Economic and Environmental Strategic Water Management in the Shale Gas Industry: Application of Cooperative Game Theory

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Abstract

In this work, a Mixed-Integer Linear Programming (MILP) model is developed to address optimal shale gas water management strategies among shale gas companies that operate relatively close. The objective is to compute a distribution of water-related costs and profit among shale companies to achieve a stable agreement on cooperation among them that allows increasing total benefits and reducing total costs and environmental impacts. We apply different solution methods based on cooperative game theory: The Core, the dual Core, the Shapley value and the minmax Core. We solved different case studies including a large problem involving 4 companies and 207 wells. In this example, individual cost distribution (storage cost, freshwater withdrawal cost, transportation cost and treatment cost) assigned to each player is included. The results show that companies that adopt cooperation strategies improve their profits and enhance the sustainability of their operations through the increase of recycled water.

Topical Heading: Process System Engineering.

Keywords: Cooperative game theory, shale gas, optimization, water management, MILP.
INTRODUCTION

In recent years, the development of shale gas extraction has generated continuous growth in the production of natural gas, which is expected to increase in the coming years. In fact, the exploitation of shale gas in the United States has experienced rapid growth during the 2010s, accounting from 8% of total natural gas production in 2000 to 49.8% in 2015. This fast increase in natural gas production from shale formations is due to recent advances in technologies, such as horizontal drilling and hydraulic fracturing. However, these techniques entail some environmental risks and involves a significant water footprint. Specifically, during the hydraulic fracturing from 7500 to 38000 m³ of freshwater is consumed. After fracturing a well, a large amount of flowback water and produced water are generated as highly contaminated water. Therefore, proper management of wastewater is needed to deal with those large volumes of water.

Current water management strategies include disposal of wastewater through Class II disposal wells, transfer to an onsite/centralized water treatment facility or direct reuse in the drilling of subsequent wells, and the reuse in new drilling and fracturing operations. From the environmental point of view, the best option is the direct reuse of the flowback water because it allows reducing the environmental problems associated with water management, such as transportation, disposal or treatment.

Several publications have focused on the design and operation of shale gas supply chains for optimal water management. Alternatively, other studies have focused on the minimization of water consumption during shale gas production. In addition, mathematical models for shale water management have been developed to minimize expenses (i.e., costs for the freshwater, treatment, storage, disposal, and transportation), freshwater usage and wastewater discharge. However, all these works have focused on studying water management considering that all wellpads are exploited by a single company, whereas in practice, there are typically different companies operating relatively close to each other in a given shale gas play as shown in Figure 1.
Figure 1. Companies operating on the Marcellus shale play.20

Companies that are working on the same shale play, and their shale pads are relatively close, could develop possible cooperation activities, such as sharing onsite water treatment facilities and wastewater among different wellpads (owned by different companies) that reduce the total demand for freshwater and the storage capacity in some wellpads and, consequently, the transportation costs. Additionally, these activities allow companies to reduce the environmental impact of their operations.

This work studies possible cooperative strategies among companies that allow reducing both costs and environmental impacts of water management in shale gas production. The result of cooperation could be the same as the result obtained using simultaneous optimization between companies. However, the question is how to distribute costs or profit among the cooperating companies, what allows them to choose if they want to cooperate or not depending on their interests. In this work, to distribute the total payoff among the members, different solution method based on cooperative game theory, such as Core, dual Core, the Shapley value and the minmax Core are applied.

Contrary to non-cooperative games, which do not analyze the coalitions and assume that each company acts independently to maximize its utility, in cooperative games companies interact with a common purpose and analyze the formation of coalitions among the members of a game.21
Regarding this area, Gao and You studied non-cooperative game theory considering a particular class of games, specifically, leader-follower Stackelberg game structure for the entire shale gas supply chain.\textsuperscript{22,23} The objective of this work is to compute the optimal operating conditions and to determine the distribution of the payoff among the different companies in order to achieve a stable agreement on cooperation among them. Operating conditions include the time, place and amount of freshwater acquired by each company, the number and size of water storage tanks, the drilling and fracturing schedule of each wellpad, the schedule of water reuse, and the characteristics of onsite treatment facilities.

The rest of this paper is organized as follows. The next section gives a general description of the cooperative game theory and its applications. Then, the problem statement is described. Different case studies are proposed in order to show the benefits of cooperative games in shale gas water management, and finally, conclusions are drawn.

**COOPERATIVE GAME THEORY**

Cooperative game theory predicts rational strategic behaviors of individuals in cooperating situations, i.e., it studies the interaction among coalitions of players. This theory has been applied to a wide variety of situations where costs and benefits resulting from cooperation are allocated to the “players”\textsuperscript{24-28} For example, some works have studied game theory in the management of water resources,\textsuperscript{29-33} and others have shown that game theory can help resolve conflicts over water acquisition.\textsuperscript{34,35}

Generally, a cooperative game is defined by a set of players $N = \{ 1, 2, ..., n \}$ and any subset of cooperation players $S \subseteq N$ is called “coalition”. When all players cooperate in a unique coalition, it is called the “grand coalition” $S = N = \{ 1, 2, ..., n \}$. Note that, the function that assigns the quantifiable unit to each coalition (e.g. profit, cost) is called “characteristic function” ($\nu(S)$).

This quantifiable unit can be interpreted according to stakeholder interest. In this work, we deal
with profit, environmental and cost games. In a profit game, players favor a higher outcome for themselves, whereas in environmental and cost games, they prefer lower amounts.

In general, a coalition is formed when the cooperation leads to additional value. It is also possible to define the dual value of a coalition. This is the value that the great coalition $N$ loses if the coalition $S$ does not cooperate with the grand coalition (Eq. (1)).

$$\nu^*(S) = \nu(N) - \nu(N \setminus S)$$

(1)

The main question in cooperative game theory is as follows: given the sets of feasible payoffs for each coalition, what payoffs will be given to each player? First, the properties that each payoff has to satisfy are described. Then, the allocation methods in cooperative game theory applied in this paper to allocate whatever quantifiable unit (cost, profit or environmental impact) of the grand coalition among the players are described in detail.

Payoff allocation properties

Players are willing to form the grand coalition given a fair allocation of the profit among the players. Otherwise, the outcome will be ineffective, and the players will not want to cooperate.

The allocation of whatever quantifiable unit is denoted by $\pi_i$ and defines the portion of the unit that is allocated to each player. The following important properties should be achieved (they are written for a profit game):

- Efficiency guarantees that the total profit of the grand coalition must be equal to the sum of the profit share of each player $N$:

$$\nu(S) = \sum_{i \in N} \pi_i$$

(5)

- Individual rationality describes that the profit of the player that acts alone must be lower or equal than the profit of that player cooperating:

$$\pi_i \geq \nu(\{i\}) \quad i \in N$$

(6)
Coalitional rationality. It extends the individual rationality to coalitions, and establishes
that the profit of a coalition must be lower or equal than the profit of that coalition when
it is part of the grand coalition:
\[ \sum_{i \in S} \pi_i \geq \nu(S) \quad S \subset N, \; S \neq \emptyset \] (7)

Note that, in environmental and cost games, the characteristic function in individual and
coalitional rationality (Eqs. (6-7)) will be higher than or equal to the corresponding outcome.

An imputation \( \pi \) strongly dominates an imputation \( \tau \) over a set \( S \) (written \( \pi \succ_S \tau \)) if:
\[ \pi_i > \tau_i \quad \forall i \in S, \]
\[ \sum_{i \in S} \pi_i < \nu(S) \] (2)

These equations state that if all players in a coalition \( S \) get strictly more in the imputation \( \pi \) than
in \( \tau \), and they can change from \( \pi \) to \( \tau \), then imputation \( \pi \) strongly dominates \( \tau \) over \( S \).

We say that an imputation \( \pi \) weakly dominates an imputation \( \tau \) over a coalition \( S \) (written
\( \pi \succeq_S \tau \)) if:
\[ \pi_i \geq \tau_i \quad \forall i \in S, \]
\[ \sum_{i \in S} \pi_i < \nu(S), \quad \sum_{i \in S} \tau_i \geq \nu(S) \] (3)

It is said that an imputation \( \pi \) dominates an imputation \( \tau \) dually over a coalition \( S \) (written
\( \pi \succ_S \tau \)) if:
\[ \pi_i \geq \tau_i \quad \forall i \in S, \]
\[ \sum_{i \in S} \pi_i \leq \nu(S), \quad \sum_{i \in S} \tau_i < \sum_{i \in N \setminus S} \pi_i \geq \nu(N \setminus S) \] (4)

Note that “strong domination” implies “weak domination” and, in turn it implies “dual
domination”. Detailed information about dominations and their properties can be found in
Stolwijk (2010).
Allocation methods in cooperative game theory

The Core

The Core is a central concept in game theory\(^3\) formed by all the imputations for which there is no sub-coalition that can obtain better results than the grand coalition. The Core is then formed by the set of imputations that are efficient and stable. An imputation is efficient if the total profit is distributed among all the partners, and it is stable if the principles of individual rationality and coalitional rationality are met. Therefore, the Core combines the three properties mentioned above and is defined as follows:

\[
C(N,c) := \left\{ \pi \in \mathbb{R}^N \mid \sum_{i \in N} \pi_i = \nu(N) \text{ and } \sum_{i \in S} \pi_i \geq \nu(S) \text{ for all } S \subset N, S \neq \emptyset \right\} \tag{8}
\]

Basically, the Core includes all the points that are not strongly dominated. The core is also the set of all not weakly dominated imputations (see Stolwijk\(^3\) for a proof).

Let us illustrate the concept of Core with a small example. Assume a three player game in which the individual players get the following profits: \(\nu(\{1\}) = 10; \nu(\{2\}) = 15; \nu(\{3\}) = 12\) the collaboration between two partners will produce the following profits for each coalition: \(\nu(\{1,2\}) = 30; \nu(\{1,3\}) = 25; \nu(\{2,3\}) = 30\), Finally the grand coalition (the three players cooperating) will produce a profit \(\nu(\{1,2,3\}) = 48\).

The set of Core imputations \((\pi_i, i = 1, 2, 3)\) is formed by all the solutions to the following set of constrains:

\[
\begin{align*}
\pi_1 + \pi_2 + \pi_3 &= \nu(\{1,2,3\}) = 48 \quad \text{Efficiency} \\
\pi_1 &\geq \nu(\{1\}) = 10 \quad \text{Individual rationality} \\
\pi_2 &\geq \nu(\{2\}) = 15 \\
\pi_3 &\geq \nu(\{3\}) = 12 \\
\pi_1 + \pi_2 &\geq \nu(\{1,2\}) = 30 \quad \text{Coalitional rationality} \\
\pi_1 + \pi_3 &\geq \nu(\{1,3\}) = 25 \\
\pi_2 + \pi_3 &\geq \nu(\{2,3\}) = 30 \\
\pi_1, \pi_2, \pi_3 &\in \mathbb{R}
\end{align*}
\tag{9}
\]

An example at a solution to Eq. (9) would be \(\pi_1 = 13, \pi_2 = 19\) and \(\pi_3 = 16\).
The allocation in the Core is fair in a weak sense because one player can benefit more than others. In addition to the Core, there are many Core variants that try to determine a fair profit allocation.  

- **The Dual Core**

The key concept in the Core definition is a strong dominance. An imputation not strongly dominated is also not weakly dominated and vice versa. If we replace strong domination by weak domination, the set stays the same. However, if instead of «not strongly dominated» we use «not dually dominated» we could get a different set of imputations. The Dual Core is the set of all imputations not dually dominated. That means that if a coalition $S$ leaves the grand coalition, either at least one member of $S$ will have to pay a price, or no player in $S$ has to pay a price and no player in $N \setminus S$ has to pay a price. The Dual Core can be defined as follows:

$$DC(N,c) := \left\{ \pi \in \mathbb{R}^{|N|} : \sum_{i \in S} \pi_i = v(S) \forall S | v^*(S) = v(S), \sum_{i \in S} \pi_i > v(S) \forall S | v^*(S) \neq v(S) \right\}$$

In the Core, it is eventually possible that imputations appear such that there is a sub-coalition $S$ that makes it necessary to cooperate in the grand coalition to improve the benefit $N \setminus S$. But at the same time, coalition $S$ does not improve its benefit by this cooperation. The Dual Core does not have that problem. Therefore, the Dual Core is a subset of imputations in the Core that are more stable (fairer). Thus, the Dual Core is a solution concept that has better rational properties than the Core. If the Dual Core exists, imputations in the Dual Core are more rational (fair) than imputations in the rest of the Core.

In non-cooperative games, the solution is usually given in terms of Nash equilibrium. Although Nash equilibrium is a non-cooperative concept, it has also been applied to cooperative games. Maybe the most interesting result is that the Dual Core is the set of all strict Nash equilibria and the Core is the set of all weak Nash equilibria. A detailed discussion on the relation of Nash equilibrium and Core / Dual Core is out of the scope of this work. The interested reader can find a comprehensive discussion in the literature.
In general, for the kind of problems that we deal in this work, the Dual Core and the Core are coincident. Therefore, the set of imputations in the Core are also the set of strict Nash equilibria solutions.

- **Minmax Core**

Another variant of the Core that guarantees a rational, efficient and fair profit allocation is the minmax Core. This solution concept is based on the relative benefit in percentage of $\nu(S)$, i.e., the greater the benefit, the higher the profit assigned to a subcoalition $S$. The mathematical formulation is similar to the Core formulation. In this case, the coalitional profit is multiplied by $\eta$, which ensures that no coalition has a profit allocation greater than $\eta \cdot \nu(S)$:

$$\min_{\pi_i} \eta \\
\text{s.t.} \sum_{i \in N} \pi_i = \nu(N)$$

$$\sum_{i \in S} \pi_i \leq \eta \nu(S) \ \forall S \subset N, S \neq \emptyset$$

$$\pi_i \in \mathbb{R} \ \forall i \in N$$

$$\eta \in \mathbb{R}$$

In the three players example presented above the minmax Core produce the following imputations by optimizing (11): $\pi_1 = 12.97, \ \pi_2 = 19.46, \ \pi_3 = 15.57$

**The Shapley value**

The Shapley value maybe is the most used solution concept that produces a unique imputation in cooperative game theory.

While the Core in most of the cases represents a set of possible allocations with specific properties, the Shapley value (Eq. (12)) provides a unique solution for every game in coalitional form:

$$\pi_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!}{|N|!} \frac{(|N|-|S|-1)!}{|S|-1} \left[ \nu(S \cup \{i\}) - \nu(S) \right]$$

The Shapley value can be interpreted as follows: Let a coalition be formed by a player at a time. When the new player joins the coalition, he/she would like to receive his/her contribution
\[ \nu(S \cup i) - \nu(S) \]. The Shapley value is the average value of this contribution taking into account all the different possible permutations in which a coalition can be formed.

The solution among the players follows three axioms (symmetry, efficiency and additivity – see Shapley (1953) for a detailed description) that are derived from properties that should be satisfied by such an allocation.

In general, the Shapley value is considered as a good answer in cooperative game theory, since it is based on those who contribute more to the groups should receive more.

In the three players’ example, the Shapley value yields the following imputations:

\[
\pi_1 = \frac{1}{3}(\nu(1) - \nu(\emptyset)) + \frac{1}{6}(\nu(1,2) - \nu(2)) + \frac{1}{6}(\nu(1,3) - \nu(3)) + \frac{1}{3}(\nu(1,2,3) - \nu(2,3)) = 14
\]

\[
\pi_2 = \frac{1}{3}(\nu(2) - \nu(\emptyset)) + \frac{1}{6}(\nu(1,2) - \nu(1)) + \frac{1}{6}(\nu(2,3) - \nu(3)) + \frac{1}{3}(\nu(1,2,3) - \nu(1,3)) = 19
\]

\[
\pi_3 = \frac{1}{3}(\nu(3) - \nu(\emptyset)) + \frac{1}{6}(\nu(1,3) - \nu(1)) + \frac{1}{6}(\nu(2,3) - \nu(2)) + \frac{1}{3}(\nu(1,2,3) - \nu(1,2)) = 15
\]

\[\text{(13)}\]

**PROBLEM DESCRIPTION**

In this work, as mentioned before, we focus on cooperative game theory to allocate a quantifiable unit (cost, profit or environmental impact) to each one of the companies which work in the same shale play. Companies will be able to follow different strategies, such as forming a ‘joint venture’ accepting the allocation of costs/benefits or environmental impacts that come from game theory, or establishing contracts (e.g., water sharing) that result in imputation of costs/benefits equal to that obtained from cooperative game theory.

To formulate the shale water management problem, we use mathematical programming techniques. The target is to find an optimal solution (maximizing or minimizing an objective function) subject to a set of equality and inequality constraints. Specifically, our planning problem is formulated as a Mixed-Integer Linear Programming (MILP) problem and is composed of parameters (i.e., known input data) and continuous and discrete variables.
Supply chain network description

Any shale gas water management model available in literature can be eventually used and extended with cooperative game theory concepts. In this work, we adapted the model presented by Carrero-Parreño et al.\textsuperscript{42}

The superstructure addressed in this work (see Figure 2) comprises wellpads (i.e., companies, player) \( p \), unconventional shale gas wells \( w \), centralized water treatment technologies (CWT) \( k \), natural freshwater sources \( f \), and disposal wells \( d \).

![Figure 2. Supply chain network of shale gas water management operations.](image)

Natural freshwater needed for hydraulic fracturing is obtained from an uninterruptible freshwater source and is stored in freshwater tanks (FWT). After hydraulic fracturing, the water that comes out, called flowback water, is stored onsite in fracturing tanks (FT) before pre-treatment (removing suspended solids, oil and grease, bacteria and certain ions) in mobile units, or else is transported to CWT facility, to a neighboring wellpad or to a Class II disposal well. It is assumed that each company has its own freshwater and fracturing tanks and its own pretreatment. After pre-treatment, the flowback and produced water stored in fracturing tanks can be recycled as a fracturing fluid in the same wellpad, or it can be desalinated in portable onsite treatment.

The following assumptions are made for the formulation of the model:
1. A fixed time period is discretized into weeks as time intervals.

2. Water transportation is only executed by trucks (the model can be easily extended to deal with transportation by pipes as well).

3. The volume of water used to fracture a well must be available when needed—this includes the possibility of storage in tanks or a ‘just in time water availability’—, including water required in drilling, construction and completion.

Qualitative mathematical model description

The mathematical model is outlined in Eq. (14) and comprises assignment constraints, logic constraints, shale gas and flowback water production, well water demands, mass balances in storage tanks, onsite and offsite treatments, treatment and storage capacity constraints and objective functions. The MILP in Eq. (14) is described in detail in the Supplementary Information, Section S.1.

\[
\begin{align*}
\text{max} & \quad \text{profit}\left(f_{i,p,w}^{w}, f_{i,p,w}^{\text{gas}}, y_{i,p,w}^{\text{on}}, y_{i,p,w}^{\text{hf}}, y_{i,p,w}^{\text{fb}}\right), \\
\text{subject to} & \quad \text{assignment constraints} \\
& \quad \text{logic constraints} \\
& \quad \text{shale gas and flowback water production} \\
& \quad \text{well water demands} \\
& \quad \text{mass balances in storage tanks, onsite and offsite treatments} \\
& \quad \text{treatment and storage capacity constraints} \\
& \quad f_{i,p,w}^{w}, f_{i,p,w}^{\text{gas}} \in \mathbb{R}^n \\
& \quad y_{i,p,w}^{\text{on}}, y_{i,p,w}^{\text{hf}}, y_{i,p,w}^{\text{fb}} \in \{0,1\} \\
& \quad p \in S \subseteq N
\end{align*}
\]

In Eq. (14), \( f \) are the continuous variables representing flowrates, \( y \) are the binary variables that involve discrete decisions, and the subscripts \( t, p \) and \( w \) are the time period, wellpad and well, respectively. The problem is implemented in GAMS 25.0.1.43 and solved using Gurobi 7.5.2.44. Depending on the objective function considered, the mathematical model will identify the best water management strategy for maximizing the profit, or minimizing the water-related costs or environmental impact (depending on the interests of companies) considering any number of
players. The gross profit to be maximized includes revenue from shale gas, and expenses for wellpad construction and preparation, shale gas production and water-related costs (i.e., wastewater disposal cost, freshwater withdrawal, friction reducer cost, onsite and offsite treatment cost, wastewater and freshwater transportation cost and storage tank cost). The cost objective function to be minimized includes the aforementioned water-related cost. The environmental objective minimizes the environmental impacts associated with water withdrawal, treatment and transportation. Environmental impacts are evaluated according to the principles of Life Cycle Impact Assessment (LCIA) using the ReCiPe methodology (see Supplementary Information, Section S.2).

CASE STUDIES AND DISCUSSION

Benefits of cooperation

Before focusing on applying the solution methods for cost or profit allocation described above, we study the benefits that are obtained when companies work together, and therefore there is interaction among them.

The benefits from the absence of cooperation to full cooperation among players is explored in a motivating example composed of a three-player game (i.e., companies, wellpad) working relatively close. Data of the problem based on Marcellus play–cost coefficients and model parameters– and ReCiPe indicators database are given in the Supplementary Information, Sections S.3.1 and S.3.2, respectively.

The time horizon of one year is discretized into weeks since most of the shale gas water is extracted during the first month after the well is drilled. However, this time period might be extended until the exploitation ends (10 – 20 years) with the renewal of the contract. The optimization model also includes one interruptible freshwater source, one centralized water treatment facility (CWT), one class II disposal well and three wellpads. Wellpads 1, 2 and 3 are composed of five, four and six wells, respectively. Each wellpad belongs and is operated by different companies with their own fracturing crew. The MILP model is implemented in GAMS
and solved using Gurobi on a computer with 3GHz Intel Zeon Processor and 32 GB RAM running on Windows 7.

In the case of the absence of cooperation, companies work independently, without sharing water recycled among different wellpads and onsite water treatment facilities. Hence, the mathematical model is solved for each individual company. Then, the total profit is equal to the sum of the individual profits. In contrast, when cooperation is carried out, the interaction between companies is allowed, therefore the mathematical model is solved including all companies. In this cooperative situation, companies can adapt the fracturing schedule to achieve additional advantages in order to maximize revenue and water reuse and reduce water management costs.

However, we also analyze the situation in which each company is willing to cooperate but it does not want to change its fracturing schedule that maximizes its revenue.

First, to show the benefits of cooperation, we maximize the gross profit considering absence of cooperation, full cooperation, and cooperation with a fixed fracturing schedule for shale water management strategies of three companies (i.e., wellpads). Figure 3 shows the optimal strategies obtained in each situation. When each company works independently (Figure 3 (a)) the total profit is $59.54M. In this case, the water that each company uses in drilling operations is the freshwater that comes from an external source and the water generated from the fractured wells belonging to its company. In this case, the total withdrawal of water increases to 160752 m$^3$.

Additionally, each company must lease an onsite treatment to manage the water when there are no more wells to fracture at the end of the total time horizon. When companies cooperate (Figure 3 (b)) the total profit is $60.48M. In this case, the best strategic solution is to install an onsite treatment in wellpad 1. In this case, the optimal schedule obtained tries to maximize the total water reused (115263 m$^3$). Note that, freshwater withdrawal decreases to 128856 m$^3$, that is, around 19.8% lower. Note also that company 3 only uses 18638 m$^3$ of freshwater for its fracturing operations. This is because wellpad 3 is the furthest away from the freshwater source. As transportation is the highest individual cost, this strategy leads to significant savings compared to the other two cases, where it is not possible to reuse the same amount of water. Additionally, when companies cooperate but they are interested in maintaining their schedule fixed the total
profit is $60.13M. In this case, reused water is limited to 90816 m$^3$, which increases the total water treated. This implies the need of installing an extra onsite treatment in wellpad 2, which increases the water treatment cost. Moreover, more freshwater is needed, increasing to 158302 m$^3$; that is, around 18.6 % higher than in the full cooperation case.

Figure 3. Optimal solution for: (a) absence of cooperation, (b) full cooperation, and (c) cooperation with a fixed fracturing schedule for shale water management strategies of three companies (i.e., wellpads).

To further demonstrate the benefit of cooperation, the previous example is expanded considering also the environmental objective function. We apply the epsilon-constraint method Pareto frontier to this bi-criteria optimization problem, obtaining the Pareto set of solutions, as shown in Figure 4, which indicates the existing trade-off between both objectives. Reductions of the LCIA can only be achieved by compromising the gross profit.
Figure 4. Pareto set of solutions (blue circles) for the bi-criteria optimization problem that maximizes the gross profit and minimizes the life cycle impact assessment (LCIA). Cooperative solutions are displayed by circles (○) and the absence of cooperation by triangles (▲). Extreme solutions A and B correspond with the cases where shale companies minimize the LCIA, whereas in extreme solutions D and E companies focus on maximizing gross profit. Solution C has the fracturing schedule fixed in advance and each company maximizes its shale gas revenue cooperating in shale gas water management costs.

In Figure 4 the following cases are displayed: cooperative solution when companies minimize the LCIA (Point A), cooperative solution when companies maximize the gross profit (Point D), no cooperative solution when companies minimize the LCIA (Point B), the fracturing schedule is fixed in advance and each company maximizes its revenue cooperating to reduce water management costs (Point C), and no cooperative solution when companies maximize the gross profit (Point E).

On the one hand, taking into consideration the environmental objective (points A and B), a reduction of 62.5% in the environmental impact is achieved (0.79 to 0.3) when all players work together and, additionally, the gross profit when all players cooperate is slightly higher.

On the other hand, taking into consideration the economic objective (points C, D and E), besides the profit increment of $942K when companies cooperate, a reduction of 41.1% in environmental impact is achieved. In the case where companies cooperate without changing their fracturing...
schedule, the gross profit increases by $590K compared to the absence of cooperation ($59.54M to $60.13M). However, setting the schedule limits the possibilities of cooperation, which the gross profit being 7.4% lower than in the cooperative solution ($60.13M vs $60.48M).

Additionally, the disaggregated water-related cost contribution and total shale gas revenue for all the cases is displayed Figure 5. As can be seen, reusing wastewater for fracturing operations reduces water transportation impact since companies are working in the same area. Therefore, they do not have to transport the water from freshwater sources located far away from the shale play. On the other hand, although shale gas revenue is higher when a company works independently than cooperating, the gross profit that each company obtains when it works cooperating is higher than when it works independently. This is because adapting the fracturing schedule in a cooperation situation to maximize the total water recycled; it is possible to significantly reduce water-related costs.

![Figure 5. Disaggregated water-related cost contribution (left axis) and total shale gas revenue (right axis) for cases A-E of shale water management strategies of three companies (i.e., wellpads). Case A (cooperation) and Case B (absence of cooperation) correspond to the cases in which shale companies minimize the LCIA, whereas in Case D (cooperation) and Case E (absence of cooperation) companies focus on maximizing the gross profit. Case C (cooperation) has the fracturing schedule fixed in advance.](image)

An additional analysis of the environmental impacts was made in order to show that the total emissions from the water management vary greatly among the five cases (see Figure 6 (a)). On
the one hand, in the cases focused on minimizing the environmental impacts (cases A and B), the LCIA is 49.6 % lower (0.66 to 0.33) when companies cooperate. On the other hand, in the cases focused on maximizing the profit (cases D and E), the LCIA is also lower when companies work together; in this case, it is around 31.7 % lower (0.80 to 0.55). The case when the schedule is fixed in advance (case C) has an environmental impact 27.9 % higher than case D (when the schedule can change), but it is around 5.4 % lower (0.80 to 0.76) than case E (when companies work independently).

Additionally, as climate change is the contribution with the highest impact in the endpoint category (see Section S.3.3.1 of the Supplementary Information), its corresponding midpoint indicator, the Global Warming Potential (GWP), is selected for the analysis. As can be seen in Figure 6 (b), in the cases focused on minimizing the LCIA (cases A and B), the GWP decreases around 50.3 % (2.54 to 1.26 kT CO$_2$-eq) when companies cooperate, while cost also decreases around 38.0 % ($3.72M/year to $2.31M/year), respectively. In the cases focused on maximizing the profit (cases D and E), GWP also decreases around 32.2 % (3.07 to 2.08 kT CO$_2$-eq) when companies work together, and the cost also decreases by 32.9 % ($4.20M/year to $2.81M/year). It should be noted that the cost follows the same trend as the environmental impact, basically because transportation and electricity are the most influential factors in economic and environmental terms.
Figure 6. (a) Environmental impact of the different life cycle stages using ReCiPe Endpoint (H,A) normalized between 0 and 1, and (b) comparison of the total GWP (using ReCiPe Midpoint (H)) and cost between case studies A, B, C, D and E. Left axis indicates the total GWP (in kT CO₂-eq) while right axis specifies the total cost of water management (in million dollars per year).

Clearly throughout this analysis, it has been shown that full cooperation between companies brings potential economic and environmental benefits.

**Profit and environmental impact allocation in a three-player game**

In this section, we explain how to allocate the corresponding profit or environmental impact (depending on players’ interest) among the players of the grand coalition. As mentioned before, the Core, Dual Core, Shapley value and minmax Core are prominent solution concepts to allocate the profit (or environmental impact) in cooperative game theory.

First, to calculate an imputation inside the Core, the characteristic function of each player and sub-coalition have to be computed. The characteristic function assigns a profit value (maximizing the gross profit in the shale gas water management model) or an environmental impact value (minimizing the LCIA) to each possible coalition. They are calculated solving the planning model as many times as coalitions are. In case of three-player game, the number of possible coalitions is equal to eight, including the empty set. **Table 1** displays the characteristic values obtained, where υ is the characteristic function when the gross profit is maximized and μ is the characteristic
function when the LCIA is minimized. Note that, for instance, the sum of \( \{\nu(1), \nu(2), \nu(3)\}\)
($59.54M) corresponds to point E (absence of cooperation) and the characteristic function
\( \{\nu(1,2,3)\}\) ($60.48M) refers to point D (cooperation) in Figure 4.

Table 1. Characteristic function for the three-player game focused on (a) the maximization of gross
profit (kS) and (b) minimization of LCIA (points).

<table>
<thead>
<tr>
<th></th>
<th>(\nu(1))</th>
<th>(\nu(2))</th>
<th>(\nu(3))</th>
<th>(\nu(1,2))</th>
<th>(\nu(1,3))</th>
<th>(\nu(2,3))</th>
<th>(\nu(1,2,3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>21314</td>
<td>15080</td>
<td>23146</td>
<td>36673</td>
<td>45149</td>
<td>38629</td>
<td>60478</td>
</tr>
<tr>
<td>(b)</td>
<td>118054</td>
<td>115689</td>
<td>158639</td>
<td>95558</td>
<td>118943</td>
<td>142664</td>
<td>148319</td>
</tr>
</tbody>
</table>

As can be seen in Table 1, the gross profit obtained when the three companies cooperate is the
highest ($60.5M) and it cannot be obtained if the companies worked independently ($59.5M).
The same behavior occurs when minimizing the LCIA, since the minimum LCIA is obtained
when all the companies work together.

Then, the constraint satisfaction problem (the Core) described in Eq. (15) must be solved to
determine the profit allocation among players. The Core ensures a stable coalition (Pareto-
efficient) and combines the properties of efficiency and individual and coalitional rationality.

Note that if the interest of stakeholders is to minimize LCIA, the environmental impact allocation
in individual and coalitional rationality will be lower than or equal to the characteristic function.

\[
\text{min}\ z = 1
\]
\[\text{s.t.}\quad \pi_1 + \pi_2 + \pi_3 = \nu(\{1,2,3\}) = 60478\quad \] Efficiency
\[\pi_1 \geq \nu(\{1\}) = 21314\quad \] Individual rationality
\[\pi_2 \geq \nu(\{2\}) = 15080\]
\[\pi_3 \geq \nu(\{3\}) = 23146\]
\[\pi_1 + \pi_2 \geq \nu(\{1,2\}) = 36673\quad \] Coalitional rationality
\[\pi_1 + \pi_3 \geq \nu(\{1,3\}) = 45149\]
\[\pi_2 + \pi_3 \geq \nu(\{2,3\}) = 38629\]
\[\pi_1, \pi_2, \pi_3 \in \mathbb{R}\]
where $\upsilon$ is the optimal profit of each coalition and $\pi_1$, $\pi_2$ and $\pi_3$ define the portion of the profit that is allocated to each player. Notice that since Eq. (15) is a feasibility problem we define a dummy objective function ($z=1$).

The geometrical interpretation of the Core of the three-player game is graphically illustrated in a ternary plot in Figure 7. However, in the case of profit allocation, the feasible region that defines the core results in a small area, being difficult to observe it in the plot. That is, the unique payoff division obtained with the Shapley value and the extreme points of the convex polyhedron that define the feasible core region are very close.

In the case of environmental impact allocation, the Core is graphically illustrated in Figure 7. Each individual and coalitional rationality constraint divides the space into two regions one being the region feasible with the Core allocation (the direction of the arrows points out into the feasible region). The compact convex polyhedron formed by the intersection of all half-spaces is the Core. The Core contains an infinite number of stable imputations (i.e., any sub-coalition could not arise to reach a better result than in the grand coalition). It is important to highlight that the non-empty Core of three players is guaranteed in advance if the following sub-additive property is satisfied:

$$\upsilon(\{1,2\}) + \upsilon(\{1,3\}) + \upsilon(\{2,3\}) \leq 2\upsilon(N).$$

The non-empty core guarantees that no conflicts are captured by the characteristic function, satisfying all players simultaneously. Figure 7 also displays the unique imputation obtained applying the Shapley value and the minmax Core solution method. As can be seen, both solutions correspond to stable imputation inside the Core.
In Table 2 (a), the marginal benefit of each player considering the profit allocation (obtained by using the Shapley value, minmax Core and the extreme allocation profit of the polyhedron that shapes the Core) is displayed. The marginal benefit solution for the Core extreme points captures the weak fairness of the Core for player 2. That is, if the companies decide to choose the allocation profit provided by the Core extreme points b and c, company 2 does not lose, but it does not benefit from joining the grand coalition either. There are always imputations that do not violate the individual or coalitional rationality constraints in which the player does not increase its benefit. Hence, in the Core some allocations might not be considered inherently fair in a strong sense because some players (or sub-coalitions) benefit more than others do.

Table 2 (b) shows the environmental impact reduction comparing the allocated impact of each player obtained with the three different solution concept and the environmental impact of absence of cooperation.
Table 2. (a) Marginal benefit (k$) of each player estimating the profit allocation based on the Shapley value, the Core and the minmax Core concepts, and (b) environmental impact reduction (%) in the cooperative game case compared to the absence of cooperation for each player, estimating the environmental impact allocation based on the Shapley value, the Core and the minmax Core concepts.

<table>
<thead>
<tr>
<th></th>
<th>Solution concept</th>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Shapley Value</td>
<td>339.3</td>
<td>196.7</td>
<td>400.8</td>
</tr>
<tr>
<td></td>
<td>Minmax Core</td>
<td>335.7</td>
<td>237.5</td>
<td>364.6</td>
</tr>
<tr>
<td></td>
<td>The Core - extreme points in the polyhedron of three companies game*</td>
<td>a</td>
<td>534.2</td>
<td>249.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>278.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
<td>534.2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d</td>
<td>29.5</td>
<td>249.0</td>
</tr>
<tr>
<td>(b)</td>
<td>Shapley Value</td>
<td>73.5</td>
<td>63.7</td>
<td>52.6</td>
</tr>
<tr>
<td></td>
<td>Minmax Core</td>
<td>74.7</td>
<td>57.2</td>
<td>56.5</td>
</tr>
<tr>
<td></td>
<td>Extreme points in the polyhedron of three companies game**</td>
<td>a'</td>
<td>70.0</td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b'</td>
<td>95.2</td>
<td>74.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c'</td>
<td>43.9</td>
<td>74.6</td>
</tr>
</tbody>
</table>

*a, b, c, d are the extreme points of the polyhedron. Note that this polyhedron is not displayed in any figure because it is difficult to observe its geometrical interpretation due to the proximity of points.

**a’, b’, c’ are the extreme points of the polyhedron displayed in Figure 7.

How to find allocations for games with a large number of players

In a three-player game, the number of coalitions is equal to eight—including the empty set—. However, the number of coalitions rises exponentially ($2^{|N|}$) with an increasing number of players. For example, in case of eight-player game the number of coalitions increases to 256. Hence, computing the characteristic function of all possible coalitions to formulate the constraint satisfaction problem and calculate the Shapley value or the minmax Core will require extensive time and effort because the planning model should be solved as many times as coalitions. Therefore, if the number of players increases, it is not feasible (or at least practical) to solve an optimization problem for each sub-coalition. Due to that fact, a row generation algorithm was suggested to tackle the problem.41

The main idea of the algorithm (detailed in Table 3) is to avoid testing the constraints for all possible coalitions to find an element in the Core. First, a master problem (Table 3 – Point 2) is...
solved including only the coalitions formed by individual players and the grand coalition. The solution of the master problem provides a possible imputation. Then, fixing the imputation obtained in master problem, we solve a subproblem (Table 3 – Point 4) that searches for a coalition that violates the most any stability constraint. If such a coalition exists, the master problem is updated, and the procedure is repeated until we get an imputation inside the core. The algorithm presented only ensures a solution inside the Core. Note, however, that it is straightforward to add constraints that force fairer imputations. For example, for computing an element in the minmax Core we only need to adapt the master problem for the set of ‘active sub-coalitions’ $S$:

\[
\text{Master}(S)\]

\[
\begin{align*}
\min_{\pi_i \in \mathbb{R}} \quad & \eta \\
\text{s.t.} \quad & \sum_{i \in N} \pi_i = \nu(N) \\
& \sum_{i \in S} \pi_i \leq \eta \nu(S) \quad \forall S \\
& \pi_i \in \mathbb{R} \quad \forall i \in N \\
& \eta \in \mathbb{R}
\end{align*}
\]
Table 3. Row generation algorithm.

1. Set $\mathcal{S}$; e.g., $\mathcal{S} = \{\{\}, \{1\}, \{2\}, \ldots, \{|N|\}\}$. Compute the individual costs $c(S)$ for those coalitions $S \in \mathcal{S}$ and the total cost $c(N)$ for the coalition $N$.

\[
\min \ w \\
\text{s.t.,} \quad \sum_{i \in N} \pi_i = c(N) \\
\sum_{i \in S} \pi_i - w \leq c(S) \quad S \in \mathcal{S} \\
\pi_i \in \mathbb{R} \quad i \in N
\]

2. Solve the master problem (LP)

3. If $w > 0$, STOP (the instance has an empty core).

4. Otherwise, find a coalition $S' \not\in (S)(S' \neq \emptyset)$ for which allocation is not in the core $\sum_{i \in S'} \pi_i > c(S')$, i.e., find the most violated core constraint fixing the cost allocation provided by the previous master problem $\pi^*$.

Sub-problem (MILP)

\[
\max \ \mu \\
\text{s.t.,} \quad \text{Assignment constraints} \\
\text{Shale gas water recovered} \\
\text{Water demand} \\
\text{Mass balance in storage tanks} \\
\text{Mass balance in onsite treatment and CWT plant} \\
\text{Treatment and storage capacity constraints} \\
\sum_{i \in S'} \pi^*_i x_i + c(S') = \mu, \quad S' := \{i \in N \mid x_i = 1\} \\
\mu \in \mathbb{R} \\
y_{i,p}^{on, h}, y_{i,p,w}^{on, h}, y_{i,p,w}^{on, h}, x_i \in \{0,1\} \quad p \in S \subseteq N
\]

5. If no such coalition $S'$ can be found, then STOP the algorithm because the allocation found is in the core.

6. Otherwise, compute the total cost $c(S')$ for this coalition, add the constraint $\sum_{i \in S'} \pi_i - w \leq c(S')$ to the master problem (i.e., update $S = S \cup \{S'\}$) and go to STEP 2.

Computing cost allocation in an eight-player game

To show the efficiency of the algorithm, an eight-player game is solved. In this case, we focus on the minimization of water-related cost, minimizing at the same time environmental impacts related to transportation and water withdrawal. Thus, the problem is tackled by applying a row
generation algorithm, following the steps detailed in Table 3. A total of 30 wells are allocated among the eight wellpads. Besides, three different freshwater natural sources are considered in this example.

First, we compute the optimal individual water related cost (shown in Figure 8, solution for the absence of cooperation) and the grand coalition cost (when all companies cooperate), which is equal to $2.9M. Then, we start the iteration process to allocate the cost among the players without computing the cost for each coalition. The iterative process to allocate the cost is detailed in Table 4, displaying in the last row the cost allocated to each stakeholder.

As can be seen in Figure 8, each player obtains significant savings cooperating. Moreover, the sum of total water management cost when the eight companies work separately is equal to $5.4M, which is 46% higher than the optimal cost obtained when all companies cooperate ($2.9M).

Figure 8. Optimal water-related cost of each player in the eight-player game (cooperating and in the absence of cooperation).
Table 4. Iteration process of row generation algorithm for an eight-player game.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\pi_1^*$</th>
<th>$\pi_2^*$</th>
<th>$\pi_3^*$</th>
<th>$\pi_4^*$</th>
<th>$\pi_5^*$</th>
<th>$\pi_6^*$</th>
<th>$\pi_7^*$</th>
<th>$\pi_8^*$</th>
<th>Subproblem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1865.2</td>
<td>521.9</td>
<td>1040.2</td>
<td>784.1</td>
<td>619.4</td>
<td>820.0</td>
<td>435.3</td>
<td>555.5</td>
<td>$S = {2,3,4,5,6,7,8}$</td>
</tr>
<tr>
<td>2</td>
<td>622.1</td>
<td>-1965.4</td>
<td>1040.2</td>
<td>784.1</td>
<td>619.4</td>
<td>820.0</td>
<td>435.3</td>
<td>555.5</td>
<td>$S = {1,3,4,5,6,7,8}$</td>
</tr>
<tr>
<td>3</td>
<td>622.1</td>
<td>289.3</td>
<td>1040.2</td>
<td>784.1</td>
<td>-1635.3</td>
<td>820.0</td>
<td>435.3</td>
<td>555.5</td>
<td>$S = {1,2,3,4,6,7,8}$</td>
</tr>
<tr>
<td>4</td>
<td>622.1</td>
<td>521.9</td>
<td>1040.2</td>
<td>784.1</td>
<td>619.4</td>
<td>820.0</td>
<td>435.3</td>
<td>-1931.8</td>
<td>$S = {1,2,3,4,5,6,7}$</td>
</tr>
<tr>
<td>5</td>
<td>375.5</td>
<td>289.3</td>
<td>-541.9</td>
<td>784.1</td>
<td>619.4</td>
<td>820.0</td>
<td>435.3</td>
<td>129.5</td>
<td>$S = {1,2,4,5,6,7}$</td>
</tr>
<tr>
<td>6</td>
<td>375.5</td>
<td>289.3</td>
<td>577.2</td>
<td>451.2</td>
<td>413.9</td>
<td>239.3</td>
<td>435.3</td>
<td>129.5</td>
<td>$S = {1,2,4,6,7}$</td>
</tr>
<tr>
<td>7</td>
<td>375.5</td>
<td>289.3</td>
<td>577.2</td>
<td>451.2</td>
<td>413.9</td>
<td>428.0</td>
<td>246.7</td>
<td>129.5</td>
<td>$S = {1,2,4,6,7}$</td>
</tr>
<tr>
<td>8</td>
<td>375.5</td>
<td>289.3</td>
<td>577.2</td>
<td>451.2</td>
<td>453.0</td>
<td>388.9</td>
<td>246.7</td>
<td>129.5</td>
<td>$S = {2,3,5,7}$</td>
</tr>
<tr>
<td>9</td>
<td>375.5</td>
<td>289.3</td>
<td>577.2</td>
<td>471.7</td>
<td>453.0</td>
<td>388.9</td>
<td>226.2</td>
<td>129.5</td>
<td><strong>No coalition found</strong></td>
</tr>
<tr>
<td>10</td>
<td>375.5</td>
<td>289.3</td>
<td>577.2</td>
<td>471.7</td>
<td>453.0</td>
<td>388.9</td>
<td>226.2</td>
<td>129.5</td>
<td><strong>No coalition found</strong></td>
</tr>
</tbody>
</table>

The larger resulting optimization problem is given when the eight companies are working together and consists of 7680 constraints, 11177 continuous variables and 848 binary variables. Gurobi provides a solution with an optimality gap equal to 3 % after 1244 s of CPU time. The master and subproblem defined in the algorithm are solved in less than 100 s of CPU time for the master problem with optimality gap of 0 % and 1 % for the subproblem.

**Eight-player game strategies and environmental analysis**

The optimal strategic solution of the cooperative game theory for eight companies (i.e., wellpads) is displayed in Figure 9. As can be seen, companies 1 and 4 drill the wells using flowback water coming from the same and neighboring wellpads, while companies 7 and 8 only use freshwater from source 1 for fracturing operations. Company 6 withdraws water from the freshwater source 3, while companies 2, 3 and 4 from the freshwater source 2. Additionally, only the installation of one onsite treatment in wellpad 5 is required. Besides, the total water withdrawal cooperating (241764 m$^3$) decreases by around 27 % with respect to the absence of cooperation (329608 m$^3$).
Figure 9. Optimal shale water management solution of the cooperative game theory of eight companies (i.e., wellpads).

We quantify the emissions embodied in water management when companies cooperate and in the absence of cooperation. As can be seen in Figure 10, the environmental impact when the eight companies cooperate is around 58.0% lower (0.34 vs. 0.81) than the environmental impact when the companies work separately. This is mainly due to the reduction of water sent to onsite treatment. Further analysis of this solution is displayed in the Supplementary Information, Section S.3.4.
How to distribute individual cost to each player

In this last example, we try to approximate a real world case study. For that reason, we consider that 4 companies (i.e., players) control a specific area. A total of 207 wells are distributed among 13 different wellpads where each company owns 3-4 of them. Each player fixes its fracturing schedule in advance, hence the objective function is focused on minimizing the water-related cost.

We consider that each company, apart from knowing the total allocated cost of water management when they are cooperating (as shown in previous examples), wants to know how much it has to pay for storage, water withdrawal, transportation, treatment and disposal.

Thus, this example also analyzes the individual cost distribution (storage cost, desalination cost, transportation cost, etc.) to each company and the strategic interaction among them. We consider that each shale gas company must pay for its own cost of storage, water withdrawal, transportation, treatment and disposal. The interaction among them is reflected by sharing water agreements in the impaired water that is sent from one to another company.

In this case, we only contemplate the fair solution, therefore, the ‘minmax Core’ is applied. To do that, the following approach is implemented.

**Step 1.** Compute the characteristic function (solving the water planning model) of each possible coalition (Table 5).

**Step 2.** Determine the grand coalition cost.

**Step 3.** Fix the individual expenses to each player and the impaired water flowrate sent among companies obtained from the previous problem.

**Step 4.** Determine the payoff of each player and the strategic interaction among them solving the following minmax Core problem (Eq. (15)).
\[
\min \ z = \eta \\
s.t., \quad \sum_{i \in S} \pi_i = c(N) \\
\quad \quad \quad + \sum_{S \subset N, S \neq \emptyset} \pi_i \leq \eta \cdot c(S) \\
\pi_i = E_i^{sto} + E_i^{source} + E_i^{trans} + E_i^{source} + E_i^{dis} + E_i^{dis}
\]
\[\sum_{\alpha \in i} F_{i,i'}^{imp} \cdot \alpha_{i,i'} - \sum_{\alpha \in i} F_{i,i'}^{imp} \cdot \alpha_{i,i'} \forall i \in N \]
\[
\alpha_{i,i'} = \alpha_{i,i'} \in \mathbb{R}^n
\]

where \( \pi_i \) is the allocation cost, \( \eta \) ensures that no coalition \( S \) has a cost share greater than \( \eta \) percentage and \( \alpha_{i,i'} \) represents the cost coefficient that player \( i \) must pay to player \( i' \). For instance, if \( \alpha_{1,2} \) is negative means that player 2 have to pay to player 1 the water that player 2 receives. Therefore, player 1 reduces its total allocation cost proportional by the water sent.

**Table 5. Characteristic function for the four-player game focused on minimizing the water-related costs (kS).**

<table>
<thead>
<tr>
<th></th>
<th>{1}</th>
<th>{2}</th>
<th>{3}</th>
<th>{4}</th>
<th>{1,2}</th>
<th>{1,3}</th>
<th>{1,4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c(1) )</td>
<td>10196</td>
<td>9841</td>
<td>13827</td>
<td>9815</td>
<td>17171</td>
<td>7253</td>
<td>19985</td>
</tr>
<tr>
<td>( c(2) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c(3) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c(4) )</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( c(1,2) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c(1,3) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c(1,4) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The total water-related cost when companies cooperate (grand coalition cost) is equal to $34.3M, 21% lower than the cost when companies work independently ($43.7M). The cost allocated to each player is equal to $8479K, $6266K, $10153K and $9448K, respectively. The individual cost distribution can be found in Table 6.

**Table 6. Individual cost allocated to each player (kS).**

<table>
<thead>
<tr>
<th>Cost</th>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>Player 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage</td>
<td>242</td>
<td>325</td>
<td>239</td>
<td>244</td>
</tr>
<tr>
<td>Friction reducers</td>
<td>182</td>
<td>150</td>
<td>295</td>
<td>112</td>
</tr>
<tr>
<td>Water withdrawal</td>
<td>1096</td>
<td>1699</td>
<td>111</td>
<td>1678</td>
</tr>
<tr>
<td>Transport</td>
<td>5036</td>
<td>7992</td>
<td>1515</td>
<td>9364</td>
</tr>
<tr>
<td>Pretreatment</td>
<td>640</td>
<td>532</td>
<td>828</td>
<td>421</td>
</tr>
<tr>
<td>Desalination</td>
<td>682</td>
<td>495</td>
<td>-</td>
<td>471</td>
</tr>
</tbody>
</table>
Companies interact with each other due to the water sent from one company to another one. For example, in a cooperative situation, as company 3 is the farthest away from the freshwater source, the solution reveals that company 3 must fracture its wellpads using the wastewater produced by the other companies. However, that means it is an important saving for company 3, which has to pay to company 2 for the water received. Table 7 shows the income and cost interaction among companies and Figure 11 displays the impaired water exchange among different wellpads when companies are cooperating and in the absence of cooperation where only the interaction among wellpads that belongs to a specific company is allowed.

Table 7. Impaired cost interaction among companies (k$).

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
<th>Player 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Player 2</td>
<td>-</td>
<td>-</td>
<td>-7166.134</td>
</tr>
<tr>
<td>Player 3</td>
<td>-</td>
<td>7166.134</td>
<td>-</td>
</tr>
<tr>
<td>Player 4</td>
<td>-601.554</td>
<td>-2239.925</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 11. Impaired water and freshwater distribution among wellpads considering (a) absence of cooperation, and (b) full cooperation among companies. In the diagram, the companies 1, 2, 3 and 4 are denoted by C1, C2, C3 and C4 before the number of the wellpad, indicated by letter p. The source water withdrawal is denoted by pink circle arcs, where the inner circle refers to the total water in cubic meters.
sent to each wellpad. In the case of absence of cooperation, impaired water exchange is only allowed by wellpads that belong to the same company. Contrary, full cooperation allows impaired water exchange among all wellpads.

The larger resulting problem is solved in Step 2, when the grand coalition is determined and the four companies are working together, and therefore, the 13 wellpads are interacting. In that case, the model has 81967 equations, 119939 continuous variables and 13 binary variables. The CPU time did not exceed few seconds to find the optimal solution and, in general, the model is solved in less than five seconds for all subproblems.

CONCLUSIONS

The current study highlights the importance of cooperation in shale gas industry to increase the profit and reduce the cost and environmental impact. The objective of this work is to investigate how to allocate whatever quantifiable unit in shale gas water management (costs, profit or environmental impact) among stakeholders when all companies work together. To do this, we use the cooperative game theory that provides a framework to calculate imputations that should be the basis of a negotiation among different companies. Specifically, we apply three important solution concepts in cooperative game theory, the Core, the minmax Core and Shapley value. First, a motivating example composed of a three-player game shows the benefits of full cooperation that shale gas water management exhibits under different indicators, the gross profit and the LCIA, respectively. An interesting fact that we found is that while the individual revenue decreases in the cooperative solution, the water management cost is decreased to a point where the profit is actually increased. A detailed procedure of how to allocate both profit and environmental impact allocation of this motivation example is presented. Then, a larger example composed of an eight-player game focused on minimizing water-related cost is analyzed to show that it is possible to efficiently solve these problems by means of a row generation algorithm.
Finally, to further demonstrate the applicability of the proposed approach for a real world, a case study composed of 4 companies cooperating is analyzed. In addition, the individual cost distribution (storage cost, desalination cost, transportation cost, etc.) to each company and the strategic interaction among them is analyzed.

The results obtained with the three case studies reveal savings of 30-50% when all companies work together instead of working independently. The major economic saving is due to the increase of water reused, reducing at the same time water withdrawal and transportation. Regarding environmental concerns, this water management alternative helps to reduce the water footprint and emissions.

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Appendix
Additional material containing methods and complementary results is available in the Supplementary Information.

References


