A comparative study of continuous-time modelings for scheduling of crude oil operations

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Abstract

This work presents a comparative analysis of multiple time-grid representations and formulations for the crude oil scheduling problem. We compare the event-based model, the unit slot model and the multi-operations sequence (MOS) model. Pros and cons of different models are highlighted based on modeling and numerical experiments. We also propose several extensions of the previous models. The MOS model exhibits promising computational performance compared with the other two models, shedding a light on its efficient performance for industrial problems.
Keywords:
Crude oil scheduling; event-based model; unit slot model; multi-operations sequence model

1. INTRODUCTION

The crude oil short-term scheduling problem is the first and critical stage of the crude oil refining process. The problem involves crude oil unloading from marine vessels to storage tanks, transfers and mixings in charging tanks, and a charging schedule for each crude oil mixture to the crude distillation units (CDUs). As there are many varieties of crude in the market, varying widely in properties, processing difficulties and product yields, most refineries procure and process several types of crude, yielding various products and a wide range of profit margins. Optimal crude oil scheduling enables cost reduction by using cheaper types of crude intelligently and minimizing

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crude changeovers, e.g., large economic and operability benefits associated with better crude oil blending scheduling are reported in Kelly and Mann (2003a,b).

The crude oil scheduling problem requires simultaneous selection of crude flows, allocations of vessels to tanks, tanks to CDUs, and calculation of crude compositions. It is closely related to batch process scheduling. Many researchers have developed models and solution techniques for the crude oil scheduling problem, primarily mixed-integer linear or nonlinear programming (MIP or MINLP). A general classification for optimization models of batch processes, based on time representation, mass balances, event representation, and objective function, is presented in Méndez et al. (2006). Depending on whether the events of the schedule can only take place at some predefined time points, or can occur at any point in time during the time horizon, models can be classified into discrete and continuous time formulations. A comprehensive survey of discrete-time models and continuous-time models is summarized in Floudas and Lin (2004).

Models based on discrete time representation. Shah (1996) decomposed the problem into an upstream subproblem and a downstream subproblem in discrete-time MIP models with the objective to minimize the tank heel. Lee et al. (1996) proposed a discrete time model in which the linearity of the bilinear constraints is maintained by replacing bilinear terms with individual component flows, which can lead to composition discrepancy. To circumvent this problem, Wenkai et al. (2002) proposed an iterative MIP-NLP algorithm. Reddy et al. (2004b) also proposed an iterative discrete-time MIP model to overcome the composition discrepancy.

Models based on continuous time representation. Unlike the discrete-time model that requires a large number of slots and increases the size of the problem, the continuous-time representation requires fewer time events or time slots, and reduces the size of the model, especially in terms of fewer binary variables. These models are further categorized as single (global) time-grid models and multiple (unit-specific) time-grids models. Reddy et al. (2004a) established a single time-grid continuous-time model based on the previous discrete-time model in Reddy et al. (2004b). Moro and Pinto (2004) presented an MINLP model of the crude oil inventory management problem that relies on a continuous-time formulation and adopted a discretization procedure for the inventory levels of the tank farm to generate an MIP problem for the original MINLP problem. Jia et al.

(2003) and Jia and Ierapetritou (2004) applied the event-based multiple time-grid continuous-time models to the crude scheduling problem. Later, Hu and Zhu (2007) extended the event-based model of Jia et al. (2003) and Jia and Ierapetritou (2004) to the unit slot model. By eliminating the redundant event points on other units, the multiple time-grids models require fewer event points or unit slots to represent the same schedule, reducing the size of the model, and hence the solution time of the problem.

Recent extensions and developments. A number of extensions of the aforementioned pioneering works have been developed in recent years. Consistent improvements have been made based on the models in Reddy et al. (2004b,a), including enhancing the robustness and efficiency of the iterative MIP algorithm (Li et al. (2007)), heuristic (Adhitya et al. (2007a)) and model-based (Adhitya et al. (2007b)) rescheduling to manage supply chain disruptions, and robustness measures (Karri et al. (2009)) for operation schedules.

Saharidis et al. (2009) proposed to discretize the time horizon based on events such as vessel arrivals and the change in blend composition required by CDUs instead of discretizing by hours. However, in real-life plants such external events do not always coincide with operational activities, especially when there are intermediate events, such as the transfer from storage tanks to charging tanks of an inland refinery. In Saharidis and Ierapetritou (2009), the authors developed a discrete time MIP model to provide not only the optimal schedule of loading and unloading of crude oil, but also the optimal type of mixture preparation. Linearity is maintained by discretizing the percentage of the total quantity stored in the tanks unloaded towards the CDUs. A moving horizon strategy is proposed in Yüzgeē et al. (2010) to maintain an optimal operation by updating control decisions based on the disturbance prediction. Shah and Ierapetritou (2011) presented a comprehensive integrated optimization model based on continuous-time formulation for the scheduling problem of production units and end-product blending problem incorporating quantity, quality, and logistics decisions related to real-life refinery operation. Robertson et al. (2011) presented an integrated approach for refinery production scheduling and unit operation optimization problems. For the CDU model, the multiple linear regression of the individual crude oil flow rates within the crude oil percentage range allowed by the facility is used to derive linear refining cost and revenue functions.

Mouret et al. (2011) applied Lagrangian decomposition to solve each problem separately and efficiently integrate the refinery planning and the crude oil operations scheduling.

Karuppiah et al. (2008) developed an MINLP model that relies on a continuous time representation making use of transfer events and proposed a global optimization algorithm to the crude oil scheduling problem. A new continuous-time formulation based on the representation of a crude-oil schedule by a single sequence of transfer operations (called the single-operation sequencing (SOS) model) was introduced in Mouret et al. (2009a). In Mouret et al. (2009b), constraint programming (CP) was explored to tighten the linear relaxation of the MINLP model for crude oil operations. Recently Mouret et al. (2010) proposed a multi-operations sequence (MOS) model, the details of which are given in Appendix C and extended in section 2.4. Li et al. (2011) made several extensions to the event-based model and applied recently developed global optimization techniques to the model.

Industrial applications and other approaches. Más and Pinto (2003) addressed short-term crude oil scheduling problems in a distribution complex that contains ports, refineries and a pipeline infrastructure. They developed an aggregate-detailed decomposition strategy based on large-scale MIP continuous-time models. Magalhães and Shah (2003) developed a continuous time MIP model minimizing the deviation from the planning targets for a system composed of a terminal, a pipeline, a refinery crude storage area and its crude units. Guyonnet et al. (2008) explored the benefits of integrating the oil uploading and the product distribution problems that have traditionally been solved separately. Lee et al. (2009) also take into account previously addressed sub-problems separately and solve them simultaneously to increase the overall efficiency. The problem is concerned with delivering materials from suppliers to plants, unloading and storing in storage tanks, and mixing the materials before directly feeding into main processes. In Fagundez et al. (2009, 2010) the authors used complementarity constraints to represent scheduling decisions so as to obtain a nonconvex NLP formulation. However, the generality of this technique is limited in that it cannot represent multiple operations in the same period. Zou et al. (2010) introduced an event-tree based modeling method, where events triggered by rules change the states of the system. The primary difficulty is that rules in real-life plants are difficult to collect, maintain and update.

Due to the complexity of the crude scheduling operations, large-scale MIP or MINLP problems cannot be effectively solved. Applying mathematical models to industrial scheduling problems still remains a major challenge. Industrial practitioners and some academic researchers resort to heuristic methods. Bok et al. (2002) presented a hybrid refinery scheduling system that combines the MIP models for crude oil movement between units with an expert system dealing with qualitative issues concerning crude vessel unloading operations. Kelly developed a chronological decomposition heuristic (Kelly (2002)), a smooth-and-dive accelerator (Kelly (2003)) and a flowsheet decomposition heuristic (Kelly and Mann (2004)). Chryssolouris et al. (2005) addressed the crude oil scheduling problem with the arrangement of the temperature cut-points for each distillation unit and the refinery operation modeled as a pooling problem. The proposed approach adopts a random-search method, which allows for controlling search depth, breadth and solution quality, as well as computational effort. Pan et al. (2009b,a) set up an MINLP formulation for the crude oil scheduling problem and proposed some heuristic rules collected form expert experience to linearize bilinear terms and fix some binary variables in the MINLP model, resulting into an MIP model with fewer binary variables. Wu et al. (2007a,b, 2008, 2009, 2010b,a, 2011) put forward a Petri net-based heuristic to check the realizability of a refining schedule by pre-assigning a number of charging tanks to each CDU and incorporated many practical operational constraints.

Uncertainties in the crude oil scheduling have also attracted the attention of researchers. Gupta and Zhang (2009) considered the uncertainty in the crude oil availability, its transfer to storage tanks and charging schedule for each crude oil mixture to crude distillation units. Several recent papers applied chance constrained programming models to the refinery short-term crude oil scheduling problem (Wang and Rong (2009); Cao and Gu (2006); Cao et al. (2009, 2010)). However these models are of limited application in real-life plants.

In this work, a comparative analysis of the state-of-the-art models for the crude oil scheduling problem, i.e., the event-based model, the unit slot formulation, and the recent multi-operations sequence formulation, are presented, implemented, analyzed and modified to further improve the efficiency. Our motivation is to compare the computational performance of different models to obtain a better understanding of the pros and cons of these models. Section 2 describes the problem and the three different formulations and their extensions. Computational results are presented in section 3. In section 4 we draw conclusions and present several remarks.

2. PROBLEM DESCRIPTION AND FORMULATIONS

2.1. Problem description

Four examples of the crude oil scheduling problem of inland refineries, which we will denote as problems Lee1 to Lee4, were reported in Lee et al. (1996). Problem Lee1 is shown as Fig. 1, where the system consists of vessels, storage tanks, charging tanks, and crude distillation units (CDUs). A number of vessels carrying various types of crude oil are scheduled to arrive, with arrival dates of the vessels along with the crude oil types and composition known in advance. In Fig. 1, we have two vessels pa_1 and pa_2, two storage tanks t_1 and t_2, two charging tanks t_3 and t_4, and one CDU cdu_1. During the scheduling horizon, different types of crude oil are first unloaded from vessels into storage tanks, then transferred from storage tanks to charging tanks, and finally charged into the CDUs. In Fig. 1, v_1 and v_2 represent unloading operations, $v_3 - v_6$ are transfer operations, and $v_7 - v_8$ are charging operations.

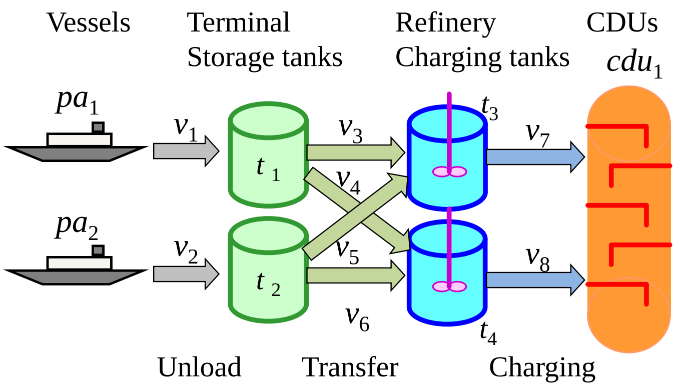

Figure 1: Configuration of the inland refinery in problem Lee1 Lee et al. (1996).

The problem can be summarized as follows:

- *Given*:
  - terminal and refinery infrastructure;
  - scheduled vessel arrival times;

- initial tanks inventory and composition;
- distillation specifications and demands.

- *Determine*:
  - detailed schedule of the vessel unloading, the terminal and refinery tank allocation, and the CDU charging operations;
  - decisions include required operations, timing decisions and transfer of volumes.

- *Objective*:
  - maximize the crude refining profit; and/or
  - minimize operational cost, including vessel unloading and sea waiting cost, inventory cost and CDU switchover cost.

- *Subject to*:
  - operational rules, including: simultaneous inlet and outlet operations on tanks are forbidden, CDUs must be operated continuously throughout the scheduling horizon, etc.
  - material and key component balance.

Two different objectives are reported in the literature, namely the minimization of the operational cost in Lee et al. (1996) and Jia et al. (2003), and the maximization of crude oil refining profit in Mouret et al. (2010). The operational cost in Lee et al. (1996) includes the unloading cost, the sea waiting cost, the inventory cost and the CDU switchover cost. We experiment on the both objectives for a comprehensive comparison.

Important features of the problems in Lee et al. (1996) are listed as below.

- The objective is to minimize the operating cost consisting of the unloading cost for the crude vessels, the cost for vessel waiting in the sea, the inventory cost for storage and charging tanks, and the CDU changeover cost;

- A vessel carries only one crude parcel;

- Unlike charging tanks, storage tanks are dedicated, thus simultaneous inlet and outlet operations of storage tanks are allowed in principle. However, in real-life plant, it is in general forbidden to feed and to withdraw a tank simultaneously due to operational and safety issues.

- Any operation is restricted to transfer crude from at most one original unit to at most one destination unit, except for the transfer operation of which multiple storage tanks can connect to one charging tank at the same time;

- All the flow rate limits are based on operations instead of physical units;

- The bilinear terms are linearized to maintain the linearity of the model.

Test cases for the model in Lee et al. (1996) and the model in Mouret et al. (2010) can be retrieved from `http://newton.cheme.cmu.edu/interfaces/crudeoil/main.html`. While the model in Lee et al. (1996) assumes that the storage tank is guaranteed to be dedicated for a specific type of crude oil without mixing, the Lee3 instance intends to deal with the scenario for which the storage tank is modeled as a blending tank, for instance when there are more types of crude oil imported to than the total number of storage tanks. We find, however, that the refinery topology and parameters of the Lee3 instance tested in subsequent works Jia et al. (2003) (denoted as Lee3b) and Mouret et al. (2010) (denoted as Lee3a) were different from the original paper Lee et al. (1996), as shown in Fig. 2. For the Lee3a example, previous literature obtained feasible solutions because they did not enforce the property specifications constraints of *storage tanks* requiring that concentrations of key components in storage tanks should be within specified range. After we impose the property specifications constraints for storage tanks (c.f. equation (*A*.24d) and (19)), problem Lee3a becomes infeasible of all formulations and is hence excluded from our experiments. For the Lee3b example, it is possible for a parcel to unload into multiple storage tanks, necessitating particular constraints to ensure continuous operations of the parcel and to compute the unloading time of the parcel. Formulation extensions for the Lee3b example are discussed in the subsections of section 2. In section 3 we test the Lee1-Lee4 examples and the Lee3b example to present a benchmark study of different models.

Several models based on different representations have been developed during the last fifteen

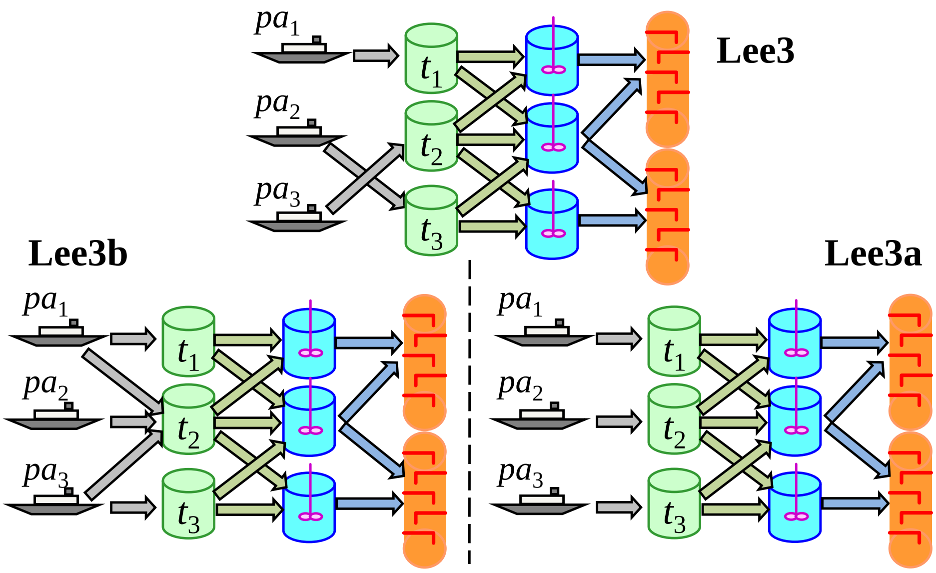

Figure 2: Refinery topology of the Lee3 example. Top: Lee3 in Lee et al. (1996); Bottom left: Lee3b Jia et al. (2003); Bottom right: Lee3a in Mouret et al. (2010)

years. Here we focus on the event-based model of Jia et al. (2003), the unit slot model of Hu and Zhu (2007) and the multi-operations sequence (MOS) model of Mouret et al. (2010), which are presented in detail in Appendices B, A and C, respectively. These models have proved so far to be the most effective formulations reported in the literature. We rephrase key ideas of the three formulations in the succeeding subsections and list the detailed models in the appendices. To make a complete and fair comparison of the three formulations, we add a variety of extensions and enhancements to the models.

- The event-based model
  - extending the model to forbid simultaneous inlet and outlet operations of storage tanks;
  - formulating a parcel unloading into multiple storage tanks in different event points;
  - correcting and improving the objective function of minimizing operational cost.
- The unit slot model
  - modeling the continuous operations of parcel unloading;

  - postulating constraints to formulate the objective function of maximizing crude oil distillation profit.

- The MOS model
  - adding resource-based property specifications constraints in addition to operations-based property specifications constraints;
  - computing the objective function of minimizing operational cost;
  - further reducing the number of slots for the MOS model.

2.2. The event-based model

The event-based formulation in Jia et al. (2003) and Jia and Ierapetritou (2004) traces back to the basic idea of a series of papers by Ierapetritou et al., namely Ierapetritou and Floudas (1998a) for multipurpose batch processes, Ierapetritou and Floudas (1998b) for continuous and semicontinuous processes, and Ierapetritou et al. (1999) for multiple intermediate due dates. The authors proposed to decouple the task events from the unit events instead of displaying the unit subscript explicitly in the triple indexed variables, claiming that it leads to smaller and simpler MIP models which exhibit fewer binary and continuous variables. Nonetheless, Sundaramoorthy and Karimi (2005) noticed that by hiding the unit information behind tasks, the formulation of decoupled double indexed variable does not actually decrease the number of binary variables. This observation is clearly reflected in the crude oil scheduling problem, as an operation or task involves two units instead of only one. In Jia et al. (2003) the start and end time variables are defined on the set of origin unit, destination unit and event point. The authors used the state-task network framework (STN, see Kondili et al. (1993)), although their tasks require a redefinition due to the *unit-to-unit* feature of crude oil operations. The more reasonable understanding here is that *tasks* take place on the unit-to-unit connections, and *states* correspond to different crude oil mixtures in the parcels, tanks, and CDUs, with pipelines connecting parcels, tanks and CDUs the real *units*. Therefore, the crude oil scheduling problem resembles a multistage, multipurpose structure with sharing intermediate storages and product mixing features.

The limitation of the event-based model lies in that all the timing variables are triple indexed as (unit, unit, time event). Accordingly, the timing and sequence constraints are not so intuitive. This is because every timing variable is associated with the *event* or the *unit-to-unit* time grid type, leading to more obscure comprehension and more complex constraints. These key timing variables and sequence constraints are shown in Appendix B. As the event-based formulation shares most of the same parameters, variables and constraints with the unit slot model, we do not present the full event-based model. Besides the timing variables and sequencing constraints listed in Appendix B, the rest part of the model is readily available from Jia et al. (2003) or Appendix A. To properly state the timing and mass balance constraints while maintaining succinctness, initial parameters are assigned to variables with timing subscript '0' whenever necessary. When there are no initial parameters for such variables, the corresponding equations are excluded automatically. For instance, the $Vt(t, n-1)$ term in the mass balance constraint (A.15) equals to $Vt_0(t)$ when $n - 1 = 0$ holds, and equation (2) is defined only on $n - 1 > 0$. Note also that as each vessel carries only one crude parcel, the subscript parcel p is used interchangeably with vessel v. More rigorously, we always use the subscript p and establish timing constraints between them when parcel v carries p, (c.f. equation (A.11)).

2.2.1. Formulating a parcel unloading into different tanks

In Jia et al. (2003), allocation constraints include only equations (A.4a) and (A.8). The model allows simultaneous inlet and outlet operations of storage tanks, which in general is not allowed in real refinery plants, especially when storage tanks are not not dedicated for a specific crude oil type, as the Lee3 example in Lee et al. (1996). We impose additional allocation constraint (1) and timing constraint (2) to amend it. We also require that each parcel unloads to one storage tank at a time as equation (A.6).

$$X(p, i, n) + \sum_{j \in J_i} Y(i, j, n) \leq 1, \forall p \in P, i \in I, n \in N. \tag{1}$$

$$
\begin{aligned}
&Txs(p,i,n) \geq Tyf(i,j,n-1) \\
&\qquad -H\left[1 - Y(i,j,n-1)\right],
\end{aligned}
\qquad \forall i \in I, p \in P_i, j \in J_i, n \in N, \tag{2a}
$$

$$
\begin{aligned}
&Tys(j,l,n) \geq Txf(p,i,n-1) \\
&\qquad -H\left[1 - X(p,i,n-1)\right],
\end{aligned}
\qquad \forall i \in I, p \in P_i, j \in J_i, n \in N. \tag{2b}
$$

The operating rule that parcels arriving later cannot start unloading until previous parcels finish and leave, is stated in Jia et al. (2003) as equation (3a). The condition $p' > p$ represents that parcel p' arrives later than parcel p, i.e., $Tarr(p') > Tarr(p)$. Equation (3a) holds only when a parcel discharges completely into one storage tank at a time. The constraint fails, however, to capture the operating rule when parcel p' and p unload to different storage tanks in the Lee3b instance. That is, the tank-by-tank time ordering of a parcel is not enough. Also the summation of timing variables over all events encounters difficulties when a parcel is allowed to unload into different tanks, or the unloading of a parcel can occupy multiple events. Instead we force the entire time-grid of parcel p' beyond parcel p using equation (3b). Nonetheless, constraint (3b) introduces many big-M inequalities to the model in order to activate triple indexed timing variables before postulating constraints on them. This reveals the typical disadvantage of modeling resource-based constraints on the *unit-to-unit* time grid.

$$
\sum_n Tpst(p',i,n) \geq \sum_n Tpft(p,i,n), \qquad \forall i \in I, p \in P_i, p' \in P_i, p' > p, \tag{3a}
$$

$$
\begin{aligned}
&Txs(p',i',n') + H\left[1 - X(p',i',n')\right] && \forall i,i' \in I, n,n' \in N, \\
&\geq Txf(p,i,n) - H\left[1 - X(p,i,n)\right], && p \in P_i, p' \in P_{i'}, p' > p.
\end{aligned} \tag{3b}
$$

Normally a parcel is required to unload continuously. A simple way to model the continuity of parcel unloading is to impose that the parcel should unload completely at one time as in equation (4), the same constraints adopted in the MOS model. However, its generality is limited. If a parcel is allowed to unload into different storage tanks, as the Lee3 case in Jia et al. (2003), we propose constraint (5) to ensure the continuous unloading operations of the parcel. Specifically, equation (5a) orders the start and end time of loading into different storage tanks in consecutive events. Equation (5b) enforces empty event intervals if there is no such unloading operation. Equation (5c) states that for events $n' > n$, if there are no unloading operations between event n and event n', then the unloading operation in event n' should start right after the unloading operation in event n.

$$\sum_{i \in I_p, n \in N} X(p,i,n) = 1, \qquad \forall p \in P, \tag{4a}$$

$$Bx(p,i,n) \geq Vv_0(p) X(p,i,n), \qquad \forall p \in P, i \in I_p, n \in N. \tag{4b}$$

$$Txs(p,i',n) \geq Txf(p,i,n-1), \qquad \forall (p,i) \in PI, (p,i') \in PI, n \in N, \tag{5a}$$

$$Txf(p,i,n) - Txs(p,i,n) \leq H \times X(p,i,n), \qquad \forall (p,i) \in PI, n \in N, \tag{5b}$$

$$\begin{aligned} &Txs(p,i',n') - H\left[1 - X(p,i',n')\right] \leq Txf(p,i,n) + \\ &\quad H\left[1 - X(p,i,n) + \sum_{i'' \in I_p} \sum_{n < n'' < n'} X(p,i'',n'')\right], \end{aligned} \qquad \forall (p,i) \in PI, (p,i') \in PI, n' > n. \tag{5c}$$

We would also like to point out that the sea waiting cost item in the objective of Jia et al. (2003) is not correctly stated. Instead of $\sum_p \sum_{i \in I_p} \sum_n \left[Tpst(p,i,n) - Tarr(p)\right]$, it should be $\sum_p \left[\sum_{i \in I_p} \sum_n Tpst(p,i,n) - T$
The objective function of operational cost is listed in equation (6).

$$
\begin{aligned}
(EVENT) \quad \text{minimize } COST_{EVENT} = {} & C_{\text{sea}} \sum_{p} \left[\sum_{i \in I_p} \sum_{n} Tpst(p,i,n) - Tarr(p) \right] \\
& + C_{\text{unload}} \sum_{p} \sum_{i \in I_p} \sum_{n} [Tpft(p,i,n) - Tpst(p,i,n)] \\
& + C_{\text{set}} \left[\sum_{j} \sum_{l \in L_j} \sum_{n \in N} Z(j,l,n) - NCDU \right] \\
& + H \sum_{i \in I} C_{\text{inv}}(i) \times \frac{[\sum_{n \in N} Vt(i,n) + Vt_0(i)]}{NE + 1} \\
& + H \sum_{j \in J} C_{\text{inv}}(j) \times \frac{[\sum_{n \in N} Vt(j,n) + Vt_0(j)]}{NE + 1}
\end{aligned} \tag{6}
$$

Also, summing up timing variables over all events to obtain the sea waiting and parcel unloading time can not be extended to the Lee3b instance of a parcel unloading into multiple storage tanks. An alternative and more general expression of the start and end time of parcel unloading is utilized by imposing constraint (7). Equations (7a) and (7b) compute $Tps(p)$, the start time of unloading parcel p; equations (7c) and (7d) calculate $Tpf(p)$, the end time of unloading parcel p. This is achieved by identifying the first and last event point of unloading operations of parcel p in equation (7b) and equation (7d), respectively. Constraint (7e) enforces the sequential unloading of parcels. Note that all the big-M item H in previous equations can be tightened by using hard bounds of variables, for instance H can be replaced by $Tpall(p) - Bx^L/Fx^U$ in equation (7a), where $Tpall(p)$ is the latest time before when parcel p should discharge completely.

$$Tps(p) \leq Txs(p,i,n) + H\left[1 - X(p,i,n)\right], \qquad \forall p \in P, i \in I)p, n \in N, \tag{7a}$$

$$Tps(p) \geq Txs(p,i,n) - H\left[1 - X(p,i,n) + \sum_{(i',n'),i' \in I_p, n'<n} X(p,i',n')\right], \qquad \forall p \in P, i \in I_p, n \in N, \tag{7b}$$

$$Tpf(p) \geq Txf(p,i,n) - H\left[1 - X(p,i,n)\right], \qquad \forall p \in P, i \in I)p, n \in N, \tag{7c}$$

$$Tpf(p) \leq Txf(p,i,n) + H\left[1 - X(p,i,n) + \sum_{(i',n'),i' \in I_p, n'>n} X(p,i',n')\right], \qquad \forall p \in P, i \in I_p, n \in N, \tag{7d}$$

$$Tps(p) \geq Tpf(p-1), \qquad \forall p \in P. \tag{7e}$$

The objective function of minimizing operational cost for the event-based model is summed up in equation (8). In the computational section, equation (6) is employed for the Lee1-Lee4 examples, and equation (8) is adopted for the Lee3b example. In fact, all the extensions dealing with unloading a parcel into different storage tanks, such as constraints (3), (5), and (7), are developed to deal with problem Lee3b. We left the formulation of the objective function of maximizing crude distillation profit in subsection 2.3, as it is basically the same as the unit slot formulation.

$$\begin{aligned}(EVENT) \quad \text{minimize } COST_{EVENT} = {} & C_{\text{sea}} \sum_{p} \left[Tps(p) - Tarr(p)\right] \\ & + C_{\text{unload}} \sum_{p} \left[Tpf(p) - Tps(p)\right] \\ & + C_{\text{set}} \left[\sum_{j} \sum_{l \in L_j} \sum_{n \in N} Z(j,l,n) - NCDU\right] \\ & + H \sum_{i \in I} C_{\text{inv}}(i) \times \frac{\left[\sum_{n \in N} Vt(i,n) + Vt_0(i)\right]}{NE+1} \\ & + H \sum_{j \in J} C_{\text{inv}}(j) \times \frac{\left[\sum_{n \in N} Vt(j,n) + Vt_0(j)\right]}{NE+1}\end{aligned} \tag{8}$$

2.3. *The unit slot model*

The unit slot model Hu and Zhu (2007) is an extension of the event-based model in Jia et al. (2003). Both of the models use the ordering of slots to synchronize different time-grids. The difference is that the unit slot model clearly defines timing variables on the set of (unit, time slot) and aligns timing variables of different units *via the slot index*, being a "real" unit-specific or multiple time-grid model. A recent paper Susarla et al. (2010) employed the same synchronization technique to deal with material transfers between processing units and storage tanks in multipurpose batch plants. They demanded that if a unit receives or delivers a material to a storage device, then the corresponding unit slot on both the unit and the storage device must have *the same index*. Typically, if an operation transfers materials (e.g., crude oil in this study) from unit u_{org} to unit u_{dest} takes place in time slot n, a formulation would enforce that the start and end time of slot n for the two units coincide. The mathematical expressions would be $Tts(u_{\text{org}}, n) = Tts(u_{\text{dest}}, n)$ and $Ttf(u_{\text{org}}, n) = Ttf(u_{\text{dest}}, n)$. In the unit slot model, however, fewer slots are needed to represent the same schedule by not strictly synchronizing the nth time-grids of the two units. An example is presented below to illustrate this feature. The detailed unit slot model is listed in Appendix A.

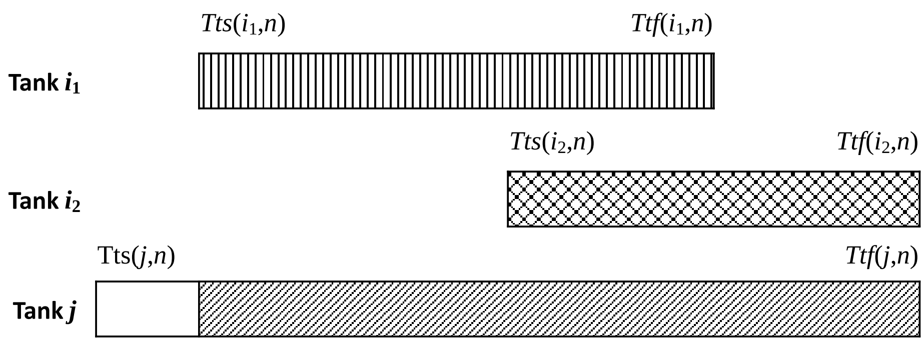

Figure 3: Illustrative example for the unit slot modeling approach.

In Fig. 3, storage tanks i_1 and i_2 transfer crude oil to charging tank j during time slot n. The timing constraints are expressed in equation (9).

$$Tts(j, n) \leq Tts(i_1, n) + H\left[1 - Y(i_1, j, n)\right], \tag{9a}$$

$$Ttf(j, n) \geq Ttf(i_1, n) - H\left[1 - Y(i_1, j, n)\right], \tag{9b}$$

$$Tts(j, n) \leq Tts(i_2, n) + H\left[1 - Y(i_2, j, n)\right], \tag{9c}$$

$$Ttf(j, n) \geq Ttf(i_2, n) - H\left[1 - Y(i_2, j, n)\right]. \tag{9d}$$

Here the binary variable $Y(i_1, j, n) = 1$ indicates that tank j is fed by storage tank i_1 during the time slot n. In the unit slot model, each unit uses its own time grid. For instance, $Tts(j, n)$ represents the start time of the nth time slot of charging tank j, and $Ttf(i_1, n)$ represents the end time of the nth time slot of storage tank i_1. For the same unit j, the time-grid is ordered as $Tts(j, n-1) \leq Tts(j, n) \leq Tts(j, n+1)$, while the ordering of timing variables on different units, for instance $Tts(j, n_j)$ and $Tts(i_1, n_{i_1})$, are not determined. However, if $n_j = n_{i_1} = n$, then the previous timing constraints (9a) and (9b) hold, stating that the nth time slot of tank j ”contains” the nth time slot of storage tank i_1, i.e., the nth time slot of tank j starts earlier than storage tank i_1, and ends later than storage tank i_2. This is utilized to reduce the number of slots, by using flow rate times $(Ttf(i_1, n) - Ttf(i_1, n))$ when calculating the volume of crude oil transfers to tank j from tank i_1 during time slot n. In other words only the period of the nth time slots of storage tank i_1 (or i_2) ”counts”. Therefore, only one slot is enough to model the following two events: tank j receives crude oil from tank i_1 and storage tank i_2 during time slot n.

To model the continuous operations of parcel unloading, one can either enforce constraint (4) for the case of one parcel unloading into one tank, or employ constraint (10) similar to (5) for the general case of Lee3b.

$$Tps(p,n) = Tpf(p,n-1), \qquad \forall p \in P, i \in N, \tag{10a}$$

$$Tpf(p,n) - Tps(p,n) \leq H \times \sum_{i \in I_p} X(p,i,n), \qquad \forall p \in P, n \in N, \tag{10b}$$

$$\begin{aligned} &Tps(p,n') - H\left[1 - X(p,i',n')\right] \leq Tpf(p,n) \\ &+H\left[1 - X(p,i,n) + \sum_{i'' \in I_p} \sum_{n<n''<n'} X(p,i'',n'')\right], \end{aligned} \qquad \begin{aligned} &\forall p \in P, \\ &i, i' \in I_p, n' > n. \end{aligned} \tag{10c}$$

2.3.1. The objective function: maximization of crude oil distillation profit

The cost minimization model in Hu and Zhu (2007) utilizes the key component concentration representation instead of the crude-by-crude representation. Additional crude content based parameters, variables and constraints are added to modify the objective function as a profit maximization form. Basically any parameters, variables and constraints with a key component k subscript are duplicated with a crude type c subscript. Formally, using subscript k is called the total flows and compositions representation. Alternatively, using subscript c is called the individual flows and split fractions representation. These are the two major ways to model the optimization problem with blending. Correspondence between the two representations is established via the parameter $pck(k,c)$, the concentration of key component k of crude type c. According to Karuppiah and Grossmann (2006), variables bounds of using subscript c (individual flows) often differ significantly in magnitude and the optimization is more likely to run into numerical difficulties.

The objective function would be equation (11a) or equation (11b), depending on whether the distillation profit of the same crude type c differs in each CDU l. To make a fair comparison with the MOS model, equation (11a) is employed.

$$(UNIT) \qquad \text{maximize} \quad PROFIT_{UNIT} = \sum_{c} C_{\text{prof}}(c) Bzn(c) \tag{11a}$$

$$(UNIT) \qquad \text{maximize} \quad PROFIT_{UNIT} = \sum_{(l,c) \in LC} C_{\text{prof}}(l,c) Bzuc(l,c) \tag{11b}$$

The following crude composition based mass balance constraints in equations (12) and (13) are added to the unit slot model and the event-based model as well.

$$Vt(t,n) = \sum_{c} Vtc(t,c,n), \qquad \forall (t,c) \in IC, n \in N, \tag{12a}$$

$$By(i,j,n) = \sum_{c} Byc(i,j,c,n), \qquad \forall (i,j) \in IJ, (i,c) \in IC, (j,c) \in JC, n \in N, \tag{12b}$$

$$Bz(j,l,n) = \sum_{c} Bzc(j,l,c,n), \qquad \forall (j,l) \in JL, (j,c) \in JC, (l,c) \in LC, n \in N, \tag{12c}$$

$$Bzuc(l,c) = \sum_{j \in J_l, (j,c) \in JC} \sum_{n} Bzc(j,l,c,n), \qquad \forall l \in L, (l,c) \in LC, \tag{12d}$$

$$Bzn(c) = \sum_{j,(j,c) \in JC} Bzuc(j,c), \qquad \forall c \in C. \tag{12e}$$

$$\begin{aligned} Vtc(i,c,n) = Vtc(i,c,n-1) - \sum_{j \in J_i} Byc(i,j,c,n) \\ + \sum_{p \in P_i, (p,c) \in PC} Bx(p,i,n) fpc(p,c), \end{aligned} \qquad \forall (i,c) \in IC, n \in N, \tag{13a}$$

$$\begin{aligned} Vtc(j,c,n) = Vtc(j,c,n-1) - \sum_{l \in L_j} Bzc(j,l,c,n) \\ + \sum_{i \in I_j, (i,c) \in IC} Byc(i,j,c,n), \end{aligned} \qquad \forall (j,c) \in JC, n \in N. \tag{13b}$$

Next, we impose the crude composition based quality constraints and develop bounding inequalities by linearizing them using lower and upper bounds of crude fractions. The crude fraction from the charging tank should be equal to the crude fraction inside the same tank, as stated in equation (14). Constraint (14a) is replaced by equation (15a), as the crude fractions in storage and charging tanks should be within a certain range. Constraint (14b), which introduces bilinearity to the model, is replaced by the bounding inequalities (15b).

$$Vtc(j,c,n) = Vt(j,n)ftc(j,c,n), \qquad \forall j \in J, c \in C, n \in N, \tag{14a}$$

$$Bzc(j,l,c,n) = Bz(j,l,n)ftc(j,c,n-1), \qquad \forall j \in J, l \in L_j, c \in C, n \in N. \tag{14b}$$

$$\begin{aligned} Vt(t,n)ftc^L(t,c) &\leq Vtc(t,c,n) \\ &\leq Vt(t,n)ftc^U(t,c), \end{aligned} \qquad \forall t \in J, c \in C, n \in N, \tag{15a}$$

$$\begin{aligned} Bz(j,l,n)ftc^L(j,c) &\leq Bzc(j,l,c,n) \\ &\leq Bz(j,l,n)ftc^U(j,c), \end{aligned} \qquad \forall j \in J, l \in L_j, c \in C, n \in N. \tag{15b}$$

The same crude fraction consistency constraints (16) hold for storage tanks in problem Lee3 (from Lee et al. (1996)) and problem Lee3b (from Jia et al. (2003)). They are replaced by their bounding inequalities (17).

$$Vtc(i,c,n) = Vt(i,n)ftc(i,c,n), \qquad \forall i \in I, c \in C, n \in N, \tag{16a}$$

$$Byc(i,j,c,n) = Bz(j,l,n)ftc(i,c,n-1), \qquad \forall i \in I, j \in J_i, c \in C, n \in N. \tag{16b}$$

$$\begin{aligned} Vt(t,n)ftc^L(t,c) &\leq Vtc(t,c,n) \\ &\leq Vt(t,n)ftc^U(t,c), \end{aligned} \qquad \forall t \in I, c \in C, n \in N, \tag{17a}$$

$$\begin{aligned} By(i,j,n)ftc^L(i,c) &\leq Byc(i,j,c,n) \\ &\leq By(i,j,n)ftc^U(i,c), \end{aligned} \qquad \forall i \in I, j \in J_i, c \in C, n \in N. \tag{17b}$$

The key component based representation is associated with the crude type based representation by equations (18a) and (18b).

$$Bzk(j, l, k, n) = \sum_{c} Bzc(j, l, c, n)pck(c, k), \qquad \begin{aligned} &\forall (j, l) \in JL, (j, c) \in JC, \\ &(l, c) \in LC, k \in K, n \in N, \end{aligned} \tag{18a}$$

$$Byk(i, j, k, n) = \sum_{c} Byc(i, j, c, n)pck(c, k), \qquad \begin{aligned} &\forall (i, j) \in IJ, (i, c) \in IC, \\ &(j, c) \in JC, k \in K, n \in N. \end{aligned} \tag{18b}$$

2.4. The multi-operations sequence model

The MOS model can be seen as a dual representation of the unit time-grid based models. That is, while the unit time-grid models locate the start and the end time of operations via allocating operations to different units, the MOS model matches timings of different units to operations by postulating a pre-defined number of priority-slots. Here *operations* denote unload, transfer and charging operations, while vessel parcels, storage and charging tanks, and CDUs are treated as units or *resources*. In the operations based MOS representation, several operations can be assigned to each priority slot as long as they are allowed to overlap with each other. For instance, in Fig. 1 on page 6 operations v_3 and v_8 are allowed to overlap and can be assigned to the same priority-slot. However, as simultaneous inlet and outlet operations on tank t_3 are forbidden, operations v_3 and v_7 cannot overlap and consequently should be assigned to different priority-slots. If two non-overlapping, operations v and v' are assigned to priority-slots i and j, respectively, such that $i < j$, then operation v' must be executed after operation v. For instance, in Fig. 1 operation v_3 assigned to priority-slot 3 should be executed after operation v_7 assigned to priority-slot 2. The detailed model formulation is presented in Appendix C. An implementation of the MOS model is available from Mouret (2010). In the following we present extensions of the MOS model.

In addition to the operations-based property specifications constraints (*C*.6b), resource-based property specifications constraints (19) are also necessary. In this way the concentrations of key components of both resource $r \in R_T$ and the outlet operation $v \in O_r$ of the resource are within specified range. It should be noted that in Mouret et al. (2010) the crude composition of blends in tanks is tracked instead of their properties. The distillation specifications are later enforced by calculating a posteriori the properties of the blend in terms of its composition.

$$\underline{x_{rk}} \cdot L^t_{ir} \leq \sum_{c\in C} x_{ck} L_{irc} \leq \overline{x_{rk}} \cdot L^t_{ir} \qquad i \in T, r \in R_T \tag{19}$$

2.4.1. The objective function: minimization of operational cost

While the MOS model is operations-based, various operational costs are on the basis of resources. Below we present the formulation of different operational costs of the MOS model, which is not included in Mouret et al. (2010).

- The sea waiting and unloading cost

  To compute the sea waiting and unloading cost of vessels or parcels, the start and end unloading time of each parcel of the MOS model is extracted in (20) and (21), respectively. The underlying assumptions are: (1) each vessel carries only one parcel; thus, the set of parcel and vessel can be used interchangeably; and (2) each parcel corresponds to only one operation; in other words it is only allowed to be unloaded into a specific tank. We call it the *one-to-one* correspondence between a parcel and its unloading operation.

$$\begin{aligned} Trs_r &\geq S_{iv} - H\left[1 - Z_{iv} + \sum_{j<i} Z_{jv}\right], r \in R_P, i \in T, v \in W_U, v \in O_r, \\ Trs_r &\leq S_{iv} + H\left[1 - Z_{iv} + \sum_{j<i} Z_{jv}\right], r \in R_P, i \in T, v \in W_U, v \in O_r; \end{aligned} \tag{20}$$

$$\begin{aligned} Trf_r &\geq E_{iv} - H\left[1 - Z_{iv} + \sum_{j>i} Z_{jv}\right], r \in R_P, i \in T, v \in W_U, v \in O_r, \\ Trf_r &\leq E_{iv} + H\left[1 - Z_{iv} + \sum_{j>i} Z_{jv}\right], r \in R_P, i \in T, v \in W_U, v \in O_r. \end{aligned} \tag{21}$$

  If a parcel is allowed to unload into different tanks in different slots, the terms $\sum_{j<i} Z_{jv}$ and $\sum_{j>i} Z_{jv}$ in equation (20) and equation (21) are replaced by $\sum_{j<i}\sum_{v'\in O_r} Z_{jv'}$ and $\sum_{j>i}\sum_{v'\in O_r} Z_{jv'}$ in equation (22) and equation (23), respectively. In addition, constraint (24) is imposed to perform the seamless unloading operations of parcels, i.e., unloading operations of a parcel should be continuous and sequential.

$$\begin{aligned}
Trs_r &\geq S_{iv} - H\left[1 - Z_{iv} + \sum_{j<i}\sum_{v'\in O_r} Z_{jv'}\right], r \in R_P, i \in T, v \in W_U, v \in O_r,\\
Trs_r &\leq S_{iv} + H\left[1 - Z_{iv} + \sum_{j<i}\sum_{v'\in O_r} Z_{jv'}\right], r \in R_P, i \in T, v \in W_U, v \in O_r;
\end{aligned} \tag{22}$$

$$\begin{aligned}
Trf_r &\geq E_{iv} - H\left[1 - Z_{iv} + \sum_{j>i}\sum_{v'\in O_r} Z_{jv'}\right], r \in R_P, i \in T, v \in W_U, v \in O_r,\\
Trf_r &\leq E_{iv} + H\left[1 - Z_{iv} + \sum_{j>i}\sum_{v'\in O_r} Z_{jv'}\right], r \in R_P, i \in T, v \in W_U, v \in O_r.
\end{aligned} \tag{23}$$

$$Trf_r - Trs_r = \sum_{i,v\in O_r} D_{iv}, \forall r \in R_P. \tag{24}$$

The total sea waiting and unloading cost of the MOS model is summed up in (25), where the item $\sum_{v\in W_U, v\in O_r} \underline{S_v}$ is the arrival time of the vessel carries parcel $r \in R_P$.

$$COST_{SEA} = C_{\text{sea}} \sum_{r\in R_P}\left[Trs_r - \sum_{v\in W_U, v\in O_r} \underline{S_v}\right] + C_{\text{unload}} \sum_{r\in R_P} [Trf_r - Trs_r] \tag{25}$$

- The inventory cost

  The inventory cost item of the objective function was first introduced in Lee et al. (1996). However, we decide to exclude the inventory cost from the objective function because of two reasons. Firstly, as observed by Reddy et al. (2004b) and Lee et al. (2009), in real-life refinery plant the inventory cost is determined by the long-term production and procurement decisions. Once purchased, the inventory cost of crude is incurred. The second reason is that according to our computational experience, with the denominator $NS + 1$ of the last two items in equation ($A.1$) increasing, the approximation of the the average inventory tends to decline in the continuous time representation. This drives the model to use more time events or slots to minimize the cost, which is undesirable.

- The CDU switchover cost

  The total number of switchovers of all CDUs in the unit slot model is calculated as equation (26), whereas in the MOS model it is counted as in (27). The notation $|R_U|$ is the number of CDUs of the refinery. Note that equation (27) holds due to the one-to-one correspondence between the charging tank and the CDU, i.e., each CDU would be fed by exactly on charging tank in any slot, and each charging tank feeds to at most one CDU simultaneously.

$$CO_{SUM} = \sum_{l \in L} \sum_{n \in N} CO(l, n), \tag{26}$$

$$CO_{SUM} = \sum_{i \in T} \sum_{v \in W_u} Z_{iv} - |R_U|\,. \tag{27}$$

- The total cost

  In summary, the cost minimization objective function of the MOS model is given below.

$$(MOS) \quad \text{minimize} \quad COST_{MOS} = COST_{SEA} + C_{\text{set}} CO_{SUM} \tag{28}$$

2.4.2. To further reduce the number of slots for the MOS model

In the following we exploit a way that possibly represents the same schedule with fewer slots. As there is only one berth at the terminal, at most one parcel can unload at one time. In problem Lee1, parcel pa_2 should unload after parcel pa_1, thus operations v_1 and v_2 should not overlap. In the MOS model, the two operations should be assigned to different slots. However in the unit slot model, it would not be a problem for operations v_1 and v_2 to occupy the same slot, as long as they do not overlap in the time domain. This observation motivates us to eliminate the non-overlap constraint for operations v_1 and v_2, as the following precedence constraints can be used to ensure non-overlapping without increasing the number of slots. Mathematically, instead of assigning operations v_1 and v_2 to different slots, i.e.,

$$Z_{iv_1} + Z_{iv_2} \leq 1, \forall i \in T, \tag{29}$$

we only need to impose the following constraint:

$$E_{i_1v_1} \leq S_{i_2v_2} + H(2 - Z_{i_1v_1} - Z_{i_2v_2}) \tag{30}$$

where i_2 can be possibly equal to i_1, reducing the number of slots. Here constraint (29) simply says that operations v_1 and v_2 cannot be assigned to the ith priority-slot simultaneously. It depicts the non-overlapping feature between operations v_1 and v_2 by assigning different priority-slots to them. In doing so, operations v_2 is required to take place *before or after* v_1. Yet constraint (30) ensures that operation v_2 should start exactly *after* the end of operation v_1, regardless of the priority-slots they are assigned to. The constraint is based on the operating rule that parcels that arrive later cannot start unloading until previous parcels finish and leave. In this occasion, more information of precedence between unloading operations is utilized in the model. Computational experiments show that it is possible to represent the same or very similar schedule with fewer slots by postulating constraint (30) instead of (29).

3. COMPUTATIONAL RESULTS

The three models in the previous section were implemented and tested on the four examples Lee1-Lee4 from Lee et al. (1996) and the Lee3b example (see Fig. 2) from Jia et al. (2003) using GAMS/CPLEX 23.6 on an Intel Pentium 3.39 GHz PC with 1.00 GB of RAM. Primary results of *minimizing operational costs* and *maximizing refining profit*, denoted as *mincost* and *maxprof*, for all three formulations are presented in Tables 1 and 2, respectively. For the MOS formulation, both the original model and the extended model with the best performance are reported. The detailed computational results are presented in Table D.1 – Table D.6, where the notations are: `Cases`-problem, `Nb`-number of events or slots, `Vars`-number of variables, `DVars`-number of binary variables, `Eqns`-number of equations, `Node`-number of nodes explored, `Iter`-number of iterations, `CPU`-CPU time in seconds, `MIP`-MIP objective value, and `RMIP`-Relaxed LP solution. For consistency in the comparisons we solved all the models with the number of events or slots `Nb` from 1 to 7 and report results for those for which the MIP is feasible. The limit of CPU time is set as 10,000 seconds. Relative and absolute gaps of the MIP models are given below the tables if the solution times exceed the limit.

To quantitatively evaluate the composition discrepancy results from the linearization of the bilinear term (see Wenkai et al. (2002) and Reddy et al. (2004b)), we resort to the following statistics: `NT`-number of transfer operations, `ET`-average composition discrepancy of transfer operations from storage tanks to charging tanks ($\times 10^{-6}$), `NC`-number of charging operations, and `EC`-average composition discrepancy of charging operations from charging tanks to CDUs ($\times 10^{-6}$). Note that `ET` can be nonzero only when storage tanks are not dedicated, e.g., problems Lee3 and Lee3b. Equation (31) is for the event-based model and the unit slot model, and equation (32) is adopted to evaluate the MOS model.

$$NT = \sum_{i} \sum_{j \in J_i} \sum_{n} Y(i, j, n), \tag{31a}$$

$$ET = \frac{\sqrt{\sum_{(i,j,k,n),Y(i,j,n)=1} [Byk(i, j, k, n)/By(i, j, n) - ftk(i, k, n-1)]^2}}{NT}, \tag{31b}$$

$$NC = \sum_{l} \sum_{j \in J_l} \sum_{n} Z(j, l, n), \tag{31c}$$

$$EC = \frac{\sqrt{\sum_{(j,l,k,n),Z(j,l,n)=1} [Bzk(j, l, k, n)/Bz(j, l, n) - ftk(j, k, n-1)]^2}}{NC}. \tag{31d}$$

$$NT = \sum_{v \in W_T} \sum_{i} Z_{iv}, \tag{32a}$$

$$ET = \frac{\sqrt{\sum_{v \in W_T, Z_{iv}=1, v \in O_r} \left(V_{ivk}/V_{iv}^t - L_{irk}/L_{ir}^t\right)^2}}{NT}, \tag{32b}$$

$$NC = \sum_{v \in W_D} \sum_{i} Z_{iv}, \tag{32c}$$

$$EC = \frac{\sqrt{\sum_{v \in W_D, Z_{iv}=1, v \in O_r} \left(V_{ivk}/V_{iv}^t - L_{irk}/L_{ir}^t\right)^2}}{NT}. \tag{32d}$$

We also adopt the same MIP-NLP procedure as in Mouret et al. (2009a) to fix the discrete decisions of the MIP solution and solve the corresponding reduced NLP problem using GAMS/CONOPT 3.14. The rest of the notations are: `rS`-model status of reduced NLP (F local optimum, I locally infeasible), `rNLP`-objective value of reduced NLP (possibly infeasible), `rT`-CPU time of reduced NLP, `NZ`-number of nonzero elements of MIP, `Nlin`-number of nonlinear items of reduced NLP, `rT`-CPU time of reduced NLP, `Gap(%)`-relative gap of MIP objective and reduced NLP solution (equation (33)).

$$Gap = \frac{|MIP - rNLP|}{rNLP} \times 100\%. \tag{33}$$

The MOS formulation in Mouret et al. (2010) adopts the unloading and distillation cardinality constraints (C.2) to tighten the model. Constraint (C.2a) requires that each vessel (parcel) unloads completely at one time, which is not true when a parcel is allowed to unload into different tanks. We have already discussed the issue in section 2.4 and presented a variety of extensions of the MOS model to deal with it. Constraint (C.2b) enforces bounds on the number of distillations, of which the bounds are given manually. In order to make a fair comparisons with the other two models, we test the MOS model *with cardinality constraints* and *without cardinality constraints* (C.2b), denoted as *card* / *nocard* respectively. For the assignment of unloading operations to priority-slots, we employ the non-overlapping assignment constraint, e.g., equation (29) in the *without precedence constraints* setting, denoted as *prec*, and the precedence enforcement constraint, e.g., equation (30) in the *with precedence constraints* setting, denoted as *assign*.

The following observations are made from the tables of computational results.

The objective function. The maximization of profit is not necessarily a good objective for the crude oil scheduling problem. First of all, the exclusive maximization of profit tends to "ignore" other factors, such as undesirably postponing the unloading time of vessels once the amount of crude for the current scheduling period is satisfied, or allowing frequent CDU switchovers, which

Table 1: Summary of the results of three formulations: minimizing operational costs

		event-based							unit slot							MOS original							MOS best						
	Nb	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap
Lee1	4								0.39	49.5	37	I	-	0	-														
	5	4.53	48.6	22	F	49	0.1	0.84	2.06	44.5	37	F	**44.5**	0	0	0.64	**44.5**	42	F	**44.5**	0.03	0	0.64	**44.5**	42	F	**44.5**	0.03	0
	6	34.2	44.5	22	F	**44.5**	0.1	0	7.63	44.5	37	F	44.5	0	0	0.95	47.7	42	F	47.7	0.06	0							
	7	685.97	44.5	22	F	44.5	0.1	0	30.3	44.5	37	F	44.5	0	0	1.78	47.7	42	F	47.7	0.06	0							
Lee2	4								2.78	70	53	F	70	0	0														
	5	974.58	69	40	F	69.27	0.2	0.39	40.56	69	53	F	69	0.1	0	1.52	72.7	63	F	72.7	0.22	0							
	6	10000	66	40	I	-	0.3	-	2343.38	66	53	F	66	0.1	0	4.03	**65**	63	F	**65**	0.52	0	4.03	**65**	63	F	**65**	0.52	0
	7	10000.1	67.6	40	F	*67.83*	0.3	0.41	4942.09	65	53	F	**65**	0.1	0	8.94	65	63	F	65	0.99	0							
Lee3	3								0.34	210	110	I	-	0	-	0.09	**210**	210	F	**210**	0.05	0	0.09	**210**	210	F	**210**	0.05	0
	4	87.27	210	40	F	**210**	0.1	0	2.02	210	110	I	-	0.1	-	0.25	210	210	I	-	0.06	-							
	5	1871.86	210	40	I	-	0.1	-	10.27	210	110	I	-	0	-	0.98	210	210	F	210	0.27	0							
	6	10170	210	40	F	210	0.1	0	87.86	210	110	I	-	0.1	-	2.72	210	210	I	-	0.36	-							
	7	10000.2	210	40	F	210	0.2	0	538.81	210	110	I	-	0	-	6.3	210	210	F	210	0.61	0							
Lee4	4	60.17	243	0	F	243	0.1	0	15.48	183	93	F	**183**	0.1	0														
	5	9581.84	183	0	I	-	0.2	-	125.95	183	93	F	183	0.1	0	0.55	**183**	183	F	183.25	0.28	0.14							
	6	10000.1	183	0	F	**183**	0.2	0	2932.7	183	93	I	-	0.1	-	0.78	183	183	F	**183**	1.17	0	0.78	**183**	183	F	**183**	1.17	0
	7	10000.1	183	0	I	-	0.3	-	8768.34	183	93	I	-	0.1	-	6.33	188.5	183	F	188.5	0.36	0							
Lee3b	3								0.84	210	92	I	-	0	-								0.42	**210**	110	F	**210**	0.11	0
	4	39.27	210	40	F	**210**	0.1	0	58.7	210	92	I	-	0.1	-	0.27	**210**	210	I	-	0.11	-							
	5	2586.58	210	40	F	210	0.1	0	1738.2	210	92	I	-	0.1	-	1.06	210	210	I	-	0.28	-							
	6	10000.1	210	40	I	-	0.2	-	10000	210	92	I	-	0.1	-	2.55	210	210	I	-	0.53	-							
	7	10000.1	210	40	I	-	0.3	-	10000.1	210	92	I	-	0.1	-	5.14	210	210	I	-	1.13	-							

`Nb`-number of events or slots,
`CPU`-CPU time in seconds,
`MIP`-MIP objective value,
`RMIP`-Relaxed LP solution,
`rS`-model status of reduced NLP (F local optimum, I locally infeasible),
`rNLP`-objective value of reduced NLP (possibly infeasible),
`rT`-CPU time of reduced NLP,
`rT`-CPU time of reduced NLP, and
`Gap`(%)-relative gap of MIP objective and reduced NLP solution (equation (33)).

Table 2: Summary of the results of three formulations: maximizing refining profit

		event-based							unit slot							MOS original							MOS best						
	Nb	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap	CPU	MIP	RMIP	rS	rNLP	rT	Gap
Lee1	5	3.06	79.75	80	I	-	0.1	-	1.89	79.9	80	I	-	0.1	-	0.42	**79.75**	80	F	**79.75**	0.03	0	0.42	**79.75**	80	F	**79.75**	0.03	0
	6	6.63	79.9	80	I	-	0.1	-	5	79.9	80	F	79.43	0.1	0.59	0.98	79.35	80	F	79.26	0.06	0.12							
	7	33.81	79.9	80	I	-	0.2	-	8.95	79.9	80	F	77.44	0.1	3.18	1.92	79.35	80	F	79.26	0.09	0.12							
Lee2	4								9.28	90	103	F	90	0.3	0														
	5	199.48	97.67	103	I	-	0.7	-	53.41	99.75	103	F	97.6	0.4	2.2	1.53	91.6	103	F	91.6	0.16	0							
	6	1497	101.99	103	F	100.7	0.6	1.3	277.94	102.7	103	F	101.6	0.7	1.11	3.8	**95.26**	103	F	**94.85**	0.34	0.44							
	7	1463	102.76	103	I	-	2.2	-	4883.42	102.8	103	I	-	2.2	-	9.72	95.26	103	F	94.85	0.44	0.44	83.78	**102.7**	103	F	**102.11**	0.81	0.57
Lee3	3								0.72	82.5	100	F	78.01	0.1	5.76	0.09	82.4	100	F	82.4	0.05	0							
	4	7	88.33	100	F	82.79	0.2	6.69	5.81	88.33	100	F	81.91	0.2	7.84	0.94	82.5	100	F	**82.5**	0.09	0							
	5	30.17	90.08	100	I	-	0.4	-	23.25	90.17	100	F	87.16	0.3	3.45	5.28	**84.5**	100	F	81.41	0.16	3.79	22.44	90.17	100	F	**86.26**	0.45	4.53
	6	112.02	91.72	100	F	*89.38*	0.8	2.62	173.23	91.73	100	F	85.39	0.8	7.42	13.95	84.5	100	F	81.99	0.38	3.06							
	7	2364.7	92.1	100	I	-	1	-	2641.56	92.1	100	F	**87.46**	1.3	5.31	22.06	84.5	100	F	82	0.42	3.05							
Lee4	4	17.91	132.13	132.6	F	132.1	0.6	0	44.44	132.4	132.6	I	-	0.3	-														
	5	323.75	132.55	132.6	F	**132.6**	0.9	0	43.63	132.6	132.6	I	-	0.7	-	0.41	**132.22**	132.58	F	131.57	0.48	0.5							
	6	394.75	132.57	132.6	I	-	1.9	-	516.97	132.6	132.6	I	-	0.6	-	2.5	132.22	132.59	F	**132.22**	0.81	0	2.42	132.57	132.59	F	**132.56**	1.11	0
	7	10000	132.57	132.6	I	-	1.3	-	874.39	132.6	132.6	I	-	1.3	-	4.88	132.06	132.59	F	131.91	1.34	0.12							
Lee3b	3								1.01	82.87	100	I	-	0.1	-														
	4	13.92	84.97	100	I	-	0.4	-	13.61	91.06	100	I	-	0.1	-	0.38	82.7	100	I	-	0.27	-	31.28	92.73	100	F	**87.16**	0.34	6.39
	5	162.13	86.44	100	I	-	0.3	-	70.42	92.23	100	I	-	0.4	-	10.42	**85.75**	100	I	-	0.27	-							
	6	228.95	89.05	100	F	87.09	0.8	2.25	1519.78	93.95	100	I	-	0.6	-	53.67	85.62	100	I	-	0.45	-							
	7	5651.6	89.41	100	I	-	1.6	-	10001.4	94.88	100	F	**91.59**	0.9	3.59	213.5	84.05	100	I	-	0.83	-							

`Nb`-number of events or slots,
`CPU`-CPU time in seconds,
`MIP`-MIP objective value,
`RMIP`-Relaxed LP solution,
`rS`-model status of reduced NLP (F local optimum, I locally infeasible),
`rNLP`-objective value of reduced NLP (possibly infeasible),
`rT`-CPU time of reduced NLP,
`rT`-CPU time of reduced NLP, and
`Gap`(%)-relative gap of MIP objective and reduced NLP solution (equation (33)).

bring difficulties of scheduling the following periods. Secondly, scheduling decisions focus on operational activities, with the profit objective passed down from planning decisions. In the four examples reported by Lee et al. (1996), the demand of crude oil mixture for each charging tank is fixed, with tight restriction of key components, leaving limited space for the maximization of crude distillation profit. In fact, as the profit of each crude is given (parameter $C_{\text{prof}}(c)$ of the event-based model and the unit slot model, and parameter G_c of the MOS model), the total refining profit of the refinery plant is fixed in the long run. More exactly, it is determined by the amount of crude purchased. Lastly, when maximizing refining profit, with the number of event points or time slots increasing, the objective value seems keep increasing for most of the problems. However, this is because the objective does not take into consideration of the switchover cost, and consequently the system would switch to crude mixture with higher profitability whenever possible. As the refinery runs the scheduling in a rolling horizon way, myopically refining too much highly profitable oil in the current period means deteriorating the quality of the crude for later periods, potentially causing sub-optimality or even infeasibility. Note, however, that the underlying assumption of the preceding statements is that the amount of procured crude is *fixed in the long-term decisions*. If, on the contrary, crude is purchased *from the spot market*, then the long run refinery profit is determined by future crude purchases, which are determined by current crude usages. In this way, the optimal choice would be to process as soon as possible crude with high profit margin.

Computational performance. Generally speaking, the unit slot model requires fewer slots to be feasible, but obtaining the optimal solution requires more slots. The number of nonzero elements and the number of nonlinear equations of the MOS model are much higher than the other two models. Nevertheless, as the number of slots increases, the CPU times of both the event-based and the unit slot model increase significantly. Even though optimal solutions are obtained within CPU time limit (10,000 seconds), it takes much longer time for the event-based model and the unit slot model to close the optimality gap. The computational time of the MOS model, on the other hand, only increases moderately. On solution quality, the original MOS model in Mouret et al. (2010) obtains relatively inferior MIP objective values in some problems. Nevertheless, it gives comparatively low concentration discrepancy. For the reduced NLP problem after fixing binary variables to the MIP solution, many problems of the event-based model and the unit slot model become locally infeasible. In contrast, most problems of the MOS model are feasible, i.e., the schedules obtained by the MOS model are more realistic and implementable. Moreover, by choosing different options of the extended MOS model, equivalent or better solutions are obtained with far less CPU time, as shown in the "*MOS best*" block of Tables 1 and 2.

Variants and discussions of the MOS model. For the sake of reducing the length of the paper, we placed the computational results of variants of the MOS model in the supplementary material.

- The model *with cardinality constraints* (*C*.2) reduces the solution time significantly, especially for the objective of profit maximization. Furthermore, it does not deteriorate the solution of cost minimization as the manually postulated bounds on the number of unloading operations and distillations are quite reasonable for the refinery plant. For maximizing refining profit, as discussed previously, the model tends to ignore the switchover cost and report artificially high profit. The cardinality constraints used in Mouret et al. (2010) can somewhat alleviate such an effect.

- Although the model *with precedence constraints* (see section 2.4.2) requires fewer slots to obtain a feasible solution in some examples, the *with precedence constraints* and *without precedence constraints* settings do not dominate each other in terms of CPU time and for obtaining superior solutions.

- The reduced NLP problems after fixing binary variables to the MIP solution of problem Lee3b tend to be infeasible if the resource-based property specification constraint (19) is not enforced. If constraint (19) is postulated, then the employment of precedence constraints tend to give infeasible reduced NLP problems, or even infeasible MIP problems of problem Lee3b.

- From the modeling perspective, the MOS model is operations-based, therefore it is more difficult to express resource-based constraints and objectives. For instance, it is not intuitive to express the continuous operations of CDUs, CDU switchovers, the unloading and sea-waiting cost of parcels, etc. This might be further complicated by the fact that the correspondence between the origin and the destination units for an operation is not one to one but many to many. Also, in the many to many correspondence cases, the capacity and flow rate limits, as well as the timing synchronization constraints on shared resources, are much more complicated. Therefore, it would be interesting to further exploit the MOS model to deal with the aforementioned issues.

4. CONCLUSIONS AND FUTURE WORK

Three state-of-the-art formulations of the crude oil scheduling models, namely the event-based formulation, the unit slot formulation, and the recent multi-operations sequence (MOS) formulation, have been reviewed, analyzed, modified and implemented to further improve the efficiency. The experimental results on examples Lee1-Lee4 from Lee et al. (1996) and the Lee3b example (see Fig. 2) from Jia et al. (2003) have shown that the MOS model (original and extended) is the fastest, although the original MOS model was observed to fail to find the best solutions of some problems.

For future work, as the MOS shows promising computational performance, extensions are needed to further incorporate more complex practical logistics constraints readily applicable to industrial scheduling problems, for instance the many to many correspondence between the origin and the destination units of an operation.

A. The unit-specific slot continuous time model

In this section, the unit slot model of the crude oil scheduling problem from Hu and Zhu (2007) is presented.

A.1. Nomenclature of the unit slot model

Indices and sets	
$v \in V$	Vessels
$p \in P$	Parcels
P_v	all parcels of vessel v
FP_v	the firstly unloaded parcel of vessel v
LP_v	the lastly unloaded parcel of vessel v
$i \in I$	Storage tanks
$j \in J$	Charging tanks
$t \in I \cup J$	Storage and charging tanks
$l \in L, u \in U$	CDUs
$n \in N$	Slots (Event points)
$k \in K$	Key components
$c \in C$	Crude oil types
$b \in B$	Unloading berths
I_p	set of storage tanks which can unload crude from parcel p
P_i	set of parcels which can feed crude oil to storage tank i
I_j	set of storage tanks which can transfer crude to charging tank j
J_i	set of charging tanks which can be fed by storage tank i
J_l	set of charging tanks which can charge crude to CDU l
L_j	set of CDUs which can be fed by charging tank j
$PI(p, i)$	denotes if parcel p can unload crude oil into storage tank i
$IJ(i, j)$	denotes if storage tank i can transfer crude oil to charging tank j
$JL(j, l)$	denotes if charging tank j can charge the crude-oil mix to CDU l
$PC(p, c)$	denotes if parcel p is composed of crude type c
$IC(i, c)$	denotes if crude type c can be stored in storage tank i
$JC(j, c)$	denotes if crude type c can be stored in charging tank j
$LC(l, c)$	denotes if crude type c can be distilled in CDU l
Parameters	
H	scheduling time horizon
TBS	brine settling time for storage tanks
NE/NS	number of events/slots utilized
NST	total number of storage tanks
NCH	total number of charging tanks
$NCDU$	number of CDUs
C_{set}	unit changeover cost
C_{sea}	unit sea waiting cost
C_{unload}	unit unloading cost

$C_{\text{inv}}(i)$	unit inventory cost of storage tank i
$C_{\text{inv}}(j)$	unit inventory cost of charging tank j
$C_{\text{prof}}(c)$	distillation profit of crude type c
$C_{\text{prof}}(l,c)$	distillation profit of crude type c in CDU l
$Tvarr(v)$	arrival time of vessel v
$Tparr(p)$	arrival time of parcel p
$Tvall(v)$	the latest time before when vessel v should leave
$Tpall(p)$	the latest time before when parcel p should discharge completely
$Vv_0(p)$	initial volume of parcel p
$Vt_0(t)$	initial volume of crude oil in tank t
$Vt^L/Vt^U(t)$	minimum/maximum volume of crude oil in tank t
$DM(j)$	demand of crude oil mix for charging tank j
$DM(l)$	demand of crude oil mix for CDU l
Bx^L/Bx^U	minimum/maximum volume of crude oil being unloaded
By^L/By^U	minimum/maximum volume of crude oil being transferred
Bz^L/Bz^U	minimum/maximum volume of crude oil being charged
Fx^L/Fx^U	minimum/maximum volume unloading rate
Fy^L/Fy^U	minimum/maximum volume transfer rate
Fz^L/Fz^U	minimum/maximum volume charging rate
$Vvk_0(p,k)$	initial volume of key component k in parcel p
$fpk(p,k)$	initial concentration of key component k in parcel p
$ftk_0(k,t)$	initial concentration of key component k in tank t
$Vtk_0(k,t)$	initial volume of key component k in tank t
$ftk^L/ftk^U(k,t)$	minimum/maximum concentration of key component k in tank t
$pck(k,c)$	concentration of component k of crude type c
$Vvc_0(p,c)$	initial volume of crude type c in parcel p
$fpc(p,c)$	initial concentration of crude type c in parcel p
$ftc_0(c,t)$	initial concentration of crude type c in tank t
$Vtc_0(c,t)$	initial volume of crude type c in tank t
$ftc^L/ftc^U(c,t)$	minimum/maximum concentration of crude type c in tank t
Variables	
Binary variables	
$X(p,i,n)$	=1 when parcel p is loading tank i in slot n
$Y(i,j,n)$	=1 when tank i is feeding tank j in slot n
$Z(j,l,n)$	=1 when tank j is charging CDU l in slot n
$A(v,b)$	= 1 when vessel v uses berth b to unload
Continuous variables	
$CO(l,n)$	0-1 continuous variables, =1 when there is switchover of CDU l in slot n
$XT(i,n)$	0-1 continuous variables, =1 when storage tank i is fed by parcels in slot n
$DC(v)$	sea waiting cost of vessel v
$Tvs/Tvf(v)$	start/end time of unloading vessel v

$Tps/Tpf(p,n)$	start/end time of parcel p in slot n
$Tts/Ttf(i,n)$	start/end time of storage tank i in slot n
$Tts/Ttf(j,n)$	start/end time of charging tank j in slot n
$Tus/Tuf(l,n)$	start/end time of CDU l in slot n
$Bx(p,i,n)$	volume of crude oil that parcel p unloads into storage tank i in slot n
$By(i,j,n)$	volume of crude oil that storage tank i transfers to charging tank j in slot n
$Bz(j,l,n)$	volume of crude oil that charging tank j charges into CDU l in slot n
$Vv(p,n)$	volume of crude parcel p at the end of slot n
$Vt(t,n)$	volume of crude oil in tank t at the end of slot n
$Vtk(t,k,n)$	volume of key component k in tank t at the end of slot n
$Byk(i,j,k,n)$	volume of key component k transfers from storage tank i to charging tank j in slot n
$Bzk(j,l,k,n)$	volume of key component k charges from charging tank j to CDU l in slot n
$ftk(t,k,n)$	concentration of key component k in tank t at the end of slot n
$Vtc(t,c,n)$	volume of crude oil c in tank t at the end of slot n
$Byc(i,j,c,n)$	volume of crude c transfers from storage tank i to charging tank j in slot n
$Bzc(j,l,c,n)$	volume of crude c charges from charging tank j to CDU l in slot n
$ftc(t,c,n)$	concentration of crude type c in tank t at the end of slot n
$Bzuc(l,c)$	volume of crude c distilled by CDU l during the scheduling horizon
$Bzn(c)$	volume of crude c distilled during the scheduling horizon

A.2. The unit slot model

- Objective function

  The objective function is to minimize the operational cost defined by ($A.1$), as adopted in Lee et al. (1996) and Jia et al. (2003). The alternative objective is to maximize profit defined by (A.2), as in Reddy et al. (2004b,a).

$$
\begin{aligned}
(UNIT) \quad \text{minimize} \quad COST_{UNIT} = {} & C_{\text{sea}} \sum_{v\in V} [Tvs(v) - Tvarr(v)] \\
& + C_{\text{unload}} \sum_{v\in V} [Tvf(v) - Tvs(v)] \\
& + C_{\text{set}} \left[\sum_{l\in L} \sum_{n\in N} CO(l,n) \right] \\
& + H \sum_{i\in I} C_{\text{inv}}(i) \times \frac{[\sum_{n\in N} Vt(i,n) + Vt_0(i)]}{NS + 1} \\
& + H \sum_{j\in J} C_{\text{inv}}(j) \times \frac{[\sum_{n\in N} Vt(j,n) + Vt_0(j)]}{NS + 1}
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
(UNIT) \quad \text{maximize} \quad PROFIT_{UNIT} = {} & \sum_{l\in L} \sum_{i\in I_l} \sum_{c\in C_i} \sum_{n\in N} C_{\text{prof}}(c) Bzc(i,l,c,n) \\
& - C_{\text{set}} \sum_{l\in L} \sum_{n\in N} CO(l,n) - \sum_{v\in V} DC(v)
\end{aligned} \tag{A.2}
$$

$$
DC(v) \geq C_{\text{sea}} \times [Tvf(v) - Tvall(v)], \qquad \forall v \in V. \tag{A.3}
$$

- Constraints of task assignment and operational rules
  - Each CDU would be fed by at most or exactly one charging tank in any slot.

    The event-based model in Jia et al. (2003) takes constraint (A.4a). In Hu and Zhu (2007), equation (A.4b) is adopted, because it leads to a more direct and general way to process the constraints of CDU continuity. It also readily facilitates calculating CDU switchovers when CDUs are allowed to be fed by multiple charging tanks simultaneously or charging tanks are allowed to charge into multiple CDUs at the same time. Was equation (A.4a) adopted in the unit slot model, additional constraint (A.5) is added to enforce empty slots of CDUs without operations.

$$
\sum_{j\in J_l} Z(j,l,n) \leq 1, \qquad \forall l \in L, n \in N, \tag{A.4a}
$$

$$
\sum_{j\in J_l} Z(j,l,n) = 1, \qquad \forall l \in L, n \in N. \tag{A.4b}
$$

$$
Tuf(l,n) - Tus(l,n) \leq H \sum_{j\in J_l} Z(j,l,n), \qquad \forall l \in L, n \in N. \tag{A.5}
$$

– Each parcel can unload to one storage tank at a time.

$$\sum_{i\in I_p} X(p,i,n) \leq 1, \qquad \forall p \in P, n \in N. \qquad (A.6)$$

– A storage tank can transfer crude to at most one charging tank at the same time.

$$\sum_{j\in J_i} Y(i,j,n) \leq 1, \qquad \forall i \in I, n \in N. \qquad (A.7)$$

– Loading and unloading operations of the same charging tank should not overlap.

$$Y(i,j,n) + \sum_{l\in L_j} Z(j,l,n) \leq 1, \qquad \forall j \in J, i \in I_j, n \in N. \qquad (A.8)$$

Note that the above constraint not only forbids simultaneous inlet and outlet of charging tanks, but also guarantees that each charging tank feeds to at most one CDU.

- Timing constraints

  – Timing constraints for each unit

$$Tps(p,n) \geq Tpf(p,n-1) \geq Tps(p,n-1), \qquad \forall p \in P, n \in N \qquad (A.9a)$$
$$Tts(i,n) \geq Ttf(i,n-1) \geq Tts(i,n-1), \qquad \forall i \in I, n \in N \qquad (A.9b)$$
$$Tts(j,n) \geq Ttf(j,n-1) \geq Tts(j,n-1), \qquad \forall j \in J, n \in N \qquad (A.9c)$$
$$\begin{aligned} Tus(l,n) &= Tuf(l,n-1) \\ &\geq Tus(l,n-1) + Bz^L/Fz^U, \end{aligned} \qquad \forall l \in L, n \in N. \qquad (A.9d)$$

  – Timing constraints between two units

$$\begin{aligned} Tts(i,n) &\leq Tps(p,n) \\ &+[H - Tparr(p)][1 - X(p,i,n)], \end{aligned} \qquad \forall p \in P, i \in I_p, n \in N, \qquad (A.10a)$$
$$\begin{aligned} Ttf(i,n) &\geq Tpf(p,n) \\ &-Tpall(p)[1 - X(p,i,n)], \end{aligned} \qquad \forall p \in P, i \in I_p, n \in N; \qquad (A.10b)$$
$$Tts(i,n) \geq Tts(j,n) - H[1 - Y(i,j,n)], \qquad \forall j \in J, i \in I_j, n \in N, \qquad (A.10c)$$
$$Ttf(i,n) \leq Ttf(j,n) + H[1 - Y(i,j,n)], \qquad \forall j \in J, i \in I_j, n \in N; \qquad (A.10d)$$
$$Tts(j,n) \leq Tus(l,n) + H[1 - Z(j,l,n)], \qquad \forall l \in L, j \in J_l, n \in N, \qquad (A.10e)$$
$$Ttf(j,n) \geq Tuf(l,n) - H[1 - Z(j,l,n)], \qquad \forall l \in L, j \in J_l, n \in N. \qquad (A.10f)$$

– Timing constraints for vessel unloading.

$$Tps(p,1) \geq Tpf(p-1, NE), \qquad \forall v \in V, p, p-1 \in P_v, \qquad (A.11a)$$
$$Tvs(v) = Tps(p,1), \qquad \forall v \in V, p \in FP_v, \qquad (A.11b)$$
$$Tvf(v) = Tpf(p, NS), \qquad \forall v \in V, p \in LP_v, \qquad (A.11c)$$
$$Tvs(v) \geq Tvf(v-1), \qquad \forall v, v-1 \in V. \qquad (A.11d)$$

– CDU should process continuously.

$$\sum_{n \in N} [Tuf(l,n) - Tus(l,n)] = H, \qquad \forall l \in L. \qquad (A.12)$$

– Brine settling time constraints.
Reddy et al. (2004b,a) mentioned that each tank needs some time for brine settling and removal after receiving crude. The settling time constraint is modeled as below.

$$XT(i,n) \geq X(p,i,n), \qquad \forall i \in I, p \in P_i, n \in N, \qquad (A.13a)$$
$$XT(i,n) \leq \sum_{p \in P_i} X(p,i,n), \qquad \forall i \in I, n \in N, \qquad (A.13b)$$
$$\begin{aligned} Tts(i,n) &\geq Ttf(i,n-1) \\ +TBS \cdot &[XT(i,n-1) - XT(i,n)], \end{aligned} \qquad \forall i \in I, n \in N. \qquad (A.13c)$$

- Mass balance constraints

  – Each parcel should unload its crude completely.

$$Vv_0(p) = \sum_{i \in I_p} \sum_{n \in N} Bx(p,i,n), \qquad \forall p \in P, \qquad (A.14a)$$
$$Vv(p,n) = Vv(p,n-1) - \sum_{i \in I_p} Bx(p,i,n), \qquad \forall p \in P, n \in N. \qquad (A.14b)$$

  – Mass balance constraints for storage tanks.

$$Vt(i,n) = Vt(i,n-1) + \sum_{p \in P_i} Bx(p,i,n) - \sum_{j \in J_i} By(i,j,n), \qquad \forall i \in I, n \in N. \qquad (A.15)$$

  – Mass balance constraints for charging tanks.

$$Vt(j,n) = Vt(j,n-1) + \sum_{i \in I_j} By(i,j,n) - \sum_{l \in L_j} Bz(j,l,n), \qquad \forall j \in J, n \in N. \qquad (A.16)$$

  – The amount the crude fed to CDU should satisfy the demand requirement. Equation

(A.17a) is based on charging tanks, while equation (A.17b) is based on CDUs.

$$\sum_{l \in L_j} \sum_{n \in N} Bz(j,l,n) = DM(j), \qquad \forall j \in J, \qquad (A.17a)$$

$$\sum_{j \in J_l} \sum_{n \in N} Bz(j,l,n) = DM(l), \qquad \forall l \in L. \qquad (A.17b)$$

- Key component balance constraints
  - Key component balance constraints for storage tanks

$$\begin{aligned} Vtk(i,k,n) &= Vtk(i,k,n-1) \\ &+ \sum_{p \in P_i} Bx(p,i,n) fpk(p,k) - \sum_{j \in J_i} Byk(i,j,k,n), \end{aligned} \qquad \forall i \in I, k \in K, n \in N. \qquad (A.18)$$

  - Key component balance constraints for charging tanks

$$\begin{aligned} Vtk(j,k,n) &= Vtk(j,k,n-1) \\ &+ \sum_{i \in I_j} Byk(i,j,k,n) - \sum_{l \in L_j} Bzk(j,l,k,n), \end{aligned} \qquad \forall j \in J, k \in K, n \in N. \qquad (A.19)$$

- Flow rate and capacity limits constraints
  - Flow rate existence constraints for unload, transfer and charging operations

$$X(p,i,n)Bx^L \le Bx(p,i,n) \le X(p,i,n)Bx^U, \qquad \forall p \in P, i \in I_p, n \in N, \qquad (A.20a)$$

$$Y(i,j,n)By^L \le By(i,j,n) \le Y(i,j,n)By^U, \qquad \forall i \in I, j \in J_i, n \in N, \qquad (A.20b)$$

$$Z(j,l,n)Bz^L \le Bz(j,l,n) \le Z(j,l,n)Bz^U, \qquad \forall j \in J, l \in L_j, n \in N. \qquad (A.20c)$$

  - Flow rate limits for unload, transfer and charging operations

$$\begin{aligned} &[Tpf(p,n) - Tps(p,n)]\, FRx^L - Bx^U\,[1 - X(p,i,n)] \\ &\quad \le Bx(p,i,n) \le [Tpf(p,n) - Tps(p,n)]\, FRx^U, \end{aligned} \qquad \forall (p,i) \in PI, n \in N. \qquad (A.21a)$$

$$\begin{aligned} &[Ttf(i,n) - Tts(i,n)]\, FRy^L - By^U\,[1 - Y(i,j,n)] \\ &\quad \le By(i,j,n) \le [Ttf(i,n) - Tts(i,n)]\, FRy^U, \end{aligned} \qquad \forall (i,j) \in IJ, n \in N. \qquad (A.21b)$$

$$\begin{aligned} &[Tuf(l,n) - Tus(l,n)]\, FRz^L - Bz^U\,[1 - Z(j,l,n)] \\ &\quad \le Bz(j,l,n) \le [Tuf(l,n) - Tus(l,n)]\, FRz^U. \end{aligned} \qquad \forall (j,l) \in JL, n \in N. \qquad (A.21c)$$

- Quality or specification constraints
  - The concentration of the component from the charging tank should be equal to the

concentration of the key component inside the same tank.

$$Vtk(j,k,n) = Vt(j,n)ftk(j,k,n), \qquad \forall j \in J, k \in K, n \in N, \qquad (A.22a)$$
$$Bzk(j,l,k,n) = Bz(j,l,n)ftk(j,k,n-1), \qquad \forall j \in J, l \in L_j, k \in k, n \in N. \qquad (A.22b)$$

The above constraint which brings bilinearity to the model is linearize as follows.

$$Vt(t,n)ftk^L(t,k) \leq Vtk(t,k,n) \leq Vt(t,n)ftk^U(t,k), \qquad \forall t \in J, k \in K, n \in N, \qquad (A.23a)$$
$$Bz(j,l,n)ftk^L(j,k) \leq Bzk(j,l,k,n) \leq Bz(j,l,n)ftk^U(j,k), \qquad \forall j \in J, l \in L_j, k \in k, n \in N. \qquad (A.23b)$$

The same concentration consistency constraints hold for storage tanks in the Lee3 example (from Lee et al. (1996)) and the Lee3b example (from Jia et al. (2003)).

$$Vtk(i,k,n) = Vt(i,n)ftk(i,k,n), \qquad \forall i \in I, k \in K, n \in N, \qquad (A.24a)$$
$$Byk(i,j,k,n) = By(j,l,n)ftk(i,k,n-1), \qquad \forall i \in I, j \in J_i, k \in k, n \in N, \qquad (A.24b)$$
$$Vt(t,n)ftk^L(t,k) \leq Vtk(t,k,n) \leq Vt(t,n)ftk^U(t,k), \qquad \forall t \in I, k \in K, n \in N, \qquad (A.24c)$$
$$By(i,j,n)ftk^L(i,k) \leq Byk(i,j,k,n) \leq By(i,j,n)ftk^U(i,k), \qquad \forall i \in I, j \in J_i, k \in k, n \in N. \qquad (A.24d)$$

- Other constraints
  - CDU switchover calculation constraints

$$CO(l,n) \geq Z(j,l,n) - Z(j,l,n-1), \qquad \forall l \in L, j \in J_l, n > 1, \qquad (A.25a)$$
$$CO(l,n) \geq Z(j,l,n-1) - Z(j,l,n), \qquad \forall l \in L, j \in J_l, n > 1. \qquad (A.25b)$$

  - Constraints handling with multiple berths

$$\sum_{b \in B} A(v,b) = 1, \qquad \forall v \in V, \qquad (A.26a)$$
$$Tvs(v) \geq Tvf(v') - H\left[2 - A(v,b) - A(v',b)\right], \qquad \forall b \in B, v, v' \in V, v > v'. \qquad (A.26b)$$

  - Variables bounds constraints

$$Tvarr(v) \leq Tvs(v) \leq Tvall(v), \qquad \forall v \in V, \qquad (A.27a)$$
$$Tvarr(v) \leq Tps(p,n) \leq H, \qquad \forall v \in V, p \in FP_v, n \in N, \qquad (A.27b)$$
$$0 \leq Tts(i,n)/Ttf(i,n) \leq H, \qquad \forall i \in I, n \in N, \qquad (A.27c)$$
$$0 \leq Tts(j,n)/Ttf(j,n) \leq H, \qquad \forall j \in J, n \in N, \qquad (A.27d)$$
$$0 \leq Tus(l,n)/Tuf(l,n) \leq H, \qquad \forall l \in L, n \in N. \qquad (A.27e)$$

B. The event-based formulation

The event-based model is from Jia et al. (2003). In this section, only the key timing and sequence constraints that are different from the unit slot model in Appendix A are formulated.

B.1. Nomenclature of the event-based formulation

Variables	
Continuous time variables	
$Txs(p, i, n)$	start time of parcel p unloading crude oil into storage tank i at event point n
$Txf(p, i, n)$	end time of parcel p unloading crude oil into storage tank i at event point n
$Tpst(p, i, n)$	time that parcel p starts unloading into storage tank i at event point n
$Tpft(p, i, n)$	time that parcel p finishes unloading into storage tank i at event point n
$Tps(p)$	start time of unloading parcel p
$Tpf(p)$	end time of unloading parcel p
$Tys(i, j, n)$	start time of storage tank i transferring crude oil to charging tank j at event point n
$Tyf(i, j, n)$	end time of storage tank i transferring crude oil to charging tank j at event point n
$Tzs(j, u, n)$	start time of charging tank j charging the crude oil mix into CDU l at event point n
$Tzf(j, u, n)$	end time of charging tank j charging the crude oil mix into CDU l at event point n

B.2. The event-based model

- Timing constraints for vessel parcels
  The start and end time of unloading parcel p into storage tank i are $Tpst(p, i, n) = Txs(p, i, n)X(p, i, n)$ and $Txf(p, i, n)X(p, i, n)$ involving bilinear terms (continuous times binary). Linearity can be p-reserved by applying Glover's transformation to the two constraints.

$$\begin{aligned} &Txs(p, i, n) - H\,[1 - X(p, i, n)] \leq \\ &\qquad Tpst(p, i, n) \leq Txs(p, i, n), \end{aligned} \qquad \forall p \in P, i \in I_p, n \in N, \tag{B.1a}$$

$$Tpst(p, i, n) \leq H \times X(p, i, n), \qquad \forall p \in P, i \in I_p, n \in N, \tag{B.1b}$$

$$\begin{aligned} &Txf(p, i, n) - H\,[1 - X(p, i, n)] \\ &\quad \leq Tpft(p, i, n) \leq Txf(p, i, n), \end{aligned} \qquad \forall p \in P, i \in I_p, n \in N, \tag{B.1c}$$

$$Tpft(p, i, n) \leq H \times X(p, i, n), \qquad \forall p \in P, i \in I_p, n \in N. \tag{B.1d}$$

- Timing and sequence constraints for the unloading operation

$$
\begin{aligned}
&Txs(p,i,n) \geq Tarr(v)X(p,i,n), && \forall v \in V, p \in P_v, i \in I_p, n \in N, && (B.2a)\\
&Txf(p,i,n) \leq H, && \forall p \in P, i \in I_p, n \in N, && (B.2b)\\
&Txs(p,i,n) \geq Txf(p,i,n-1) - H\left[1 - X(p,i,n-1)\right], && \forall p \in P, i \in I_p, n \in N, && (B.2c)\\
&Txs(p,i,n) \geq Txs(p,i,n-1), && \forall p \in P, i \in I_p, n \in N, && (B.2d)\\
&Txf(p,i,n) \geq Txf(p,i,n-1), && \forall p \in P, i \in I_p, n \in N. && (B.2e)
\end{aligned}
$$

- Timing and sequence constraints for the transfer operation

$$
\begin{aligned}
&Tys(i,j,n) \geq Tyf(i,j,n-1) - H\left[1 - Y(i,j,n-1)\right], && \forall (i,j) \in IJ, n \in N, && (B.3a)\\
&Tys(i,j,n) \geq Tys(i,j,n-1), && \forall (i,j) \in IJ, n \in N, && (B.3b)\\
&Tyf(i,j,n) \geq Tyf(i,j,n-1), && \forall (i,j) \in IJ, n \in N. && (B.3c)
\end{aligned}
$$

- Timing and sequence constraints for the charging operation

$$
\begin{aligned}
&Tzs(j,l,n) \geq Tzs(j,l,n-1), && \forall (j,l) \in JL, n \in N, && (B.4a)\\
&Tzf(j,l,n) \geq Tzf(j,u,n-1), && \forall (j,l) \in JL, n \in N, && (B.4b)\\
&Tzs(j,l,n) \geq Tzf(j,l,n-1) - H\left[1 - Z(j,l,n-1)\right], && \forall (j,l) \in JL, n \in N, && (B.4c)\\
&Tzs(j,l,n) \geq Tzf(j,l',n-1) - H\left[1 - Z(j,l',n-1)\right], && \forall j \in J, l, l' \in L_j, n \in N, && (B.4d)\\
&Tzs(j,l,n) \geq Tzf(j',l,n-1) - H\left[1 - Z(j',l,n-1)\right], && \forall l \in L, j, j' \in J_l, n \in N, && (B.4e)\\
&Tzs(j,l,n) \leq Tzf(j',l,n-1) + H\left[1 - Z(j',l,n-1)\right], && \forall l \in L, j, j' \in J_l, n \in N, && (B.4f)\\
&\sum_{n \in N} \sum_{j \in J_l} \left[Tzf(j,l,n) - Tzs(j,l,n)\right] = H, && \forall l \in L. && (B.4g)
\end{aligned}
$$

Two additional timing constraints that are not considered in Jia et al. (2003) are enforced to forbid simultaneous inlet and outlet operations of charging tanks.

$$
\begin{aligned}
&Tys(i,j,n) \geq Tzf(j,l,n-1) - H\left[1 - Z(j,l,n-1)\right]; && \forall j \in J, i \in I_j, l \in L_j, n \in N, && (B.5a)\\
&Tzs(j,l,n) \geq Tyf(i,j,n-1) - H\left[1 - Y(i,j,n-1)\right]; && \forall j \in J, i \in I_j, l \in L_j, n \in N. && (B.5b)
\end{aligned}
$$

- Flow rate limits for unload, transfer and charging operations

$$[Txf(p,i,n) - Txs(p,i,n)]\, FRx^L - Bx^U\, [1 - X(p,i,n)] \\ \leq Bx(p,i,n) \leq [Txf(p,i,n) - Txs(p,i,n)]\, FRx^U, \qquad \forall (p,i) \in PI, n \in N, \qquad (B.6a)$$

$$[Tyf(i,j,n) - Tys(i,j,n)]\, FRy^L - By^U\, [1 - Y(i,j,n)] \\ \leq By(i,j,n) \leq [Tyf(i,j,n) - Tys(i,j,n)]\, FRy^U, \qquad \forall (i,j) \in IJ, n \in N, \qquad (B.6b)$$

$$[Tzf(j,l,n) - Tzs(j,l,n)]\, FRz^L - Bz^U\, [1 - Z(j,l,n)] \\ \leq Bz(j,l,n) \leq [Tzf(j,l,n) - Tzs(j,l,n)]\, FRz^U, \qquad \forall (j,l) \in JL, n \in N. \qquad (B.6c)$$

C. The multi-operations sequence model

In this section, the MOS model of the crude oil scheduling problem from Mouret et al. (2010) is presented.

C.1. Nomenclature of the MOS model

Indices and sets	
$T = \{1, \ldots, n\}$	is the set of priority-slots
W	is the set of all operations: $W = W_U \cup W_T \cup W_D$ ($W = \{v_1..v_8\}$ for problem Lee1, see Fig. 1)
$W_U \subset W$	is the set of unloading operations ($W_U = \{v_1, v_2\}$ for problem Lee1)
$W_T \subset W$	is the set of tank-to-tank transfer operations ($W_T = \{v_3, v_4, v_5, v_6\}$ for problem Lee1)
$W_D \subset W$	is the set of distillation operations ($W_D = \{v_7, v_8\}$ for problem Lee1)
R	is the set of resources (i.e. vessels, parcels, tanks, or CDUs): $R = R_V \cup R_P \cup R_S \cup R_C \cup R_D$
$R_V \subset R$	is the set of vessels
$R_P \subset R$	is the set of parcels
$R_S \subset R$	is the set of storage tanks
$R_C \subset R$	is the set of charging tanks
$R_T = R_S \cup R_C$	is the set of storage and charging tanks
$R_D \subset R$	is the set of distillation units
$I_r \subset W$	is the set of inlet transfer operations on resource r
$O_r \subset W$	is the set of outlet transfer operations on resource r
C	is the set of products (i.e. types of crude)
K	is the set of key components or product properties (e.g. crude sulfur concentration)
Parameters	
H	is the scheduling horizon
$[\underline{V_v^t}, \overline{V_v^t}]$	are bounds on the total volume transferred during transfer operation v ; in all instances, $\underline{V_v^t} = 0$ for all operations except unloadings for which $\underline{V_v^t} = \overline{V_v^t}$ is the volume of crude in the marine vessel
$[\underline{N_D}, \overline{N_D}]$	are the bounds on the number of distillations
$[\underline{FR_v}, \overline{FR_v}]$	are flowrate limitations for transfer operation v
$\underline{S_v}$	is the minimum start time of unloading operation $v \in W_U$ (i.e. arrival time of the corresponding vessel)
$[\underline{x_{vk}}, \overline{x_{vk}}]$	are the limits of property k of the blended products transferred during operation v
$[\underline{x_{vc}}, \overline{x_{vc}}]$	are the limits of crude type c of the blended products transferred during operation v

$[\underline{x_{rk}}, \overline{x_{rk}}]$	are the limits of property k of resource r
$[\underline{x_{rc}}, \overline{x_{rc}}]$	are the limits of crude type c of resource r
x_{ck}	is the value of the property k of crude c
$[\underline{L_r^t}, \overline{L_r^t}]$	are the capacity limits of tank r
L_{0r}^t	is the initial total level in tank r
L_{0rc}	is the initial crude level in tank r for crude c
$[\underline{D_r}, \overline{D_r}]$	are the bounds of the demand on products to be transferred out of the charging tank r during the scheduling horizon
G_c	is the individual gross margin of crude c
$NO_{v_1 v_2}$	is 1 if operations v_1 and v_2 must not overlap, 0 if they are allowed to overlap
$G_{NO} = (W, E)$	is the *non-overlapping graph*, an undirected graph where the set of vertices W is the set of operations and the set of edges is defined by $E = \{\{v, v'\}$ s.t. $NO_{vv'} = 1\}$.

Variables

Assignment variables	
$Z_{iv} = 1$	if operation v is assigned to priority-slot i, $Z_{iv} = 0$ otherwise.
Time variables	
S_{iv}	is the start time of operation v if it is assigned to priority-slot i, $S_{iv} = 0$ otherwise.
D_{iv}	is the duration of operation v if it is assigned to priority-slot i, $D_{iv} = 0$ otherwise.
E_{iv}	is the end time of operation v if it is assigned to priority-slot i, $E_{iv} = 0$ otherwise.
Operation variables	
V_{iv}^t	is the total volume of crude transferred during operation v if it is assigned to priority-slot i, $V_{iv}^t = 0$ otherwise.
V_{ivc}	is the volume of crude c transferred during operation v if it is assigned to priority-slot i, $V_{ivc} = 0$ otherwise.
Vfc_{ivc}	is the fraction of crude c transferred during operation v if it is assigned to priority-slot i, $Vfc_{ivc} = 0$ otherwise.
V_{ivk}	is the volume of key component k transferred during operation v if it is assigned to priority-slot i, $V_{ivk} = 0$ otherwise.
Vfk_{ivk}	is the concentration of key component k transferred during operation v if it is assigned to priority-slot i, $Vfk_{ivk} = 0$ otherwise.
Resource variables	
L_{ir}^t	is the total *accumulated* level of crude in tank $r \in R_S \cup R_C$ before the operation assigned to priority-slot i.
L_{irc}	is the *accumulated* level of crude c in tank $r \in R_S \cup R_C$ before the operation assigned to priority-slot i.
Lfc_{irc}	is the fraction of crude c in tank $r \in R_S \cup R_C$ before the operation assigned to priority-slot i.
L_{irk}	is the *accumulated* level of key component k in tank $r \in R_S \cup R_C$ before the operation assigned to priority-slot i.

Lfk_{irk}	is the concentration of key component k in tank $r \in R_S \cup R_C$ before the operation assigned to priority-slot i.

C.2. The multi-operations sequence model

- The objective is to maximize the gross margins of the distilled crude blends. Using the individual gross margins G_c, it is written as follows.

$$\max \sum_{i \in T} \sum_{r \in R_D} \sum_{v \in I_r} \sum_{c \in C} G_c \cdot V_{ivc}$$

- The following variable bound and time constraints (C.1) are used.

$$S_{iv} \geq \underline{S_v} \cdot Z_{iv} \qquad i \in T, v \in W_U \qquad (C.1a)$$
$$E_{iv} \leq H \cdot Z_{iv} \qquad i \in T, v \in W \qquad (C.1b)$$
$$E_{iv} = S_{iv} + D_{iv} \qquad i \in T, v \in W \qquad (C.1c)$$

- The following unloading and distillation cardinality constraints (C.2) are used.

$$\sum_{i \in T} \sum_{v \in O_r} Z_{iv} = 1 \qquad r \in R_V \qquad (C.2a)$$
$$\underline{N_D} \leq \sum_{i \in T} \sum_{v \in W_D} Z_{iv} \leq \overline{N_D} \qquad (C.2b)$$

- The following unloading precedence constraints (C.3) are used to make sure that crude vessels unload their content according to their respective order of arrival at the refinery. The notation $r_1 < r_2$ denotes that vessel r_1 is scheduled to arrive at the refinery before vessel r_2.

$$\sum_{i \in T} \sum_{v \in O_{r_1}} E_{iv} \leq \sum_{i \in T} \sum_{v \in O_{r_2}} S_{iv} \qquad r_1, r_2 \in R_V, r_1 < r_2 \qquad (C.3a)$$
$$\sum_{\substack{j \in T \\ j < i}} \sum_{v \in O_{r_1}} Z_{jv} \geq \sum_{\substack{j \in T \\ j \leq i}} \sum_{v \in O_{r_2}} Z_{jv} \qquad i \in T, r_1, r_2 \in R_V, r_1 < r_2 \qquad (C.3b)$$

- The following constraint (C.4) states that each CDU must be operated without interruption throughout the scheduling horizon. As CDUs perform only one operation at a time, the continuous operation constraint is defined by equating the sum of the duration of distillations to the time horizon.

$$\sum_{i \in T} \sum_{v \in I_r} D_{iv} = H \qquad r \in R_D \qquad (C.4)$$

- The following variable constraints (C.5) are directly derived from the definition of volume and level variables.

$$V^t_{iv} \leq \overline{V^t_v} \cdot Z_{iv} \qquad i \in T, v \in W \qquad (C.5a)$$
$$V^t_{iv} \geq \underline{V^t_v} \cdot Z_{iv} \qquad i \in T, v \in W \qquad (C.5b)$$
$$V^t_{iv} = \sum_{c \in C} V_{ivc} \qquad i \in T, v \in W \qquad (C.5c)$$
$$L^t_{ir} = L^t_{0r} + \sum_{j \in T, j<i} \sum_{v \in I_r} V^t_{iv} - \sum_{j \in T, j<i} \sum_{v \in O_r} V^t_{iv} \qquad i \in T, r \in R \qquad (C.5d)$$
$$L_{irc} = L_{0rc} + \sum_{j \in T, j<i} \sum_{v \in I_r} V_{ivc} - \sum_{j \in T, j<i} \sum_{v \in O_r} V_{ivc} \qquad i \in T, r \in R, c \in C \qquad (C.5e)$$
$$L^t_{ir} = \sum_{c \in C} L_{irc} \qquad i \in T, r \in R \qquad (C.5f)$$

- The following operation constraints ($C.6$) include:
  1. flowrate limitations that link volume and duration variables
  2. property specifications, assuming that the mixing rule is linear
  3. composition constraints, which are nonlinear

$$\underline{FR_v} \cdot D_{iv} \leq V^t_{iv} \leq \overline{FR_v} \cdot D_{iv} \qquad i \in T, v \in W \qquad (C.6a)$$
$$\underline{x_{vk}} \cdot V^t_{iv} \leq \sum_{c \in C} x_{ck} V_{ivc} \leq \overline{x_{vk}} \cdot V^t_{iv} \qquad i \in T, v \in W, k \in K \qquad (C.6b)$$
$$V_{ivc} \cdot L^t_{ir} = L_{irc} \cdot V^t_{iv} \qquad i \in T, r \in R, v \in O_r, c \in C \qquad (C.6c)$$

It has been shown Quesada and Grossmann (1995) that processes including both mixing and splitting of streams cannot be expressed as a linear model. Mixing occurs when two streams are used to fill a tank and is expressed linearly in constraints ($C.5$d- $C.5$e). Splitting occurs when partially discharging a tank, resulting in two parts: the remaining content of the tank and the transferred products. This constraint is nonlinear. The composition of the products transferred during a transfer operation must be identical to the composition of the origin tank. Note that constraint ($C.6$c) is a bilinear reformulation of the original constraint ($C.7$) and is correct even when operation v is not assigned to priority-slot i, as then $V^t_{iv} = V_{ivc} = 0$.

$$\frac{L_{irc}}{L^t_{ir}} = \frac{V_{ivc}}{V^t_{iv}} \qquad i \in T, r \in R, v \in O_r, c \in C \qquad (C.7)$$

- The following resource constraints ($C.8$) models inventory capacity limitations. As simultaneous charging and discharging of tanks is forbidden, these constraints are sufficient.

$$\underline{L_r^t} \leq L_{ir}^t \leq \overline{L_r^t} \qquad i \in T, r \in R_S \cup R_C \qquad (C.8a)$$

$$0 \leq L_{irc} \leq \overline{L_r^t} \qquad i \in T, r \in R_S \cup R_C, c \in C \qquad (C.8b)$$

$$\underline{L_r^t} \leq L_{0r}^t + \sum_{i\in T}\sum_{v\in I_r} V_{iv}^t - \sum_{i\in T}\sum_{v\in O_r} V_{iv}^t \leq \overline{L_r^t} \qquad r \in R_S \cup R_C \qquad (C.8c)$$

$$0 \leq L_{0rc} + \sum_{i\in T}\sum_{v\in I_r} V_{ivc} - \sum_{i\in T}\sum_{v\in O_r} V_{ivc} \leq \overline{L_r^t} \qquad r \in R_S \cup R_C, c \in C \qquad (C.8d)$$

- The following demand constraints (C.9) defines lower and upper limits, $\underline{D_r}$ and $\overline{D_r}$, on the total volume of products transferred out of each charging tank r during the scheduling horizon.

$$\underline{D_r} \leq \sum_{i\in T}\sum_{v\in O_r} V_{iv}^t \leq \overline{D_r} \qquad r \in R_C \qquad (C.9)$$

- Strengthened constraints

  The maximum cliques of G_{NO}, the non-overlapping graph are used to derive the following assignment and scheduling constraints.

$$\sum_{v\in W'} Z_{iv} \leq 1 \qquad i \in T, W' \in \text{clique}(G_{NO}) \qquad (C.10)$$

$$\sum_{v\in W'} E_{i_1 v} + \sum_{\substack{i\in T \\ i_1<i<i_2}}\sum_{v\in W'} D_{iv} \leq \sum_{v\in W'} S_{i_2 v} + H\cdot(1-\sum_{v\in W'} Z_{i_2 v}) \qquad i_1, i_2 \in T, i_1 < i_2, W' \in \text{clique}(G_{NO}) \qquad (C.11)$$

  Bicliques of G_{NO} can also be used to generate non-overlapping constraints.

$$\sum_{v\in W_1} E_{i_1 v} \leq \sum_{v\in W_2} S_{i_2 v} + H\cdot(1-\sum_{v\in W_2} Z_{i_2 v}) \qquad i_1, i_2 \in T, i_1 < i_2, (W_1; W_2) \in \text{biclique}(G_{NO}) \qquad (C.12a)$$

$$\sum_{v\in W_2} E_{i_1 v} \leq \sum_{v\in W_1} S_{i_2 v} + H\cdot(1-\sum_{v\in W_1} Z_{i_2 v}) \qquad i_1, i_2 \in T, i_1 < i_2, (W_1; W_2) \in \text{biclique}(G_{NO}) \qquad (C.12b)$$

- Equation (C.13) is the symmetry breaking constraint. It states that an operation v cannot be assigned to priority-slot i if no other non-overlapping operation is assigned to priority-slot $i-1$. To avoid redundant search, equation (C.14) rejects any solution that do not make use of all priority-slots.

$$Z_{iv} \leq \sum_{\substack{v' \in W \\ NO_{vv'}=1}} Z_{(i-1)v'} \qquad i \in T, i \neq 1, v \in W \qquad (C.13)$$

$$\sum_{v \in W} Z_{iv} \geq 1 \qquad i \in T \qquad (C.14)$$

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i \in T} \sum_{r \in R_D} \sum_{v \in I_r} \sum_{c \in C} G_c \cdot V_{ivc} \\
\text{s.t.} \quad & \text{Variable bound and time constraints (} C.1\text{)} \\
& \text{Cardinality constraints (} C.2\text{)} \\
& \text{Precedence constraints (} C.3\text{)} \\
& \text{Continuous distillation constraints (} C.4\text{)} \\
& \text{Variable constraints (} C.5\text{)} \\
& \text{Operation constraints (} C.6\text{)} \\
& \text{Resource constraints (} C.8\text{)} \\
& \text{Demand constraint (} C.9\text{)} \\
& \text{Clique-based assignment constraint (} C.10\text{)} \\
& \text{Clique-based non-overlapping constraint (} C.11\text{)} \\
& \text{Symmetry breaking constraint (} C.13\text{)} \\
& \text{Slots occupation constraint (} C.14\text{)} \\
& S_{iv}, D_{iv}, E_{iv}, V_{iv}^t, V_{ivc}, L_{ir}^t, L_{irc} \geq 0 \qquad i \in T, v \in W, c \in C, r \in R \\
& Z_{iv} \in \{0, 1\} \qquad i \in T, v \in W
\end{aligned}
$$

D. Detailed computational results

Table D.1: Results of the event-based formulation: minimizing operational costs

Cases	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	5	319	40	841	2020	39652	4.53	48.59	22	6	0	3	288.68	F	49	0.05	2332	91	0.84
	6	382	48	1031	15752	371635	34.2	44.5	22	5	0	3	2905.18	F	44.5	0.05	2921	111	0.00
	7	445	56	1225	189257	6563497	685.97	**44.5**	22	7	0	3	2914.23	F	**44.5**	0.11	3550	131	0.00
Lee2	5	690	70	1812	232228	9466284	974.58	69	40	14	0	10	2319.66	F	69.27	0.19	4909	311	0.39
	6	827	84	2221	1243775	72731158	10000.03[a]	66.02	40	16	0	10	995.7	I	-	0.3	6150	381	-
	7	964	98	2639	1282102	71798052	10000.06[b]	67.55	40	16	0	10	379.66	F	**67.83**	0.3	7478	451	0.41
Lee3	4	485	56	1172	55143	962291	87.27	210	40	3	1938.85	5	7071.07	F	**210**	0.05	3188	286	0.00
	5	605	70	1512	876835	15159658	1871.86	**210**	40	3	7746.28	5	7071.07	I	-	0.08	4210	371	-
	6	725	84	1861	3131594	62455529	10169.96[c]	210	40	5	9720.93	5	7071.07	F	210	0.13	5310	456	0.00
	7	845	98	2219	1981697	59856723	10000.2[d]	210	40	3	9256.4	5	7071.07	F	210	0.2	6497	541	0.00
Lee4	4	800	76	1749	22951	627126	60.17	243	0	8	2099.11	9	811.12	F	243	0.14	4651	171	0.00
	5	999	95	2243	1920036	74332156	9581.84	183	0	8	2981.67	7	744.34	I	-	0.22	6114	221	-
	6	1198	114	2746	1511292	66521992	10000.05[e]	**183**	0	7	2460.22	7	2016.59	F	**183**	0.22	7691	271	0.00
	7	1397	133	3258	1396156	59107734	10000.08[f]	183	0	7	2464.15	7	1206.84	I	-	0.27	9400	321	-
Lee3b	4	493	64	1448	17130	307703	39.27	210	40	3	3883.73	5	7071.07	F	210	0.08	4250	286	0.00
	5	615	80	1903	743372	17053195	2586.58	**210**	40	5	4515.06	5	7071.07	F	**210**	0.14	5833	371	0.00
	6	737	96	2383	1701633	58421016	10000.06[g]	210	40	4	6405.43	5	944.97	I	-	0.17	7642	456	-
	7	859	112	2888	1143662	51434185	10000.05[h]	210	40	4	4396.56	5	7071.34	I	-	0.25	9704	541	-

Gap:
a: relative gap: 20.48%; absolute gap: 13.52
b: relative gap: 33.66%; absolute gap: 22.74
c: relative gap: 18.54%; absolute gap: 38.93
d: relative gap: 62.08%; absolute gap: 130.38
e: relative gap: 47.55%; absolute gap: 87.01
f: relative gap: 65.95%; absolute gap: 120.7
g: relative gap: 33.25%; absolute gap: 69.83
h: relative gap: 56.48%; absolute gap: 118.6

Table D.2: Results of the event-based formulation: maximizing refining profit

Cases	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	5	547	40	1009	731	21366	3.06	79.75	80	6	0	4	250	I	-	0.13	3180	91	-
	6	654	48	1231	1329	40652	6.63	79.9	80	8	0	5	3849.84	I	-	0.13	3941	111	-
	7	761	56	1457	10552	270765	33.81	**79.9**	80	6	0	6	2718.51	I	-	0.16	4742	131	-
Lee2	5	1408	70	2245	50481	1555546	199.48	97.67	103	16	0	12	2629.27	I	-	0.69	7695	311	-
	6	1685	84	2737	261088	10759223	1497.03	101.99	103	18	0	16	345.12	F	100.68	0.56	9498	381	1.30
	7	1962	98	3238	170316	9159867	1463	102.76	103	22	0	22	2168.34	I	-	2.2	11388	451	-
Lee3	4	1146	56	1497	1031	38012	7	88.33	100	5	18.79	6	1973.12	F	82.79	0.24	5337	286	6.69
	5	1426	70	1913	4419	154838	30.17	90.08	100	6	1570.53	8	1767.77	I	-	0.36	6903	371	-
	6	1706	84	2338	17588	625713	112.02	91.72	100	6	2155.17	9	4180.39	F	**89.38**	0.81	8547	456	2.62
	7	1986	98	2772	297883	8694824	2364.72	92.1	100	7	1079.9	10	5913.23	I	-	1.03	10278	541	-
Lee4	4	2472	76	2405	6654	116130	17.91	132.13	132.59	7	0	9	0	F	132.13	0.58	9209	171	0.00
	5	3081	95	3055	141983	2414252	323.75	132.55	132.58	10	0	10	103.52	F	**132.55**	0.91	11825	221	0.00
	6	3690	114	3714	128688	2809242	394.75	132.57	132.59	9	0	10	362.78	I	-	1.92	14555	271	-
	7	4299	133	4382	3151556	63204112	10000.09[a]	132.57	132.58	11	0	14	433.46	I	-	1.31	17417	321	-
Lee3b	4	1154	64	1773	1606	69679	13.92	84.97	100	5	3434.68	6	2624.45	I	-	0.39	6407	286	-
	5	1436	80	2304	22229	917445	162.13	86.44	100	6	0	6	2566.66	I	-	0.34	8536	371	-
	6	1718	96	2860	27184	1087884	228.95	89.05	100	6	93.86	8	6055.65	F	87.09	0.77	10891	456	2.25
	7	2000	112	3441	505839	25342781	5651.58	89.41	100	7	4724.56	9	4884.77	I	-	1.55	13499	541	-

Gap:
a: relative gap: 0.01%; absolute gap: 0.02

Table D.3: Results of the unit slot formulation: minimizing operational costs

	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	4	189	32	462	142	2696	0.39	49.5	37	4	0	4	3004.63	I	-	0.02	1283	71	-
	5	237	40	583	1751	24046	2.06	44.5	37	5	0	3	2943.21	F	**44.5**	0.02	1649	91	0.00
	6	285	48	706	3168	63367	7.63	**44.5**	37	6	0	3	2963.55	F	44.5	0.03	2039	111	0.00
	7	333	56	831	18241	318177	30.3	44.5	37	7	0	3	2892.54	F	44.5	0.03	2457	131	0.00
Lee2	4	385	56	971	1341	25876	2.78	70	53	12	0	12	2623.54	F	70	0.03	2598	241	0.00
	5	484	70	1222	21402	446142	40.56	69	53	14	0	10	2034.73	F	69	0.06	3332	311	0.00
	6	583	84	1476	939387	24353380	2343.38	66.02	53	18	0	10	1346.08	F	66.02	0.08	4114	381	0.00
	7	682	98	1733	1495198	45443814	4942.09	65	53	16	0	10	958.31	F	**65**	0.13	4953	451	0.00
Lee3	3	235	42	543	186	2827	0.34	210	110	2	6795.11	5	1175.8	I	-	0.03	1486	201	-
	4	317	56	731	1239	18714	2.02	**210**	110	2	6563.85	5	2253.47	I	-	0.05	2040	286	-
	5	399	70	922	6738	79809	10.27	210	110	4	7921.81	5	7072.04	I	-	0.03	2633	371	-
	6	481	84	1116	63587	854859	87.86	210	110	9	12121.38	5	7071.07	I	-	0.08	3274	456	-
	7	563	98	1313	309300	4558337	538.81	210	110	3	11079.42	5	7071.07	I	-	0.03	3972	541	-
Lee4	4	467	76	1048	10254	158061	15.48	183	93	6	2333.22	7	0	F	**183**	0.05	2898	171	0.00
	5	591	95	1319	87020	1527789	125.95	**183**	93	8	4302.91	7	1250.91	F	183	0.06	3755	221	0.00
	6	715	114	1593	1362443	26803328	2932.7	183	93	6	2658.74	7	285.13	I	-	0.05	4696	271	-
	7	839	133	1870	3681306	69799220	8768.34	183	93	9	4799.4	7	1062.22	I	-	0.08	5739	321	-
Lee3b	3	245	48	603	362	7793	0.84	210	92	3	8093.52	5	944.97	I	-	0.03	1732	201	-
	4	329	64	823	30407	655568	58.7	**210**	92	4	7652.11	5	1743.43	I	-	0.05	2464	286	-
	5	413	80	1052	699962	17047097	1738.2	210	92	5	6341.22	5	1881.55	I	-	0.05	3313	371	-
	6	497	96	1290	2081768	82079262	10000.03[a]	210	92	4	6934.72	5	7071.07	I	-	0.13	4306	456	-
	7	581	112	1537	1637860	69719083	10000.06[b]	210	92	3	8093.52	5	1738.45	I	-	0.06	5470	541	-

Gap:
a: relative gap: 35.71%; absolute gap: 75
b: relative gap: 49.82%; absolute gap: 104.62

Table D.4: Results of the unit slot formulation: maximizing refining profit

Cases	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	5	465	40	751	725	13094	1.89	79.9	80	6	0	5	2064.06	I	-	0.11	1959	91	-
	6	557	48	906	1517	32860	5	**79.9**	80	7	0	5	2155.26	F	**79.43**	0.11	2497	111	0.59
	7	649	56	1063	3893	80343	8.95	79.9	80	7	0	6	1786.26	F	77.44	0.09	3059	131	3.18
Lee2	4	963	56	1321	2547	67937	9.28	90	103	14	0	12	1067.02	F	90	0.25	3649	241	0.00
	5	1202	70	1655	15816	427250	53.41	99.75	103	16	0	16	285.34	F	97.6	0.41	4822	311	2.20
	6	1441	84	1992	94119	2297505	277.94	102.68	103	22	0	18	2264.14	F	**101.55**	0.69	6118	381	1.11
	7	1680	98	2332	1313711	27370745	4883.42	102.81	103	20	0	20	844.56	I	-	2.2	7462	451	-
Lee3	3	736	42	792	276	4815	0.72	82.5	100	4	1285.71	6	529.21	F	78.01	0.09	8863	201	5.76
	4	978	56	1056	1180	39405	5.81	88.33	100	4	1785.71	6	2041.24	F	81.91	0.16	3091	286	7.84
	5	1220	70	1323	5168	143390	23.25	90.17	100	6	2037.85	8	3888.44	F	87.16	0.31	4189	371	3.45
	6	1462	84	1593	33515	913245	173.23	91.73	100	8	2996.16	9	4574.44	F	85.39	0.77	5326	456	7.42
	7	1704	98	1866	339616	13373764	2641.56	92.1	100	9	1717.26	8	4925.27	F	**87.46**	1.28	6511	541	5.31
Lee4	4	2139	76	1704	37691	371441	44.44	132.43	132.59	9	0	9	0	I	-	0.27	7753	171	-
	5	2673	95	2131	31417	310936	43.63	132.57	132.59	8	0	11	389.89	I	-	0.67	9466	221	-
	6	3207	114	2561	203978	4080972	516.97	**132.57**	132.59	12	0	11	539.28	I	-	0.61	11560	271	-
	7	3741	133	2994	426071	6349046	874.39	132.57	132.59	7	0	12	516.35	I	-	1.33	13756	321	-
Lee3b	3	746	48	852	220	6352	1.01	82.87	100	3	4506.43	5	0	I	-	0.09	3343	201	-
	4	990	64	1148	1146	66585	13.61	91.06	100	4	500	7	2192.54	I	-	0.13	4621	286	-
	5	1234	80	1453	10125	454564	70.42	92.23	100	5	2519.83	7	3340.25	I	-	0.41	6016	371	-
	6	1478	96	1767	259458	7036814	1519.78	93.95	100	6	2497.31	10	4327.27	I	-	0.55	7555	456	-
	7	1722	112	2090	442768	21224244	10001.36[a]	94.88	100	9	0	12	3382.57	F	91.59	0.86	9265	541	3.59

Gap:
a: relative gap: 4.00%; absolute gap: 3.79

Table D.5: Results of the MOS formulation: minimizing operational costs, with cardinality constraints, without precedence constraints

Cases	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	6	647	48	1572	72	6783	1.14	49.75	42	7	0	3	0	F	49.75	0.06	6637	481	0
	7	754	56	1855	68	9710	1.97	**49.75**	42	8	0	3	0	F	**49.75**	0.08	8496	561	0
Lee2	6	1393	84	3106	89	17428	4.86	67.39	63	22	0	10	0	F	68.8	1.06	14989	1441	2.05
	7	1624	98	3655	179	43482	11.55	**67.39**	63	24	0	10	0	F	**68.8**	1.52	19062	1681	2.05
Lee3	3	775	42	1626	0	101	0.09	210	210	3	1649.57	5	0	F	**210**	0.14	5894	2311	0
	4	1031	56	2139	0	379	0.34	**210**	210	3	2969.23	5	0	I	-	0.08	8669	3081	-
	5	1287	70	2669	0	395	0.66	210	210	4	1428.57	5	38.01	F	210	0.23	11908	3851	0
	6	1543	84	3216	90	9898	2.75	210	210	6	3738.12	5	22.36	I	-	0.31	15649	4621	-
	7	1799	98	3780	310	39649	10.44	210	210	6	3493.59	5	544.95	F	210	0.52	19930	5391	0
Lee4	5	1962	95	3984	0	720	0.38	183	183	7	0	7	19.8	F	183	0.17	17618	2401	0
	6	2353	114	4782	0	1068	1.47	**183**	183	11	0	7	25.2	F	**183**	0.75	23011	2881	0
	7	2744	133	5600	105	22643	7.41	188.5	183	11	0	7	25.2	F	188.5	1.09	29134	3361	0
Lee3b	3	847	48	1744	0	216	0.19	210	210	4	1428.57	5	0	F	**210**	0.05	6646	2311	0
	4	1127	64	2305	0	526	1.05	**210**	210	4	2751.79	5	5293.85	I	-	0.24	9865	3081	-
	5	1407	80	2888	40	6341	2.78	210	210	5	1688.07	5	0	I	-	0.31	13658	3851	-
	6	1687	96	3493	24	5042	2.8	210	210	7	4363.91	5	1739.16	F	210	0.48	18072	4621	0
	7	1967	112	4120	4921	565671	189.94	210	210	9	5295.58	5	356.44	F	210	0.61	23154	5391	0

Table D.6: Results of the MOS formulation: maximizing refining profit, with cardinality constraints, without precedence constraints

Cases	Nb	Vars	DVars	Eqns	Node	Iter	CPU	MIP	RMIP	NT	ET	NC	EC	rS	rNLP	rT	NZ	Nlin	Gap(%)
Lee1	6	647	48	1572	69	4902	1.19	72.32	80	7	0	3	0	F	**72.32**	0.14	6669	481	0
	7	754	56	1855	95	7615	1.81	**72.32**	80	8	0	3	0	F	72.32	0.06	8534	561	0
Lee2	6	1393	84	3106	150	21578	5.17	90.68	103	22	0	10	0	F	**90.13**	0.63	15103	1441	0.61
	7	1624	98	3655	289	52603	12.08	**90.68**	103	24	0	10	0	F	90.13	0.99	19196	1681	0.61
Lee3	3	775	42	1626	0	159	0.13	82.4	100	3	1649.57	5	0	F	**82.4**	0.06	5960	2311	0
	4	1031	56	2139	73	4888	1.01	82.5	100	5	3606.56	5	231.32	F	80.86	0.14	8759	3081	2.03
	5	1287	70	2669	258	26042	4.45	84.5	100	6	3164.72	5	1955.52	F	82.21	0.48	12022	3851	2.79
	6	1543	84	3216	559	83669	15.31	**84.5**	100	7	3134.66	5	2966.41	F	80.29	0.45	15787	4621	5.24
	7	1799	98	3780	1374	249183	67.38	84.5	100	7	2915.28	5	2140.24	F	82.1	0.45	20092	5391	2.92
Lee4	5	1962	95	3984	10	981	0.44	132.22	132.59	11	0	7	0	F	131.57	0.49	17822	2401	0.50
	6	2353	114	4782	84	12382	3.61	**132.22**	132.58	11	0	7	0	F	**131.91**	1.02	23257	2881	0.24
	7	2744	133	5600	122	16360	5.41	132.06	132.58	9	0	7	134.99	F	131.76	1.47	29422	3361	0.23
Lee3b	3	847	48	1744	42	1568	0.44	82.91	100	3	1371.42	5	39.23	F	**82.71**	0.11	6712	2311	0.24
	4	1127	64	2305	334	25688	4.76	86.06	100	5	4496.62	5	5311.15	F	81.63	0.25	9955	3081	5.43
	5	1407	80	2888	1930	241607	51.3	**86.06**	100	6	4462.16	5	6074.04	F	81.36	0.45	13772	3851	5.78
	6	1687	96	3493	4596	669222	183.69	86.06	100	6	2945.58	5	6941.64	F	81.75	0.69	18210	4621	5.27
	7	1967	112	4120	6474	1210856	390.3	83.39	100	7	5622.38	5	4625.47	F	81.74	0.77	23316	5391	2.02

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