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Hierarchical decomposition heuristic for crude oil scheduling: a SINOPEC case

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- This work addresses the large-scale crude oil scheduling problem of China's SINOPEC Maoming refinery importing various types of crude oil from two terminals via long-distance pipelines. A hierarchical decomposition approach is employed to construct a two-stage MILP model of the practical refinery operations. In the upper level model, storage and charging tanks are aggregated to determine the inflows and outflows among the two terminals and the refinery. Detailed loading and unloading operations located on storage and charging tanks are solved in the lower level sub-models. In order to further improve the computational efficiency, we develop a practical rule-based tank selection strategy to efficiently obtain feasible detailed schedules. While state-of-the-art commercial solvers cannot obtain a feasible solution of the relaxed monolithic MILP model within reasonable time limit, the decomposition heuristic can obtain near-optimal and flexible scheduling results with much less computational effort compared to the one obtained manually.

Key words: Crude oil scheduling, hierarchical decomposition, multiple terminals, long-distance pipelines
History:

1. Introduction

Crude oil scheduling is the first stage of the crude oil refining process. It involves crude oil unloading from marine vessels to storage tanks, transfers and mixings in charging tanks, and a charging schedule for each crude oil mixture to the crude distillation units (CDUs). Consequently, the scheduling decision requires simultaneous selection of crude flows, allocations of vessels to tanks, tanks to CDUs, and calculation of crude compositions.

Optimal crude oil scheduling is critical to refineries. There are many varieties of crude (e.g., light and heavy crude) in the market nowadays, varying widely in properties, processing difficulty and product yields. Most refineries procure and process several types of crude that yield various products and a wide range of profit margins. Optimal crude oil scheduling enables cost reduction by using cheaper types of crude intelligently and minimizing crude changeovers. Kelly and Mann from Honeywell reported very large economic and operability benefits associated with better crude oil blending scheduling (Kelly and Mann 2003a,b). Up to now, nevertheless, schedulers in most Chinese refineries rely largely on their experience to make these decisions.

The Maoming Petrochemical Company, a subsidiary of China Petroleum & Chemical Corporation (referred to as SINOPEC in short), is the biggest petrochemical base in South China with an annual crude oil processing capacity of 96.5 million barrels and annual ethylene processing capacity of 300,000 million tons, as well as complete supporting systems of power supply, port handling, railway transport, crude and product oil transfer pipelines and a 300,000 tonnage SBM (single buoy mooring) offshore crude loading and unloading system. It mainly makes 30-odd types of products including gasoline, kerosene, diesel oil, lube oil, solvent oil, naphtha, asphalt, ethylene, toluene, PP, ethylene glycol, styrene, SBL and SBS. Fig. 1 depicts the refinery configuration. The refinery branch consists of the Zhanjiang (Z) terminal, the Beisanling (B) terminal, the Maoming (M) refinery plant and two crude transfer pipelines connecting two terminals and the plant, respectively the Zhan-mao (ZM) pipeline and the Bei-mao (BM) pipeline. Terminal Z is state-owned and thus its storage capacity for refinery M is limited. Terminal B, although with much higher storage capacity, is not a natural deep-water terminal, hence the refinery built a SBM 30 kilometers away from terminal B. Fifteen tanks located in terminal Z, terminal B and refinery M, respectively. Pipeline ZM connecting terminal Z and refinery M is 110 kilometers long with the capacity of the pipeline be about 143,000 barrels. Pipeline BM connecting terminal B and refinery M is 64 kilometers long with the capacity of the pipeline be about 86,000 barrels. As both pipelines are long-distance, the same flow pack injecting to and stripping out of pipeline ZM and BM are asynchronous. Moreover, while pipeline BM is unidirectional, pipeline ZM is bidirectional, the reason of which is illustrated in section Crude oil scheduling in the SINOPEC Maoming Company. Four CDUs and fifteen charging tanks feeding crude oil to CDUs are located at the inland refinery plant M. Each CDU is designed to process different types of crude with different specifications. In general, crude from terminal Z feeds CDU1 and CDU2, while crude from terminal B charges CDU3 and CDU4. In case of crude from terminal Z being not sufficient, CDU1 and CDU2 would process crude from terminal B. To maintain steady production process, unless for planned temporary shutdown of production, each CDU should be operating continuously and the processing rate should be within the lower and the upper limit. Frequent changeovers and process rate fluctuations should also be avoided.

2. Crude oil scheduling in the SINOPEC Maoming Company

2.1. Features of the refinery

Distinct features and practical operational constraints of the refinery are as follows.

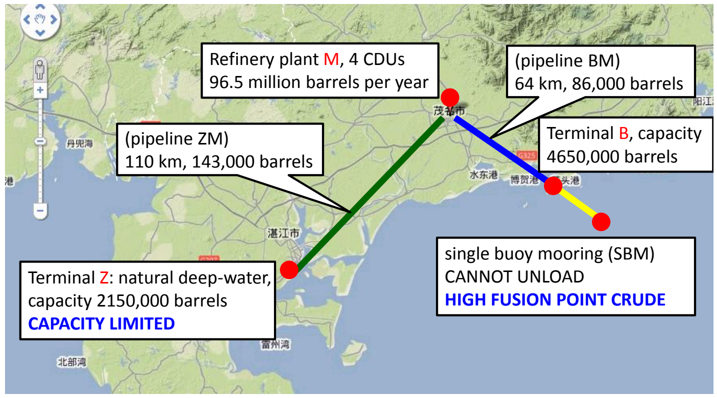

Figure 1 Configuration of the SINOPEC Maoming refinery.

- Crude storage segregation mode

—Unlike the illustrative examples in the literature, in our case it is required by the refinery that tanks and pipelines should avoid undesirable mixture of different classes of types of crude before feeding CDUs.

—It is ideal that only tanks storing the same type of crude or empty tanks receive crude from vessels or pipelines. The detailed reason is explained in section Perspectives from the practitioners.

—In general, tanks and CDUs usually store or process only specific classes of types of crude, as different types of crude vary significantly in properties and processability.

- Special treatment of crude oil transportation

—High pour point and high viscosity crude oil (denoted as HPHVC) cannot be stored in storage tanks or be transported via the long-distance pipeline without special treatment (such as heating up, blending with a specified proportion of light crude, or both).

—Thanks to its above-ground unloading lines and heating devices for storage tanks, HPHVC can be unloaded at terminal Z. Due to the submarine pipeline connecting the SBM and terminal B, however, HPHVC cannot unload at the SBM. Instead, other categories of crude oil including light oil are mainly unloaded at terminal B.

—To safely transfer certain types of HPHVC from terminal Z to plant M, it is mandatory to mix HPHVC with light crude in case of pipeline freezing. To be more specific, each type of high pour point, high viscosity or high sulfur crude must be blended with a specified type of proportion of light crude, determined by the laboratory branch of the refinery. The blending process is commonly finished inside the long-distance pipeline, that is, multiple tanks storing different types of crude would feed the pipeline simultaneously by taking into account the operational constraints. The mixture of multiple types of crude inside the pipeline is denoted as a new type of crude.

- Bidirectional long distance pipeline

—Similar to the system in Reddy et al. (2004b,a), different types of crude arrive in either large multi-parcel tankers or small single-parcel vessels. The capacity of terminal Z is limited, causing this terminal to be the bottleneck of the system. Accordingly, very large scale vessels (VLCCs) carrying large amounts of light crude from Middle East cannot unload to terminal Z. Therefore, light crude needed in terminal Z is transferred from terminal B to plant M via pipeline BM, and then from plant M to terminal Z via pipeline ZM conversely. The transfer operation from plant M to terminal Z is called the *backward transfer* or the *reverse transfer*, which takes place about once monthly. It follows that pipeline ZM is bidirectional.

—While pipeline BM is always operating as the transfer rate can be adjusted according to the inventory level of the Plant M, pipeline ZM would occasionally shutdown because the capacity of terminal Z is much smaller. Once shutdown, pipeline ZM should not fill with high viscosity crude in order to avoid pipeline freezing. To insure that, an operation called *cleaning transfer* would inject certain amount light crude into pipeline ZM to push out all the HPHVC inside the pipeline. This segment of light crude, which would backtrack in case the follow-up operation is backward transfer, is named as the *cleaning crude*.

—Since light crude seldom unloads at terminal Z, unless cleaning pipeline ZM or blending with high pour point, high viscosity, and high sulfur crude, terminal Z would commonly not feed light crude into pipeline ZM.

The overall process of the forward transfer, the pipeline stoppage, and the backward transfer is shown in Fig. 2.

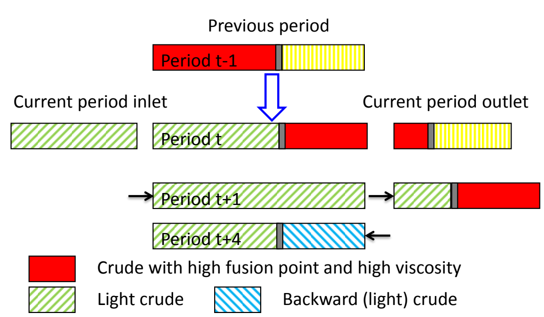

Figure 2 Transportation process of the bidirectional pipeline ZM.

2.2. Decision procedure of the refinery

Similar to the decision process described in Shah et al. (2009), there are two decision levels in refinery M, namely the quarterly plan level and the ten-day plan level. Given demand forecasts, the quarterly plan determines the variety and the volume of crude oil needed for the upcoming three months, as well as the type and estimated quantities of final products to be ordered. Based on the quarterly plan results, the ten-day plan determines the detailed schedule of unloading crude oil from vessels into storage tanks, transporting crude oil among terminals and the plant, and feeding the CDUs at various rates over time according to the production plan by taking into account the operational constraints.

The current manual ten-day plan has the following shortcomings. First of all, it is complicated and time consuming and thus not flexible enough to reschedule when supply chain disruptions occur. Usually it takes hours to make the detailed crude oil schedule. Human errors and inconsistencies are easily brought into the schedule. Secondly, since the schedule is made in a user-driven simulation environment with the Aspen ORION system in the SINOPEC Maoming refinery, the overall process cannot be optimized from a systematic perspective. Therefore, it was not rare to see that the CDUs were running out of crude while the transportation of HPHVC was delayed because of the lack of light crude, or the total amount of crude was sufficient but some of the CDUs had to process the "downgraded" type of crude. Besides, the manual scheduling results rely heavily on the scheduler's experience and skills, sometimes not easy to be used by shopfloor operators.

2.3. Problem definition

With the above introduction, we now state the multi-regional crude oil scheduling problem addressed in the paper.

- *Given*:

— configuration of the refinery and the two terminals (numbers of CDUs, storage tanks, berths, and interconnections among them), as shown in Fig. 1;

— builtin attributes of the system, including the set of types of crude, crude segregation mode of storage and processing, upper and lower inventory levels of tanks, etc.;

— operational parameters of the system, including the start and the end time of the scheduling horizon, the minimum crude settling time, limits on the number of simultaneously connected tanks for all operations, flowrate limits for all operations and resources, etc.;

— initial state of the system, involving initial distribution of crude inside pipeline ZM (BM) and the initial transfer direction, initial crude types and initial inventory levels of terminal and refinery tanks, initial crude type of distillation and processing rate of each CDU, etc.;

— input of the system: information of vessels at each terminal, including arrival times, crude types, volumes, and the unloading sequence of their parcels, updated in real time according to crude suppliers and shipping companies;

— output of the system: production plan of each CDU passed down from higher level planning decisions;

— economic parameters such as unit costs of different operations and crude distillation profits.

- *Determine*:

— detailed unloading schedule for each vessel;

— inventory levels and composition profiles of storage and charging tanks during the scheduling horizon;

— detailed transfer schedule of pipeline ZM/BM, including the cleaning and reverse transfer operations;

— detailed crude feeding profiles for CDUs.

- *Objectives*:

— maximize the total crude distillation profit;

— minimize the waiting time, and consequently the demurrage cost of each vessel;

— reduce the number of changeovers of tanks, piplines and CDUs;

— reduce the number of undesirable blending of different varieties of crude;

- *Operational rules*:

— There are broadly five categories of operating rules (constraints): safety rule, interconnection rules, sequence rules of operations, mass and crude composition balance, and capacity limits. Some of the detailed constraints are described in Appendix A.

In sum, we study a scheduling scheme (Fig.1) that is quite different from previous work reported in the literature. The system consists of multiple regions distributing and sharing crude through long-distance, possibly bidirectional pipelines. Mixing different varieties of crude oil should be avoided (except for required blending) before delivering to CDUs. HPHVC can only be unloaded, transferred, or processed with special treatment.

The above mentioned practical constraints along with the long-distance pipeline feature are very difficult to *model*. Furthermore, the inherent large-scale combinatorial feature poses a challenge for *solving* the model. The various types of crude processed (up to 20), the number of CDUs (4 CDUs, each is designed to distill different crude types), and the number of storage and charging tanks (45 tanks in total) are very large in this case, making the problem computationally intractable. Hence, we aim at obtaining satisfactory and flexible solutions of the problem robustly (within tolerable time limits).

3. Perspectives from the practitioners

Crude oil scheduling in real-life plants exhibits features that are very different from the common assumptions used in the literature. In this section, we present some of our industrial collaborators' viewpoints based on their operating and management experiences of real-life plants.

3.1. The objective function

The crude oil scheduling problem is inherently multi-objective. The pioneering work Lee et al. (1996) tried to minimize operational cost including sea waiting cost and unloading cost of marine vessels, inventory cost, and CDU changeover cost. Magalhães and Shah (2003) minimized the deviation of the planned and the scheduled amounts for crude when solving the crude oil scheduling problem in PETROBRAS' REFAP refinery, Brazil. Mouret et al. (2010) focused on maximizing crude distillation profit for the current scheduling period, as required by TOTAL, France. Reddy et al. (2004b,a) studied the crude oil scheduling problem of coastal refineries based on the information provided by the Singapore Refining Company, Singapore. They took into account of both profit and operational cost. When executing the obtained schedules, more operational costs are incurred, such as the changeover costs of tanks and pipelines, utility costs of storing and transferring crude oil, waste costs the result from undesirable mixture of crude, etc. It is in general unrealistic to include them all in the objective function, as many of these objectives are difficult to be expressed in economic terms. For instance, in a recent work (Shah and Ierapetritou 2011), the crude oil scheduling problem incorporating with logistic constraints is considered. However, one can observe that too many penalty items are added to the objective function, in which the weight coefficients are difficult to determine in real-life plants. Other than that, assuring continuous and smooth production of the refinery is considered to be the primary goal. That is to say, were CDU demands not be satisfied, the objectives aiming at reducing operational cost, such as the minimization of pipeline changeovers and the minimization of undesirable crude mixtures, are not relevant. From the viewpoint of the plant manager, a schedule that satisfies all the demands with minor unnecessary mixture is undoubtedly preferable than another schedule that avoids mixture without fulfilling production demands. This reflects the feature that more *soft* constraints exhibit in real world problems instead of purely hard mathematical constraints. The insight is that these objectives are hierarchical with different priorities. As a matter of fact, it would not be appropriate to optimize them simultaneously. Rather, different objectives with different priorities should be optimized within different levels.

- Safety issues should always be the highest priority. For instance, continuous operations of units should be guaranteed, the temperature of the crude inside the pipeline should not below its fusion point, etc.

• Demand or customer orders should be satisfied, including the demand flow among the inner sub-systems. With this as the prerequisite, different types of crude should be distilled intelligently to make as much profit as possible.

• Operational costs incurred from unload, storage, transfer, and charge operations should be minimized. This includes vessel demurrage cost, utility cost, undesirable crude mixtures cost, changeover cost, etc.

Moreover, feasibility besides global optimality is one of the major concerns in practice. Feasibility can be defined as "to satisfy the planning decisions" or "to distill different types of crude on their corresponding most profitable CDUs". Consequently a more straightforward way is to firstly balance and optimize the amount of crude inflow and outflow *globally*, of which the mathematical model is able to efficiently obtain global optimal solutions. Detailed scheduling with a large number of discrete decisions, of which the current linear programming based optimization method cannot give optimal or even feasible solutions with limited computation effort, can be solved *locally* or heuristically to yield near-optimal solutions within distinctly less CPU time. This kind of methodology mimics the practical two level *planning-scheduling* decision process. Therefore, it is more intuitive to schedulers. The key ingredient lies on how to reasonably incorporate the relaxed scheduling constraints into the upper level planning model in order to avoid over-optimization of the planning layer and infeasibility of the lower level problem.

3.2. Time representation

During the last two decades, research in the process system engineering (PSE) field has developed various methods of representing temporal constraints. These methods are broadly categorized into the discrete time representation and the continuous time representation, depending on whether the events of the schedule can only take place at certain predefined time points, or can occur at any moment during the time horizon. A comprehensive review is given in Méndez et al. (2006). As pointed out in Wassick and Ferrio (2011), industrial practitioners tend to use the discrete-time representation as it is simple and applicable to general processes. Also, according to our industrial collaborators, in large-scale systems such as the SINOPEC Maoming refinery, start and end times of tasks or operations given by continuous time models cannot be executed exactly at these time points. Based on our computational experience, we also favor the discrete time representation for complex systems more than its continuous time representation counterparts.

3.3. Handling with crude oil blending

In the following we explain why the SINOPEC Maoming refinery tends to avoid crude blending before feeding CDUs, which introduces more discrete decision variables to the model, making the

Table 1 **The crude-by-crude setting: a typical refining schedule for a CDU**

crude	Crude A	Crude A/C 50%/50%	Crude B+D mixture
Tank %	T127	T127,50: T182,50,	T129
Tank vol	5722		
start	2011-7-29 5:15	2011-7-29 22:00	2011-8-1 19:40
end	2011-7-29 22:00	2011-7-31 14:23	2011-8-5 13:38

model more difficult to solve. Literature models usually utilize the *blending* setting to impose constraints on tolerable upper and lower limits of the key component concentration, as stated in equation (1) from Reddy et al. (2004b), where xk_{kc} is a parameter of key component $k \in \mathcal{K}$ in crude $c \in \mathcal{C}$, $FCTU_{iuct}$ represents the amount of crude $c \in \mathcal{C}$ delivered by tank $i \in \mathcal{I}$ to CDU $u \in \mathcal{U}$ during period $t \in \mathcal{T}$, and $xk_{ku}^{L/U}$ are limits on the concentration of key component $k \in \mathcal{K}$ in feed to CDU $u \in \mathcal{U}$, with $\mathcal{IU}$ the set of pairs (tank i, CDU u) such that i can feed crude to CDU u and $\mathcal{IC}$ the set of pairs (tank i, crude c) such that i can hold crude c.

$$xk_{ku}^{L} FU_{ut} \leq \sum_{l} \sum_{c} FCTU_{iuct} xk_{kc} \leq xk_{ku}^{U} FU_{ut}, \quad (i,u) \in \mathcal{IU}, (i,c) \in \mathcal{IC}. \tag{1}$$

In the SINOPEC Maoming refinery, however, our industrial collaborators tend to use the *crude-by-crude* setting. A typical refining schedule for a CDU is shown in Table 1, for which we need to define the binary variable y_{iuct} to indicate whether CDU u distills crude c from tank i during time period t or not. Despite the large increase of number of binary variables, the advantage of the crude-by-crude setting is that it conforms to the scheduler's experience. When there are supply chain disruptions, for instance late arrival of crude vessels, it would be much more intuitive and much safer to modify the current schedule. On the contrary, although the blending setting incurs no extra binary variables, the result given by the blending setting is confusing to the scheduler. It is known that the mathematical programming models tend to set the variables to their bounds in order to obtain better economic objective values. Once disruptions occur, for instance the insufficiency of a certain type of crude or key component cause by the arrival delay of a vessel, the scheduler would have to decrease the processing rate or even stop one of the units, incurring production stability or even safety issues.

4. The decomposition framework

The crude oil scheduling problem is closely related to the batch process scheduling problem. Many researchers have developed models and solution techniques for this class of problem, primarily mixed integer linear or nonlinear programming (MILP or MINLP) models (Méndez et al. 2006). Due to the complexity of the crude scheduling operations, however, large-scale MILP or MINLP problems cannot be effectively solved. In the very beginning, we tried to apply different representations from the literature to the problem, for instance the discrete-time representation (Reddy et al. 2004b), the synchronous continuous-time representation (Reddy et al. 2004a), and the unit slot representation (Hu and Zhu 2007) (an extended formulation from the event-based model (Jia et al. 2003)). We excluded constraints that make the problem difficult to model and solve, such as the bidirectional feature of pipeline ZM, bilinear terms results from crude oil blending, and unavoidable tank heels. Despite these simplifications, the state-of-the-art commercial MILP solvers cannot provide a feasible solution for the large-scale monolithic model in several hours. In fact, we were unable to obtain a feasible ten-day schedule of the refinery sub-problem without taking the two terminals into consideration.

Although industrial applications based on mathematical programming techniques have been reported in the literature (Más and Pinto 2003, Magalhães and Shah 2003), due to the complexity of the crude scheduling operations, other exact and heuristic approaches have also been proposed, such as the NLP approach (Fagundez et al. 2009), the event-tree search method (Zou et al. 2010), combination of the MILP models and expert systems (Bok et al. 2002, Pan et al. 2009), model based decomposition heuristics (Kelly 2002, Kelly and Mann 2004), random search method (Chryssolouris et al. 2005), and Petri net-based heuristics that pre-assign a number of charging tanks to each CDU (Wu et al. 2007, 2010). In this work, we resort to a hierarchical decomposition approach to efficiently obtain a satisfactory solution of the large-scale and complex process.

To reduce the computational time, we break the monolithic model into two levels: (1) The upper level problem that involves the transfer schedule of the long-distance pipelines and the charging schedule of CDUs; (2) The lower level problem consists of three sub-problems, wherein the Z problem involves the feeding and receiving schedule of tanks, and the unloading schedule of parcels at terminal Z, the B problem involves the feeding and receiving schedule of tanks and the unloading schedule of parcels at terminal B, and finally the M problem involves the feeding and receiving schedule of tanks at refinery plant M. In principle, the upper level model makes high level decisions of allocating crude resources among the refinery plant and two terminals in a tank-aggregated way, while the lower level problem distributes these crude resources to specific tanks. In practice we found that the upper level model for industrial problems can still not be readily solved in reasonable time limit, notwithstanding its tank-aggregation simplification. This motivates us to further separate the upper level model into pipeline transportation decisions, and

CDU charging decisions based on the observation that the two decisions are loosely linked. In this way the upper level model consists of the *Pipeline* model and the *CDU* planning model. The decomposition framework is shown in Fig. 3.

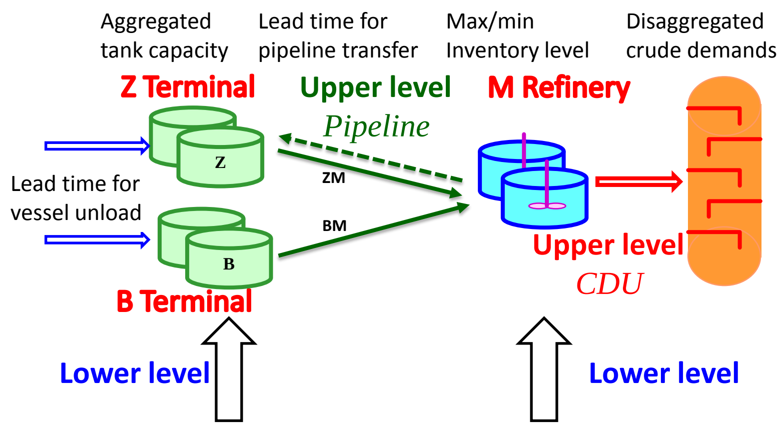

Figure 3 Decomposition framework of the SINOPEC refinery.

As in the upper level model, only the resource balance (among two terminals and the refinery plant M) is considered without involving detailed operations of the storage and charging tanks. The key point is to reasonably aggregate the demands of the refinery and two terminals. The basic ideas of the upper level model are as follows.

- The storage and charging tank inventory capacity constraints are incorporated approximately in a tank-aggregated way.
- A buffer time parameter of vessel unloading is employed to aggregate the supply of the terminal.
- We introduce the lead time parameters of pipeline transportation and changeover to implicitly handle with complex constraints of pipelines. The parameter of pipeline transportation *lead time* denotes that the timing of each crude pack transported from the terminal to the refinery is adjusted to take account of the transportation time, similar to Shah (1996). Complex constraints of pipelines include that the HPHVC cannot stay inside the pipeline upon pipeline stoppage, and that the amount and specifications of the reversely transferred light crude need to be determined.

Note that the choice of two parameters, namely VTr, the maximal inventory ratio for all the charging tanks in the refinery, and $SULBM/SULZM$, the estimated vessel unloading lead time, has a great impact on the aggregation of the terminal supply. With respect to the issue of determining these parameters, we were notified by the refinery that: (1) the experienced scheduler of the refinery would be able provide the model with fairly accurate parameters, in that restrictions are imposed via the higher level production plan decisions, and that the scheduler is expert at the production and scheduling details; (2) it provides more flexibility to the scheduler in the matter of adjusting those parameters to accommodate production variations and supply disruptions, thus obtaining more implementable, flexible, and profitable schedules. In fact, by adjusting these parameters and leaving the generation of detailed schedules to computers, the scheduler can focus more on high-level decisions. Instead of manually generating only one schedule in hours, the schedule is able to generate several schedules in a matter of minutes and to choose the best one.

After the ”resource balance” stage, the primary objective of the lower level sub-problems is to determine detailed tank operations, i.e., to allocate the (already known) resource demands on certain tanks. Therefore, feasibility, other than optimality, is the major concern in the lower level subproblem in that high level optimal decisions are obtained from the upper level model. As there are a large number of discrete decision variables with triple indices (crude type c, tank i, time period t), increasing the computational burden, a heuristic tank allocation policy is proposed based on experience of the scheduler from the M refinery to obtain satisfactory and reliable results in seconds. The proposed method did meet our industrial collaborators' requirements by providing them with near-optimal, flexible, and more implementable scheduling results in a short time.

The complete upper level MILP model is presented in Appendix A. The uniform discrete-time representation is used. That it, the scheduling horizon is partitioned into NT identical periods ($t = 1, 2, \dots, NT$) with the time interval of each period be 6 hours. The main assumptions are as follows: the change over time between different tasks at each unit is negligible; the crude flow in any pipeline is plug flow; and crude mixing is perfect in long-distance pipelines. Furthermore, crude is normally stored in floating roof tanks to minimize evaporation losses. Such a tank requires a minimum crude level (or heel) to avoid damage to the roof, when the tank goes empty. Because of the presence of the heel, crude usually accumulates in the tank over time. However, a crude type with negligible volume fraction does not significantly affect the overall quality. Thus, to model the rule that each tank can store at most one type of crude simultaneously while maintaining the heel, the initial inventory in each tank in the proposed model equals the real inventory minus the heel, and the maximum storage capacity equals the real capacity minus the heel, with the minimum crude level be zero. In this way, linearity is maintained and the developed model corresponds to an MILP problem.

Due to space issues, we do not include the lower level sub-problems. They are available upon request. According to the schedule obtained by the upper level model, we can deduce when, where, what type of crude and the amount of crude each region will receive or feed, by taking account to the time lags of flow in and out of long-distance pipelines. We can also find out on the type and amount of crude that each CDU processes during each period. Next, we solve the lower level sub-models to obtain the detailed schedules, respectively.

Further decomposition. Despite the tank-aggregation simplification, the solution time of the upper level model is still not acceptable for industrial problems. To further reduce the solution time, we employ a spatial-based decomposition approach to the upper level model. The basic idea is to solve the pipeline problem and the CDU planning problem separately. Firstly, the pipeline problem treats the refinery as a demand node accumulating crude without consuming it. Then, the CDU model allocates the accumulated crude resources to each CDU. To avoid crude shortage of the CDU model, the deviation from the CDU production demand plan is minimized in the pipeline problem.

Rule-based tank allocation algorithm. The lower level sub-problems can be solved by commercial MILP solvers. However, the solution time is usually too long for industrial applications. Therefore, we design rule-based tank allocation algorithms to solve the lower level sub-problems. Applying these rule-based heuristics is a more practical choice for the industry. On the one hand, it has limited impact on the optimality of the solution as economic objectives and demand balance are the main issues in the upper level model. On the other hand, rule-based algorithms are able to optimize lower level scheduling decisions, which sometimes are not readily modeled or solved. From the perspective of practitioners, the rule-based heuristic algorithm is much more flexible than mathematical models. The problem is solved with far less computational effort. In practice the scheduling process or the operational rules might change, the scheduler can easily adjust the algorithm interactively by adding new rules or deleting violated rules. Furthermore, due to tank capacity shortage, the assumption that one tank can only store at most one type of crude at the same time might not hold. Mathematical models based on this assumption would probably not be computationally feasible. On the contrary, the proposed algorithm can be easily modified to satisfy production demand by allowing minor crude mixtures in the same tank. For instance, if the inventory level in a tank is relatively low and no other tank is available, the proposed algorithm allows the tank to receive other types of crude. In this way, more flexible and more pragmatic results are obtained.

5. Solutions and applications

A scheduling software implementing the approaches described in this work was implemented using Excel VBA for the refinery. Two basic outputs of the software system are the Gantt charts and the inventory profiles of vessels and tanks. Here we present the results of a real instance of the M refinery (the first ten day period of December, 2009) that was solved on a Pentium M processor with 1.60 GHz and 1.99GB memory using the upper level model, and the lower level sub-models or the heuristic tank allocation algorithm. The data of the instance is available upon request. In this scheduling period, CDU3 can be possibly shut down for annual inspection or keep processing. The plant manager would like to analyze different scenarios to facilitate the decision making. This provides an opportunity for the proposed method to easily generate two different schedules. The actual schedule of the refinery made by the scheduler with the help of the spreadsheet-based Aspen ORION system is to shut down CDU3.

1. All CDUs operate continuously, and each model is solved with XPRESS-MP 2008 with a gap smaller than 1%. The upper level model consists of 9075 equations, 4133 variables and 1265 discrete variables. Note that the number of equations and discrete variables increases approximately linearly with the number of types of crude (10 for this case) and the number of periods (40 for this case) used. The total CPU time of the four models is 1,873.8s (31.2min). Since we can concurrently solve the three lower level sub-models, the total CPU time for solving the problem is 1,361.7s (22.7min).
2. The upper level model is solved with XPRESS-MP 2008 for at most 600s, while the lower level sub-problems are solved using the heuristic tank allocation algorithm.
   - All CDUs operate continuously. The total CPU time is about 603s (the upper level model: 600s, the heuristic tank allocation algorithm: about 3s);
   - CDU3 is shut down, and pipeline ZM does not transfer in reverse (the actual schedule of the refinery): the total CPU time is about 603s (the upper level model 600s, and the heuristic tank allocation algorithm about 3s). Fig. 4 shows the detailed schedule of refinery plant M obtained by the proposed approach. An example of inventory level charts of tanks is displayed in Fig. 5.

For the Gantt chart figures (implemented by Excel VBA), a cell block in the same color indicates that during these periods, both the type and the amount of crude fed/received by the unit (parcel, tank or CDU) do not change. Numbers in the cell indicate the flowrate, i.e., the amount of crude transferred each period. F denotes Feed, R denotes Receive, U denotes Unload, Line 1 denotes pipeline ZM, and Line 2 denotes pipeline BM. Different colors represent different types of crude.

As the solution time for large-scale problems is still too long, we apply the further decomposition models and rule-based tank allocation algorithms to three scheduling horizons of the M refinery, namely the last ten days of August, 2009, the middle and the last ten days of December, 2009.

Comparing to the currently adopted manual schedules, which would take an experienced scheduler about one day with the help of the Aspen ORION system, the solution time of our approach is far more efficient. The total number of changeovers of pipelines, tanks and CDUs reduces 19.05%.

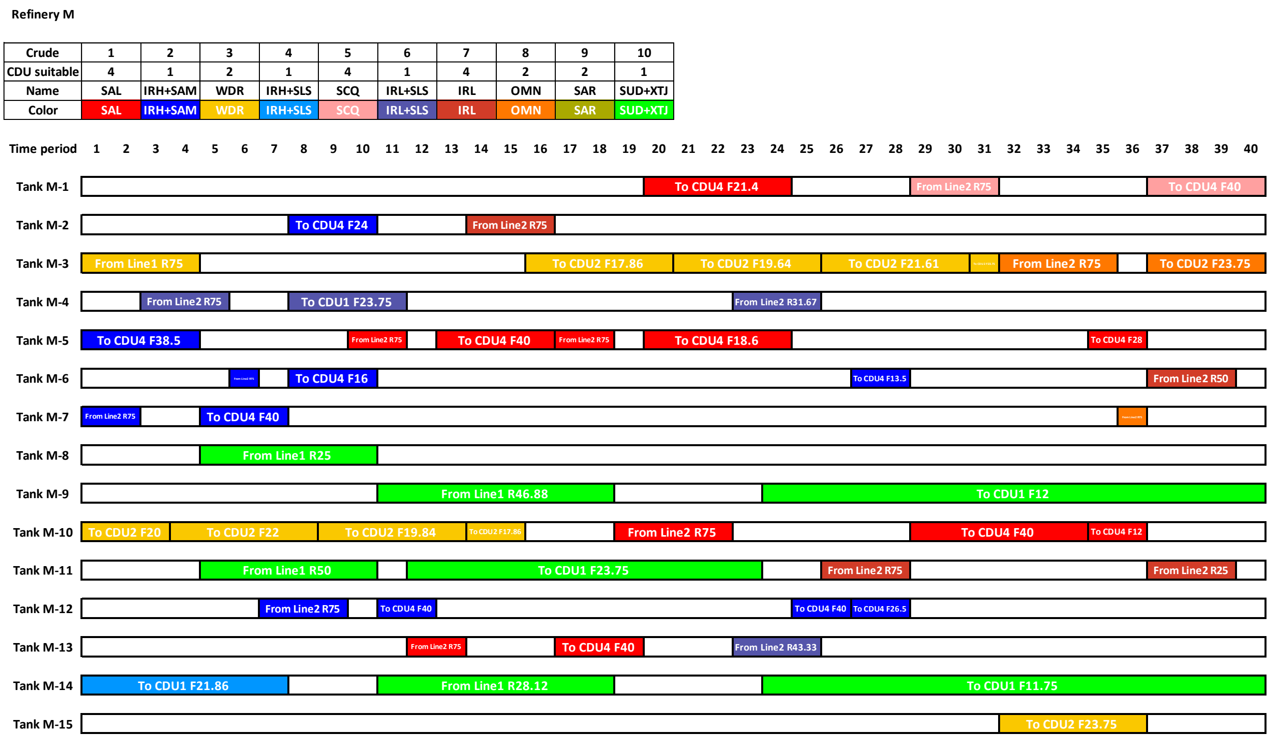

Figure 4 **Gantt chart of refinery plant M for case 2.**

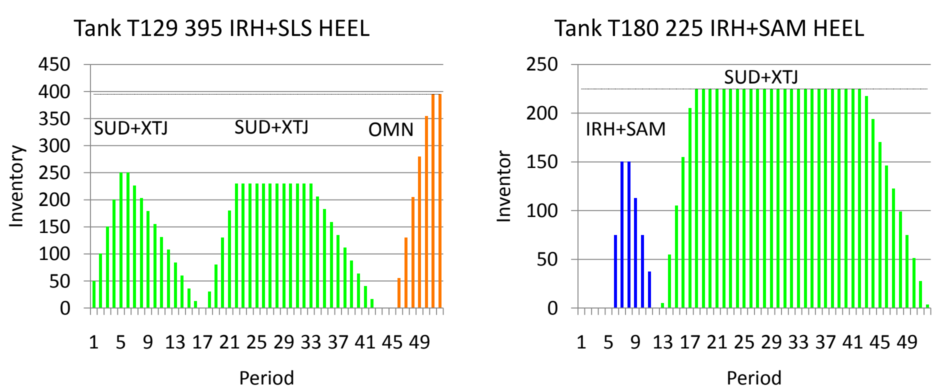

Figure 5 **Example of the inventory level chart.**

6. Conclusions

This study has focused on a multi-regional crude oil scheduling problem of the SINOPEC Maoming refinery, which is quite different from case studies reported in the literature. A hierarchical decomposition strategy for solving large scale problems was presented. We proposed a general two-stage decomposition scheme that generates smaller tractable subproblems. Based on a uniform discrete-time representation, the upper level model allocates different types of crude oil among multiple regions to obtain the pipeline transportation plan and the CDU processing plan. The lower level sub-models are solved next to obtain detailed schedules within each region. In order to further improve the solution efficiency, we further decompose the upper level model into the pipeline transportation plan model and the CDU processing plan model. A rule-based tank allocation algorithm was adopted to obtain near-optimal schedules of the lower level sub-models. In a user-driven simulation environment using the Aspen ORION system, a schedule was obtained by experienced schedulers in several hours. In contrast, the proposed method provided the refinery with near-optimal, flexible, and more implementable scheduling results in minutes. Hence, the scheduler can focus on high-level decisions and leave the generation of detailed schedules to computers.

Acknowledgments

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Appendices

Appendix A The upper level model

We now present the upper level model using the decomposition framework shown in Fig. 3.

A.1 Nomenclature

Indices and Sets	
B	Set of tank b at terminal B
C	Set of crude c
C_L	Set of light crude c_L, $C_L \subseteq C$
C_H	Set of heavy crude c_H, $C_H \subseteq C$
C_u	Set of crude that can be processed by CDU u
M	Set of tank m at refinery plant M
P_A	Set of parcel p_A carried on vessels arriving at Jetty A of terminal Z
P_B	Set of parcel p_B carried on vessels arriving at Jetty B of terminal Z
P_C	Set of parcel p_C carried on vessels arriving at terminal B
P_AC	Set of (p_A, c) satisfying that parcel p_A carries crude c; P_BC and P_CC are similar

P_AT	Set of (p_A, t) satisfying that PAT_{p_A} is no larger than t; P_BT and P_CT are similar
P_AV_A	Set of (p_A, v_A) satisfying that p_A is carried on v_A; P_BV_B and P_CV_C are similar
T	Set of period t, $t=1, 2, \ldots, NT$
U	Set of CDU u
V_A	Set of vessel v_A arriving at Jetty A of terminal Z
V_B	Set of vessel v_B arriving at Jetty B of terminal Z
V_C	Set of vessel v_C arriving at terminal B
V_AT	Set of (v_A, t) satisfying that VAT_{v_A} is no larger than t; V_BT and V_CT are similar
Z	Set of tank z at terminal Z
Parameters	
$BLrZM_{c_1,c_2}$	Proportion of c_2 to c_1 when pipeline ZM is transferring c_1 (e.g. c_1 is a blend of c_2 and some other type of crude, see Table 3 for an example)
$PROFIT_{uc}$	Unit profit of processing crude c in CDU u
$CLRECZ_{t't}$	Percentage of the crude fed to pipeline ZM by terminal Z in period t', which will be ejected back in period t, obtained by the upper level model; $CLRECM_{t't}$ is similar
${CR_u}^U$	Upper limit on the fluctuation of processing rates of CDU u in two adjacent periods
CVV_B	Unit inventory cost of crude carried on vessels arriving at terminal B
$FBM^{L/U}$	Lower and upper limit of the amount of crude transferred each period if in pipeline BM is transferring
$FCIBM_{ct}$	Amount of crude c that must be fed to pipeline BM by tank yard at terminal B during period t and will arrive at Refinery M, obtained by the upper level model; $FCIMZ_{ct}$, $FCIZM_{ct}$, $FCOBM_{ct}$, $FCOMZ_{ct}$, and $FCOZM_{ct}$ are similar
$FCLZM$	Amount of light oil fed to pipeline ZM each period when forwardly cleaning pipeline ZM; $FCLMZ$ is similar
FCU_{uct}	Amount of crude c fed to CDU u during period t, obtained from the upper level model
${FIBM_b}^{L/U}$	Lower and upper limit of the amount of crude fed each period by tank b if b connects to pipeline BM
${FIMZ_m}^{L/U}$	Lower and upper limit of the amount of crude fed each period by tank m if m connects to pipeline ZM
${FIZM_z}^{L/U}$	Lower and upper limit of the amount of crude fed each period by tank z if z connects to pipeline ZM
$FMZ^{L/U}$	Lower and upper limit of the amount of crude transferred each period if in pipeline ZM is reversely transferring
${FOBM_m}^{L/U}$	Lower and upper limit of the amount of crude received each period by tank m if m connects to pipeline BM
${FOMZ_z}^{L/U}$	Lower and upper limit of the amount of crude received each period by tank z if z connects to pipeline ZM
${FOZM_m}^{L/U}$	Lower and upper limit of the amount of crude received each period by tank m if m connects to pipeline ZM
${FPT_A}^{L/U}$	Lower and upper limit of the amount of crude unloaded each period if Jetty A at terminal Z is unloading; ${FPT_B}^{L/U}$ and ${FPT_C}^{L/U}$ are similar

${FPT_{p_A}}^U$	Upper limit of the amount of crude unloaded each period if p_A is unloading; ${FPT_{p_B}}^U$ and ${FPT_{p_C}}^U$ are similar
${FPT_b}^{L/U}$	Lower and upper limit of the amount of crude received each period by tank b if b connects to unloading line; ${FPT_z}^{L/U}$ and similar
${FTU_{mu}}^{L/U}$	Lower and upper limit of the amount of crude fed to CDU u by tank m each period if m connects to u
${FU_u}^{L/U}$	Lower and upper limit of the amount of crude processed by CDU u
FU_{u0}	Amount of crude processed by CDU u in the last period of the previous scheduling horizon
$FZM^{L/U}$	Lower and upper limit of the amount of crude transferred each period if pipeline ZM is forwardly transferring
$MAXULTZ$	Maximum time lag between the arriving of a vessel at terminal and its fully unloading
NCT	Minimum number of periods during which the type of crude transferred in a long-distance pipeline does not change
$NCXT_B$	Maximum number of changeovers on each tank at terminal B; $NCXT_M$ and $NCXT_Z$ are similar
NLL	Maximum number of tanks at a tank yard connecting to a long pipeline at the same time
NPT	Maximum number of tanks at a tank yard connecting to an unloading line at the same time
NTU	Maximum number of tanks at refinery connecting to a CDU at the same time
PAT_{p_A}	Arrival time of parcel p_A, PAT_{p_B} and PAT_{p_C} are similar
PS_{p_A}	Size of parcel p_A, PS_{p_B} and PS_{p_C} are similar
SBM	Safety time lag between flow in and out of pipeline BM ($SBM < NT$)
$SCLZM$	Number of periods needed when cleaning pipeline ZM
SCT_u	Safety time lag between two changeovers on CDU u
SMZ	Safety time lag between reverse flow in and out of pipeline ZM ($SMZ < NT$)
ST_b	Safety settling time of tank b after receiving crude; ST_m and $ST_z$are similar
ST_M	Largest safety settling time of each tank at Refinery M after receiving crude; ST_Z is similar
$SULBM$	Safety transfer time of crude from any vessel just arriving at terminal B to tank yard at Refinery M; $SULZM$ is similar
SV_M	Safety inventory level for Refinery M at the end of the scheduling horizon
SV_u	Lower limit on the total inventory of crude which are suitable for processing in CDU u at Refinery M at the end of the scheduling horizon
$SVLC_Z$	Safety inventory of light crude for terminal Z at the end of the scheduling horizon
SZM	Safety time lag between forward flow in and out of pipeline ZM ($SZM < NT$)
VAT_{v_A}	Arrival time of vessel v_A; VAT_{v_B} and VAT_{v_C} are similar
VCT_{bc}	Initial inventory of crude c in tank b; VCT_{mc} and VCT_{zc} are similar
VS_{v_A}	Size of vessel v_A; VS_{v_B} and VS_{v_C} are similar
${VT_b}^{L/U}$	Lower and upper limit on the inventory level of tank b; ${VT_m}^{L/U}$ and ${VT_z}^{L/U}$ are similar
VTr	Upper limit on the utilization ratio of the total storage capacity of Refinery M
Variables	

Binary variables

xbm_t	If refinery M is receiving crude from pipeline BM during period t and the crude comes from terminal B, , then it equals 1, otherwise 0; xzm_t (forwardly) and xmz_t (reversely) are similar
$xfcimz_{ct}$	If refinery M is feeding crude c reversely to pipeline ZM during period t in order to transfer it to terminal Z, then it equals 1, otherwise 0; $xfcobm_{ct}$ and $xfcozm_{ct}$ are similar
$xfctu_{uct}$	If refinery M is feeding crude c to CDU u, then it equals 1, otherwise 0
$xroizm_t$	If during the latest transferring period before t (including t), pipeline ZM is forwardly transferring, then it equals 1, otherwise 0
0-1 continuous variables	
$cxtu_{ut}$	If there is a changeover on CDU u at the start of period t, then it equals 1, otherwise 0
$cxfcimz_t$	If the type of crude fed to pipeline ZM by refinery M changes at the start of period t, then it equals 1, otherwise 0; $cxfcobm_t$ and $cxfcozm_t$ are similar
$cxfcu_{ut}$	If the type of crude processed in CDU u changes at the start of period t, then it equals 1
$cxfimz_t$	If refinery M starts or stops to feed crude to pipeline ZM at the start of period t, then it equals 1, otherwise 0; $cxfobm_t$ and $cxfozm_t$ are similar
Continuous variables	
$fcibm_{ct}$	Total amount of crude c fed by tank yard at terminal B to pipeline BM during period t; $fcimz_{ct}$, $fcizm_{ct}$, $fcobm_{ct}$, $fcomz_{ct}$, and $fcozm_{ct}$ are similar
fcu_{uct}	Total amount of crude c fed to CDU u during period t
$fibm_t$	Total amount of crude fed by tank yard at terminal B to pipeline BM during period t; $fimz_t$, $fizm_t$, $fobm_t$, $fomz_t$, and $fozm_t$ are similar
fu_{ut}	Total amount of crude fed to CDU u during period t
$rtimz_t$	Latest period before t (including t) at which refinery M is reversely feeding crude to pipeline ZM in order to transfer it to terminal Z
$rtozm_t$	Latest period before t (including t) at which refinery M is receiving crude which comes from terminal Z from pipeline ZM
$vctm_{ct}$	Total inventory of crude c in tank yard at refinery M at the end of period t

A.2 Model

In the upper level model, the scheduling objective is to maximize the crude processing profit and to minimize changeovers of two pipelines. The coefficient C_{PIPESET} in equation (A-2) is the estimated cost per pipeline changeover.

$$\text{minimize} \sum_{u\in U}\sum_{c\in C}\sum_{t\in T} PROFIT_{uc}\cdot fcu_{uct} + C_{\text{PIPESET}}\sum_{t\in T}(cxfozm_t + cxfobm_t) \tag{A-2}$$

Constraints of the upper level model are listed below.

1. To insure pipeline transportation stability, once a new type of crude started to transfer in pipeline ZM or pipeline BM, the transfer operation of this type of crude should last for at least NCT periods. If the transfer state (i.e., forward transfer, reverse transfer, and stoppage) of pipeline ZM (BM) changed at the beginning of period t, then $cxfozm_t$ ($cxfobm_t$) equals to 1.

$$\sum_{k=0}^{\min\{NCT,NT-t\}} cxfcozm_{t+k} \leq 1 \qquad t \in T \qquad (A\text{-}3a)$$

$$\sum_{k=0}^{\min\{NCT,NT-t\}} cxfcobm_{t+k} \leq 1 \qquad t \in T \qquad (A\text{-}3b)$$

$$\sum_{k=0}^{\min\{NCT,NT-t\}} cxfcimz_{t+k} \leq 1 \qquad t \in T \qquad (A\text{-}3c)$$

2. For feeding crude to CDUs, continuous variable fcu_{uct} is defined as the amount of crude c delivered to CDU u during time period t. It follow that fu_{ut}, the total amount of crude feeds to CDU u during time period t should be within the processing limits of CDU u.

$$fu_{ut} = \sum_{c \in C} fcu_{uct} \qquad u \in U,\ t \in T \qquad (A\text{-}4a)$$

$$FU_u{}^L \leq fu_{ut} \leq FU_u{}^U \qquad u \in U,\ t \in T \qquad (A\text{-}4b)$$

3. To maintain smooth production state, the total amount feeds to CDU u during each time period is allowed to vary within the limits $(\pm CR_u{}^U)$ from the previous period. The parameter FU_{u0} is the total feed to CDU u during the last period of the previous scheduling horizon.

$$\left(1 - CR_u{}^U\right) fu_{ut} \leq fu_{u(t+1)} \leq \left(1 + CR_u{}^U\right) fu_{ut} \qquad u \in U,\ 1 < t < NT \qquad (A\text{-}5a)$$

$$\left(1 - CR_u{}^U\right) FU_{u0} \leq fu_{u1} \leq \left(1 + CR_u{}^U\right) FU_{u0} \qquad u \in U \qquad (A\text{-}5b)$$

4. During each time period, CDU u is allowed to process only one type of crude. Binary variable $xfctu_{uct}$ being equal to 1 represents that CDU u processes crude type c during time period t.

$$FU_u{}^U \cdot xfctu_{uct} \geq fcu_{uct} \qquad u \in U,\ c \in C,\ t \in T \qquad (A\text{-}6a)$$

$$\sum_{c \in C} xfctu_{uct} \leq 1 \qquad u \in U,\ t \in T \qquad (A\text{-}6b)$$

5. The following constraints require that the processing of a type of crude should last for at least SCT_u time periods in CDU u before switching to another type. We further constrain that the processing rate cannot change when processing the same type of crude to stabilize the production process.

$$cxfcu_{ut} \geq xfctu_{uct} - xfctu_{uc(t-1)} \qquad u \in U,\ c \in C,\ t > 1 \qquad (A\text{-}7a)$$

$$cxfcu_{ut} \geq xfctu_{uc(t-1)} - xfctu_{uct} \qquad u \in U,\ c \in C,\ t > 1 \qquad (A\text{-}7b)$$

$$cxfcu_{u1} = 1 \qquad u \in U \qquad (A\text{-}7c)$$

$$\sum_{k=0}^{\min\{SCT_u - 1, NT-t\}} cxfcu_{u(t+k)} \leq 1 \qquad u \in U,\ t \in T \qquad (A\text{-}7d)$$

$$fu_{ut} - fu_{u(t-1)} \leq FU_u{}^U \cdot cxfcu_{ut} \qquad u \in U,\ t > 1 \tag{A-8a}$$

$$fu_{u(t-1)} - fu_{ut} \leq FU_u{}^U \cdot cxfcu_{ut} \qquad u \in U,\ t > 1 \tag{A-8b}$$

6. Before the end of period t, the total amount of crude in the tank yard at refinery M cannot exceed the total maximum available storage capacity of all tanks. Parameter VTr denoting the maximal utilization of the overall storage capacity at refinery M is adopted to ensure feasibility of the lower level problem. Parameter VCT_{mc} is the initial inventory of crude type c in tank m.

$$vctm_{ct} = \sum_{m \in M} VCT_{mc} + \sum_{k=1}^{t} \left(fcozm_{ck} + fcobm_{ck} - fcimz_{ck} - \sum_{u \in U} fcu_{uck} \right) \quad c \in C,\ t \in T \tag{A-9}$$

$$\sum_{c \in C} vctm_{ct} \leq VTr \cdot \sum_{m \in M} VT_m{}^U \qquad t \in T \tag{A-10}$$

7. Practically, the accumulated inventory for crude type c at refinery M at the end of time period t, defined as the left hand side (LHS) of (A-11), should be above zero. For production robustness, it is further required that the accumulated inventory at the end of each time period should be sufficient to make sure that the processing in CDU u last for another SCT_u time periods without crude refill. Accordingly, the summation of item $\sum_{u \in U} fcu_{ucs}$ of the right hand side (RHS) of (A-11) is from 1 to $t + SCT_u - 1$. Since each tank needs some time (ST_M) for brine settling and removal after receiving crude in the detailed lower level decisions, both items $fcozm_{ck}$ and $fcobm_{ck}$ are summed from 1 to $t - ST_M$.

$$\sum_{m \in M} VCT_{mc} + \sum_{k=1}^{t-ST_M} (fcozm_{ck} + fcobm_{ck}) - \sum_{j=1}^{t} fcimz_{cj} \geq \sum_{s=1}^{\min\{t+SCT_u-1,\ NT\}} \sum_{u \in U} fcu_{ucs} \qquad c \in C,\ t > ST_M \tag{A-11}$$

$$\sum_{m \in M} VCT_{mc} - \sum_{j=1}^{t} fcimz_{cj} \geq \sum_{s=1}^{\min\{t+SCT_u-1,\ NT\}} \sum_{u \in U} fcu_{ucs} \quad c \in C,\ t \leq ST_M \tag{A-12}$$

8. Similar to the refinery setting, before the end of time period t, the total amount of each type of crude received from terminal Z (terminal B) should not exceed the maximum available supply at terminal Z (terminal B). For terminal Z, an extra constraint is required to guarantee that

the total amount of light crude transferred reversely to terminal Z should be enough to ensure the safety transfer of high pour point, high viscosity, and high sulfur crude in pipeline ZM.

$$\sum_{c'\in C}\sum_{k=1}^{t} fcozm_{c'k}\cdot BLrZM_{c'c} \le \sum_{z\in Z} VCT_{zc} + \sum_{\substack{(p_A,c)\in P_AC,\\ (p_A,t-SULZM)\in P_AT}} PS_{p_A} + \sum_{\substack{(p_B,c)\in P_BC,\\ (p_B,t-SULZM)\in P_BT}} PS_{p_B} + \sum_{s=1}^{t-SMZ-ST_Z} fcimz_{cs} \quad c\in C,\ t > SMZ+ST_Z \tag{A-13}$$

$$\sum_{c'\in C}\sum_{k=1}^{t} fcozm_{c'k}\cdot BLrZM_{c'c} \le \sum_{z\in Z} VCT_{zc} + \sum_{\substack{(p_A,c)\in P_AC,\\ (p_A,t-SULZM)\in P_AT}} PS_{p_A} + \sum_{\substack{(p_B,c)\in P_BC,\\ (p_B,t-SULZM)\in P_BT}} PS_{p_B}, \quad c\in C,\ t \le SMZ+ST_Z \tag{A-14}$$

$$\sum_{k=1}^{t} fcobm_{ck} \le \sum_{b\in B} VCT_{bc} + \sum_{\substack{(p_C,c)\in P_CC,\\ (p_C,t-SULBM)\in P_CT}} PS_{p_C} \quad c\in C,\ t\in T \tag{A-15}$$

Since terminal Z may receive light crude from refinery M while terminal B does not, the extra term $BLrZM_{c'c}$ in constraints (*A*-13) and (*A*-14) is not included in constraint (*A*-15). Parameter $BLrZM_{c'c}$ is the specified proportion of the amount of crude type c blending with crude type c' when transferring crude c'. If c' and c are the same type of crude, then $BLrZM_{c'c}$ equals to 1. If c' is not light crude and c is light crude, then $BLrZM_{c'c}$ equals to the specified proportion of the amount of c blending with the amount of c' to guarantee transportation safety. In other cases, $BLrZM_{c'c}$ equals to 0. For crude type c which is *pure*, parameters VCT_{zc}, VCT_{bc}, PS_{p_A}, PS_{p_B}, and PS_{p_C} are the initial crude inventory levels or parcel sizes at terminal Z or terminal B. For crude type c which is a *blend*, parameters VCT_{zc}, VCT_{bc}, PS_{p_A}, PS_{p_B}, and PS_{p_C} are computed as adjusted crude inventory levels or adjusted parcel sizes by taking into account the proportion of each pure type of crude in the blend. A typical instance of parameter $BLrZM_{c'c}$ is listed in Table 3, where crude type SUD_XTJ is obtained by blending 70% of crude SUD with 30% of crude XTJ.

9. Before the end of period t, the total inventory level of light crude at terminal Z should be above the safety inventory for cleaning pipeline ZM. There are three cases in which we need to clean pipeline ZM: (1) forward transfer - stop - forward transfer, (2) forward transfer - reverse transfer, and (3) forward transfer – stop until the end of the scheduling horizon. Since pipeline ZM is allowed to transfer reversely at most once during the scheduling horizon, case (3) can

also happen at most once. We set the safety inventory level for cleaning pipeline ZM to be the amount of light crude ($SCLZM \cdot FCLZM$) for cleaning once. Besides, at the end of the scheduling horizon, the total inventory level of light crude at terminal Z cannot be lower than the safety inventory of light crude ($SVLC_Z$) for terminal Z (light crude unloaded at terminal Z is omitted).

$$\begin{aligned}\sum_{c\in C_L}\sum_{k=1}^{t} fcimz_{ck} \geq \sum_{c\in C_L}\sum_{c'\in C}\sum_{s=1}^{t} fcozm_{c's} BLrZM_{c'c}\\ -\sum_{c\in C_L}\sum_{z\in Z} VCT_{zc} + SCLZM\cdot FCLZM \qquad t\in T\end{aligned} \tag{A-16}$$

$$\sum_{c\in C_L}\sum_{t\in T}\left(fcimz_{ct} - \sum_{c'\in C} fcozm_{c't}\cdot BLrZM_{c'c}\right) + \sum_{z\in Z}\sum_{c\in C_L} VCT_{zc} \geq SVLC_Z \tag{A-17}$$

10. At the end of the scheduling horizon, the total inventory level of the types of crude for CDU u cannot be below than the safety inventory SV_u, and the total inventory level of all types of crude cannot be under the total safety inventory SV_M.

$$\sum_{c\in C_U} vctm_{cNT} \geq SV_u \qquad u\in U \tag{A-18a}$$

$$\sum_{c\in C} vctm_{cNT} \geq SV_M \tag{A-18b}$$

11. Pipeline ZM can transfer forwardly at most one type of crude, pure or blended, during period t, and the total amount of crude forwardly transferred during period t must be within the transfer limits.

$$FZM^U \cdot xfcozm_{ct} \geq fcozm_{ct} \qquad c\in C, t\in T \tag{A-19a}$$

$$\sum_{c\in C} xfcozm_{ct} \leq 1 \qquad t\in T \tag{A-19b}$$

$$fozm_t = \sum_{c\in C} fcozm_{ct} \qquad t\in T \tag{A-19c}$$

$$FZM^L \cdot xzm_t \leq fozm_t \leq FZM^U \cdot xzm_t \qquad t\in T \tag{A-19d}$$

12. Pipeline BM can transfer at most one type of crude during period t, and the total amount of crude transferred during period t must be within the transfer limits.

$$fobm_t = \sum_{c\in C} fcobm_{ct} \qquad t\in T \tag{A-20a}$$

$$FBM^L \cdot xbm_t \leq fobm_t \leq FBM^U \cdot xbm_t \qquad t\in T \tag{A-20b}$$

$$FBM^U \cdot xfcobm_{ct} \geq fcobm_{ct} \qquad c\in C, t\in T \tag{A-20c}$$

$$\sum_{c\in C} xfcobm_{ct} \leq 1 \qquad t\in T \tag{A-20d}$$

13. Pipeline ZM can transfer reversely at most one type of light crude during period t, and the total amount of crude reversely transferred during period t must be within the transfer limits.

$$fimz_t = \sum_{c \in C_L} fcimz_{ct} \qquad t \in T \qquad (A\text{-}21a)$$

$$FMZ^L \cdot xmz_t \leq fimz_t \leq FMZ^U \cdot xmz_t \qquad t \in T \qquad (A\text{-}21b)$$

$$FIMZ^U \cdot xfcimz_{ct} \geq fcimz_{ct} \qquad c \in C, t \in T \qquad (A\text{-}21c)$$

$$\sum_{c \in C_L} xfcimz_{ct} \leq 1 \qquad t \in T \qquad (A\text{-}21d)$$

$$\sum_{c \in C_H} xfcimz_{ct} = 0 \qquad t \in T \qquad (A\text{-}21e)$$

14. Continuity constraints to assure the safe transportation of pipelines: When pipeline ZM (BM) transfers continuously, the total amount of crude transferred during each period should be the same. That is, if both xzm_t and xzm_{t-1} equal to 1, then $fozm_t$ equals $fozm_{t-1}$.

$$fozm_t - fozm_{t-1} \leq FZM^U \cdot (2 - xzm_t - xzm_{t-1}) \qquad t > 1 \qquad (A\text{-}22a)$$

$$fozm_{t-1} - fozm_t \leq FZM^U \cdot (2 - xzm_t - xzm_{t-1}) \qquad t > 1 \qquad (A\text{-}22b)$$

$$fobm_t - fobm_{t-1} \leq FBM^U \cdot (2 - xbm_t - xbm_{t-1}) \qquad t > 1 \qquad (A\text{-}22c)$$

$$fobm_{t-1} - fobm_t \leq FBM^U \cdot (2 - xbm_t - xbm_{t-1}) \qquad t > 1 \qquad (A\text{-}22d)$$

$$fimz_t - fimz_{t-1} \leq FMZ^U \cdot (2 - xmz_t - xmz_{t-1}) \qquad t > 1 \qquad (A\text{-}22e)$$

$$fimz_{t-1} - fimz_t \leq FMZ^U \cdot (2 - xmz_t - xmz_{t-1}) \qquad t > 1 \qquad (A\text{-}22f)$$

15. If refinery M is feeding pipeline ZM during period t, then refinery M cannot receive any crude from terminal Z via pipeline ZM during periods $t+1$, $t+2$, $\cdots$, and $t+SMZ+SZM$. This constraint is used to approximate the bi-directional feature of pipeline ZM instead of directly model the movement of flows inside it.

$$(SMZ + SZM) \cdot xmz_t + \sum_{k=0}^{\min\{SMZ+SZM, NT-t\}} xzm_{(t+k)} \leq SMZ + SZM \qquad t \in T \qquad (A\text{-}23)$$

16. Variables $cxfozm_t$, $cxfobm_t$, $cxfcozm_t$, $cxfcobm_t$, and $cxfcimz_t$ are defined as in the following examples. If xzm_t does not equal to xzm_{t-1} (pipeline ZM changes the transfer state from forward transfer to stoppage (or reverse transfer), or from stoppage (or reverse transfer) to forward transfer, at the beginning of period t), then $cxfozm_t \geq 1$. If $xfcozm_{ct}$ does not equal to $xfcozm_{c(t-1)}$ for any c (pipeline ZM changes the type of crude forwardly transferring at the beginning of period t), then $cxfcozm_t \geq 1$.

$$cxfozm_t \geq xzm_t - xzm_{t-1} \qquad t > 1 \qquad (A\text{-}24a)$$

$$cxfozm_t \geq xzm_{t-1} - xzm_t \qquad t>1 \qquad \text{(A-24b)}$$

$$cxfozm_1 = 0 \qquad \text{(A-24c)}$$

$$cxfobm_t \geq xbm_t - xbm_{t-1} \qquad t>1 \qquad \text{(A-24d)}$$

$$cxfobm_t \geq xbm_{t-1} - xbm_t \qquad t>1 \qquad \text{(A-24e)}$$

$$cxfobm_1 = 0 \qquad \text{(A-24f)}$$

$$cxfimz_t \geq xmz_t - xmz_{t-1} \qquad t>1 \qquad \text{(A-24g)}$$

$$cxfimz_t \geq xmz_{t-1} - xmz_t \qquad t>1 \qquad \text{(A-24h)}$$

$$cxfimz_1 = 0 \qquad \text{(A-24i)}$$

$$cxfcozm_t \geq xfcozm_{ct} - xfcozm_{c(t-1)} \qquad c\in C, t>1 \qquad \text{(A-24j)}$$

$$cxfcozm_t \geq xfcozm_{c(t-1)} - xfcozm_{ct} \qquad c\in C, t>1 \qquad \text{(A-24k)}$$

$$cxfcozm_1 = 0 \qquad \text{(A-24l)}$$

$$cxfcobm_t \geq xfcobm_{ct} - xfcobm_{c(t-1)} \qquad c\in C, t>1 \qquad \text{(A-24m)}$$

$$cxfcobm_t \geq xfcobm_{c(t-1)} - xfcobm_{ct} \qquad c\in C, t>1 \qquad \text{(A-24n)}$$

$$cxfcobm_1 = 0 \qquad \text{(A-24o)}$$

$$cxfcimz_t \geq xfcimz_{ct} - xfcimz_{c(t-1)} \qquad c\in C, t>1 \qquad \text{(A-24p)}$$

$$cxfcimz_t \geq xfcimz_{c(t-1)} - xfcimz_{ct} \qquad c\in C, t>1 \qquad \text{(A-24q)}$$

$$cxfcimz_1 = 0 \qquad \text{(A-24r)}$$

17. Pipeline ZM can reversely transfer at most once during the scheduling horizon.

$$\sum_{t\in T} cxfimz_t \leq 1 + (1 - xmz_1) \qquad \text{(A-25)}$$

18. If pipeline ZM has just switched the transfer state from stoppage (stop after forward transfer) to forward transfer, then the crude firstly ejected from pipeline ZM must be light crude. If forward transfer lasts for less than $SCLZM$ periods, then all the crude ejected from pipeline ZM must be light crude, otherwise the type of crude ejected from pipeline ZM at the first $SCLZM$ periods should be light crude.

$$\sum_{c\in C_L} fcozm_{c(t+k)} \leq FCLZM \cdot \left(xzm_{t+k} - cxfozm_{t+1} + 1\right) \qquad \text{(A-26)}$$
$$t < NT,\ k = 1, 2, \ldots, \min\{SCLZM, NT - t\}$$

$$\sum_{c\in C_L} fcozm_{c(t+k)} \geq FCLZM \cdot \left(xzm_{t+k} + cxfozm_{t+1} - 2 + xroizm_t\right) \qquad \text{(A-27)}$$
$$t < NT,\ k = 1, 2, \ldots, \min\{SCLZM, NT - t\}$$

Here $xroizm_t$ is a binary variable defined as below: if pipeline ZM is forwardly transferring during the latest period of transfer operations before period t (including t), then $xroizm_t$ equals 1, otherwise 0.

$$rtimz_1 = xmz_1 \quad (A\text{-}28a)$$
$$rtimz_t \geq rtimz_{t-1} \qquad t > 1 \quad (A\text{-}28b)$$
$$rtimz_t \leq rtimz_{t-1} + t \cdot xmz_t \qquad t > 1 \quad (A\text{-}28c)$$
$$rtimz_t \geq t \cdot xfimz_t \qquad t > 1 \quad (A\text{-}28d)$$
$$rtimz_t \leq t \qquad t > 1 \quad (A\text{-}28e)$$
$$rtozm_1 = xzm_1 \quad (A\text{-}28f)$$
$$rtozm_t \geq rtozm_{t-1} \qquad t > 1 \quad (A\text{-}28g)$$
$$rtozm_t \leq rtozm_{t-1} + t \cdot xzm_t \qquad t > 1 \quad (A\text{-}28h)$$
$$rtozm_t \geq t \cdot xzm_t \qquad t > 1 \quad (A\text{-}28i)$$
$$rtozm_t \leq t \qquad t > 1 \quad (A\text{-}28j)$$
$$NT \cdot xroizm_t \geq rtozm_t - rtimz_t \qquad t \in T \quad (A\text{-}28k)$$
$$NT \cdot (1 - xroizm_t) \geq rtimz_t - rtozm_t \qquad t \in T \quad (A\text{-}28l)$$

19. Since pipeline ZM and BM are initially non-empty in the beginning of the scheduling horizon, we have to take into account the initial holdup inside the long-distance pipelines. In light of the fact that the model always ensures a safety inventory for each CDU in refinery M, initial holdup in pipelines are not in emergent need by CDUs and we omit the initial holdup in the upper level model. Decisions about when and where the initial holdup in long-distance pipelines will be ejected are considered in lower level sub-models. However, we do have to restrain that during the first SZM (SBM) periods of the scheduling horizon, refinery M cannot receive any crude which is fed by terminal Z (B) during the scheduling horizon.

$$xzm_t = 0 \qquad t \leq SZM \quad (A\text{-}29a)$$
$$xbm_t = 0 \qquad t \leq SBM \quad (A\text{-}29b)$$

To sum up, the upper level model consists of expressions (A-2) ˜ (A-29b).

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Table 3 **Example blending proportion $BLrZM_{c'c}$ for the pure and blend type of crude**

crude	SUD	XTJ	SUD_XTJ	IRH	SAM	SAM_IRH
SUD	0	0	0.7	0	0	0
XTJ	0	0	0.3	0	0	0
SUD_XTJ	0.7	0.3	0	0	0	0
IRH	0	0	0	0	0	0.5
SAM	0	0	0	0	0	0.5
SAM_IRH	0	0	0	0.5	0.5	0

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