OPTIMIZATION TECHNIQUES FOR BLENDING AND SCHEDULING OF OIL-REFINERY OPERATIONS

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Abstract

This paper presents a novel MILP formulation that addresses the simultaneous optimization of the short-term scheduling and blending problem in oil-refinery applications. Depending on the problem characteristics as well as the required flexibility in the solution, the model can be based on either a discrete or a continuous-time domain representation. In order to preserve the model's linearity, an iterative procedure is proposed to effectively deal with non-linear gasoline properties and variable recipes for different product grades. Thus, the solution of a large MINLP formulation is replaced by a sequential MILP approximation. Instead of predefining fixed component concentrations for products, preferred blend recipes can be forced to apply whenever it is possible. The proposed optimization approach is oriented towards providing an effective and integrated solution for both the scheduling and the blending problem. In order to provide convenient solutions for all circumstances, different alternatives to cope with infeasible problems are presented in detail. The new method is illustrated by solving several real world problems with very low computational requirements.

1. INTRODUCTION

The gasoline short-term scheduling and blending are critical aspects in oil refinery operations. The economic and operability benefits associated with obtaining better-quality and less expensive gasoline blends, and at the same time making a more effective use of the available resources, are numerous and significant. The main objective in oil refining is to convert a wide variety of crude oils into valuable final products such as gasoline, jet fuel and diesel. The major challenge lies on generating profits for a large process with high volumes and small margins. Figure 1 shows a diagram of a standard refinery system. The general structure of this particular process comprises three major sections: (1) crude oil unloading and blending, (2) production unit scheduling and (3) product blending and delivery of final products. The first sub-problem involves the crude oil unloading from vessels, its transfer to storage tanks and the charging schedule for each crude oil mixture to the distillation units. The second sub-problem consists of the production unit scheduling, which includes both fractionation and reaction processes. Reactions sections alter the molecular structure of hydrocarbons, in general to improve octane number, whereas fractionation sections separate the reactor effluent into streams of different properties and values. Lastly, the third sub-problem is related to the scheduling, blending, storage and delivery of final products, which is generally agreed as being the most important and complex sub-problem. Its importance comes from the fact that gasoline can yield 60-70% of total refinery's profit. On the other hand, the complexity mainly arises from the large number of product demands and quality specifications for each final product, as well as the limited number of available resources that can be used to reach the production goals. This paper is focused on the gasoline blending and the short-term scheduling problem of oil refinery operations.

Mathematical programming techniques have been extensively used for long-term planning as well as the short-term scheduling of refinery operations. For planning problems, most of the computational tools have been based on successive linear programming models, such as RPMS from Honeywell, Hi-Spec Solutions (Bonner and Moore, 1979) and PIMS from Aspen Technology (Betchel Corp., 1993). On the other hand, scheduling problems have been addressed through linear and non-linear mathematical approaches that make use of binary variables (MILP and MINLP codes) to explicitly model the discrete decisions to be made (Grossmann et al., 2002 ; Shah, 1998). Short-term scheduling problems have been mainly studied for batch plants. Extensive reviews can be found in Reklaitis (1992), Pinto and Grossmann (1998) and Ierapetritou and Floudas (1998). Much less work has been devoted to continuous plants. Lee et al. (1996) addressed the short-term scheduling problem for the crude-oil inventory management problem. Nonlinearities of mixing tasks were reformulated into linear inequalities with which the original MINLP model was converted to a MILP formulation that can be solved to global optimality. This exact linear reformulation was possible because only mixing operations were considered (see Quesada and Grossmann, 1995). The objective function was the minimization of the total operating cost, which comprises waiting time cost of each vessel in the sea, unloading cost for crude vessels, inventory cost and changeover cost.

Several examples were solved to highlight the computational performance of the proposed model. Moro et al. (1998) developed a mixed-integer nonlinear programming planning model for refinery production. The model assumes that a general refinery is composed of a number of processing units producing a variety of input/output streams with different properties, which can be blended to satisfy different specifications of diesel oil demands. Each unit belonging to the refinery is defined as a continuous processing element that transforms the input streams into several products. The general model of a typical unit is represented by a set of variables such as feed flowrates, feed properties, operating variables, product flowrates and product properties. The main objective is to maximize the total profit of the refinery, taking into consideration sales revenue, feed costs and the total operating cost. Kelly and Mann (2002) highlight the importance of optimizing the scheduling of an oil-refinery's crude-oil feedstock from the receipt to the charging of the pipestills. The use of successive linear programming (SLP) was proposed for solving the quality issue in this problem. More recently, Kelly (2004) analyzed the underlying mathematical modeling of complex nonlinear formulations for planning models of semi-continuous facilities, where the optimal operation of petroleum refineries and petrochemical plants was mainly addressed.

The gasoline blending problem has also been addressed with several optimization tools. The main objective is to find the best way of mixing different intermediates products from the refinery and some additives in order to minimize blending cost subject to meeting the quality and demand requirements of different final products. The term quality refers to meeting given product specifications. Rigby et al. (1995) discussed successful implementation of decision support systems for offline multi-period blending problems at Texaco. Since these software packages are restricted to solving the blending problem, resource and temporal decisions must be made a priori either manually or by using a special method. To solve both sub-problems simultaneously, Glismann and Gruhn (2001) proposed a two-level optimization approach where a mixed-integer linear model (MILP) is utilized for the scheduling problem whereas a nonlinear model is run for the recipe optimization. The proposed decomposition technique for the entire optimization problem is based on solving first the nonlinear model aiming at generating the optimal solution of the blending problem, which is then incorporated into the MILP scheduling model as fixed decisions for optimizing only resource and temporal aspects. In this way, the solution of a large MINLP model is replaced by sequential NLP and MILP models. Jia and Ierapetritou (2003) proposed a solution strategy based on decomposing the overall refinery problem in three subsystems: (a) the crude-oil unloading and blending, (b) the production unit operations, and (c) the product blending and delivery. In order to solve each one of these sub-problems in the most efficient way, a set of mixed-integer linear models (MILPs) were developed, which take into account the main features and difficulties of each case. In particular, fixed product recipes were assumed in the third sub-problem, which means that blending decisions were not incorporated into this model. The MILP formulation was based on a continuous time representation and the notion of event points. The mathematical formulation proposed to solve each sub-problem involves material balance constraints, capacity constraints, sequence constraints, allocation constraints, demand constraints, and a specific objective function. Continuous variables are defined to represent flowrates as well as starting and ending times of processing tasks. Binary variables are principally related to allocation decisions of tasks to event points, or to some specific aspect of each sub-problem.

To conclude, it is worth to mention that a variety of mathematical programming approaches are currently available to the short-term scheduling and blending problem. However, in order to reduce the inherent problem difficulty, most of them rely on special assumptions that generally make the solution inefficient or unrealistic for real world cases. Some of the common assumptions are: (a) fixed recipes for different product grades are predefined, (b) component and product flow-rates are known and constant and (c) all product properties are assumed to be linear. On the other hand, more general Mixed-Integer Non-Linear Programming (MINLP) formulations consider the majority of the problem features, but the complexity and the size of the model are greatly increased, making the problem intractable for large or even medium size problems. The major issue here is related to non-linear and non-convex constraints with which the computational performance strongly depends on the initial values and bounds assigned to the variables. Taking into account the principal weaknesses of the available mathematical approaches, the major goal of this work is to develop a novel mixed-integer linear programming (MILP) formulation for the simultaneous gasoline short-term scheduling and blending problem of oil refinery operations. Non-linear property specifications based on variable and preferred product recipes can be effectively handled through the proposed iterative linear procedure, which allows the model to generate near-optimal solutions with modest computational effort.

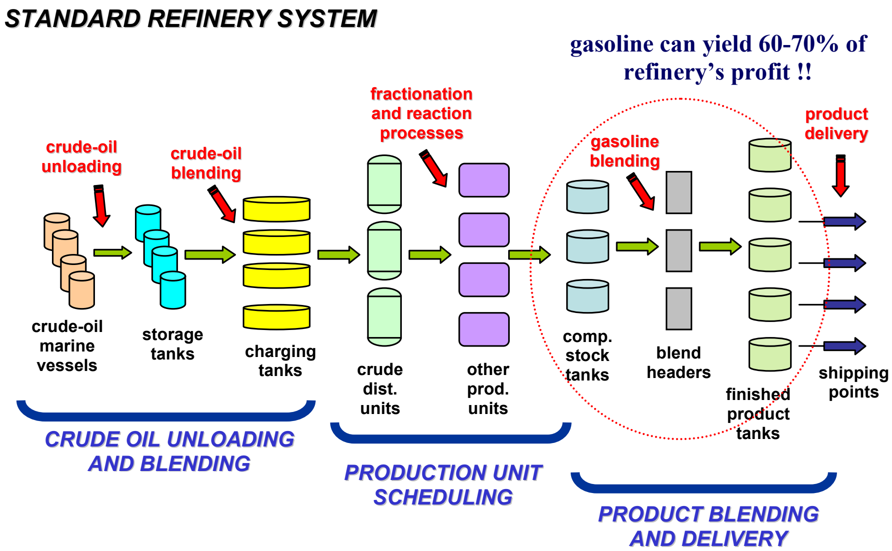

Figure 1. Illustration of a standard refinery system

2. MODELING ISSUES

The gasoline blending and short-term scheduling problem takes into account two major issues. The first one is related to aspects of production logistics, which mainly involves multiple production demands with different due dates, inventory pumping constraints for products and components as well as different logistic and operating rules. Most of these features are part of typical scheduling problems and are usually modeled as discrete and continuous decisions in an optimization framework. On the other hand, the second issue is the production quality, which represents an additional difficulty for standard scheduling problems. This second issue is also known as the blending problem and takes into account variable product recipes and property specifications such as minimum octane number, maximum sulfur and aromatic content, etc. Its main objective is to produce on-spec blends at minimum cost, where product specifications are stringent and constantly changing in most of the markets. Product qualities are usually predicted through complex correlations that depend on the concentration and the properties of the components used in the blend. Depending on the product property, non-linear correlations may include linear, bilinear, trilinear and exponential terms.

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To deal with this challenging problem, different optimization techniques have been developed, which are based on different assumptions and mathematical methods. For instance, several approaches try to solve the entire problem in one step. However these approaches usually introduce several simplifications to the real problem, such as considering fixed recipes instead of variable product recipes (Jia and Ierapetritou, 2003). While these approaches can be computationally effective, their solutions may be unrealistic for real industrial problems. Other approaches have used the idea of decomposing the problem into different sub-problems. The best example of this strategy is to solve the logistics aspect first, then fix the temporal and resource decisions to solve for the quality part by adjusting the flows appropriately to meet all product specifications. Although this method is currently used in several industrial optimization tools, it has the disadvantage that it may generate infeasible solutions, particularly when hard constrains are imposed.

Non-linear optimization models have been developed to circumvent the problem of infeasibilities by explicitly modeling non-linear properties for final products. However, as pointed out by several authors solving logistics and quality aspects for large-scale problems is not possible in a reasonable time with current mixed integer non-linear programming (MINLP) codes and global optimization techniques (Kelly et al. 2002 ; Jia et al., 2003). For this reason linear approximations are commonly used for handling the nonlinear properties of final products. As a result, near-optimal solutions for industrial problems can be generated with modest computational requirements.

On the other hand, a major aspect of any scheduling model is related to timing decisions. Mathematical formulations are based on either a discrete or continuous time domain representation. The discrete time representation only allows events to occur at certain time points, which correspond to the boundaries of a set predefined time intervals. The main advantage of using a discrete time grid is that mass balance and inventory constraints become easier to handle but at the same time the solution loses flexibility, unless smaller time intervals are used, which may significantly decrease the computational performance of the method. In contrast, continuous time representations are capable of generating more flexible solutions, although with higher CPU time requirements. Also, inventory and mass balance constraints generally become more difficult to model since they have to be checked at any time during the scheduling horizon in order to ensure that a feasible solution will be generated. Based on the above issues we will state more precisely the problem addressed in this paper.

3. PROBLEM STATEMENT

The general topology for the short-term scheduling and blending problem of oil refinery operations corresponds to a multistage system composed of component storage tanks, blend headers and product storage tanks. Specifically, we assume that we are given the following items:

1. A predefined scheduling horizon, typically 7 to 10 days
2. A set of intermediate products from the refinery (components)
3. A set of dedicated storage tanks for each component with minimum and maximum capacity restrictions
4. Initial stocks for components
5. Component supplies with known flowrates
6. Properties or qualities for components
7. Minimum and maximum flowrates between component tanks and blend headers
8. A set of final products with predefined minimum and maximum quality specifications
9. A set of equivalent blend headers working in parallel that can be allocated to each final product
10. A set of correlations, mostly non-linear, for predicting the values of properties of each blend.
11. Minimum and maximum component concentrations in final products
12. Preferred product recipes

The main goal is to determine:

a) The allocation of blenders to final products
b) The inventory levels of components and products in storage tanks
c) The volume fraction of components included in each product
d) The total volume of each product
e) The pumping rates for components and products
f) The optimal timing decisions for production and storage tasks

The objective is to maximize the production profit while satisfying the process constraints, final product demands and quality specifications. The objective function includes the total product value, the raw material cost and penalties for deviation from preferred recipes. Additional terms involving slack variables for handling infeasible solutions can also be incorporated into the objective function to provide effective solutions for all circumstances.

4. PROPOSED OPTIMIZATION APPROACH

The main features of the proposed approach can be summarized in the following points:

- A multiperiod optimization method that is able to deal with multiple product demands with different due dates and quality specifications.
- Discrete or continuous time domain representations can be used, depending on the problem characteristics.
- Linear approximations are used together with an iterative procedure to get better predictions of all product properties, even those naturally non-linear such as the octane number.
- Simultaneous solution of the production logistics and quality specifications.
- Fixed or variable product recipes as well as minimum and maximum limits on component concentration.
- Binary variables are used to represent allocation decisions as well as any other logistic or production rule found in the problem.

In order to describe the main model variables, Figure 2 illustrates a simple example of a gasoline scheduling and blending problem, which has traditionally been treated as two separate problems. The solution of the scheduling problem defines the way in which the products are processed with respect to time and available equipment. On the other hand, the solution of the blending problem defines how the available components are blended or mixed together to produce on-spec products with minimum cost.

The key decision variables involved in a standard problem are the following: The continuous variable $F^{I}_{i,p,t}$ defines the volumetric flow of component *i* being transferred to product *p* during the time interval *t* whereas $F^{P}_{p,t}$ denotes the volumetric flow of product *p* being blended during each time interval *t*. The continuous variables $V^{I}_{i,t}$ and $V^{P}_{p,t}$ define the amount of component and product being stored at each time point *t*, respectively. Finally, the discrete variable $A_{p,t}$ defines which products are allocated to blenders in each time interval *t.* Additional continuous and discrete variables can be included into the mathematical model to tackle particular problem characteristics and operating constraints.

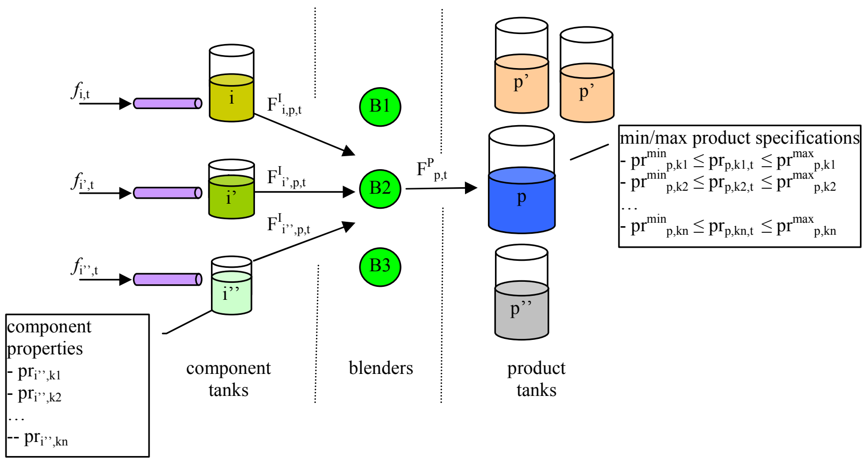

Figure 2. Illustration of the meaning of the principal model variables

5. PRODUCT PROPERTY PREDICTION

Before describing the proposed models, we present in this section an iterative scheme for predicting the properties of the products.

A significant number of gasoline properties can be directly predicted by using a volumetric average as shown in equation (a):

$$pr_{p,k,t} = \sum_i pr_{i,k} v^I_{i,p,t} \qquad \forall p,k,t \qquad \text{(a)}$$

where $v^I_{i,p,t}$ is the volume fraction of component *i* in product *p* at time *t*, $pr_{p,k,t}$ defines the value of the property *k* for product *p* in time *t* and $pr_{i,k}$ is the value of the property *k* for component *i*.

The volume fraction variable v^I is linked to the volumetric flow variables $F^I_{i,p,t}$ *and* $F^P_{p,t}$ through the following equality (b),

$$v^{I}_{i,p,t} F^{P}_{p,t} = F^{I}_{i,p,t} \qquad \forall p,k,t \qquad \text{(b)}$$

Taking into account that volumetric flowrate variables are required to control inventory levels in tanks and volume fraction variables are needed to predict product properties, the general mathematical model for the scheduling and blending problem is bilinear, even if only linear product properties are considered. However, in order to preserve the linearity of the model, the original equality (a) can be expressed in an alternative way by multiplying it by $F^{P}_{p,t}$

$$pr_{p,k} F^{P}_{p,t} = \sum_{i} pr_{i,k} V^{I}_{i,p,t} F^{P}_{p,t} \qquad \forall p,k,t \qquad \text{(c)}$$

Then equality (b) can be incorporated into equation (c), yielding the linear equation (d)

$$pr_{p,k} F^{P}_{p,t} = \sum_{i} pr_{i,k} F^{I}_{i,p,t} \qquad \forall p,k,t \qquad \text{(d)}$$

Taking advantage of minimum and maximum property specification constraints for products, constraint (d) can then be replaced by constraint (e), in which the variable $pr_{p,k}$ is substituted by their respective minimum and maximum property values, which are problem data.

$$pr^{\min}_{p,k} F^{P}_{p,t} \le \sum_{i} pr_{i,k} F^{I}_{i,p,t} \le pr^{\max}_{p,k} F^{P}_{p,t} \qquad \forall p,k,t \qquad \text{(e)}$$

In this way, the variable $v^{I}_{i,p,t}$ is no longer required and the model remains linear. This linearization is valid only if volumetrically computed properties are considered in the blending problem. However, other gasoline properties can be approximated by adding minor changes to the previous equation. For instance, if the correlation for predicting a particular product property is based on a linear volumetric average plus additional non-linear terms, such as the case of the octane number, the non-linear part of the equation can be removed and replaced by a correction factor $bias_{p,k,t}$, as shown in equation (f),

$$pr^{\min}_{p,k} F^{P}_{p,t} + bias_{p,k,t} F^{P}_{p,t} \le \sum_{i} pr_{i,k} F^{I}_{i,p,t} \le pr^{\max}_{p,k} F^{P}_{p,t} + bias_{p,k,t} F^{P}_{p,t} \qquad \forall p,k,t \qquad \text{(f)}$$

Thus, nonlinear product properties can be approximated through the linear equation (f), which is composed of a volumetric average followed by a correction factor 'bias'. This correction factor depends on the product, property and time slot, and it is iteratively calculated by using the proposed procedure described in the following section.

It should be noted that product properties such as oxygen and sulfur content are blended gravimetrically, which means that component and product specific gravities are also taking into account for the prediction, as shown in equation (g). In this case, ρ_i and ρ_p define the specific gravity of component i and product p, respectively. Given that ρ_p is a variable that is not directly computed through the proposed linear approach and with the intention of maintaining the model's linearity, ρ_p can be substituted by an approximated value $grav_p$, which can be easily computed through the iterative procedure described in the following section.

$$pr_{p,k,t} = \frac{\sum_i pr_{i,k}\, \rho_i V^I_{i,p,t}}{\rho_p} \qquad \forall p,k,t \tag{g}$$

Therefore, the proposed linear approximation for gravimetric blending is as follows,

$$pr^{\min}_{p,k} F^P_{p,t} \le \frac{\sum_i pr^{\min}_{p,k}\, \rho_i F^I_{i,p,t}}{grav_p} \le pr^{\max}_{p,k} F^P_{p,t} \qquad \forall p,k,t \tag{h}$$

To begin illustrating the iterative procedure and the proposed linear approximation, Fig. 3 shows a comparison between the values of the linear volumetric average, the nonlinear original correlation and the proposed linear approximation for a real nonlinear product property such as the motor octane number. In this example, the blend of two components A and B is only considered. The final product property is a nonlinear function of component concentrations. As shown in Fig. 3, if 40% of component A is blended with 60% of component B, the values of the volumetric average and the real nonlinear correlation are 88.5 and 88.74, respectively. This difference arises because all non-linear terms involved in the exact motor octane correlation are not included in the linear volumetric average. In order to correct this discrepancy, the correction factor bias is calculated and used to yield a better property prediction in the next iteration. For this specific mixture of components the correction factor bias is equal to 0.24. The linear approximation comprising the volumetric average together with the correction factor bias will always predict the exact value of the property if the same component concentration is utilized the next iteration. Furthermore, it was observed that the proposed linear approximation tends to predict a very close value of the real property if component concentrations are not significantly changed in next iteration, as shown in Figure 3.

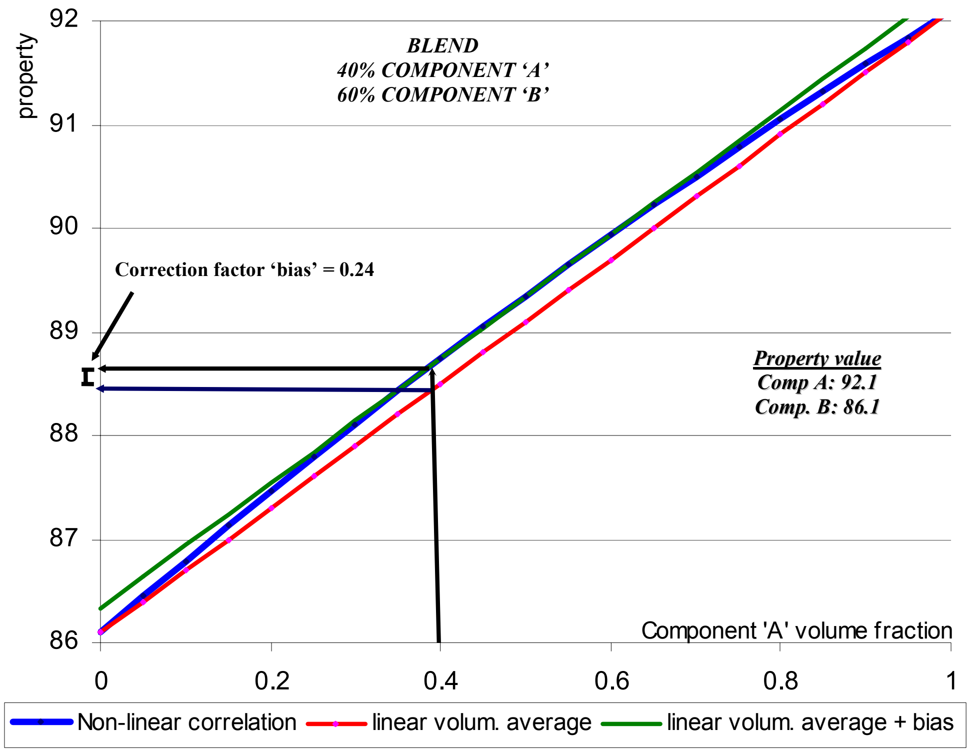

Figure 3. A non-linear property and the proposed linear approximation

The proposed iterative procedure to solve simultaneously the scheduling and blending problem using only linear equations can be summarized as seeing in Fig. 4. The first step is to find an initial recipe for all products. If preferred product recipes are known they can be proposed as initial product recipes. Preferred recipes are the best alternative for the blending issue because they satisfy all product specifications with minimum cost. However, the use of them strongly depends on the scheduling decisions, component inventories and product demands and for this reason, they should not be treated as fixed mixtures in any blending tool. On the other hand, if preferred recipes are not defined, one possibility for generating initial recipes is to solve the MILP model including only linear product properties. When initial recipes were generated, they will provide the component volume fractions used in each blend, which can then been employed as fixed parameters in more realistic non-linear correlations. The value predicted by the non-linear correlation and the linear volumetric average are both used to calculate the correction factor 'bias' (see Fig. 3). Given that we are dealing with a multiperiod optimization problem, the correction factor will be calculated for all non-linear properties, products and time slots as the difference between the value predicted by the original non-linear equation and the linear volumetric average. The specific gravity of each product and time slot is also computed. After that, the MILP model including linear approximation with the parameter bias for volumetric properties and the parameter *grav* for gravimetric properties is solved. Subsequently, the solution of this problem is revised and the product recipes for those products meeting all specifications in a specific time slot are fixed. If different recipes are used for the same product in different time slots, only those that are feasible will be fixed. This process is repeated until all product recipes are fixed or a predefined iteration limit is reached. The main objective of this iterative procedure is to progressively find feasible recipes for all products while optimizing all temporal and resource constraints. As will be shown later in the paper only few iterations are needed to get a very good solution for both sub-problems. This has also been confirmed with our experience in solving real world problems. It should be noted that the parameter bias will be equal to zero for all linear properties that can be computed volumetrically. Figure 4 depicts a diagram illustrating the iterative approach proposed as basis of the linearization technique for non-linear properties.

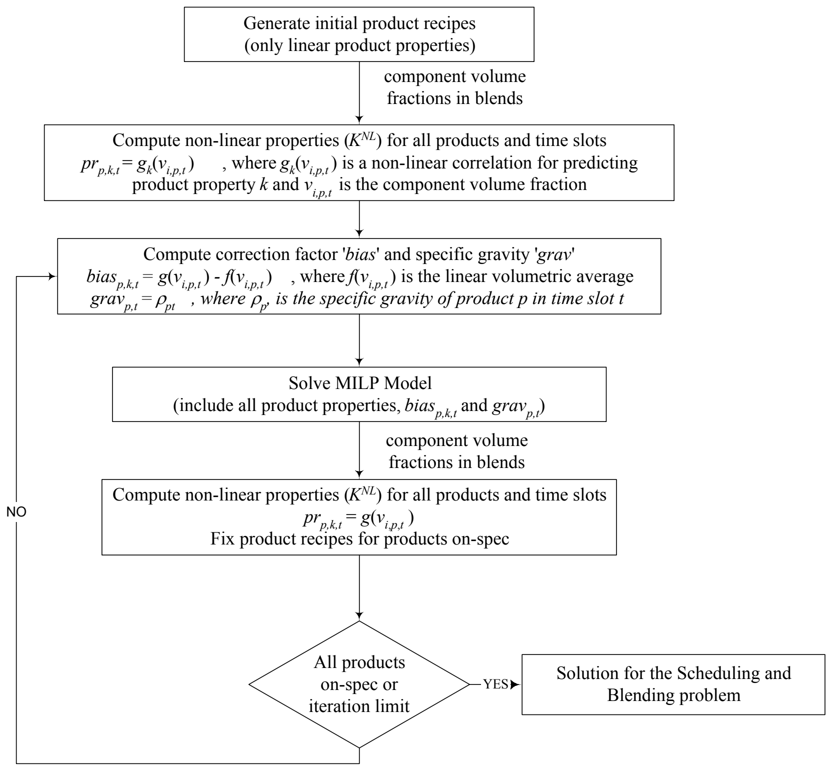

Figure 4. Proposed iterative approach

6. SCHEDULING MODEL

Before presenting the proposed mathematical models the nomenclature is as follows,

Nomenclature

Indices

d	due dates of product demands
i	intermediates or components
p	final products or gasoline grades
k	properties or qualities
t	time slots

Sets

D	set of product due dates
I	set of intermediates to be blended
P	set of demanded final products
K	set of properties for intermediates and products
T	set of time slots
T_d	set of time slots postulated for the sub-interval ending at due date *d* (continuous time)

Parameters

h	time horizon
$n^B{}_t$	maximum number of blenders that can be working in parallel in time slot t
s_t	predefined starting time of time slot t (discrete time representation)
e_t	predefined ending time of time slot t (discrete time representation)
c_i	cost of component i
sp_i	penalty for inventory of component i
sp_p	penalty for inventory of product p
$plty^{R+}{}_{ip}$	penalty for excess of component i in product p
$plty^{R-}{}_{ip}$	penalty for shortage of component i in product p
$plty^{S+}{}_{kp}$	penalty for a deviation from the minimum specification for property k
$plty^{S-}{}_{kp}$	penalty for a deviation from the maximum specification for property k
$plty^{SH}{}_i$	penalty for purchasing component i from third-party
d	demand due date
dd_{pd}	demand of product p to be satisfied at due date d
$l^{min}{}_p$	minimum time slot duration when it is allocated to product p
p_p	price of product p
inv_i	initial inventory of component i
inv_p	initial inventory of product p

V^{min}_{i}	minimum storage capacity of component i
V^{max}_{i}	maximum storage capacity of component i
V^{min}_{p}	minimum storage capacity of product p
V^{max}_{p}	maximum storage capacity of product p
rcp^{min}_{ip}	minimum concentration of component i in product p
rcp^{max}_{ip}	maximum concentration of component i in product p
$rate^{min}_{p}$	minimum flow rate of product p
$rate^{max}_{p}$	maximum flow rate of product p
rcp_{ip}	preferred concentration of component i in product p according to product recipe
pr_{ik}	value of property k for component i
pr^{min}_{pk}	minimum value of property k for product p
pr^{max}_{pk}	maximum value of property k for product p
f_i	constant flowrate of component i
$bias_{p,k,t}$	correction factor of the value of property k of product p in time slot t

Variables

$F^{I}_{i,p,t}$	amount of component i being transferred to product p during time slot t
$F^{P}_{p,t}$	amount of product p being blended during time slot t
$V^{I}_{i,t}$	amount of component i stored at the end of time slot t
$V^{P}_{p,t}$	amount of product p stored at the end of time slot t
$v^{I}_{i,p,t}$	volume fraction of component i in product p at time t
$pr_{p,k,t}$	value of the property k for product p in time t
S_t	starting time of time slot t (continuous time representation)
E_t	ending time of time slot t (continuous time representation)
$A_{p,t}$	binary variable denoting that product p is blended in time slot t
$D^{R-}_{i,p,t}$	shortage of component i that is used for product p in time slot t according to the preferred product recipe
$D^{R+}_{i,p,t}$	excess of component i that is used for product p in time slot t according to the preferred product recipe
$D^{S-}_{k,p,t}$	deviation from the minimum specification of property k for product p
$D^{S+}_{k,p,t}$	deviation from the maximum specification of property k for product p
$S_{i,t}$	amount of component i to be purchased in time slot t

7. DISCRETE TIME REPRESENTATION

In this section we present a MILP model that assumes that the entire scheduling horizon is divided into a finite number of consecutive time slots that are common for all units and can be allocated to different products, i.e. blending tasks. The proposed model has the following features:

1. A discrete time domain representation is used where the scheduling horizon is divided into a set of consecutive time slots.
2. Equivalent blenders working in parallel are available for different product grades
3. A particular product demand can be satisfied by one or more time slots whenever they are allocated to this product and finished before product due date.
4. Variable product recipes are considered and product properties are predicted by linear approximations.
5. Constant flow rate of components is assumed during the entire scheduling horizon
6. Constant flow rate of products is assumed during the allocated time slot.

MILP Formulation

Allocation constraint

Constraint (1) defines with the binary variables $A_{p,t}$ the number of final products allocated to time slot t. Given that a set of equivalent blenders are available to produce different gasoline grades simultaneously, n^B_t specifies the maximum number of units that can be working in parallel during time interval t.

$$\sum_p A_{p,t} \le n_t^B \qquad \forall t \qquad (1)$$

Product composition constraint

Every final product or gasoline grade p is a blend of different components i, as expressed by constraint (2)

$$\sum_i F^I_{i,p,t} = F^P_{p,t} \qquad \forall p,t \qquad (2)$$

Note that a significant reduction in the number of continuous variables can be obtained if equation (2) is deleted from the model and $F^P_{p,t}$ is replaced by $\sum_i F^I_{i,p,t}$. However, in order to make the model easier to understand, $F^P_{p,t}$ has been included in all model equations.

Minimum/maximum component concentration

In order to satisfy product qualities and/or market conditions, upper and lower bounds can be forced on the component concentration for specific gasoline grades. Then, constraint (3) ensures that product composition will always satisfy the predefined component specifications. Parameters $rcp_{i,p}^{min}$ and $rcp_{i,p}^{max}$ define the minimum/maximum concentration of component *i* for product *p*, respectively

$$rcp_{i,p}^{\min} F_{p,t}^{P} \leq F_{i,p,t}^{I} \leq rcp_{i,p}^{\max} F_{p,t}^{P} \qquad \forall i,p,t \qquad (3)$$

It should be noted that a fixed recipe for a particular product *p* can also be taken into consideration by fixing the values of rcp_{ip}^{min} and rcp_{ip}^{max} to the predefined concentration of component *i* for product *p*. However, the use of fixed recipes should be avoided unless they were the only possibility to produce a particular product. As a better option, preferred recipes can be proposed as an initial solution of the proposed iterative procedure. In this way the generation of infeasible solutions will be avoided.

Minimum/maximum volumetric flowrates for products

Constraint (4) specifies that minimum and maximum volumetric flow rates must be satisfied when product *p* is blended during time slot *t*. Due to the fact that a constant product flow rate is assumed in this work, the volumetric flow rate can be computed by multiplying the upper and lower flowrates by the time slot duration whenever product *p* is allocated to a particular time slot *t* ($A_{p,t}$ =1). Moreover, since a discrete time representation is used, the time slot duration is a known parameter computed through the predefined starting s_t and ending times e_t of each time slot *t*. It should be noted that if product *p* is not processed during time interval *t,* ($A_{p,t} =0$), the volumetric flow rate will be also equal to zero.

$$rate_{p}^{\min}(e_t - s_t)A_{p,t} \leq F_{p,t}^{P} \leq rate_{p}^{\max}(e_t - s_t)A_{p,t} \qquad \forall p,t \qquad (4)$$

Material balance equation for components

Given that a discrete time representation allows the blending tasks to start and finish at the same that the time slot allocated, inventory limits have only to be checked at the end of each time slot. Then, as expressed by constraint (5), the amount of component *i* being stored in tank at the end of time slot *t* is equal to the initial inventory of component *i* plus the component produced up to the end of time slot *t* minus the component transferred to blenders up to the end of time slot *t,*

$$V^I_{i,t} = ini_i + f_i \, e_t - \sum_{p,t' \le t} F^I_{i,p,t'} \qquad \forall i,t \qquad (5)$$

where the parameter f_i specifies the constant production rate of component *i* and e_t defines the ending time of time slot *t*. Given that a discrete time representation is used, both parameters are known in advance.

Component storage capacity

Constraint (6) imposes lower/upper bounds V_i^{min} and V_i^{max} on the total amount of component *i* being stored in a storage tank during the scheduling horizon. Given that constant component flowrates are assumed, a perfect coordination between the production of components and final products is required to satisfy the storage constraints through the entire scheduling horizon.

$$V_i^{\min} \le V^I_{i,t} \le V_i^{\max} \qquad \forall i,t \qquad (6)$$

Material balance equation for products

Constraint (7) computes the amount of product *p* being stored in tank at the end of time slot *t* taking into account the initial inventory, production and demands of product *p*

$$V^P_{p,t} = ini_p + \sum_{t' \le t} F^P_{p,t'} - \sum_{d \le t} dd_{pd} \qquad \forall p,t \qquad (7)$$

Product storage capacity

A minimum safety stock and a finite storage capacity is assumed for final products

$$V_p^{\min} \le V^P_{p,t} \le V_p^{\max} \qquad \forall p,t \qquad (8)$$

Minimum/maximum product qualities

Assuming that properties are volumetrically computed, constraint (9) guarantees that the value of property k for product p in time interval t will always satisfy minimum and maximum product specifications. To maintain the model's linearity, property k is not directly computed and bounds are only imposed on each property. Otherwise, non-convex bilinear equations would be generated in the model, which would then become non-linear. Although this linearization is only valid for properties volumetrically computed, the original equation (9) can be slightly modified as equation (9') to account for real-word product properties, as described previously in the paper with the used of the parameter $bias_{k,p}$. The best value of this parameter can be obtained through the proposed iterative procedure. In this way, the proposed mathematical model is able to effectively deal with the quality issue, including variable recipes and non-linear properties.

$$pr_{p,k}^{\min} F_{p,t}^{P} \leq \sum_{i} pr_{i,k} F_{i,p,t}^{I} \leq pr_{p,k}^{\max} F_{p,t}^{P} \qquad \forall p,k,t \tag{9}$$

$$pr_{p,k}^{\min} F_{p,t}^{P} \leq \sum_{i} pr_{i,k} F_{i,p,t}^{I} + bias_{k,p} F_{i,p,t}^{I} \leq pr_{p,k}^{\max} F_{p,t}^{P} \qquad \forall p,k,t \tag{9'}$$

In turn, Equation (9'') defines the proposed linear approximation for those product properties gravimetrically predicted.

$$pr_{p,k}^{\min} F_{p,t}^{P} \leq \frac{\sum_{i} pr_{p,k}^{\min} \rho_i F_{i,p,t}^{I}}{grav_p} \leq pr_{p,k}^{\max} F_{p,t}^{P} \qquad \forall p,k,t \tag{9''}$$

Note that constraints (9), (9') and (9'') are only required for those gasoline grades that can be produced using variable recipes. If a fixed recipe is enforced, product properties must be satisfied in advance through the predefined component concentrations.

Multiple product demands

Refinery operations typically require that multiple demands for the same gasoline grade be satisfied during the entire scheduling horizon. Constraint (10) denotes that the total amount of product *p* available at the end of time slot *t* must be enough to satisfy all demands of this particular product.

$$\sum_{t \le d} F^{P}_{p,t} \le \sum_{d' \le d} dd_{p,d'} \qquad \forall p,d \tag{10}$$

Objective function (Maximize net profit)

While satisfying all quality and logistic issues, the main objective of the scheduling problem is to maximize the net profit defined as the total product value minus the total component cost.

$$Max \quad \sum_{t}\sum_{p}\left(p_p F^{P}_{p,t} - \sum_{i} c_i F^{I}_{i,p,t} \right) \tag{11}$$

The formulation can also accommodate alternative objective functions. An example is equation (12), where penalties related to component and product inventories has been included in order to also reduce storage costs.

$$Max \quad \sum_{t}\sum_{p}\left(p_p F^{P}_{p,t} - \sum_{i} c_i F^{I}_{i,p,t} \right) - \sum_{p}\sum_{t} sp_p V^{P}_{p,t} - \sum_{i}\sum_{t} sp_i V^{I}_{i,t} \tag{12}$$

8. CONTINUOUS TIME REPRESENTATION

The model in the previous section is based on a discrete time domain representation. To generate more flexible schedules capable of maximizing the plant performance without significantly increasing the model size, a continuous time representation will be utilized for the model. However, special attention must be paid to the limited storage capacity since continuous time representation tends to make the inventory constraints much more difficult. The main idea here is first to partition the entire time horizon into a predefined number of sub-intervals. The size of each sub-interval will depend on the product due dates. For instance, the first sub-interval will start at the beginning of the scheduling horizon and finish at the first product due date. The second one will be extended from the first up to the second product due date. A similar idea is applied to the next sub-intervals. Then, the number of sub-intervals will be equal to the number of product due dates. In this way, the starting and ending time of each sub-interval is known in advance.

Once the sub-intervals are defined, a set of time slots with unknown duration are postulated for each one. The number of time slots for each sub-interval will depend on the sub-interval length as well as the grade of flexibility desired for the solution. Time slot starting and ending times will be new model variables, allowing the production events to happen at any time during the scheduling horizon. Figure 5 shows a diagram illustrating the main features of the proposed continuous time domain representation. In this case, four product demands with different due dates are to be satisfied, which means that 4 sub-intervals are predefined. Then, nine time slots can be postulated for the entire scheduling horizon, where two time-slots are defined for each one of the first three sub-intervals whereas three are allocated to the last one.

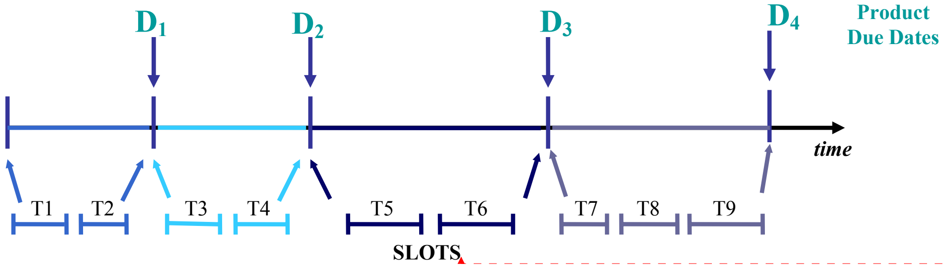

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Figure 5. Proposed continuous time representation

The proposed model has the following features:

1. A continuous time domain representation is used where the scheduling horizon is divided into a sub-intervals and a set of time slots with unknown duration are postulated for each one.
2. Equivalent blenders working in parallel are available for different product grades
3. A particular product demand can be satisfied by one or more time slots whenever they are allocated to this product and finished before product due date.
4. Final product properties are based on a volumetric average and a correction factor computed through the proposed iterative process.
5. A constant flow rate of components is assumed during the entire scheduling horizon
6. A constant flow rate of product is assumed during the allocated time slot.

MILP Formulation

When the mathematical model is based on a continuous time domain representation, starting and ending times for the time slots are new continuous decisions variables. For that reason, part of the original constraints used for discrete time representation must be updated in order to maintain model's linearity as well as to account new problem features. In this section we describe the set of constraints that must be modified as well as the new ones to be added. Constraints that are not required to change must be included into the model in the same way they were presented in the previous section, such as equations (1), (2), (3), (6), (7), (8), (9), (9'), (9''), (10), (11).

Minimum/maximum volumetric flowrates for products

Constraints (4') and (4'') replace constraint (4) when a continuous time representation is used. When product p is not allocated to time slot t, the binary variable A_{pt} is equal to zero and constraint (4'') enforces the variable $FP_{p,t}$ to be equal to zero as well. On the other hand, A_{pt} will be equal to one whenever product p is processed during time slot t. In this case, constraint (4'') becomes redundant and constraint (4') imposes minimum and maximum volumetric flow rates depending on the time slot duration.

$$rate_p^{\min}(E_t - S_t) - rate_p^{\min} h (1 - A_{p,t}) \le F_{p,t}^P \le rate_p^{\max}(E_t - S_t) \qquad \forall p,t \qquad (4')$$

$$F_{p,t}^P \le rate_p^{\max} h A_{p,t} \qquad \forall p,t \qquad (4'')$$

Material balance equation for components

To ensure that only feasible solutions are generated, the amount of component stored in tank has to be checked not only at the end but also at the beginning of each time slot. To make this possible, a new variable $V'^I_{i,t}$ is included into the model and the original equation (5) is replaced by constraints (5') and (5''). The same idea for computing the inventory of components is applied to these new constraints.

$$V_{i,t}^I = ini_i + f_i\, E_t - \sum_{p,t' \le t} F_{i,p,t'}^I \qquad \forall i,t \qquad (5')$$

$$V'^I_{i,t} = ini_i + f_i\, S_t - \sum_{p,t' < t} F_{i,p,t'}^I \qquad \forall i,t \qquad (5'')$$

Note that despite the fact that E_t and S_t are model variables, both constraints remain linear because a constant production rate f_i is assumed for components.

Component storage capacity

An additional constraint (6.1) is required to impose lower/upper bounds V_i^{min} and V_i^{max} on the total amount of component i being stored in tank at the beginning of time slot t.

$$V_i^{\min} \le V'^I_{i,t} \le V_i^{\max} \qquad \forall i,t \qquad (6.1)$$

Material balance equation for products

Constraint (7.1) computes the inventory of product p at the moment of satisfying the production demand d_p. In this way, a minimum safety stock is guaranteed at any time during the scheduling horizon, even after a product delivery is carried out.

$$V'^{P}_{p,t} = ini_p + \sum_{t<d} F^{p}_{p,t} - \sum_{d' \leq d} dd_{pd'} \qquad \forall p, d_p \tag{7.1}$$

Constraint (8.1) explicitly defines the lower bound on the new inventory variable.

$$V^{\min}_{p} \leq V'^{P}_{p,t} \qquad \forall p, t \tag{8.1}$$

Set of time slot timing constraints

Instead of defining time slot starting and ending times as fixed parameters, a continuous time representation models these decisions as additional continuous variables to be optimized. In order to allow more flexible solutions and avoid overlapping time slots, a correct order and sequence between postulated time slots must be established through the next set of constraints.

Time slot duration

Constraint (13) defines a minimum time slot duration when product p is allocated to time slot t. It is generally used to model an existing operating condition, but at the same time permits eliminating schedules using very short time slots, which are usually inefficient in practice.

$$E_t - S_t \geq l^{\min}_{p} A_{p,t} \qquad \forall p, t \tag{13}$$

To ensure that duration of a slot is zero if it is not used, equation (14) is included into the model.

$$E_t - S_t \leq h \sum_{p} A_{p,t} \qquad \forall t \tag{14}$$

Time slot sequencing

Constraint (15) establishes a sequence between consecutives time slots t and $t+1$.

$$E_t \leq S_{t+1} \qquad \forall t \qquad (15)$$

Sub-interval bounds

The set T_d comprises all time slots that are postulated for a sub-interval related to a particular due date *d*. This sub-interval begins at the previous due date *d-1* and finishes at due date *d*. Constraint (16) defines that time slots pre-allocated to this sub-interval must start after due date *d-1* whereas constraints (17) imposes that them must end before due date *d*. The main goal of this assumption is that neither additional variables nor new constraints are required to establish which time slots can satisfy a specific product demand. As a result, more flexible schedules can be obtained without increasing the complexity of inventory constraints.

$$S_t \geq d - 1 \qquad \forall t \in T_d \qquad (16)$$

$$E_t \leq d \qquad \forall t \in T_d \qquad (17)$$

Time slot allocation

Constraint (18) imposes an order for using the set of predefined time slots. In other words, a time slot *t+1* can be only allocated to a product *p* whenever the previous time slot has been used.

$$\sum_p A_{p,t+1} \leq n_t^B \sum_p A_{p,t} \qquad \forall d, (t, t+1) \in T_d \qquad (18)$$

9. TREATMENT OF INFEASIBLE SOLUTIONS

The short-term scheduling and blending of oil refinery operations is a very complex and highly-constrained problem, where even feasible solutions are difficult to find in most of the cases. For that reason, in this section we present an additional set of variables and equations that can be used together with the proposed model, which are mainly oriented towards relaxing some hard problem constraints that can generate infeasible solutions when real world problems are addressed.

Penalty for preferred recipe deviation

If a preferred combination of components is defined for a particular product through the parameter rcp_{ip}, the following constraints can be included in the model to try using the desired recipe whenever it is possible.

$$rcp_{ip} F^{P}_{p,t} + D^{R^-}_{i,p,t} \geq F_{i,p,t} \qquad \forall i,p,t \tag{19}$$

$$rcp_{ip} F^{P}_{p,t} - D^{R^+}_{i,p,t} \leq F_{i,p,t} \qquad \forall i,p,t \tag{20}$$

Where $D^{R+}{}_{i,p,t}$ and where $D^{R-}{}_{i,p,t}$ define the excess and the shortage of component i that is used in product p in time slot t, according to the preferred product recipe. Constraint (21) penalizes the slack variables $D^{R+}{}_{i,p,t}$ and $D^{R-}{}_{i,p,t}$ in the objective to ensure that deviations from the preferred recipe are minimized

$$Penalty = \sum_{t}\sum_{p}\sum_{i}\left(plty^{R^+}_{ip} D^{R^+}_{i,p,t} + plty^{R^-}_{ip} D^{R^-}_{i,p,t}\right) \tag{21}$$

Penalty for minimum/maximum specification deviation

If desired product qualities can not be fully achieved and, at the same time, they can partially be violated for certain products, the following constraints can be used in order to minimize the deviation.

$$prop^{\min}_{p,k} F^{P}_{p,t} - D^{S^+}_{k,p,t} \leq \sum_{i} pr_{i,k} F^{I}_{i,p,t} \qquad \forall p,k,t \tag{22}$$

$$prop^{\max}_{p,k} F^{P}_{p,t} + D^{S^-}_{k,p,t} \geq \sum_{i} pr_{i,k} F^{I}_{i,p,t} \qquad \forall p,k,t \tag{23}$$

where the continuous variables $D^{S+}{}_{k,p,t}$ and $D^{S-}{}_{k,p,t}$ define a value that, in some way, represents the deviation from the minimum and maximum specification for property k, respectively. If property k for product p is between minimum and maximum specification values, both variables will be equal to zero. The corresponding objective penalty terms are shown in Eq. (24)

$$Penalty = \sum_t \sum_p \sum_k \left(plty_{k,p}^{S^+} D_{k,p,t}^{S^+} + plty_{k,p}^{S^-} D_{k,p,t}^{S^-} \right) \tag{24}$$

Penalty for intermediate shortage

A common source of infeasible solutions is the lack of the minimum amount of intermediate required to satisfy either predefined component concentrations or certain market specifications. In this case, intermediate products can be purchased at higher cost from third-party. The continuous variable $S_{i,t}$ defines the amount of intermediate i needed in time slot t, which allows to relax minimum inventory constraints.

$$V_{i,t}^{I} = ini_i + prod_i \, e_t - \sum_{p,t' \le t} F_{i,p,t'}^{I} + S_{i,t} \qquad \forall i,t \tag{25}$$

The penalty term (26) includes is directly proportional to the component purchase cost.

$$Penalty = \sum_t \sum_i \left(plty_i^{SH} \, S_{i,t} \right) \tag{26}$$

10. NUMERICAL RESULTS

The performance of the proposed MILP-based approach for the scheduling and blending problem was tested with several real-world examples. The data are shown in Table 2 and 3. The basis of the example comprises nine intermediate product or components from the refinery which can be blended in different ways to satisfy multiple demands of three gasoline grades with different specifications over a 8-day scheduling horizon. Twelve key component and product properties are taken into consideration for solving the blending issue, where eight of them can be predicted by a linear volumetric average whereas the remainder is based on non-linear correlations. All the information about components such as cost, constant production rate, initial, minimum and maximum stocks and properties is shown in Table 2. Product data including price, requirements, inventory constraints, rate, recipe limits and specifications are given in Table 3. Dedicated storage tanks with limited capacities for components and products and three equivalent blend headers working in parallel are available in the refinery. The main goal is to maximize the total profit, considering component cost, product values and different penalties for component shortages and out-spec products.

Four different examples were solved with the purpose of analyzing the strong interaction between scheduling and blending decisions. In order to guarantee finding feasible solutions, slack variables for property deviations and intermediate shortages were included in all cases, which were null for all solutions generated. Example 1 is only focused on the blending problem and its solution is used as initial product recipes for next cases. Examples 2, 3 and 4 are solved using the proposed model based on both a discrete and a continuous time domain representation. When the discrete time representation is used, the scheduling horizon is divided into six consecutive time intervals, where intervals 1, 3, 4 and 6 have 1-day duration whereas intervals 2 and 6 have 2-day duration. For the continuous time representation, one time slot with unknown duration is postulated for each one of the six subintervals defined by the product due dates.

Table 2. Component data

	Component								
	C1	C2	C3	C4	C5	C6	C7	C8	C9
Cost ($/bbL)	24.00	20.00	26.00	23.00	24.00	50.00	50.00	50.00	50.00
Prod. rate (Mbbl/day)	15.00	33.00	20.00	14.00	18.00	10.00	0.00	0.00	0.00
Initial stock (Mbbl)	48.00	20.00	75.00	22.00	30.00	54.00	12.00	20.00	15.00
Min. stock (Mbbl)	5.0	5.0	5.0	5.0	5.0	5.0	0.0	0.0	0.0
Max. stock (Mbbl)	100.00	250.00	250.00	100.00	100.00	100.00	100.00	100.00	100.00
Property									
P1	93.00	104.00	104.90	94.80	87.40	118.00	87.30	95.20	93.30
P2	92.10	91.90	91.90	81.50	86.10	100.00	79.50	85.80	81.90
P3	0.7069	0.8692	0.6167	0.6731	0.6540	0.7460	0.7460	0.8187	0.7339
P4	3.60	1.00	100.00	94.90	91.50	15.00	0.00	1.30	34.30
P5	16.30	4.50	100.00	97.10	95.50	100.00	0.00	6.00	57.10
P6	94.30	93.50	100.00	100.00	100.00	100.00	0.00	93.90	95.90
P7	35.00	22.70	351.10	117.10	93.00	31.30	63.30	16.00	52.40
P8	0.007	0.00	0.00	0.009	0.0002	0.05	0.0063	0.1805	0.057
P9	0.00	88.60	0.00	2.30	0.20	0.00	43.98	65.30	21.30
P10	0.00	0.1	61.30	48.90	36.00	0.00	1.04	0.60	33.30
P11	0.00	3.30	0.00	1.10	0.10	0.00	3.33	0.90	0.80
P12	0.00	0.00	0.00	0.00	0.00	15.40	0.00	0.00	0.00

Table 3. Product data

	Product								
	G1			G2			G3		
Price ($/bbL)		31.00			31.00			31.00	
Requirement(Mbbl)	MIN	MAX	LIFT	MIN	MAX	LIFT	MIN	MAX	LIFT
Day 1 (Mbbl)	5.00	45.00	10.00	5.00	50.00	12.00	5.00	50.00	10.00
Day 3 (Mbbl)				5.00	50.00	25.00			
Day 4 (Mbbl)	5.00	45.00	25.00	5.00	50.00	23.00			
Day 5 (Mbbl)									
Day 7 (Mbbl)	5.00	45.00	30.00						
Day 8 (Mbbl)	5.00	45.00	10.00				5.00	50.00	22.00
Inventory (Mbbl)	5.00	150.00		5.00	150.00		5.00	150.00	
Rate (Mbbl/day)	5.00	45.00		5.00	50.00		5.00	50.00	
Recipe (%)	MIN	MAX		MIN	MAX		MIN	MAX	
C1	0.00	22.00		0.00	25.00		0.00	25.00	
C2	0.00	20.00		0.00	24.00		0.00	24.00	
C3	2.00	10.00		0.00	10.00		0.00	10.00	
C4	0.00	6.00		0.00	23.00		0.00	23.00	
C5	0.00	25.00		0.00	25.00		0.00	25.00	
C6	0.00	10.00		0.00	10.00		0.00	10.00	
C7	0.00	100.00		0.00	0.00		0.00	0.00	
C8	0.00	100.00		0.00	0.00		0.00	0.00	
C9	0.00	100.00		0.00	0.00		0.00	0.00	
Specifications	MIN	MAX		MIN	MAX		MIN	MAX	
P1	95.00			98.00			98.00		
P2	85.00			88.00			88.00		
P3	0.72	0.775		0.72	0.775		0.72	0.775	
P4	20.00	50.00		20.00	48.00		22.00	50.00	
P5	46.00	71.00		46.00	71.00		46.00	71.00	
P6	85.00			85.00			85.00		
P7	45.00	60.00		45.00	60.00		60.00	90.00	
P8		0.015			0.015			0.008	
P9		42.00			42.00			42.00	
P10		18.00			18.00			18.00	
P11		1.00			1.00			1.00	
P12		2.70			2.70			2.70	

10.1. Example 1 (Blending Problem)

Example 1 is focused on a single-period blending problem of three products (G1, G2, G3). Its main goal is to find the best or 'preferred' recipe for each product that minimizes blend cost and simultaneously satisfies all quality specifications. Preferred recipes are used as initial solutions for subsequent examples. For this particular problem, temporal, inventory and resource constraints coming from the scheduling problem are disregarded by assuming that enough resources, component stocks and time are available as needed to produce 1 Mbbl of each product once. In this way only the blending problem is taking into consideration. Component cost and properties, variable recipe limits and stringent product specifications are the central features to be considered for solving example 1. The proposed MILP-based iterative procedure was used to find preferred recipes for all required products. In this case, initial product recipes were generated taking into account only linear product properties. Then, the iterative procedure was performed to update the initial recipes with the purpose of satisfying all product specifications. Preferred recipes for products G2 and G3 were found by executing just one iteration of the proposed procedure, whereas an additional iteration was needed to satisfy all specifications for product G1, since the maximum sulfur content was violated both in the initial recipe as in the first iteration. In order to generate feasible recipes, component concentrations for each product were updated by the MILP model in each iteration, which gradually increased the blend cost. The recipe evolution for product G1 in terms of component concentration is presented in detail in Figure 5. Blend cost and product properties associated to each recipe are shown in Table 4. In addition to the exact values for each property predicted by nonlinear correlations, the approximations predicted by the proposed linear functions are also presented in Table 4. It should be noted that predictions of nonlinear properties tend to improve when the number of iterations is increased. Finally, best product recipes and 'bias' factors for all products are reported in Table 5.

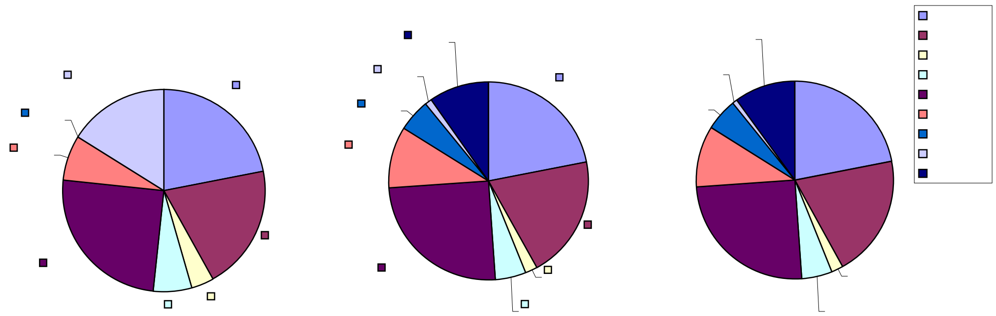


Figure 5. Convergence to preferred recipe for product G1 (iterative procedure)

Table 4. Iterative blending problem for product G1

	Min. Spec.	Initial recipe	Iteration 1		Iteration 2		Max Spec.
Blend cost ($/bbL)		29.30	29.97		29.99		
Quality		*Value*	*Value*	*approx.*	*Value*	*Approx.*	
P1	95.00	97.891	97.898	97.7737	97.893	97.8928	
P2	85.00	88.417	88.470	88.0493	88.438	88.4335	
P3	0.72	0.7418	0.7325		0.7324		0.775
P4	20.00	34.455	35.418		35.409		50.00
P5	46.00	46.00	50.80		50.833		71.00
P6	85.00	96.460	91.797		91.780		
P7	45.00	60.00	60.00		60.00		60.00
P8		0.0378	0.0152	0.0150	0.0150	0.0150	0.015
P9		28.458	22.974		22.923		42.00
P10		14.256	15.974		16.005		18.00
P11		0.8964	1.00		1.00		1.00
P12		1.1223	1.5684	1.5488	1.5687	1.5684	2.70

Table 5. Preferred product recipes

	Product		
	G1	G2	G3
Blend cost ($/bbL)	29.99	25.28	24.98
Recipe (%)			
C1	22.00	25.00	25.00
C2	20.00	23.947	24.00
C3	2.00	16.794	1.372
C4	4.847	25.00	16.636
C5	25.00	9.259	25.00
C6	10.00		7.992
C7	5.198		
C8	0.958		
C9	9.997		
Quality			
P1	97.893 (bias =1.527)	98.4122 (bias = 1.5611)	98.2214 (bias=1.5208)
P2	88.438 (bias = -0.659)	88.4594 (bias = -1.0439)	88.3310 (bias=-1.0861)
P3	0.7324	0.7305	0.7289
P4	35.409	41.3410	42.3734
P5	50.833	54.5932	54.5475
P6	91.780	97.0184	97.0150
P7	60.00	60.00	64.2465
P8	0.0150	0.0079	0.0072
P9	22.923	21.6536	21.6966
P10	16.005	17.2363	18.00
P11	1.00	1.00	1.00
P12	1.5687	1.4561	1.2597

10.2. Example 2

Example 2 addresses the original scheduling and blending example provided by ABB. Preferred product recipes found in example 1 were used as the initial solution for the proposed iterative procedure. Despite using linear approximations, the proposed MILP model was capable of finding in just one iteration the same solution generated by nonlinear optimization tools. However, although the discrete and continuous time representations obtained the same profit in terms of component cost and product value (1,611.21 $), the continuous time representation is able to find a schedule that operates the blenders at full capacity for 2.67 days less than the discrete time representation, which can significantly reduce the total operating cost. Product schedules based on a discrete and continuous time representation are reported in Tables 6 and 7, respectively. The inventory evolution of components through the scheduling horizon is shown in Figure

Table 6. Product schedule (Example 2 - discrete time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	15.02	10.00	5.02
	T2	1.00	3.00	0.00	0.00	5.02
	T3	3.00	4.00	45.00	25.00	25.02
	T4	4.00	5.00	0.00	0.00	25.02
	T5	5.00	7.00	45.00	30.00	40.02
	T6	7.00	8.00	45.00	10.00	75.02
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	1.00	3.00	50.00	25.00	63.00
	T3	3.00	4.00	50.00	23.00	90.00
	T4	4.00	5.00	0.00	0.00	90.00
	T5	5.00	7.00	0.00	0.00	90.00
	T6	7.00	8.00	0.00	0.00	90.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	1.00	3.00	0.00	0.00	40.00
	T3	3.00	4.00	0.00	0.00	40.00
	T4	4.00	5.00	0.00	0.00	40.00
	T5	5.00	7.00	0.00	0.00	40.00
	T6	7.00	8.00	50.00	22.00	68.00

Table 7. Product schedule (Example 2 - continuous time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	45.00	10.00	35.00
	T2	1.00	2.00	0.00	0.00	35.00
	T3	3.00	4.00	45.00	25.00	55.00
	T4	4.00	5.00	0.00	25.00	55.00
	T5	5.00	5.33	15.02	0.00	40.02
	T6	7.00	8.00	45.00	10.00	75.02
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	1.00	2.00	50.00	0.00	63.00
	T3	3.00	4.00	50.00	23.00	90.00
	T4	4.00	5.00	0.00	23.00	90.00
	T5	5.00	5.33	0.00	0.00	90.00
	T6	7.00	8.00	0.00	0.00	90.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	1.00	2.00	0.00	0.00	40.00
	T3	3.00	4.00	0.00	0.00	40.00
	T4	4.00	5.00	0.00	0.00	40.00
	T5	5.00	5.33	0.00	0.00	40.00
	T6	7.00	8.00	50.00	22.00	68.00

10.3. Example 3

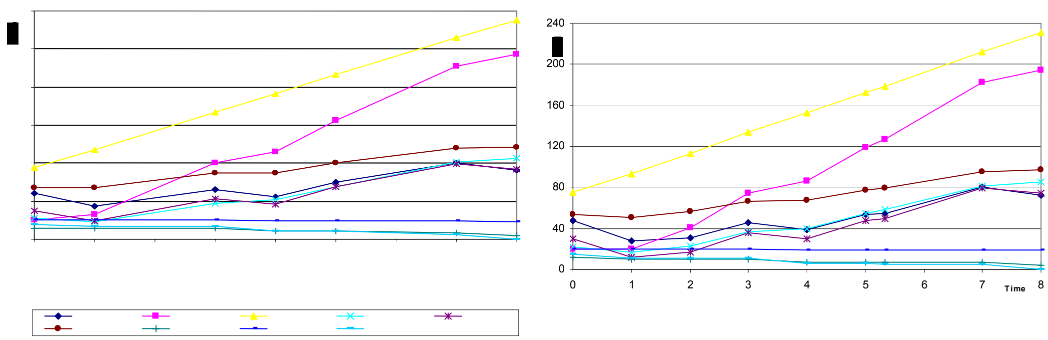
This example introduces a small change into example 2 in order to evaluate the effect of predefining minimum and maximum requirements for each time interval. In this way the amount to be produced in each time interval becomes a model variable only restricted by minimum and maximum production rates. The amount of product to be lifted at specific due dates is still a hard constraint to be satisfied. This modification allows the model to increase the total production by almost 36%, i.e. from 400.02 Mbbl to 542.02 Mbbl, which represents increasing the total profit to 2,448.05 ($), which is almost 52% increase (see Table 13). Preferred product recipes are used for all products and one iteration is only executed. Product schedules based on a discrete and continuous time representation are shown in Tables 8 and 9, respectively. In this example we note that the continuous time representation needs 2.60 days less of total operating time to reach the same production level as the discrete time model. Figure 7 shows Gantt-charts corresponding to examples 2 and 3.

Table 8. Product schedule (Example 3 - discrete time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	45.00	10.00	35.00
	T2	1.00	3.00	60.02	0.00	95.02
	T3	3.00	4.00	0.00	25.00	70.02
	T4	4.00	5.00	0.00	0.00	70.02
	T5	5.00	7.00	0.00	30.00	40.02
	T6	7.00	8.00	45.00	10.00	75.02
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	1.00	3.00	0.00	25.00	13.00
	T3	3.00	4.00	50.00	23.00	40.00
	T4	4.00	5.00	0.00	0.00	40.00
	T5	5.00	7.00	60.00	0.00	100.00
	T6	7.00	8.00	50.00	0.00	150.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	1.00	3.00	72.00	0.00	112.00
	T3	3.00	4.00	0.00	0.00	112.00
	T4	4.00	5.00	0.00	0.00	112.00
	T5	5.00	7.00	10.00	0.00	122.00
	T6	7.00	8.00	50.00	22.00	150.00

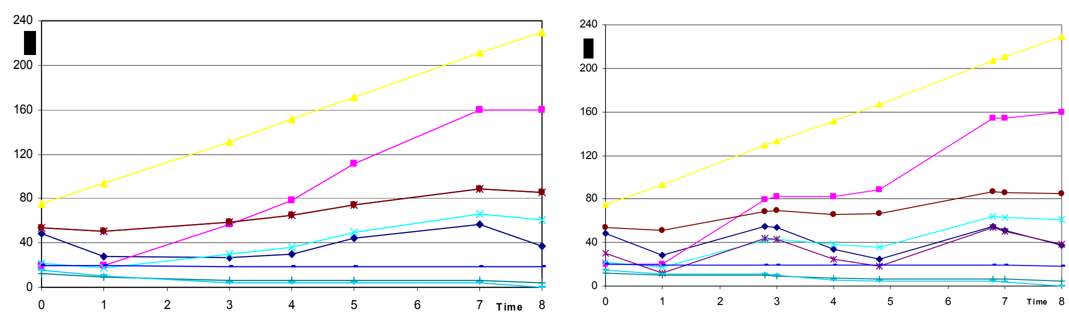
Table 9. Product schedule (Example 3 - continuous time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	45.00	10.00	35.00
	T2	2.80	3.00	9.00	10.00	44.00
	T3	3.00	4.00	45.00	35.00	64.00
	T4	4.00	4.80	4.00	35.00	68.00
	T5	6.80	7.00	9.00	65.00	47.00
	T6	7.00	8.00	38.02	75.00	75.02
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	2.80	3.00	10.00	37.00	23.00
	T3	3.00	4.00	50.00	60.00	50.00
	T4	4.00	4.80	40.00	60.00	90.00
	T5	6.80	7.00	10.00	60.00	100.00
	T6	7.00	8.00	50.00	60.00	150.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	2.80	3.00	0.00	10.00	40.00
	T3	3.00	4.00	50.00	10.00	90.00
	T4	4.00	4.80	40.00	10.00	130.00
	T5	6.80	7.00	10.00	10.00	140.00
	T6	7.00	8.00	32.00	32.00	150.00



a) Example 2 (discrete time)

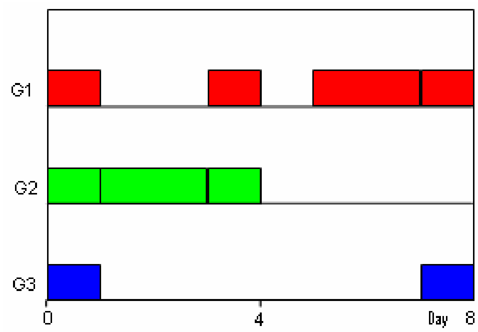
b) Example 2 (continuous time)



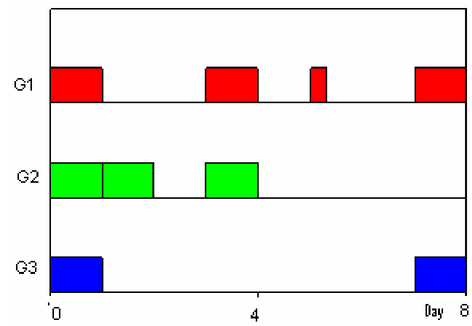
c) Example 3 (discrete time)

d) Example 3 (continuous time)

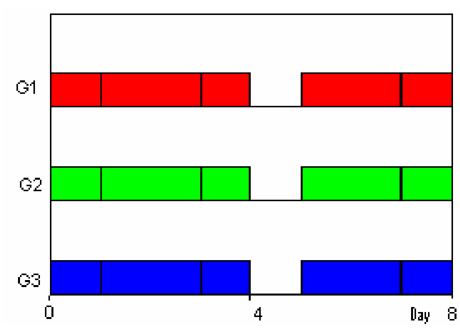
Figure 6. Evolution of component stocks (Examples 2 and 3)



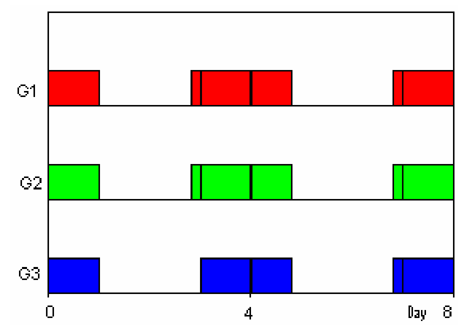
a) Example 2 (discrete time)



b) Example 2 (continuous time)



c) Example 3 (discrete time)



d) Example 3 (continuous time)

Figure 7. Gantt charts (examples 2 and 3)

10.4. Example 4

Finally, this example deals with a modified version of the original example where the following changes are introduced: (1) Properties P1 and P2 are decreased by 1 for components C1, C2, C3 and C6, (2) the price of G3 is increased to 31.05 \$/bbl, (3) component cost is increased to 27 \$/bbl and 23 \$/bbl for C1 and C2 and (4) production rates for C1 and C2 are reduced to 13 Mbbl/day and 31 Mbbl/day respectively. All other data remain as in the original example. The main goal here is to analyze the effect of these changes in the blending and scheduling decisions. Detailed product schedules for discrete and continuous time representations for Example 4 are shown in Tables 10 and 11, respectively. Regarding the blending decisions, preferred recipes found in example 1 are proposed as the initial solution. However, they have to be updated because changes in the component properties make some preferred recipes infeasible. Only one iteration is required to modify the infeasible recipes related to products G2 and G3. Preferred and updated recipes for these products are compared in Table 12. As shown, the new recipes satisfy all product specifications but at the same time, updated component concentrations increase the blend cost with which the profit is reduced to 1,234.49 ($). This difference mainly arises because component costs were increased and octane numbers were reduced. It should be noted that key properties such RON and MON are satisfied with a very small margin, which means that quality giveaway is minimized by using the proposed method.

Table 10. Product schedule (Example 4 - discrete time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	45.00	10.00	35.00
	T2	1.00	3.00	0.00	0.00	35.00
	T3	3.00	4.00	5.00	25.00	15.00
	T4	4.00	5.00	0.00	0.00	15.00
	T5	5.00	7.00	20.00	30.00	5.00
	T6	7.00	8.00	10.00	10.00	5.00
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	1.00	3.00	100.00	25.00	113.00
	T3	3.00	4.00	50.00	23.00	140.00
	T4	4.00	5.00	0.00	0.00	140.00
	T5	5.00	7.00	0.00	0.00	140.00
	T6	7.00	8.00	0.00	0.00	140.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	1.00	3.00	0.00	0.00	40.00
	T3	3.00	4.00	0.00	0.00	40.00
	T4	4.00	5.00	0.00	0.00	40.00
	T5	5.00	7.00	0.00	0.00	40.00
	T6	7.00	8.00	50.00	22.00	68.00

Table 11. Product schedule (Example 4 - continuous time representation)

Product	Period	Start	End	Prod	Lift	Inventory
G1	T1	0.00	1.00	16.00	10.00	6.00
	T2	1.00	3.00	0.00	10.00	6.00
	T3	3.00	4.00	45.00	35.00	26.00
	T4	4.00	5.00	0.00	35.00	26.00
	T5	5.00	5.20	9.00	35.00	5.00
	T6	7.00	8.00	10.00	75.00	5.00
G2	T1	0.00	1.00	50.00	12.00	38.00
	T2	1.00	3.00	100.00	37.00	113.00
	T3	3.00	4.00	50.00	60.00	140.00
	T4	4.00	5.00	0.00	60.00	140.00
	T5	5.00	5.20	0.00	60.00	140.00
	T6	7.00	8.00	0.00	60.00	140.00
G3	T1	0.00	1.00	50.00	10.00	40.00
	T2	1.00	3.00	0.00	10.00	40.00
	T3	3.00	4.00	0.00	10.00	40.00
	T4	4.00	5.00	0.00	10.00	40.00
	T5	5.00	5.20	0.00	10.00	40.00
	T6	7.00	8.00	50.00	32.00	68.00

Table 12. Updated product recipes (Example 4)

	Product			
	G2		G3	
	Preferred	Updated	Preferred	Updated
Blend cost ($/bbL)	25.28	26.92	24.98	26.67
Recipe (%)				
C1	25.00	25.00	25.00	25.00
C2	23.947	24.00	24.00	24.00
C3	16.794	0.223	1.372	3.195
C4	25.00	16.09	16.636	14.869
C5	9.259	24.831	25.00	24.269
C6		9.856	7.992	8.640
Quality				
P1	97.8204	98.0235	97.6283	98.052
P2	87.8588	88.0133	87.7294	88.0455
P3	0.7305	0.7309	0.7289	0.7285
P4	41.3408	40.831	42.3734	41.9724
P5	54.5936	54.571	54.5476	54.6305
P6	97.0184	97.015	97.015	97.015
P7	60.00	60.00	64.2473	68.1274
P8	0.0079	0.0081	0.0072	0.0074
P9	21.6533	21.6837	21.6966	21.6546
P10	17.2362	16.968	18.00	18.00
P11	1.00	0.9938	1.00	0.9799
P12	1.4562	1.5491	1.2597	1.3626

Table 1. Comparison of results

Example	Blend value	Comp. stock production	Comp. inventory build	Total Profit	Profit / BBL
2	12,400.61	22,352.00	11,562.60	1,611.21	4.03
3	16,802.61	22,352.00	7,997.44	2,448.05	4.52
4	11,785.00	23,504.00	12,953.49	1,234.49	3.25

Unit: M$

Table 2. Model size and computational requirements

Example	Binary vars, cont. vars, constraints	Objective function	CPU time[a]
1	- , 127,81	12	0.13
2.a (NLP model*)	-, 919 ,772	1,611.21	1.25
2.a	9 , 757, 679	1,611.21	0.26
2.b	9 , 841, 832	1,611.21	0.26
3.a	18 , 757, 679	2,448.05	0.23
3.b	18 , 841, 832	2,448.05	0.26
4.a	9 , 757, 679	1,234.49	0.23
4.b	9 , 841, 832	1,234.49	0.26

[a] Seconds on Pentium IV PC with GAMS 21.2/CPLEX 8.1 - * All scheduling decisions are predefined

11. COMPUTATIONAL RESULTS

Different scheduling and blending problems were solved in the previous section in order to evaluate the efficiency of the proposed method. Example 1 dealt with a pure blending problem whereas examples 2, 3 and 4 also accounted scheduling decisions. Examples 3 and 4 correspond to modified versions of the original Example 2 where minimum and maximum requirements were relaxed (Example 3) and certain changes in component properties and cost and product prices were incorporated (Example 4). Table 13 summarizes the results for examples 2, 3 and 4, while Table 14 provides the computational statistics on the four examples. As can be seen, the size of the MILP problems is not very large and involves a modest number of 0-1 variables. For this reason every single problem needs no more than 1 sec at CPU time with CPLEX 8.1, thus showing that the proposed models and the iterative MILP procedure are very efficient. The method found more economic solutions to a combined scheduling and blending optimization problem almost an order of magnitude faster than it took to solve only the blending NLP problem with a predetermined schedule.

12. CONCLUSIONS

A new MILP approach to simultaneously solve gasoline short-term scheduling and blending problems has been proposed. Although the method is able to deal with non-linear product properties and variable recipes, the use of non-linear constraints was avoided through an iterative procedure that can be based on a discrete or a continuous time mathematical formulation. As shown in the examples, the proposed model can generate very good solutions in terms of profit with very low CPU time requirements.

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