been refractured *i* times using the function in Eq. (2). We assume that the well can be refractured up to *N* times over its lifespan of length *T*.

$$p(t) = \begin{cases} k \cdot t^{-a} & 1 \le t \le \hat{t}_1 \\ 0 & \hat{t}_1 \le t \le \hat{t}_1 + rt_1 + 1 \\ \gamma_1 \cdot k \cdot t^{-a} + r_1 \cdot \left(t - \hat{t}_1 - rt_1\right)^{-a_1 - b_1 \cdot \hat{t}_1} & \hat{t}_1 + rt_1 + 1 \le t \le \hat{t}_2 \\ \dots & \\ 0 & \hat{t}_i \le t \le \hat{t}_i + rt_i + 1 \\ \gamma_i \cdot k \cdot t^{-a} + r_i \cdot \left(t - \hat{t}_i - rt_i\right)^{-a_i - b_i \cdot \hat{t}_i} & \hat{t}_i + rt_i + 1 \le t \le \hat{t}_{i+1} \\ \dots & \\ 0 & \hat{t}_N \le t \le \hat{t}_N + rt_N + 1 \\ \gamma_N \cdot k \cdot t^{-a} + r_N \cdot \left(t - \hat{t}_N - rt_N\right)^{-a_N - b_N \cdot \hat{t}_N} & \hat{t}_N + rt_N + 1 \le t \le T \end{cases} \qquad (2)$$

Maximization of the Gas Recovery

To determine the total amount of gas recovered from a shale gas well over its lifespan, we can integrate function (2) from $t = 1$ to $t = T$. This indicator is usually referred to as the estimated ultimate recovery (EUR). Its value depends on the number of restimulations and the time when they are performed.

$$EUR(\hat{t}_1,\hat{t}_2,...,\hat{t}_N)=\frac{k}{1-a}\cdot\left[\left(\hat{t}_1\right)^{1-a}-1\right]+\sum_{i\in I}\frac{\gamma_i\cdot k}{1-a}\cdot\left[\left(\hat{t}_{i+1}\right)^{1-a}-\left(\hat{t}_i+rt_i+1\right)^{1-a}\right]+\sum_{i\in I}\frac{r_i}{1-a_i-b_i\cdot\hat{t}_i}\cdot\left[\left(\hat{t}_{i+1}-\hat{t}_i-rt_i\right)^{1-a_i-b_i\cdot\hat{t}_i}-1\right] \quad (3)$$

For the last refracture (i = N) we substitute $\hat{t}_{N+1}$ by T. The EUR function in Eq. (3) assumes that: (1) $a \neq 1$, and (2) $a_i + b_i \cdot \hat{t}_i \neq 1$, $\forall i \in I$. To identify the optimal times to refracture a well such that the EUR is maximized, we propose a continuous-time nonlinear optimization model (NLP) as in expression (4).

$$max \quad EUR(\hat{t}_1,\hat{t}_2,...,\hat{t}_N)=\frac{k}{1-a}\cdot\left[\left(\hat{t}_i\right)^{1-a}-1\right]+\sum_{i\in I}\frac{\gamma_i\cdot k}{1-a}\cdot\left[\left(\hat{t}_{i+1}\right)^{1-a}-\left(\hat{t}_i+rt_i+1\right)^{1-a}\right]+\sum_{i\in I}\frac{r_i}{1-a_i-b_i\cdot\hat{t}_i}\cdot\left[\left(\hat{t}_{i+1}-\hat{t}_i-rt_i\right)^{1-a_i-b_i\cdot\hat{t}_i}-1\right] \quad (4)$$

$$s.t. \quad 1\leq\hat{t}_1,\hat{t}_1+rt_1\leq\hat{t}_2,...,\hat{t}_i+rt_i\leq\hat{t}_{i+1},...,\hat{t}_N+rt_N\leq T$$

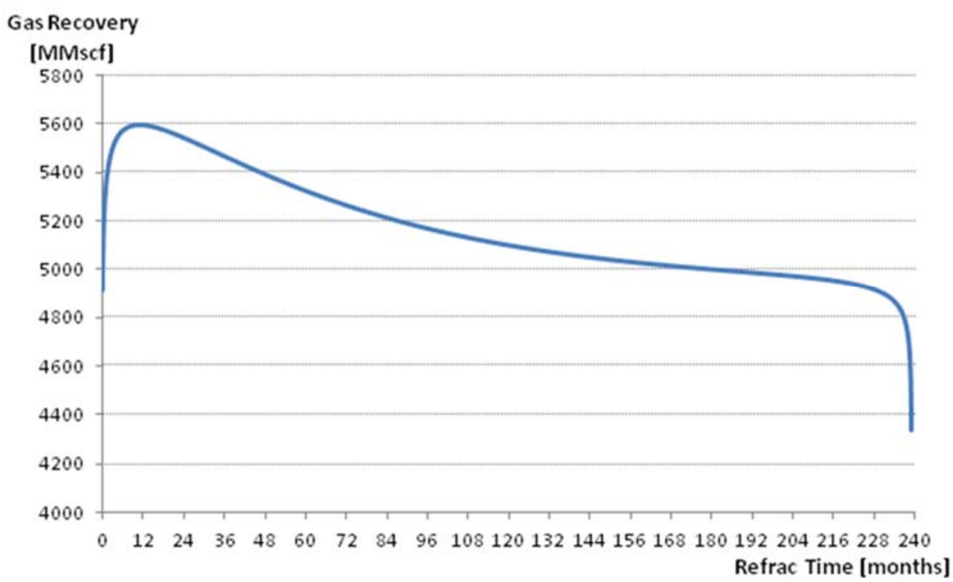

Fig. 1. *Total gas recovery vs. time of a single refracture treatment*

Fig. 1 illustrates the total gas production profile from function (4) when accounting for a single refracture treatment over a time horizon of 20 years. As discussed in previous publications[7], this function yields a nonconvex, nonlinear programming model, with singularities when $\hat{t}_i = (1-a_i)/b_i$ for any refracture $i \in I$. In such cases, the denominator of the third term in Eq. (4) is null, leading to a division by zero. It should be noted, however, that it is very unlikely that the optimal value for $\hat{t}_i$, i.e., the time to refracture the well by the *i*-th time matches the exact value $(1-a_i)/b_i$ at the optimum. Generally, $(1-a_i)/b_i > T$, which implies that the singular value for any $\hat{t}_i$ lies outside the feasible region and far beyond the expected lifespan of an ordinary shale gas well. An interesting finding from Fig. 1 is that the largest increase in the overall production is obtained when refracturing is accomplished early in the lifespan of the well. However, too early interventions may rapidly deteriorate the results (high early-life production rates are unnecessarily interrupted), while moderate delays in the restimulation treatment with regards to the optimal timing have a less negative impact.

Results for Gas Recovery Maximization

Fig. 2 shows the results of an illustrative example with up to 4 refractures, and solved to global optimality using GAMS/BARON 24.6.1.[10]

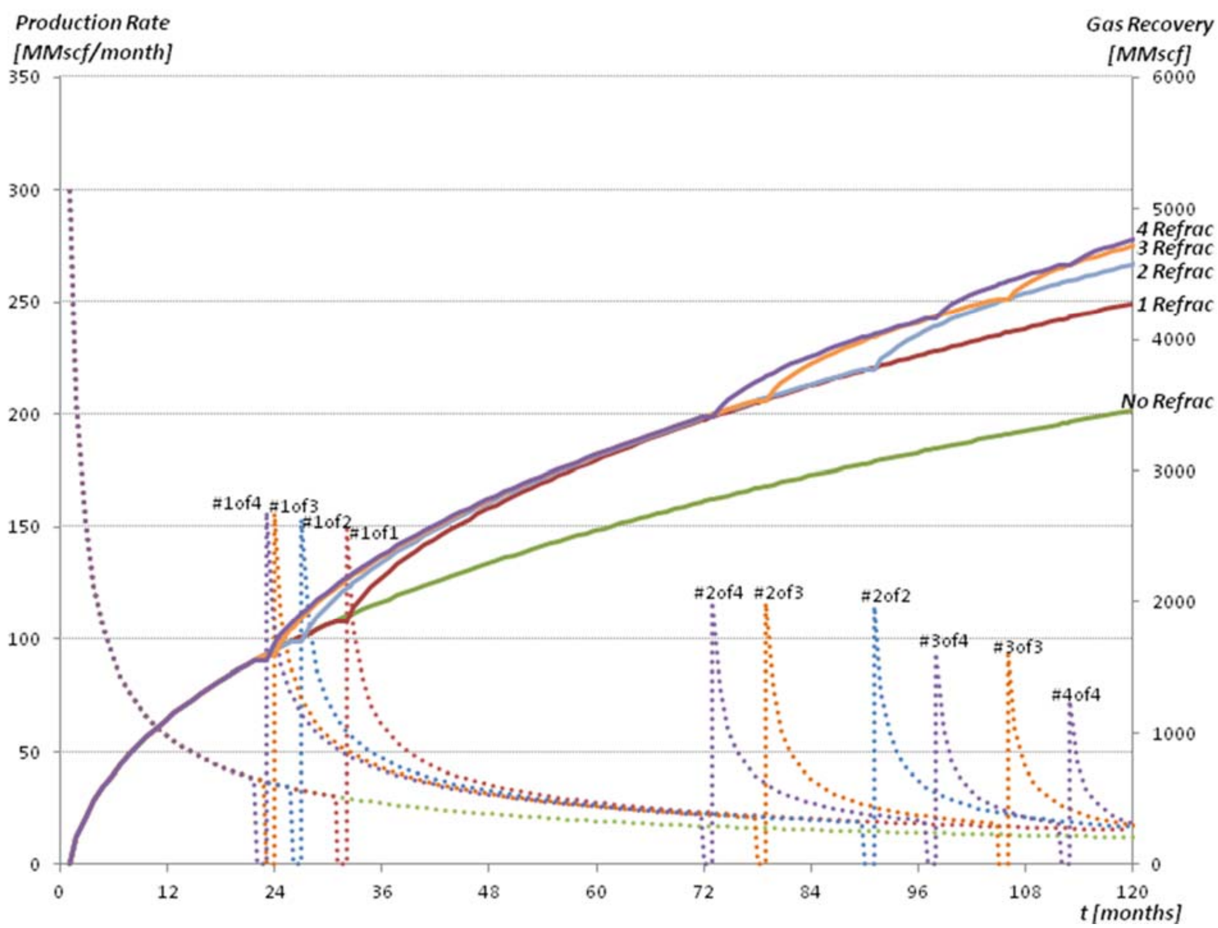

Fig. 2. *Optimization results for different numbers of refractures when maximizing the gas recovery*

In this case study, k = 299.4, $\gamma_i = (0.8)^i \ \forall\ i \in I$, $a = a_i = 0.664\ \forall\ i \in I$, $b_i = 0.0005\ \forall\ i \in I$, $rt_i = 1\ \forall\ i \in I$, and r_i = 120, 100, 80 and 60 for i = 1, 2, 3 and 4, respectively. The latter is based on the fact that the production peak after the first refracturing treatment can often match up to 40-60 % of the initial production peak[7], but the well response tends to be smaller after further restimulations. We solve the optimization model in (4) for different numbers of refractures over the lifespan of the shale gas well (T = 120 months). In all the cases, the optimal solution is found in less than 20 seconds in an Intel Xeon CPU (2.67 GHz, 16 GB RAM), with a relative optimality gap of 10^{-6}.

If the number of refractures is set to one ($I = \{1\}$), the optimization model suggests to refracture the well in month 30 (more precisely, at time $\hat{t}_1 = 29.36$), yielding a 23.1 % increase in the total amount of gas recovered when compared to the original production of the well, with no refractures. If the well can be refractured up to two times , the proposed model decides to refracture the well in months 25 and 89, i.e. after 2 and 7 years of production, respectively (see Table 1). The gain in the gas recovery is 31.2 % when compared to the no-refracture strategy, and 6.59 % with regards to the single-refracture plan.

Table 1: Comparison of refracturing strategies yielded by the proposed continuous-time models

Strategy		No Refrac	1-Refrac	2-Refracs		3-Refracs			4-Refracs			
Refracture #		-	# 1	# 1	# 2	# 1	# 2	# 3	# 1	# 2	# 3	# 4
Max EUR	$\hat{t}$	-	29.36	24.60	89.38	22.21	77.39	104.02	20.90	71.24	95.81	110.65
	EUR 10^9 scf	3.524	4.337	4.623		4.739			4.760			
	NPV 10^6 \$	3.341	6.221	6.994		7.213			7.110			
Max NPV	$\hat{t}$	-	53.43	39.20	89.39	33.46	77.07	102.96	33.46	77.07	102.96	-
	EUR 10^9 scf	3.524	4.294	4.595		4.716			4.716			
	NPV 10^6 \$	3.341	6.357	7.081		7.283			7.283			

In turn, if the well is to be refractured three times, the model decides to do so in months 22, 77 and 104, with an extra gain of merely 2.51 % with regards to the previous strategy. Finally, the optimal strategy that maximizes the gas recovery by refracturing the well four times suggests restimulations in months

21, 71, 96 and 111, yielding an extra 0.44 % from the plan with three refractures. It can be easily inferred that, based on economic criteria, refractures 3 and 4 may be not justified. In fact, the optimal number of refracture treatments will generally depend on the gas price and the cost of refracturing. To clarify this point, in the following section we present an extended version of the optimization model given in formulation (4) that includes an economic function to evaluate the impact of the refracturing strategy.

Maximization of the Net Present Value

Instead of focusing on the overall gas recovery from the shale gas well, we now propose a continuous-time optimization model designed to maximize the net present value of the well development project. Revenues from gas sales have to be maximized, whereas expenses for well development and recompletions are to be minimized, taking into account the time value of money. More precisely, given its continuous-time nature, the economic optimization model assumes that incomes and expenditures are discounted back to present value continuously, as shown in the nonlinear objective function (5). Other approximate strategies to account for discounted cash flows within continuous time horizons can be found in Lin and Floudas (2003)[11].

$$max \quad NPV(\hat{t}_1, \hat{t}_2, ..., \hat{t}_N) = \int_{t=1}^{T} gp \cdot p(t) \cdot e^{-dr \cdot t} dt - \sum_{i \in I} rc_i \cdot e^{-dr \cdot \hat{t}_i} - cc \tag{5}$$

Similar to previous contributions, it is assumed that gas production sets in approximately one month after well completion. The unit gas price (*gp*), the discount rate (*dr*), and the overall cost of completion (*cc*) and recompletions (rc_i) are assumed to be given. For simplicity, we also assume that unit gas price and discount rate are time-independent. Finally, the gas production rate *p(t)* is derived from the functions presented in (2), which depend on the refracturing strategy. As a result, integrating the first term in (5) is not a trivial matter. In fact, it can be proven that the integral of the product of decreasing power and exponential functions gives rise to incomplete gamma functions, as shown in Eqs (6) and (7).

$$\int \gamma_i \cdot k \cdot t^{-a} \cdot e^{-dr \cdot t} dt = -\gamma_i \cdot k \cdot dr^{a-1} \cdot \Gamma\left(1-a, dr \cdot t\right) + \text{constant} \tag{6}$$

$$\int r_i \cdot \left(t - \hat{t}_i - rt_i\right)^{-a_i - b_i \cdot \hat{t}_i} \cdot e^{-dr \cdot t} dt = -r_i \cdot dr^{a_i + b_i \cdot \hat{t}_i - 1} \cdot \Gamma\left(1 - a_i - b_i \cdot \hat{t}_i, dr \cdot t\right) + \text{constant} \tag{7}$$

In order to avoid including non-elementary functions like these in our optimization model (most commercial solvers cannot handle these functions), we use an asymptotic expansion of the incomplete gamma function, as described in equation (8).

$$\Gamma(s,x) = \Gamma(s) - \gamma(s,x) \qquad \gamma(s,x) = \int_0^x t^{s-1} e^{-t} dt = x^s \sum_{n=0}^{\infty} \frac{(-x)^s}{n!\,(s+n)} \tag{8}$$

It can be easily observed that the asymptotic expansion rapidly converges to the actual value of the non-elementary function. In fact, no more than ten terms ($n_{\max}$ = 10) are usually needed, due to the rapid growth of *n*! As a result, the optimization problem given in model (5), accounting for the production rates predicted in function (2), can be readily converted into a conventional NLP optimization problem, including only elementary functions, formulated in problem (9).

$$\begin{aligned} max \quad NPV(\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_N) = k \cdot dr^{a-1} \cdot \left[G_1 - G_0\right] + \sum_{i \in I} \gamma_i \cdot k \cdot dr^{a-1} \cdot \left[G_{i,3} - G_{i,2}\right] + \\ + \sum_{i \in I} dr^{a_i + b_i \cdot \hat{t}_i - 1} \cdot \left[G_{i,5} - G_{i,4}\right] - \sum_{i \in I} rc_i \cdot e^{-dr \cdot \hat{t}_i} - cc \end{aligned} \tag{9}$$

$$s.t. \qquad G_0 = dr^{1-a} \sum_{n=0}^{n_{\max}} \frac{(-dr)^{1-a}}{n!\,(1-a+n)} \qquad G_1 = \left(dr \cdot \hat{t}_1\right)^{1-a} \sum_{n=0}^{n_{\max}} \frac{(-dr \cdot \hat{t}_1)^{1-a}}{n!\,(1-a+n)}$$

$$G_{2,i} = \left(dr \cdot (\hat{t}_i + rt_i + 1)\right)^{1-a} \sum_{n=0}^{n_{\max}} \frac{\left(-dr \cdot (\hat{t}_i + rt_i + 1)\right)^{1-a}}{n!\,(1-a+n)}$$

$$G_{3,i} = \left(dr \cdot \hat{t}_{i+1}\right)^{1-a} \sum_{n=0}^{n_{\max}} \frac{(-dr \cdot \hat{t}_{i+1})^{1-a}}{n!\,(1-a+n)}$$

$$G_{4,i} = dr^{1-a_i-b_i \cdot \hat{t}_i} \sum_{n=0}^{n_{\max}} \frac{(-dr)^{1-a_i-b_i \cdot \hat{t}_i}}{n!\,(1-a_i-b_i \cdot \hat{t}_i + n)}$$

$$G_{5,i} = \left(dr \cdot (\hat{t}_{i+1} - \hat{t}_i - rt_i - 1)\right)^{1-a_i-b_i \cdot \hat{t}_i} \sum_{n=0}^{n_{\max}} \frac{\left(-dr \cdot (\hat{t}_{i+1} - \hat{t}_i - rt_i - 1)\right)^{1-a_i-b_i \cdot \hat{t}_i}}{n!\,(1-a_i-b_i \cdot \hat{t}_i + n)}$$

$$1 \le \hat{t}_1,\ \hat{t}_1 + rt_1 \le \hat{t}_2, \ldots, \hat{t}_i + rt_i \le \hat{t}_{i+1}, \ldots, \hat{t}_N + rt_N \le T$$

G_0, G_1, $G_{2,i}$, $G_{3,i}$, $G_{4,i}$ and $G_{5,i}$ are asymptotic approximations of the incomplete gamma function. We typically adopt n_{max} equal to 10 for the problem instances in the next sections. As before, the variable $\hat{t}_{N+1}$ should be substituted by the parameter T for the last refracture ($i = N$).

Results for Profit Maximization

To evaluate the economic potential of the well development and refracturing plan, we next solve the illustrating example presented in the previous sections using the NLP optimization model in (9), for different numbers of refractures over the lifespan of the shale gas well. We assume that the drilling and completion costs (cc) are 3 million USD, every refracture treatment (rc_i) amounts to 0.8 million USD $\forall i \in I$, the gas price is fixed at 3 USD/Mscf, and the monthly discount rate is 1 % (dr = 0.01). The optimal solution is found in less than 1 CPU second using GAMS/BARON 24.6.1 in all the problem instances.

As seen in Table 1, if no refracture is implemented, the NPV of the project is 3.341 million USD. When a single refracture is proposed, the optimization model suggests to refracture the well in month 54 (more precisely, at time $\hat{t}_1$ = 53.43), yielding an NPV of 6.357 million USD (90.27 % NPV increase due to refracturing!). Note that the single-refracture strategy for product maximization suggest a much earlier stimulation, in month 30, which would yield an NPV of 6.221 million USD. In order to take advantage of early gas sales during years four and five, the model suggests to postpone the intervention about two years. Table 1 presents a detailed comparison of both the gas recovery maximization and the profit maximization results. Moreover, if there is the possibility to refracture the well twice, the proposed model decides to refracture the well approximately in months 39 and 89, i.e. approximately after 3 and 7 years of production, respectively. Note that the first refracture is postponed about one year with regards to the gas recovery maximization strategy. The NPV increases to 7.081 million USD, 11.4 % more profitable than the single-refracture plan. Indeed, if the well can be refractured up to three times, the model plans optimal refractures in months 33, 77 and 103, with an NPV of 7.283 million USD (+ 2.85 % with regards to the previous strategy). Finally, as expected, if the well is refractured more than three times over the following ten years, the results demonstrate that the strategy will be not economically justified.

CONCLUSIONS

A rigorous continuous-time optimization model for planning multiple refracture treatments over the lifespan of a shale gas well has been presented. We demonstrate that the nonlinear optimization model can also be extended to account for economic objective functions like the maximization of the NPV of the well development and refracturing strategy. On the one hand, the continuous time model is very compact, comprising only N variables (N being the number of refractures). In fact, it proves to be computationally efficient given that its size does not depend on the length of the planning horizon (no time discretization). On the other hand, the NLP model is nonconvex, thus requiring global solvers to guarantee global optimality. Also, the model is not able to account for natural gas price forecast trends, like multiperiod MILP models do[8]. We are currently working to address this point.

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