Optimal Maintenance Scheduling of a

Gas Engine Power Plant

using Generalized Disjunctive Programming

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Abstract

This article presents a new continuous-time model for long-term scheduling of a gas engine power plant with parallel units. Gas engines are shutdown according to a regular maintenance plan that limits the number of hours spent online. To minimize salary expenditure with skilled labor, a single maintenance team (shared by the gas engines) is considered which is unavailable during certain periods of time. Other challenging constraints involve constant minimum and variable maximum power demands. The objective is to maximize the revenue from electricity sales assuming seasonal variations in electricity pricing, by reducing idle times and shutdowns in high tariff periods. By first

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developing a generalized disjunctive programming model and then applying both big-M and hull reformulation techniques, we reduce the burden of finding the appropriate set of mixed-integer linear constraints. We then show through the solution of a real life problem that the most efficient model involves a big-M reformulation of the set of disjunctive constraints, while gaining valuable insights about the system.

Keywords: optimization, logic, mathematical modeling, mixed-integer linear programming, planning

Introduction

All industrial sites require regular maintenance to enhance reliability of their equipments and avoid emergency shutdowns. The main concerns of maintenance scheduling are to guarantee feasible material and utility balances while minimizing payment for skilled labor[17]. As in-house skilled labors are limited and external labor is expensive, in order to minimize salary expenditure, plants will maintain separately to reduce the manpower requirement.

The maintenance scheduling of generators in power systems is one of the most significant problems in power systems operation and management[18]. In order to avoid premature aging and failure of generators leading to unplanned and costly power outages, it is important to carry out preventive maintenance at regular intervals[19]. The maintenance schedule affects many short- and long-term planning functions. For example, unit commitment, fuel scheduling, reliability calculations and production cost all have a maintenance schedule as input and so a suboptimal schedule can affect each of these function adversely[20].

In traditional power systems, the maintenance scheduling of generator units was performed by the system operator and imposed to power plants[18]. However, many concepts of the power systems changed after restructuring and in the deregulated environments. In the latter, the conventional approach for maintenance scheduling involves interaction between the independent system operator (ISO) and the generation companies (GENCOs). In this process, the objective of the GENCOs is to maximize their annual benefits, while the ISO will also try to maximize the reliability of the power

grid and increase the reserve capacity at every time interval. Hence, the ISO may return some maintenance requests for modification.

The maintenance scheduling of thermal generation units is a large-scale combinatorial optimization problem and the typical optimization methods have been applied to solve it: mathematical programming, dynamic programming, genetic algorithms, simulated annealing, tabu search, etc. The objective function is often quadratic[19-21] based on economic cost or reliability. Mathematical programming formulations involve a discrete-time representation[18-21] that uses binary variables to identify the time interval in which maintenance starts or is being performed. The maintenance time interval is one week with the time horizon being one[18-20] or 5 years[21] and four sets of linear constraints need to be enforced: (i) continuity of maintenance activities, where each unit is maintained for a specified length of time without interruption; (ii) maintenance window constraints, which define the possible times for execution of the maintenance activities; (iii) maximum and minimum power output constraints, which consider the demand and minimum reserve margins of the power system; (iv) Crew constraints, which consider the manpower availability for maintenance work; the maintenance resource is either the total number of skilled workers available[18-19], or the constraints specify a maximum number of units that can be maintained at a given time[20] or that no two units can be maintained simultaneously by the same crew[21]. Maintenance for a particular unit occurs just once in the given time horizon.

In the real life problem addressed in this article, the maintenance of the power plant involves multiple shutdowns for each generator. Maintenance is enforced between a minimum and a maximum number of run hours after the previous shutdown and so the maintenance time windows are dynamic rather than static. Generators will feature an idle mode besides the online and shutdown modes to save online hours for periods where the electricity price is higher, leading to higher revenue from electricity sales. While this is straightforward to model with a discrete-time representation, it presents a challenge for a continuous-time model, required since the shortest duration of a planned maintenance is just 12 hours, a very small value compared to a time horizon of a few years. A similar challenge is associated to time dependent resource availability constraints,

which are quite common in practice (e.g. cost for manpower higher on Sundays). The novelty is thus related to the generation of practical mixed-integer linear constraints for continuous-time models dealing with time/cost dependent resource availability and shared resources (e.g. a single maintenance team). The solution approach is to start from much simpler generalized disjunctive programming constraints[7,12], going along the lines of recent work by Castro & Grossmann[15], who have done the same for the key concepts of immediate, general precedence and multiple time grids.

Problem Statement

We consider a gas engine power plant producing electricity from natural gas. At any one time, each engine $m \in M$ can be online, in standby or shutdown mode, and is characterized by power output pw_m [MW] and number of run hours (not considering the hours spent in standby mode) before maintenance is required. Some flexibility is allowed on the shutdown schedule, so rather than considering a fixed value, the processing time is allowed to vary between given lower $p^L_{t,m}$ and upper bounds $p^U_{t,m}$ [h]. Index $t \in T$ represents an operation time period that includes a single shutdown of length $sd_{t,m}$ [h]. Typically, the required maintenance time increases with an increase in the number of total hours online.

It is assumed that there is only 1 maintenance team doing shutdowns, meaning that only one engine can be on shutdown mode at a time. To reduce the costs, it may be convenient to make the maintenance team unavailable in certain periods of time (e.g. on Sundays, Christmas season). For each such period tu, the starting u^L_{tu} and ending time u^U_{tu} [h] (with respect to the origin of the time horizon) must be given.

The amount of time spent on standby mode can be varied to stagger the shutdowns and to take advantage of higher electricity tariffs. For this particular case study, electricity sales in the winter

months are at a higher tariff than the sales in summer. Let tp represent a time period of constant electricity price ce_{tp} [$/MWh] and cp_{tp}^{L} and cp_{tp}^{U} [h] its starting and ending time, respectively.

Most of the time, all energy produced can be sold to the market (unlimited demand) but there can be periods up to a few weeks where the maximum power demand pw_{td}^{U} is limiting. The starting and ending times of time period td are given respectively by d_{td}^{L} and d_{td}^{U} [h].

The objective will be to maximize the revenue from electricity sales for a given number of operation periods $|T|$, while subject to a constant minimum power demand, pw^{L} [MW], and the above mentioned constraints.

Selection of Time Representation Concepts for Scheduling Model

In a recent review paper dealing with production scheduling models for industrial applications, Harjunkoski et al.[1] have identified the production environment and the modeling of time as the two most important features of a mathematical programming scheduling formulation. In terms of the production environment, the maintenance scheduling problem addressed here can be viewed as a sequential, single stage multiproduct continuous plant with parallel units. The multiple products are the specified operation time periods T that are scheduled following a predefined sequence with variable processing times and changeovers (the mandatory shutdowns). We also have a maintenance team, a resource that needs to be shared by the different engines that is not always available, and time dependent pricing and demand for electricity.

In view of the given processing characteristics and constraints, deciding on either a discrete or continuous-time approach is hardly straightforward. In favor of discrete-time, we have the volatile prices, maximum electricity demand and maintenance team availability, the shared resource and the minimum power demand. In favor of continuous-time, there is the very simple plant topology, the variable processing times and the fact that the changeover times can be two orders of magnitude

smaller than the processing times. In the end, the latter feature was decisive since a very large number of time slots would be required by the discrete-time formulation to handle the problem data accurately (meaning an intractable problem), whereas the shared resource and the time dependent profiles, which do not change frequently, can still be handled by a continuous-time formulation despite the use of a more inefficient set of constraints. The constant minimum power demand will be enforced by eliminating the standby mode from as many engines as those required to achieve the minimum demand, plus one.

Continuous-time models can be of different types[1]. Single time grid is preferred when in presence of shared resources[22] but is highly inefficient for single stage plants[2] when compared to multiple time grid models. In this case, it is not required to keep track of the availability of the maintenance team over time, just to make sure that the maintenance tasks do not overlap. General precedence models are known capable of modeling this constraint very efficiently and can be extended to multiple discrete resources[3,4,5]. The continuous-time model to be presented next is hybrid[5,6] in the sense that it relies on two different concepts for time representation: (i) multiple time grids, one per gas engine, to keep track of the execution of the power production tasks; (ii) sequencing variables, to handle the single maintenance team constraint. The model has also to account for events occurring at discrete points in time, which define changes in the electricity tariff, power demand and availability of the maintenance team.

Generalized Disjunctive Programming Formulation

In this section, we highlight the main elements of the scheduling formulation while providing the model constraints in their simplest form. Generalized Disjunctive Programming[7,12] (GDP) is used for this purpose, allowing us to focus on the linear constraints that are associated to each of the alternate decisions, defined by Boolean variables. In the next section, we discuss the transformation of the GDP into mixed-integer linear programming (MILP) formulations using big-M and convex hull reformulations[8].

Timing production and maintenance tasks on gas units

In order to ensure the minimum power supply constraint, gas engines M are divided into engines

that are always on (either on online or shutdown mode), M^{ON}, and those that can be idle. The latter

provide the necessary flexibility to maximize electricity production in periods of higher price.

We use the concept of multiple time grids[2] to determine the timing of the production and

maintenance tasks. Each engine will feature exactly one production and one maintenance task per

shutdown period $t \in T$, i.e. the given shutdown periods correspond to the slots of every time grid

$m \in M$, see Figure 1 and Figure 2. Given that the online time of engine m in slot t is not fixed but

allowed to vary in $[p_{t,m}^L, p_{t,m}^U]$, we need to define nonnegative continuous variables $P_{t,m}$. The other

nonnegative continuous variables are $Ts_{t,m}$ and $Te_{t,m}$, which identify the starting and ending time of

production in slot t of engine m and $Tm_{t,m}$ the beginning of the shutdown period t of engine m.
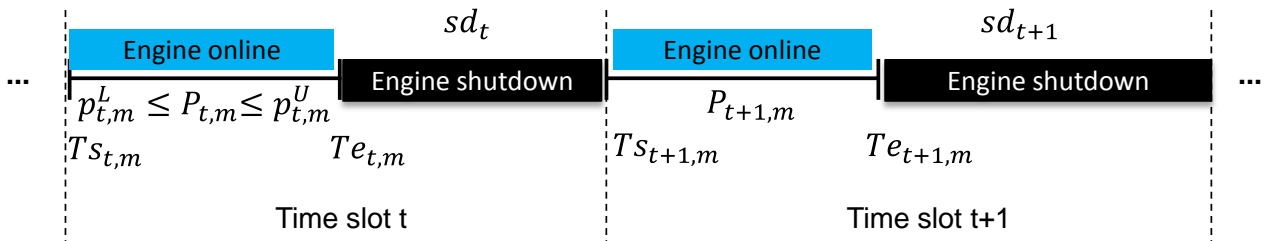

Figure 1. Events occurring within the time slots of always on gas engines.

It should be noted that the assumption of a maximum of two occurrences per time slot of the

standby (idle) mode (see Figure 2) may remove the optimal solution from the feasible space. In fact,

we started to postulate a single occurrence (before the online mode) only to find out that there were

periods of high electricity tariff with engines in standby mode. These disappeared for two
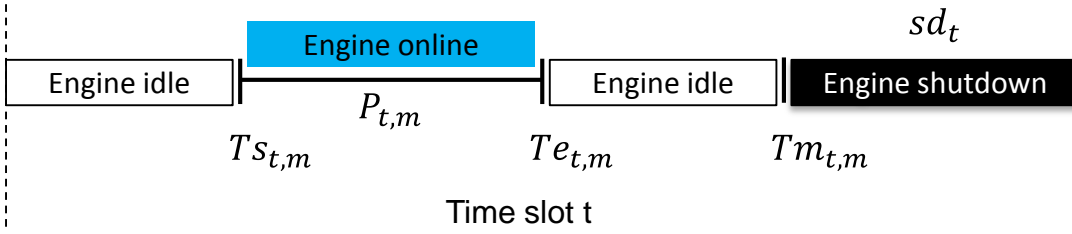
occurrences, leading to higher revenue.

Figure 2. Events occurring within the time slots of gas engines that can be idle.

From Figure 1 and Figure 2, the timing constraints are straightforward. Equation 1 states that the ending time is equal to the starting plus processing time. The beginning of the maintenance period coincides with the end of processing for always on engines, Eq. 2, or is greater than it for the others, Eq. 3. Then, the starting time of the online mode in slot $t + 1$ is either equal to (Eq. 4) or greater than (Eq. 5) the starting time of the maintenance mode in slot t plus the fixed shutdown length in t.

The starting time of the first slot for always on units must also be equal to zero, Eq. 6.

$$Te_{t,m} = Ts_{t,m} + P_{t,m} \ \forall \ t, m \tag{1}$$

$$Tm_{t,m} = Te_{t,m} \ \forall \ t, m \in M^{ON} \tag{2}$$

$$Tm_{t,m} \geq Te_{t,m} \ \forall \ t, m \in M \backslash M^{ON} \tag{3}$$

$$Ts_{t+1,m} = Tm_{t,m} + sd_{t,m} \ \forall \ t, m \in M^{ON} \tag{4}$$

$$Ts_{t+1,m} \geq Tm_{t,m} + sd_{t,m} \ \forall \ t, m \in M \backslash M^{ON} \tag{5}$$

$$Ts_{1,m} = 0 \ \forall \ m \in M^{ON} \tag{6}$$

Depending on the problem data, it is possible to reduce the domain of the model variables and improve computational performance. To facilitate interpretation of the constraints, the variable bounds are written in lower case featuring the same characters as the corresponding model variable and an extra superscript identifying if it is a lower (L) or an upper (U) bound. As an example, $ts_{t,m}^{L}$ represents the lower bound on the starting time of processing task (t, m), while $tm_{t,m}^{U}$ indicates the

upper bound on the starting time of the shutdown period t of engine m. Eqs. 7-10, thus define the

upper and lower bounds for the timing variables.

$$p_{t,m}^L \leq P_{t,m} \leq p_{t,m}^U \ \forall \ t, m \tag{7}$$

$$ts_{t,m}^L \leq Ts_{t,m} \leq ts_{t,m}^U \ \forall \ t, m \tag{8}$$

$$ts_{t,m}^L + p_{t,m}^L = te_{t,m}^L \leq Te_{t,m} \leq te_{t,m}^U = ts_{t,m}^U + p_{t,m}^U \ \forall \ t, m \tag{9}$$

$$tm_{t,m}^L \leq Tm_{t,m} \leq tm_{t,m}^U \ \forall \ t, m \tag{10}$$

Sequencing maintenance tasks performed by the single maintenance team

Enforcing the single maintenance team constraint can be done through the use of general

precedence sequencing variables[9,10]. If one considers any pair of tasks (t, m) and (t', m'), there are

only two possibilities (hence the use of the exclusive OR in Figure 3), either (t, m) before or after

(t', m'). Naturally, there can be other shutdown tasks between the pair being considered. Notice also

that there is no need to consider $m = m'$ since Eqs. 1-5 ensure that there is no overlap of shutdown
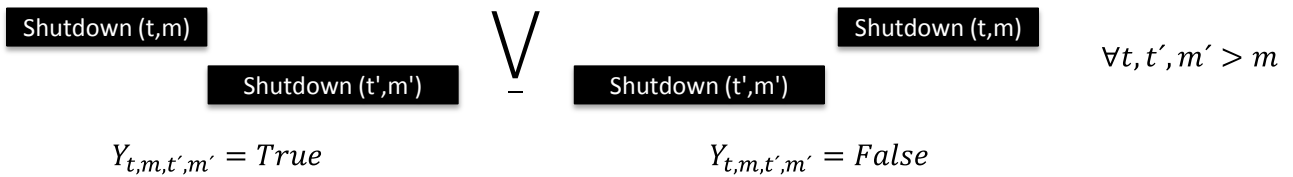
tasks belonging to different slots of the same unit.

Figure 3. Sequencing pairs of maintenance tasks through general precedence variables.

Let $Y_{t,m,t',m'}$ be Boolean variables indicating if shutdown of slot t in unit m starts before the

shutdown of slot t' in unit m'. The corresponding constraint in Disjunctive Programming form is

given by Eq. 11.

$$\left[\begin{matrix} Y_{t,m,t',m'} \\ Tm_{t,m} + sd_{t,m} \leq Tm_{t',m'} \end{matrix} \right] \bigvee \left[\begin{matrix} \neg Y_{t,m,t',m'} \\ Tm_{t',m'} + sd_{t',m'} \leq Tm_{t,m} \end{matrix} \right] \forall\, t, t', m' > m$$

(11)

Making the maintenance team unavailable in certain periods of time

Another constraint is that the maintenance team it is not available in certain periods of time. Thus, shutdown task (t,m) either ends before the start of unavailable period tu or starts after the end of tu, see Figure 4. Being $Z_{t,m,tu}$ the new set of general precedence Boolean variables, Eq. 12 results.
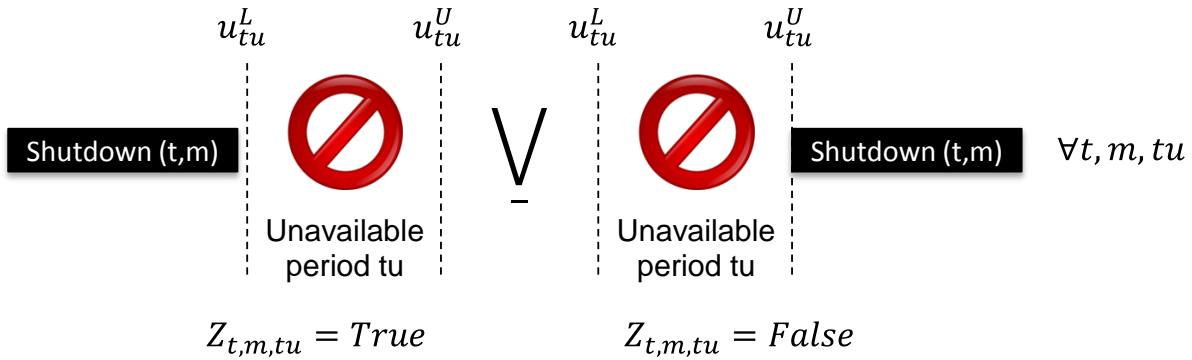

Figure 4. Shutdown tasks cannot take place over unavailable periods of the maintenance team.

$$\left[\begin{matrix} Z_{t,m,tu} \\ Tm_{t,m} + sd_{t,m} \leq u_{tu}^L \end{matrix} \right] \bigvee \left[\begin{matrix} \neg Z_{t,m,tu} \\ Tm_{t,m} \geq u_{tu}^U \end{matrix} \right] \forall\, t, m, tu$$

(12)

Calculating the revenue over the different electricity price periods

Dealing with the different electricity tariffs and computing the revenue is the most challenging part of the model. We follow the approach of Nolde and Morari[11], who have identified six different types of interactions between a processing task and a time period of constant electricity price, when addressing the electric load tracking scheduling problem of a steel plant. Constraints were derived using logic relations[12] and reformulated into mixed-integer linear programming format using the big-M technique. However, the MILP constraints were not actually shown. Also using big-M

constraints, Haït and Artigues[13] proposed a computationally more efficient model for the exact same problem, while Hadera and Harjunkoski[14] applied the concept to a more complex steel plant.
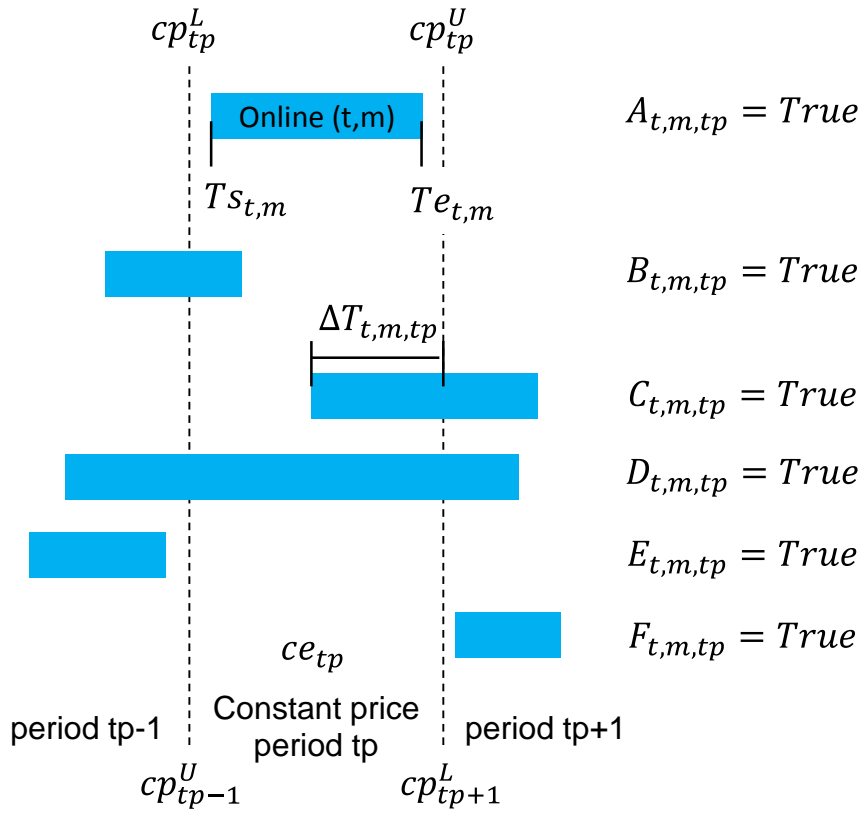

Figure 5. Interaction of processing tasks with periods of constant electricity price periods.

Let Boolean variables from $A_{t,m,tp}$ to $F_{t,m,tp}$ account for the six possible location types of

processing task (t,m) with respect to constant electricity price period tp, see Figure 5. Type A

corresponds to the full duration being located inside the price period, i.e. both the starting and ending

time of the task must be greater than the interval lower bound and lower than its upper bound. In

such case, the time factor $\Delta T_{t,m,tp}$ to consider for computing the revenue, is equal to the processing

time $P_{t,m}$, see disjunction further to the left in Eq. 13. If the tasks starts before cp_{tp}^L but ends within

tp ($B_{t,m,tp} = True$), the time factor is equal to the difference between the ending time of the task

and the interval lower bound. The same can be done for the four remaining alternatives. Note that in

the last three disjunctions, the calculation of $\Delta T_{t,m,tp}$ involves only parameters, which facilitates the convex hull reformulation.

$$
\begin{bmatrix}
A_{t,m,tp} \\
Ts_{t,m} \geq cp_{tp}^L \\
Ts_{t,m} \leq cp_{tp}^U \\
Te_{t,m} \geq cp_{tp}^L \\
Te_{t,m} \leq cp_{tp}^U \\
\Delta T_{t,m,tp} = P_{t,m}
\end{bmatrix}
\underset{-}{\vee}
\begin{bmatrix}
B_{t,m,tp} \\
Ts_{t,m} \leq cp_{tp}^L \\
(Ts_{t,m} \leq cp_{tp}^U) \\
Te_{t,m} \geq cp_{tp}^L \\
Te_{t,m} \leq cp_{tp}^U \\
\Delta T_{t,m,tp} = Te_{t,m} - cp_{tp}^L
\end{bmatrix}
\underset{-}{\vee}
\begin{bmatrix}
C_{t,m,tp} \\
Ts_{t,m} \geq cp_{tp}^L \\
Ts_{t,m} \leq cp_{tp}^U \\
(Te_{t,m} \geq cp_{tp}^L) \\
Te_{t,m} \geq cp_{tp}^U \\
\Delta T_{t,m,tp} = cp_{tp}^U - Ts_{t,m}
\end{bmatrix}
\underset{-}{\vee}
$$

$$
\underset{-}{\vee}
\begin{bmatrix}
D_{t,m,tp} \\
Ts_{t,m} \leq cp_{tp}^L \\
(Ts_{t,m} \leq cp_{tp}^U) \\
(Te_{t,m} \geq cp_{tp}^L) \\
Te_{t,m} \geq cp_{tp}^U \\
\Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L
\end{bmatrix}
\underset{-}{\vee}
\begin{bmatrix}
E_{t,m,tp} \\
(Ts_{t,m} \leq cp_{tp}^L) \\
(Ts_{t,m} \leq cp_{tp}^U) \\
Te_{t,m} \leq cp_{tp}^L \\
(Te_{t,m} \leq cp_{tp}^U) \\
\Delta T_{t,m,tp} = 0
\end{bmatrix}
\underset{-}{\vee}
\begin{bmatrix}
F_{t,m,tp} \\
(Ts_{t,m} \geq cp_{tp}^L) \\
Ts_{t,m} \geq cp_{tp}^U \\
(Te_{t,m} \geq cp_{tp}^L) \\
(Te_{t,m} \geq cp_{tp}^U) \\
\Delta T_{t,m,tp} = 0
\end{bmatrix}
\forall\, t, m, tp
$$

(13)

In Eq. 13, redundant constraints are inside parenthesis. As an example, if $C_{t,m,tp} = True$, it is not necessary to consider $Te_{t,m} \geq cp_{tp}^L$, since this is ensured by $Ts_{t,m} \geq cp_{tp}^L$ and Eq. 1. Redundant constraints are shown in order to make it easier to identify those that are shared by different location variables, something that will be explored in the next section. Overall, for every (t, m, tp) there are 14 non-redundant constraints that need to be reformulated.

Implicit in Eq. (13) is the fact that the sum over all time periods of the time factor variables must be lower than the processing time of the corresponding time period. While not strictly necessarily, Eq. (14) leads to a reduction in the integrality gap and improves computational performance by over one order of magnitude. If time periods tp are sufficiently long to allow execution of all tasks, we can use the equality instead.

$$
\sum_{tp} \Delta T_{t,m,tp} \leq P_{t,m} \quad \forall\, t, m
$$

(14)

To calculate the revenue due to processing task (t, m) in period tp, one just needs to multiply the time factor $\Delta T_{t,m,tp}$ by the electricity price ce_{tp} and the power output pw_m due to engine m. The objective function of maximizing the total revenue is thus given by Eq. 15.

$$\max \sum_t \sum_m \sum_{tp} \Delta T_{t,m,tp} \cdot ce_{tp} \cdot pw_m \tag{15}$$

Ensuring power output does not exceed maximum demand

While in general it is assumed that all energy produced can be sold to the market, it is possible to have times of low power demand during some weeks of the year. Considering that the maximum power demand pw_{td}^{max} is higher than the minimum power output pw^{min} from the always on engines, we need to account for the other units ($m \in M \backslash M^{ON}$) that are operating in low demand period td. Operating in period td does not necessarily mean starting and ending within td, i.e. any of the first four types of interaction in Figure 5 can occur. Thus, processing task (t, m) can either end before ($X_{t,m,td}^B$), be active inside or start after ($X_{t,m,td}^A$) period td, see Figure 6.
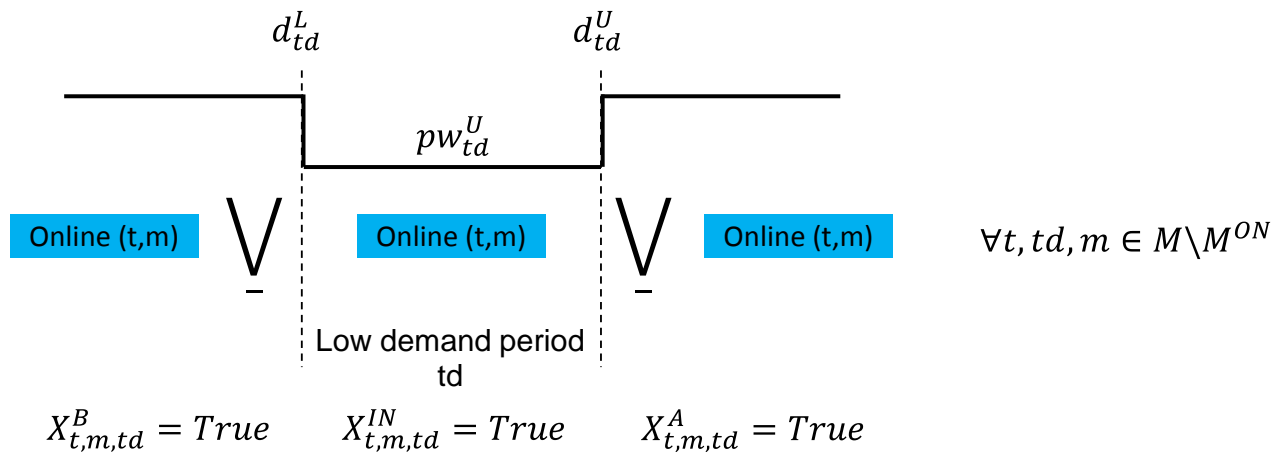

Figure 6. Interaction of processing tasks with low electricity demand periods.

The disjunctive programming constraints are given in Eq. 16. Notice that the constraints involving timing variables $Ts_{t,m}$ and $Te_{t,m}$ in disjunction $X^{IN}_{t,m,td}$ are those shared by types A, B, C, D, while before and after correspond respectively to types E and F.

$$\begin{bmatrix} X^{B}_{t,m,td} \\ Te_{t,m} \leq d^{L}_{td} \end{bmatrix} \underline{\vee} \begin{bmatrix} X^{IN}_{t,m,td} \\ Ts_{t,m} \leq d^{U}_{td} \\ Te_{t,m} \geq d^{L}_{td} \end{bmatrix} \underline{\vee} \begin{bmatrix} X^{A}_{t,m,td} \\ Ts_{t,m} \geq d^{U}_{td} \end{bmatrix} \forall \, t, td, m \in M \backslash M^{ON}$$

(16)

Eq. 17 then ensures that the maximum power demand is not exceeded.

$$\sum_{t} \sum_{m \in M \backslash M^{ON}} X^{IN}_{t,m,td} \cdot pw_m + \sum_{m \in M^{ON}} pw_m \leq pw^{U}_{td} \; \forall \, td \qquad (17)$$

Mathematical Programming Formulations

We will be deriving alternative Mathematical Programming formulations by applying the standard big-M and convex hull reformulations to the Generalized Disjunctive Programming formulation presented in the previous section. In the process, all Boolean variables are converted into binary variables, e.g. $A_{t,m,tp} = True \Leftrightarrow A_{t,m,tp} = 1$. In order to make the linear relaxations as tight as possible, we will be using information from the lower and upper bounds of the model variables (Eqs. 7-10).

The disjunctions in Eq. 13 can be reorganized so that a particular timing constraint appears only once. As will be seen in the Computational Results section, the advantage is the generation of fewer constraints, which in the case of the big-M reformulation are also tighter due to the presence of multiple binary variables. On the other hand, the quality of the hull relaxation might not be as good due to the weaker bounds on half of the constraints.

The condition that the starting time of task (t, m) is greater than lower bound of time period tp is shared by location type A, C and F, while the reverse is true for type B, D and E. This is reflected in

Eq. 18, while the six other possibilities related to variables $Ts_{t,m}$ and $Te_{t,m}$ are part of Eqs. 19-21.

Eq. 22 deals with the remaining constraints involving the $\Delta T_{t,m,tp}$ variables.

$$\begin{bmatrix} A_{t,m,tp} \vee C_{t,m,tp} \vee F_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^L \end{bmatrix} \underline{\vee} \begin{bmatrix} B_{t,m,tp} \vee D_{t,m,tp} \vee E_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^L \end{bmatrix} \forall t, m, tp$$

(18)

$$\begin{bmatrix} \neg F_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^U \end{bmatrix} \underline{\vee} \begin{bmatrix} F_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^U \end{bmatrix} \forall t, m, tp$$

(19)

$$\begin{bmatrix} E_{t,m,tp} \\ Te_{t,m} \leq cp_{tp}^L \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg E_{t,m,tp} \\ Te_{t,m} \geq cp_{tp}^L \end{bmatrix} \forall t, m, tp$$

(20)

$$\begin{bmatrix} A_{t,m,tp} \vee B_{t,m,tp} \vee E_{t,m,tp} \\ Te_{t,m} \leq cp_{tp}^U \end{bmatrix} \underline{\vee} \begin{bmatrix} C_{t,m,tp} \vee D_{t,m,tp} \vee F_{t,m,tp} \\ Te_{t,m} \geq cp_{tp}^U \end{bmatrix} \forall t, m, tp$$

(21)

$$\begin{bmatrix} A_{t,m,tp} \\ \Delta T_{t,m,tp} = P_{t,m} \end{bmatrix} \underline{\vee} \begin{bmatrix} B_{t,m,tp} \\ \Delta T_{t,m,tp} = Te_{t,m} - cp_{tp}^L \end{bmatrix} \underline{\vee} \begin{bmatrix} C_{t,m,tp} \\ \Delta T_{t,m,tp} = cp_{tp}^U - Ts_{t,m} \end{bmatrix} \underline{\vee}$$

$$\underline{\vee} \begin{bmatrix} D_{t,m,tp} \\ \Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L \end{bmatrix} \underline{\vee} \begin{bmatrix} E_{t,m,tp} \vee F_{t,m,tp} \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \forall t, m, tp$$

(22)

Overall, there are 8 sets constrains in Eqs. (18-21) as opposed to 14 in Eq. (13). It should also be highlighted that applying basic steps[16,23] to Eqs. (18-21) would yield Eq. (13) for every (t, m, tp).

Big-M reformulation

We start by reformulating Eq. 11 that avoids maintenance tasks to occur simultaneously. By following the general guidelines[8] to generate the MILP constraints and compute the tightest possible values for the big-M parameters, which were illustrated by Castro and Grossmann[15] in the context of scheduling formulations, Eqs. 23-24 result.

$$Tm_{t,m} + sd_{t,m} \leq Tm_{t',m'} + (tm_{t,m}^U - tm_{t',m'}^L + sd_{t,m}) \cdot (1 - Y_{t,m,t',m'}) \ \forall \ t, t', m' > m \tag{23}$$

$$Tm_{t',m'} + sd_{t',m'} \leq Tm_{t,m} + (tm_{t',m'}^U - tm_{t,m}^L + sd_{t',m'}) \cdot Y_{t,m,t',m'} \ \forall \ t, t', m' > m \tag{24}$$

Equations 25-26 correspond to the reformulation of the two disjunctions in Eq. 12, which ensures that the maintenance team cannot be assigned to work on unavailable periods.

$$Tm_{t,m} + sd_{t,m} \leq u_{tu}^L + (tm_{t,m}^U - u_{tu}^L + sd_{t,m}) \cdot (1 - Z_{t,m,tu}) \ \forall \ t, m, tu \tag{25}$$

$$Tm_{t,m} \geq u_{tu}^U - (u_{tu}^U - tm_{t,m}^L) \cdot Z_{t,m,tu} \ \forall \ t, m, tu \tag{26}$$

The timing constraints related to the location of the processing tasks with respect to the constant electricity time periods are given by Eqs. 27-34. They correspond to the reformulation of the inequalities inside the disjunctions in Eqs. 18-21. Note that we have used the conversion[12,15] of logic constraints of the type $W_{t,m,tp} \Leftrightarrow A_{t,m,tp} \lor C_{t,m,tp} \lor F_{t,m,tp}$, featuring the model's Boolean variables and the auxiliary variable $W_{t,m,tp}$, into MILP format $W_{t,m,tp} = A_{t,m,tp} + C_{t,m,tp} + F_{t,m,tp}$, with 0-1 variables.

$$Ts_{t,m} \geq cp_{tp}^L - (cp_{tp}^L - ts_{t,m}^L) \cdot (1 - A_{t,m,tp} - C_{t,m,tp} - F_{t,m,tp}) \ \forall \ t, m, tp \tag{27}$$

$$Ts_{t,m} \leq cp_{tp}^L + (ts_{t,m}^U - cp_{tp}^L) \cdot (1 - B_{t,m,tp} - D_{t,m,tp} - E_{t,m,tp}) \ \forall \ t, m, tp \tag{28}$$

$$Ts_{t,m} \leq cp_{tp}^U + (ts_{t,m}^U - cp_{tp}^U) \cdot F_{t,m,tp} \ \forall \ t, m, tp \tag{29}$$

$$Ts_{t,m} \geq cp_{tp}^U - (cp_{tp}^U - ts_{t,m}^L) \cdot (1 - F_{t,m,tp}) \ \forall \ t, m, tp \tag{30}$$

$$Te_{t,m} \leq cp_{tp}^L + \left(te_{t,m}^U - cp_{tp}^L\right) \cdot \left(1 - E_{t,m,tp}\right) \forall\, t, m, tp \tag{31}$$

$$Te_{t,m} \geq cp_{tp}^L - \left(cp_{tp}^L - te_{t,m}^L\right) \cdot E_{t,m,tp} \,\forall\, t, m, tp \tag{32}$$

$$Te_{t,m} \leq cp_{tp}^U + \left(te_{t,m}^U - cp_{tp}^U\right) \cdot \left(1 - A_{t,m,tp} - B_{t,m,tp} - E_{t,m,tp}\right) \forall\, t, m, tp \tag{33}$$

$$Te_{t,m} \geq cp_{tp}^U - \left(cp_{tp}^U - te_{t,m}^L\right) \cdot \left(1 - C_{t,m,tp} - D_{t,m,tp} - F_{t,m,tp}\right) \forall\, t, m, tp \tag{34}$$

It should be emphasized at this point that the equivalent constraints to Eqs. 27-34 in Hudera and Harjunkoski[14] are larger in number and weaker due to the presence of fewer binary variables (except in Eqs. 30-31) and the use of a single and hence necessarily larger big-M value in the constraints. The steel problem[14] also features fixed rather than variable processing times, allowing for the simplification of Eq. (35). Interestingly, the computation of the equivalent variables to $\Delta T_{t,m,tp}$

resembles Eq. (72) for the hull relaxation, but a different set of variables is used, which appear from the exact linearization of bilinear terms involving continuous and binary variables.

The next set of constraints calculate the time taken by processing task (t, m) in period tp. Since we are dealing with equality constraints, these first need to be divided into two inequality constraints. Taking as example the constraint inside the $A_{t,m,tp}$ disjunction in Eq. 22,

$\Delta T_{t,m,tp} = P_{t,m} \Leftrightarrow \Delta T_{t,m,tp} \geq P_{t,m} \wedge \Delta T_{t,m,tp} \leq P_{t,m}$. The values of the big-M parameters are then

$\max\left(P_{t,m} - \Delta T_{t,m,tp}\right) = p_{t,m}^U$ and $\max\left(\Delta T_{t,m,tp} - P_{t,m}\right) = 0$, leading to Eqs. 35-36.

$$\Delta T_{t,m,tp} \geq P_{t,m} - p_{t,m}^U \cdot \left(1 - A_{t,m,tp}\right) \forall\, t, m, tp \tag{35}$$

$$\Delta T_{t,m,tp} \leq P_{t,m} \,\forall\, t, m, tp \tag{36}$$

The remaining constraints are obtained in a similar fashion (Eqs. 37-42). Notice that the global constraints in Eq. (36) and Eq. (42), stating that $\Delta T_{t,m,tp}$ cannot be higher than the task's processing time neither than the time interval length, appear naturally from the derivation.

$$\Delta T_{t,m,tp} \geq Te_{t,m} - cp_{tp}^L - (te_{t,m}^U - cp_{tp}^L) \cdot (1 - B_{t,m,tp}) \ \forall \ t, m, tp \tag{37}$$

$$\Delta T_{t,m,tp} \leq Te_{t,m} - cp_{tp}^L + (cp_{tp}^L - ts_{t,m}^L) \cdot (1 - B_{t,m,tp}) \ \forall \ t, m, tp \tag{38}$$

$$\Delta T_{t,m,tp} \geq cp_{tp}^U - Ts_{t,m} - (cp_{tp}^U - ts_{t,m}^L) \cdot (1 - C_{t,m,tp}) \ \forall \ t, m, tp \tag{39}$$

$$\Delta T_{t,m,tp} \leq cp_{tp}^U - Ts_{t,m} + (te_{t,m}^U - cp_{tp}^U) \cdot (1 - C_{t,m,tp}) \ \forall \ t, m, tp \tag{40}$$

$$\Delta T_{t,m,tp} \geq (cp_{tp}^U - cp_{tp}^L) \cdot D_{t,m,tp} \ \forall \ t, m, tp \tag{41}$$

$$\Delta T_{t,m,tp} \leq cp_{tp}^U - cp_{tp}^L \ \forall \ t, m, tp \tag{42}$$

$$\Delta T_{t,m,tp} \leq \min(cp_{tp}^U - cp_{tp}^L, p_{t,m}^U) \cdot (1 - E_{t,m,tp} - F_{t,m,tp}) \ \forall \ t, m, tp \tag{43}$$

Four sets of big-M constraints are required to reformulate Eq.16. Eq. 44 states that if processing task (t, m) is executed before time period td, then the ending time must be lower than the start of the low maximum demand period. Notice that there is no need to use big-M constraints for the starting time variables due to Eq. 1. The same applies when enforcing the starting time variables to be greater than the ending time of period td whenever the processing task is executed after td, see Eq 45. If, on the other hand, part of the task takes place within td, we need to enforce the bounds in Eqs 46-47.

$$Te_{t,m} \leq d_{td}^L + (te_{t,m}^U - d_{td}^L) \cdot (1 - X_{t,m,td}^B) \ \forall \ t, td, m \in M \backslash M^{ON} \tag{44}$$

$$Ts_{t,m} \geq d_{td}^U - (d_{td}^U - ts_{t,m}^L) \cdot (1 - X_{t,m,td}^A) \ \forall \ t, td, m \in M \backslash M^{ON} \tag{45}$$

$$Ts_{t,m} \leq d_{td}^U + \left(ts_{t,m}^U - d_{td}^U\right) \cdot \left(1 - X_{t,m,td}^{IN}\right) \forall t, td, m \in M \backslash M^{ON} \tag{46}$$

$$Te_{t,m} \geq d_{td}^L - \left(d_{td}^L - te_{t,m}^L\right) \cdot \left(1 - X_{t,m,td}^{IN}\right) \forall t, td, m \in M \backslash M^{ON} \tag{47}$$

Hull reformulation

Compared to the big-M reformulation, the convex hull reformulation involves additional disaggregated variables and constraints. Hence, the benefits in solution time from a stronger linear relaxation may be surpassed by the difficulties resulting from a larger problem size[16] and is often difficult to predict the best performer. The current problem, with its four sets of disjunctive constraints (Eqs. 11-13, 16) involving different sets of variables, is a good opportunity to gain valuable knowledge concerning the identification of problems where the additional modeling effort required for the derivation of the hull reformulation may be compensated by the improved computational performance.

The first set of constraints (Eq. 11) ensures that the maintenance team is not assigned to two tasks simultaneously and involves timing variables $Tm_{t,m}$ and $Tm_{t',m'}$ for disjunction (t, t', m, m'). The convex hull reformulation for this part of the model requires disaggregated variables with 6 indices (e.g. $\widetilde{Tm}_{t',m',t,m,t',m'}^Y$) together with 12 sets of constraints, as can be seen in the Appendix. It compares with the just two sets of constraints required by the big-M reformulation, which is responsible for a significantly smaller mathematical problem and considerably better performance. These results are consistent with those for the single stage general precedence formulation in Castro and Grossmann[15].

Fortunately, the results for the other two sets of constraints are more encouraging, as will be seen later on. We start with the unavailability of the maintenance team constraint in Eq. 12, which involves a single variable ($Tm_{t,m}$) and one or two parameters in each disjunction (t, m, tu). Since the indices of the variable are shared with the constraint domain, the new disaggregated variables,

$\widetilde{Tm}^Z_{t,m,tu}$ and $\widetilde{Tm}^{\neg Z}_{t,m,tu}$, only feature one additional index compared to the original variable.

Furthermore, the single constraint inside the disjunction can be viewed as a bounding constraint, acting as an upper bound on the execution of the maintenance task for the left (Z) disjunction and as a lower bound for the right disjunction ($\neg Z$), see Eqs. 48-49. As a consequence, we will be requiring fewer additional constraints than expected. In particular, the fifth set of constraints in Eq. 50 states that the sum of disaggregated variables associated to the two disjunctions, must be equal to the original variable for every unavailable period tu.

$$tm^L_{t,m} \cdot Z_{t,m,tu} \leq \widetilde{Tm}^Z_{t,m,tu} \leq (u^L_{tu} - sd_{t,m}) \cdot Z_{t,m,tu} \ \forall \, t, m, tu \tag{48}$$

$$u^U_{tu} \cdot (1 - Z_{t,m,tu}) \leq \widetilde{Tm}^{\neg Z}_{t,m,tu} \leq tm^U_{t,m} \cdot (1 - Z_{t,m,tu}) \ \forall \, t, m, tu \tag{49}$$

$$Tm_{t,m} = \widetilde{Tm}^Z_{t,m,tu} + \widetilde{Tm}^{\neg Z}_{t,m,tu} \ \forall \, t, m, tu \tag{50}$$

The majority of the constraints required for calculating the revenue on the different electricity time periods also act as bounding constraints on the timing variables. Eqs. 18-21 also involve a single variable with a subset of the indices featured in the constraint domain, thus leading to the same five sets of constraints. More specifically, Eqs. 51-53 are the hull reformulation of Eq. 18.

$$cp^L_{tp} \cdot (A_{t,m,tp} + C_{t,m,tp} + F_{t,m,tp}) \leq \widetilde{Ts}^{ACF}_{t,m,tp} \leq ts^U_{t,m} \cdot (A_{t,m,tp} + C_{t,m,tp} + F_{t,m,tp}) \ \forall \, t, m, tp \tag{51}$$

$$ts^L_{t,m} \cdot (B_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp}) \leq \widetilde{Ts}^{BDE}_{t,m,tp} \leq cp^L_{tp} \cdot (B_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp}) \ \forall \, t, m, tp \tag{52}$$

$$Ts_{t,m} = \widetilde{Ts}^{ACF}_{t,m,tp} + \widetilde{Ts}^{BDE}_{t,m,tp} \ \forall \, t, m, tp \tag{53}$$

The MILP constraints for Eqs. 19-21 are given in Eqs 54-62.

$$ts^L_{t,m} \cdot (1 - F_{t,m,tp}) \leq \widetilde{Ts}^{\neg F}_{t,m,tp} \leq cp^U_{tp} \cdot (1 - F_{t,m,tp}) \ \forall \, t, m, tp \tag{54}$$

$$cp^U_{tp} \cdot F_{t,m,tp} \leq \widetilde{Ts}^F_{t,m,tp} \leq ts^U_{t,m} \cdot F_{t,m,tp} \ \forall \, t, m, tp \tag{55}$$

$$Ts_{t,m} = \widetilde{Ts}^{\neg F}_{t,m,tp} + \widetilde{Ts}^{F}_{t,m,tp} \ \forall \ t,m,tp \tag{56}$$

$$te^{L}_{t,m} \cdot E_{t,m,tp} \leq \widetilde{Te}^{E}_{t,m,tp} \leq cp^{L}_{tp} \cdot E_{t,m,tp} \ \forall \ t,m,tp \tag{57}$$

$$cp^{L}_{tp} \cdot (1 - E_{t,m,tp}) \leq \widetilde{Te}^{\neg E}_{t,m,tp} \leq te^{U}_{t,m} \cdot (1 - E_{t,m,tp}) \ \forall \ t,m,tp \tag{58}$$

$$Te_{t,m} = \widetilde{Te}^{E}_{t,m,tp} + \widetilde{Te}^{\neg E}_{t,m,tp} \ \forall \ t,m,tp \tag{59}$$

$$te^{L}_{t,m} \cdot (A_{t,m,tp} + B_{t,m,tp} + E_{t,m,tp}) \leq \widetilde{Te}^{ABE}_{t,m,tp} \leq cp^{U}_{tp} \cdot (A_{t,m,tp} + B_{t,m,tp} + E_{t,m,tp}) \ \forall \ t,m,tp$$

$$\tag{60}$$

$$cp^{U}_{tp} \cdot (C_{t,m,tp} + D_{t,m,tp} + F_{t,m,tp}) \leq \widetilde{Te}^{CDF}_{t,m,tp} \leq te^{U}_{t,m} \cdot (C_{t,m,tp} + D_{t,m,tp} + F_{t,m,tp}) \ \forall \ t,m,tp \tag{61}$$

$$Te_{t,m} = \widetilde{Te}^{ABE}_{t,m,tp} + \widetilde{Te}^{CDF}_{t,m,tp} \ \forall \ t,m,tp \tag{62}$$

It remains to reformulate the disjunctions associated to the calculation of time factors $\Delta T_{t,m,tp}$, Eq.

22. Notice that the three disjunctions further to the left involve three other variables, $P_{t,m}, Te_{t,m}$ and

$Ts_{t,m}$, which need to be disaggregated. However, unlike what we have seen so far, each of these

variables appears in a single constraint. Thus, rather than defining one disaggregated variable for

each disjunction, it suffices two define just two, one for the disjunction where they appear

(superscript equal to the corresponding binary variable, e.g. $\hat{P}^{A}_{t,m,tp}$) and another for the other cases (

e.g. $\hat{P}^{\neg A}_{t,m,tp}$). The constraints relating the new disaggregated variables with the binary variables and

the disjunctive programming variables are given below in Eqs. 63-71.

$$p^{L}_{t,m} \cdot A_{t,m,tp} \leq \hat{P}^{A}_{t,m,tp} \leq \min(p^{U}_{t,m}, cp^{U}_{tp} - cp^{L}_{tp}) \cdot A_{t,m,tp} \ \forall \ t,m,tp \tag{63}$$

$$p^{L}_{t,m} \cdot (1 - A_{t,m,tp}) \leq \hat{P}^{\neg A}_{t,m,tp} \leq p^{U}_{t,m} \cdot (1 - A_{t,m,tp}) \ \forall \ t,m,tp \tag{64}$$

$$P_{t,m} = \hat{P}^{A}_{t,m,tp} + \hat{P}^{\neg A}_{t,m,tp} \ \forall \ t,m,tp \tag{65}$$

$$\max(cp_{tp}^L, te_{t,m}^L) \cdot B_{t,m,tp} \leq \widetilde{Te}_{t,m,tp}^B \leq \min(cp_{tp}^U, te_{t,m}^U) \cdot B_{t,m,tp} \ \forall \ t,m,tp \tag{66}$$

$$te_{t,m}^L \cdot (1 - B_{t,m,tp}) \leq \widetilde{Te}_{t,m,tp}^{\neg B} \leq te_{t,m}^U \cdot (1 - B_{t,m,tp}) \ \forall \ t,m,tp \tag{67}$$

$$Te_{t,m} = \widetilde{Te}_{t,m,tp}^B + \widetilde{Te}_{t,m,tp}^{\neg B} \ \forall \ t,m,tp \tag{68}$$

$$\max(cp_{tp}^L, ts_{t,m}^L) \cdot C_{t,m,tp} \leq \widetilde{Ts}_{t,m,tp}^C \leq \min(cp_{tp}^U, ts_{t,m}^U) \cdot C_{t,m,tp} \ \forall \ t,m,tp \tag{69}$$

$$ts_{t,m}^L \cdot (1 - C_{t,m,tp}) \leq \widetilde{Ts}_{t,m,tp}^{\neg C} \leq ts_{t,m}^U \cdot (1 - C_{t,m,tp}) \ \forall \ t,m,tp \tag{70}$$

$$Ts_{t,m} = \widetilde{Ts}_{t,m,tp}^C + \widetilde{Ts}_{t,m,tp}^{\neg C} \ \forall \ t,m,tp \tag{71}$$

The time factor $\Delta T_{t,m,tp}$ can then be written as a sum of multiple terms, one for each constraint

associated to the disjunctions in Eq. 22.

$$\Delta T_{t,m,tp} = \hat{P}_{t,m,tp}^A + \widetilde{Te}_{t,m,tp}^B - cp_{tp}^L \cdot B_{t,m,tp} + cp_{tp}^U \cdot C_{t,m,tp} - \widetilde{Ts}_{t,m,tp}^C + (cp_{tp}^U - cp_{tp}^L) \cdot$$

$$D_{t,m,tp} \ \forall \ t,m,tp$$

$$\tag{72}$$

Finally, we have the constraints identifying the tasks executed in the low electricity demand

periods, Eqs. 73-80. Notice the three additional sets of disaggregated starting and ending time

variables.

$$ts_{t,m}^L \cdot X_{t,m,td}^B \leq \widetilde{Ts}_{t,m,td}^{XB} \leq \min\left[ts_{t,m}^U, (d_{td}^L - p_{t,m}^L)\right] \cdot X_{t,m,td}^B \ \forall \ t,td, m \in M \backslash M^{ON} \tag{73}$$

$$te_{t,m}^L \cdot X_{t,m,td}^B \leq \widetilde{Te}_{t,m,td}^{XB} \leq \min\left[te_{t,m}^U, d_{td}^L\right] \cdot X_{t,m,td}^B \ \forall \ t,td, m \in M \backslash M^{ON} \tag{74}$$

$$\max\left[ts_{t,m}^L, (d_{td}^L - p_{t,m}^U)\right] \cdot X_{t,m,td}^{IN} \leq \widetilde{Ts}_{t,m,td}^{XIN} \leq \min\left[ts_{t,m}^U, d_{td}^U\right] \cdot X_{t,m,td}^{IN} \ \forall \ t,td, m \in M \backslash M^{ON} \tag{75}$$

$$\max\left[te_{t,m}^L, d_{td}^L\right] \cdot X_{t,m,td}^{IN} \leq \widetilde{Te}_{t,m,td}^{XIN} \leq \min\left[te_{t,m}^U, (d_{td}^U + p_{t,m}^U)\right] \cdot X_{t,m,td}^{IN} \ \forall \ t,td, m \in M \backslash M^{ON}$$

$$\tag{76}$$

$$\max\left[ts_{t,m}^L, d_{td}^U\right] \cdot X_{t,m,td}^A \leq \widetilde{Ts}_{t,m,td}^{XA} \leq ts_{t,m}^U \cdot X_{t,m,td}^A \ \forall \ t,td, m \in M \backslash M^{ON} \tag{77}$$

$$\max\left[te_{t,m}^L, (d_{td}^U + p_{t,m}^L)\right] \cdot X_{t,m,td}^A \leq \overline{Te}_{t,m,td}^{XA} \leq te_{t,m}^U \cdot X_{t,m,td}^A \ \forall \, t,td,m \in M \backslash M^{ON} \tag{78}$$

$$Ts_{t,m} = \overline{Ts}_{t,m,td}^{XB} + \overline{Ts}_{t,m,td}^{XIN} + \overline{Ts}_{t,m,td}^{XA} \ \forall \, t,td,m \in M \backslash M^{ON} \tag{79}$$

$$Te_{t,m} = \overline{Te}_{t,m,td}^{XB} + \overline{Te}_{t,m,td}^{XIN} + \overline{Te}_{t,m,td}^{XA} \ \forall \, t,td,m \in M \backslash M^{ON} \tag{80}$$

Remarks

It should be highlighted that the min and max functions in Eqs 63, 66, 69 and 73-78 have the purpose of tightening the upper and lower bounds and lead to a significant improvement in the quality of the linear relaxation. As an example, if the interaction is of type *A*, the full extent of the task must be located within tp, meaning that the upper bound on the disaggregated variable in Eq. 63 is the lowest value between the maximum processing time and the duration of tp. While we can do the same in other constraints of the hull reformulation, preliminary results have shown a worse performance and no influence in the integrality gap. Hence we have opted to keep the constraints simpler.

Common constraints

The big-M and convex hull reformulation share the constraint that exactly one type of interaction of processing task (t, m) with constant electricity price period tp must be selected.

$$A_{t,m,tp} + B_{t,m,tp} + C_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp} + F_{t,m,tp} = 1 \ \forall \, t, m, tp \tag{81}$$

Similarly, processing task (t, m) is either completely before, completely after or partially within lower power demand period td.

$$X_{t,m,td}^B + X_{t,m,td}^{IN} + X_{t,m,td}^A = 1 \ \forall \, t,td,m \in M \backslash M^{ON} \tag{82}$$

Case Study

We consider a power plant with 18 identical gas engines with a generation capacity per engine $pw_m = 10$ MW $\forall m$. The minimum electricity production should be $pw^L = 80$ MW, corresponding to a minimum of $|M^{ON}| = 9$ engines online simultaneously (recall that this class is either online or in shutdown mode and that at most one engine is in shutdown mode). The schedule involves $|T| = 12$ operation time periods, with the number of online hours ranging between [2000, 2500] except for $= 8$, which ends in a major shutdown. The duration of the required maintenance shutdowns is also given in Table 1.

The production plan is to be obtained for roughly 3.5 years, comprising 8 time periods of constant electricity price, where the low tariff is intercalated with the high tariff, see Table 2. There are also 4 periods of maximum power demand lasting 3 weeks each (Table 3) and the maintenance team is unavailable one week around Christmas (Table 4).

Table 1. Bounds on time spent online and length of shutdown periods (same for all engines) [h]

t	1	2	3	4	5	6	7	8	9	10	11	12
$p^U_{t,m}$	2500	2500	2500	2500	2500	2500	2500	3000	2500	2500	2500	2500
$p^L_{t,m}$	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
$sd_{t,m}$	12	72	12	96	12	120	12	432	12	72	12	120

Table 2. Electricity cost data

tp	1	2	3	4	5	6	7	8
cp^U_{tp} [h]	3624	5832	12384	14592	21144	23352	29928	32136
cp^L_{tp} [h]	0	3624	5832	12384	14592	21144	23352	29928
ce_{tp} [$/MWh]	40	75	40	75	40	75	40	75

Table 3. Periods of maximum power demand

td	1	2	3	4
d^U_{td} [h]	2664	11424	20184	28968
d^L_{td} [h]	2160	10920	19680	28464
pw^U_{td} [MW]	140	140	140	140

Table 4. Periods of maintenance team unavailability

tu	1	2	3	4
u_{tu}^{U} [h]	8688	17448	26208	34968
u_{tu}^{L} [h]	8520	17280	26040	34800

Most of the complexity of the model arises from the 4-index binary variables $Y_{t,m,t',m'}$. Given that

the gas engines are identical, we can assume that within operation period t, the shutdown of unit m

precedes the shutdown of unit $m' > m$. Then, due to the processing time constraints, it can also be

ensured that shutdown (t, m) precedes shutdown (t', m) for $t' > t$. The general condition, given in

Eq. 83, is responsible for orders of magnitude reduction in computational time. In later periods, it

may also occur that shutdown $(t + 1, m)$ of low index units occurs before (t, m') of high index units,

but that is a decision for the optimization solver to make.

$$Y_{t,m,t',m'} = 1 \ \forall \ t' \geq t, m' > m \tag{83}$$

Bounding the model variables

Most constraints need information from the timing variables lower and upper bounds. Based on

the chosen order for the shutdown tasks, one can derive rigorous lower bounds for the start of the

maintenance tasks ($tm_{t,m}^{L}$) based on Figure 7. On the one hand, the lower bound for shutdown (t, m)

may be the preceding processing task (t, m), which lasts a minimum of $p_{t,m}^{L}$ (e.g. slot 1 for unit 1).

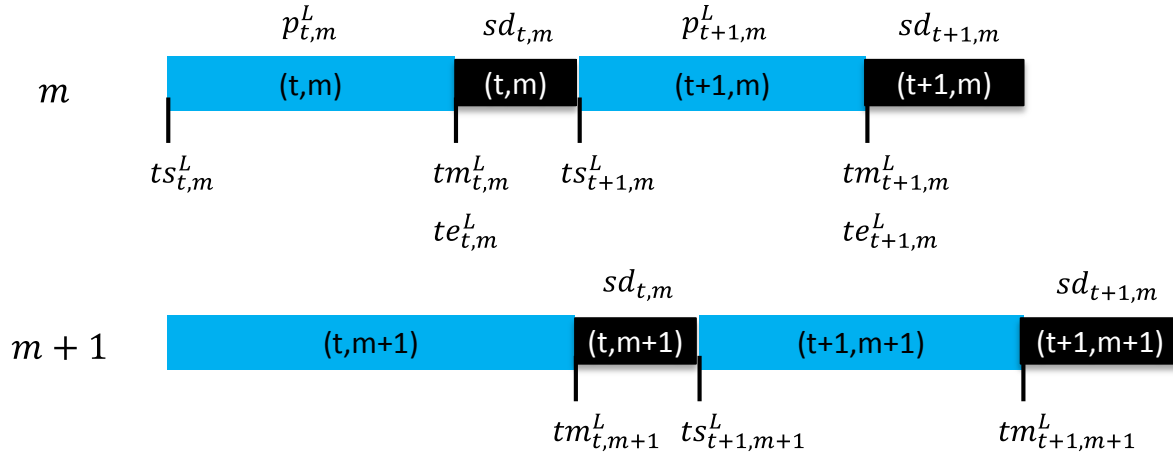On the other hand, the limiting factor for unit $m + 1$ may be the end of shutdown task (t, m).

Figure 7. Two possible situations defining the lower bounds on the start of the shutdown tasks.

The required lower bounds are calculated according to Eq. 84. As for the upper bounds, we use the same formula replacing the lower bounding parameters with their upper bounding counterparts. However, it should be highlighted that upper bounds are heuristic since both periods of low power demand and maintenance team unavailability may cause further delays, possibly compromising feasibility. The reason why this was not observed is probably due to the large flexibility of online hours before shutdown and the small number of such relatively short periods.

$$
\begin{aligned}
&\text{FOR } m = 1 \ to \ |M| \\
&\quad \text{FOR } t = 1 \ to \ |T| \\
&\quad\quad tm_{t,m}^L = \max(tm_{t,m-1}^L + sd_{t,m-1}, tm_{t-1,m}^L + sd_{t-1,m} + p_{t,m}^L) \\
&\quad\quad tm_{t,m}^U = \max(tm_{t,m-1}^U + sd_{t,m-1}, tm_{t-1,m}^U + sd_{t-1,m} + p_{t,m}^U) \\
&\quad \text{NEXT } t \\
&\text{NEXT } m
\end{aligned}
\tag{84}
$$

The bounds for the starting time variables are given by Eqs. 85-86. In Eq. 86, the right-hand side has a conditional domain since the starting time of always on units in the first slot is equal to 0, recall Eq. 6.

$$
ts_{t,m}^L = tm_{t-1,m}^L + sd_{t-1,m} \ \forall \ t, m
\tag{85}
$$

$$
ts_{t,m}^U = \max\left[\max\left(0, tm_{t,m}^U - p_{t,m}^U\right), tm_{t-1,m}^U + sd_{t-1,m}\right]\Big|_{m \in M \backslash M^{ON} \vee t > 1} \ \forall \ t, m
\tag{86}
$$

Computational Results

The mixed-integer linear programming models resulting from the big-M and hull reformulations of the generalized disjunctive programming model were implemented in GAMS 24.1 and solved by CPLEX 12.5 using a single thread and default options up to a relative optimality tolerance=10^{-6} or maximum computational time=3600 CPUs. The hardware consisted on a desktop with an Intel i7 950 (3.07 GHz) with 8 GB of RAM running Windows 7.

Table 5. Composition of four MILP formulations tested as a function reformulation strategy used for each set of disjunctions.

Model	BM-1	BM-2	Hybrid		Hull
Disjunctive constraints/Reformulation		Big-M	Big-M	Hull	Hull
Single maintenance team (Eq. 11)	✓	✓	✓		✓
Unavailability of maintenance team (Eq. 12)	✓	✓		✓	✓
Interaction with electricity price periods (Eq. 13)	✓				
Interaction with electricity price periods (Eqs. 18-22)		✓		✓	✓
Maximum power output (Eq. 16)	✓	✓		✓	✓

Computational performance of four alternative mixed-integer linear programming models involving different combinations of disjunctive constraints and reformulation methods (details given in Table 5) is illustrated through the solution of 5 test problems of varying difficulty based on the data provided in the previous section. More specifically, we consider the full problem with $|T| = 12$ operation periods, $|Tp| = 8$ constant electricity price periods, $|Td| = 4$ periods of maximum power demand and $|Tu| = 3$ periods where the maintenance team is unavailable; and four subproblems of it. The actual data is provided in Table 6 together with key computational statistics.

We can see that model BM-1 has a slightly better performance than BM-2 and Hybrid, being responsible for lower computational times and leading to a better solution for $|T| = 12$.

Nevertheless it should be highlighted that BM-2 returns a lower optimality gap than BM-1, due to a lower best possible solution at the time of termination (254.85 vs. 255.72). The hull reformulation

was the worst performer, barely failing to prove optimality for $|T| = 8$ and roughly doubling the

optimality gap for the largest problem.

Table 6. Key results for alternative reformulations of disjunctive programming model (best solutions in bold, maximum computational time in italic)

Reformulation				BM-1			BM-2			Hybrid			Hull		
$|T|$	$|Tp|$	$|Td|$	$|Tu|$	Revenue [10^6\$]	Gap (%)	CPUs	Revenue [10^6\$]	Gap (%)	CPUs	Revenue [10^6\$]	Gap (%)	CPUs	Revenue [10^6\$]	Gap (%)	CPUs
4	3	2	1	**83.64**	-	0.38	**83.64**	-	0.51	**83.64**	-	0.52	**83.64**	-	0.66
6	4	2	1	**127.31**	-	2.28	**127.31**	-	2.23	**127.31**	-	7.43	**127.31**	-	8.44
8	5	3	2	**163.17**	-	75.0	**163.17**	-	356	**163.17**	-	544	**163.17**	0.01	*3600*
10	7	4	3	**213.83**	-	119	**213.83**	-	142	**213.85**	-	142	**213.85**	-	914
12	8	4	3	**251.38**	1.73	*3600*	250.87	1.59	*3600*	250.89	1.82	*3600*	250.37	3.28	*3600*

Table 7. Computational statistics related to problem size [a]

Reformulation		BM-1			BM-2			Hybrid			Hull		
$|T|$	BV	TV	E	I.G. (%)	TV	E	I.G. (%)	TV	E	I.G. (%)	TV	E	I.G. (%)
4	2502	4537	10641	37	4537	9345	26	8137	14601	1	17929	39081	1
6	5319	9397	22125	46	9397	19533	40	16309	29577	9	38341	84657	9
8	9540	16345	38362	55	16345	34042	44	28297	51466	9	67465	149386	9
10	16065	26461	63167	50	26461	55607	46	47341	86027	11	108541	239027	11
12	22410	36937	88115	55	36937	77747	52	65017	118571	19	153145	338891	19

[a] BV=binary variables; TV=total variables; E=equations; I.G.=integrality gap calculated with respect to best known solution

Table 8. Computational statistics following increase in shutdown flexibility

	BM-1				BM-2				Hybrid				Hull			
$\|T\|$	I.G. (%)	Revenue [10^6\$]	Gap (%)	CPUs	I.G. (%)	Revenue [10^6\$]	Gap (%)	CPUs	I.G. (%)	Revenue [10^6\$]	Gap (%)	CPUs	I.G. (%)	Revenue [10^6\$]	Gap (%)	CPUs
4	41	**83.64**	-	0.60	30	**83.64**	-	0.72	7	**83.64**	-	1.30	7	**83.64**	-	1.71
6	51	**127.31**	-	1524	47	**127.31**	-	1217	14	**127.31**	-	1704	14	**127.31**	0.55	*3600*
8	58	**163.64**	3.04	*3600*	50	**163.64**	2.32	*3600*	22	163.64	2.29	*3600*	22	**163.64**	3.48	*3600*
10	53	214.20	4.24	*3600*	50	**214.21**	4.48	*3600*	26	213.88	6.51	*3600*	26	no sol.	-	*3600*
12	58	249.55	14.5	*3600*	56	**250.15**	12.8	*3600*	36	247.50	15.8	*3600*	36	no sol.	-	*3600*

The computational statistics in Table 7 help to explain the relative performance of the models. While sharing the same number of binary variables, the big-M models have the advantage of requiring the fewest variables and constraints but the disadvantage of providing the worst linear relaxation (higher integrality gaps). The results for BM-2 show that reorganizing the disjunctions in Eq. (13) into Eqs. (18-22) effectively leads to a smaller and tighter model. However, the reduction in integrality gap is considerably smaller than the one that can be achieved with the hull relaxation, which is reflected in comparable performances by BM-1 and BM-2 (confirmed by the results in Table 8). The other interesting result is that the reformulation method chosen for single maintenance team constraint has a major effect in problem size without affecting the integrality gap. More specifically, the Hybrid formulation requires less than half the number of variables and constraints than the Hull formulation. It thus represents the best tradeoff between problem size and quality of the relaxation, leading to better solutions and lower optimality gaps at the 1-hour termination time (check also Table 8).

The best found solution for $|T| = 12$ is given in Figure 8. Notice that not all engines are capable of finishing their production tasks within 32316 h, the upper bound of the last electricity cost period. Twelve engines complete the 12 periods of operation, some long before the end of the horizon, engines M13-M17 complete 11, while engine M18 completes just 10. For the overall analysis of the schedule it is thus convenient to neglect the terminal effects roughly after the 27000-h mark so we are considering 3 full years of operation.

The most interesting aspect is that there are no idle periods in the high electricity cost periods, as desired to meet the goal of maximizing revenue from electricity sales. This can be confirmed by the power output profile that shows a minimum production of 170 MW in the first three green periods, corresponding to a single engine under maintenance. Considering that the labels inside the rectangles give the processing time, a distinction can be made between always on engines (M1-M9) and the others. Non always on engines mostly operate for a time corresponding to the upper bound values given in Table 1, which maximizes productivity. In contrast, always on engines, predominantly M1,

often operate closer to the lower bound to provide enough flexibility to execute all required shutdowns. This point will be discussed further in the next subsection.
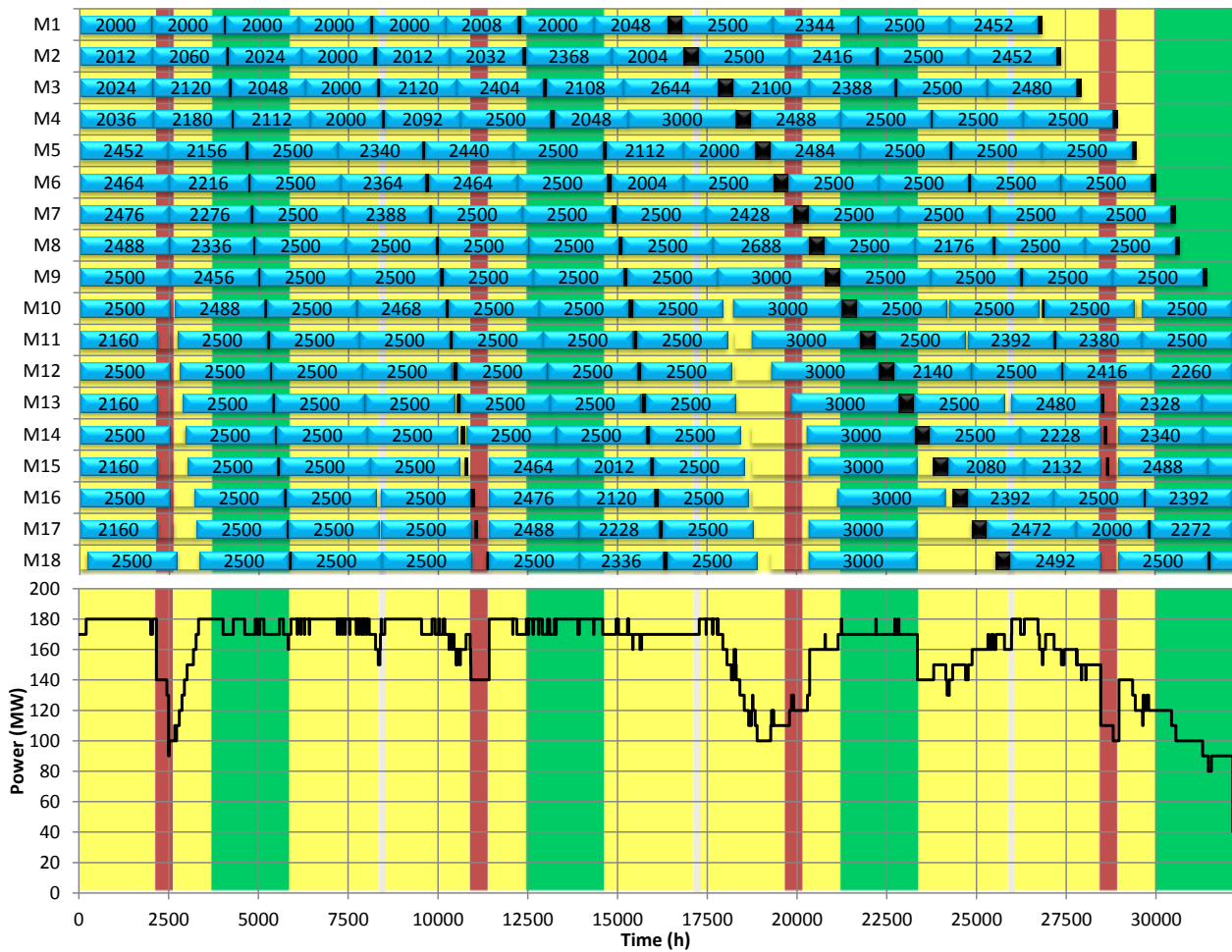

Figure 8. Best found solution for $|T| = 12$ operation periods problem (background shows periods with low tariff in yellow, periods with high tariff in green, periods with no maintenance team available in red and periods with maximum power demand in grey).

The maximum demand periods of 140 MW, in red, are frequently used to perform the mandatory maintenance tasks, as it is apparent in the second of such periods with the 96 h shutdowns of engines M16-M18. The 432 h shutdowns, which start around the 17000-h mark, are a severe bottleneck on power output. One should thus consider hiring an additional maintenance team and incorporating its cost in the objective function. This will be the subject of future work. Finally, and while difficult to see, the constraint of no maintenance team available during the periods in grey, is being respected.

Shutdown schedule flexibility

The mathematical formulation meets the minimum power demand constraint in an indirect way, by not allowing idle times for a subset of the gas engines. Given the single maintenance team

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constraint, there exists a feasible region only if the processing times are allowed to vary within a sufficiently wide range. In fact, raising the minimum electricity production to $pw^L = 90$ MW makes test problems with $|T| \in \{8,10,12\}$ infeasible, while the remaining two become infeasible for $pw^L = 140$ MW. This occurs even after increasing the values of the heuristic upper bounds in Eqs. 84 and 86.

To test the influence of the range of processing times in computational performance, we reduced the values of $p_{t,m}^L$ in Table 1 from 2000 h to 1500 ($t \neq 8$) and 1000 ($t = 8$). Compared to the base case, the problem is being relaxed and hence solution quality cannot degrade provided that sufficient time is given to the optimization solver. The results in Table 8 show no improvement in the value of the objective function for $|T| \in \{4,6\}$, but slightly higher revenues are returned by the big-M model for $|T| \in \{8,10\}$, (163.64 vs. 163.17 and 214.21 vs. 213.85 check Table 6). The drawback is that the computational time increased by at least one order of magnitude, which was difficult to predict despite the increase in integrality gap, which roughly doubled for the Hybrid and Hull formulations but did not change much for BM-1 and BM-2, given that the problem size did not change. As a consequence, good quality solutions become harder to obtain for the largest problem (notice that no feasible solution could be found in 1-h of computational time by the Hull model). We have also confirmed of the well-known capability of big-M formulations to find good solutions in the early nodes of the search tree, which, by better guiding the search, lead to smaller optimality gaps than those obtained by the Hybrid formulation.

Overall, optimality gaps around 12% can be too high in the context of revenue in the interval [250, 282] million dollars. Hence, shutdown flexibility should be kept as low as possible to ensure a good computational performance.

Conclusions

This paper has proposed a new continuous-time generalized disjunctive programming model for the optimal maintenance scheduling of a gas engine power plant. Emphasis was put on the derivation of constraints related to the availability of the single maintenance team, a resource shared by the gas engines, to the maximum demand constraints and to the calculation of the revenue from electricity sales in the given constant tariff electricity periods. By using the high-level construct of disjunctive programming, simple linear constraints could be associated to each decision variable, which were then converted into mixed-integer linear programming format using big-M and hull reformulations. In particular, we have shown that the disjunctions linked to revenue calculation can be reorganized so that a particular linear constraint appears only once. This has the advantage of leading to smaller and tighter mathematical formulations. The results have shown that the big-M reformulation of the disjunctive constraints leads to the most computationally efficient models and that the hull reformulation of the single maintenance team constraint is particularly inefficient.

Through the solution of an industrial case study featuring identical engines, we have shown that a near optimal (<2% optimality gap) maintenance plan can be derived for a time horizon of 3 years, considering seasonal variations in electricity price and other volatile, yet deterministic, resource profiles. Furthermore, we have identified that the single maintenance team becomes an important bottleneck around the time the mandatory shutdowns become longer, significantly reducing power output and revenue. Future work will thus look into the cost-benefit effect of hiring an additional team.

Acknowledgments

Appendix

For completeness, we show below the convex hull reformulation of the disjunction in Eq. 11, which is responsible for a major increase in the number of continuous variables and constraints and a significantly worse computational performance than its big-M counterpart.

Let $\widetilde{Tm}^Y_{t,m,t,m,t',m'}$ and $\widetilde{Tm}^Y_{t',m',t,m,t',m'}$ be the disaggregated variables associated to the left disjunction, which is reformulated in Eq. A1. Similarly, the right disjunction is reformulated into Eq. A2.

$$\widetilde{Tm}^Y_{t',m',t,m,t',m'} \geq \widetilde{Tm}^Y_{t,m,t,m,t',m'} + sd_{t,m} \cdot Y_{t,m,t',m'} \quad \forall \, t,t',m' > m \tag{A1}$$

$$\widetilde{Tm}^{\neg Y}_{t,m,t,m,t',m'} \geq \widetilde{Tm}^{\neg Y}_{t',m',t,m,t',m'} + sd_{t',m'} \cdot (1 - Y_{t,m,t',m'}) \quad \forall \, t,t',m' > m \tag{A2}$$

The original problem variables are related to the new disaggregated variables through Eqs. A3-A4.

$$Tm_{t,m} = \widetilde{Tm}^Y_{t,m,t,m,t',m'} + \widetilde{Tm}^{\neg Y}_{t,m,t,m,t',m'} \quad \forall \, t,t',m' > m \tag{A3}$$

$$Tm_{t',m'} = \widetilde{Tm}^Y_{t',m',t,m,t',m'} + \widetilde{Tm}^{\neg Y}_{t',m',t,m,t',m'} \quad \forall \, t,t',m' > m \tag{A4}$$

Finally, we have the bounding constraints that use the knowledge of the relative position of the two maintenance tasks under consideration (Eqs. A5-A8).

$$tm^L_{t,m} \cdot Y_{t,m,t',m'} \leq \widetilde{Tm}^Y_{t,m,t,m,t',m'} \leq \min(tm^U_{t,m}, tm^U_{t',m'} - sd_{t,m}) \cdot Y_{t,m,t',m'} \quad \forall \, t,t',m' > m \tag{A5}$$

$$\max(tm^L_{t,m}, tm^L_{t',m'} + sd_{t',m'}) \cdot (1 - Y_{t,m,t',m'}) \leq \widetilde{Tm}^{\neg Y}_{t,m,t,m,t',m'} \leq$$
$$tm^U_{t,m} \cdot (1 - Y_{t,m,t',m'}) \quad \forall \, t,t',m' > m$$

$$\tag{A6}$$

$$\max(tm^L_{t',m'}, tm^L_{t,m} + sd_{t,m}) \cdot Y_{t,m,t',m'} \leq \widetilde{Tm}^Y_{t',m',t,m,t',m'} \leq tm^U_{t',m'} \cdot Y_{t,m,t',m'} \quad \forall \, t,t',m' > m \tag{A7}$$

$$tm^L_{t',m'} \cdot (1 - Y_{t,m,t',m'}) \leq \widetilde{Tm}^{\neg Y}_{t',m',t,m,t',m'} \leq$$
$$\min(tm^U_{t',m'}, tm^U_{t,m} - sd_{t',m'}) \cdot (1 - Y_{t,m,t',m'}) \quad \forall \, t,t',m' > m$$

$$\tag{A8}$$

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