

An Optimization Model for Expansion Planning of Reliable Power Generation Systems

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Abstract

This paper aims to develop an optimization model for the expansion planning of reliable power generation systems, which can explicitly account for all possible failure states. To achieve this goal, we propose an optimization model that minimizes the total cost using Generalized Disjunctive Programming (GDP). Specifically, the model determines both investment decisions (number, size, location, and time of generators to install, retire, and decommission) and operation decisions (number of operating/backup generators, operating capacity, and expected power output) by imposing a penalty when the demand is not satisfied. The model is illustrated through an illustrative example, which consists of two regions with four power stations over five operating time periods.

Keywords: Reliability, Expansion Planning, Power systems, Optimization

1. Introduction

Expansion planning for power generation systems requires selecting the optimal number, size, location, and time to install and/or retire the power generators, while minimizing the total cost over a given time horizon (Conejo et al., 2016). Several expansion planning works have been reported. For instance, Lara et al. (2018) develop a multi-region and multi-period MILP optimization model by considering both short-term operation problems (e.g., unit commitment and ramping up/down) and investment decisions. Given the large size of the model it is solved with a Nested Benders decomposition method. Further, Lara et al. (2020) propose a multistage stochastic mixed-integer model using Stochastic Dual Dynamic Integer Programming (SDDiP) to handle power demand uncertainty. Li et al. (2022) extend the expansion model developed by Lara et al. (2018) by integrating generation and transmission in the expansion planning problem. Although studies on integrating the reliability and expansion planning have been reported (Moreira et al., 2017), there is still modest research that comprehensively accounts for the impact of reliability in expansion planning.

Reliability is the probability that a system or component can perform its required function without failure for a given time (Sherwin & Bossche, 1993). Securing high reliability in design and operation of power systems is extremely important since the goal of power generation systems is to provide uninterrupted electric power to customers. One method to improve reliability at the design phase is to add redundant or backup units, which allows the systems to operate even if one or multiple generators fail (Kim & Kim, 2017). This approach is known as ‘reliability-based design optimization,’ and various studies on this topic have been reported (Ortiz-Espinoza et al., 2021; Ye et al., 2018). However, since power systems operate dynamically due to time-varying power demand, reliability is also influenced by the operational strategies that power systems use to satisfy the load

demand. Specifically, backup units have a dual role in power generation systems. They can remain as backup units in case of low power demand, or else change to operating units when the power demand is high. This dual purpose of backup units depending on the load demand must be considered in the design of reliable power generation systems.

One of the conventional methods used to evaluate the reliability of power systems is “ $N-1$ reliability”. The $N-1$ reliability assumes that a power system can withstand an unexpected failure of a single component (Ballireddy & Modi, 2019). This implies that power systems may not function properly if multiple units fail simultaneously. The failures of multiple generators may reduce the power output but not necessarily fail the entire system. Hence, rigorous method anticipating every possible failure state and selecting the proper number and size of the backup generators should be developed for the design and planning of reliable power generation systems. The specific goal of this work is to develop an optimization model for the expansion planning of reliable power generation systems. The model is formulated using Generalized Disjunctive Programming (GDP) that determines both investment (number, size, location, and time of generators to install, retire, and decommission) and operation decisions (number of operating/backup generators, operating capacity, and expected power production level) to minimize the total cost including unmet demand penalty.

2. Problem statement

Given are regions $r \in R$, with power stations $k \in K_r$, parallel generators $j \in J_k$, and discretized capacities of generator $p \in P_k$. There are two types of power stations: existing ($k \in K_r^{EX}$) and potential ($k \in K_r^{NW}$) power stations. A set of power station designs $h \in H_k$ and corresponding operation modes $m \in M_{k,h}$, time periods (year) $t \in T$, and sub-periods (season) in each year $n \in N$ are also given. Specifically, $h = 1$ indicates that one generator is available in power stations k and $h = H$ means all generators that can be installed in power stations k are available. Likewise, $m = 1$ represents one generator is operated, $m = M$ refers to the mode in which all generators are operated. Each power station k has different failure states $s \in S_{m,k,h}$ depending on the design h and operation mode m . The failure states can also be classified into successful operation states ($S_{m,k,h}^F$) and partial operation states ($S_{m,k,h}^P$). ‘Successful operation states’ indicate the operation modes in which the power generation capacity is sufficient to satisfy the load demand, whereas ‘Partial operation states’ refer to the operation modes in which the power generation capacity is insufficient to meet the load demand as it can only produce electric power at a limited level. The major assumptions in this model are: (i) Each power station has maximum available power generators, (ii) Storage systems are not included, (iii) Detailed operation problems such as unit commitment and ramping up/down are not included.

3. Model formulation

The model is developed Generalized Disjunctive Programming (GDP) (Trespalcios and Grossmann, 2014), which can be expressed in terms of Boolean and continuous variables, algebraic constraints, disjunctions, and logic propositions. We also introduce binary variables since several investment decisions are reformulated as algebraic constraints.

3.1. Investment constraints

The binary variable $y_{j,p,k,r}^{EX}$ indicates the existence of unit j with discrete capacity p in power station k of region r . Likewise, the binary variables $y_{j,p,k,r,t}^{IN}$ and $y_{j,p,k,r,t}^{AV}$ state the

installation and availability of unit j in year t , respectively. $y_{j,p,k,r,t}^{ED}$, $y_{j,p,k,r,t}^{RT}$, and $y_{j,p,k,r,t}^{LE}$ are the binary variables indicating early decommission, retirement, and lifetime extension, respectively. The binary variables are also represented with corresponding Boolean variables ($Y_{j,p,k,r,t}^{IN}$, $Y_{j,p,k,r,t}^{AV}$, $Y_{j,p,k,r,t}^{ED}$, $Y_{j,p,k,r,t}^{RT}$, $Y_{j,p,k,r,t}^{LE}$). $IC_{j,k,r,t}$, $AC_{j,k,r,t}$, $TAC_{k,r,t}$ are installed capacity, available capacity, and total available capacity, respectively. While new generators can be installed within a given time horizon, there are three different investment decisions for the generators already installed in existing power stations: (i) early decommission, (ii) retirement due to lifetime expiration, (iii) lifetime extension. Eqns. (6) – (8) correspond to the capacity constraints, and Eqn. (9) states that only one capacity p can be selected for each generator.

$$\begin{aligned}
 & \sum_{p \in P_k} y_{j,p,k,r,t}^{EX} + \sum_{p \in P_k} \sum_{t \in T} y_{j,p,k,r,t}^{IN} \leq 1 \quad \forall j \in J_k, k \in K_r, r \in R \quad \text{1) Generator installation} \\
 & \left. \begin{aligned}
 Y_{j,p,k,r,t}^{IN} & \Rightarrow \bigwedge_{q \in [t, t+LT_{j,k,r}]} Y_{j,p,k,r,q}^{AV} \quad \forall j \in J_k, p \in P_k, k \in K_r^{NW}, r \in R, t, q \in T \\
 Y_{j,p,k,r,t}^{AV} & \Rightarrow \bigvee_{q \in [t-LT_{j,k,r}, t]} Y_{j,p,k,r,q}^{IN} \quad \forall j \in J_k, p \in P_k, k \in K_r^{NW}, r \in R, t, q \in T
 \end{aligned} \right\} \text{2) Generator in potential power stations} \\
 & y_{j,p,k,r,t}^{AV} = 1 \quad \forall (j, p, k, r) \in Pre, t = 1 \quad \text{3) Generator in existing power stations} \\
 & \left. \begin{aligned}
 y_{j,p,k,r,t-1}^{AV} & = y_{j,p,k,r,t}^{AV} + y_{j,p,k,r,t}^{ED} \quad \forall (j, p, k, r) \in Pre, 1 < t \leq RT_{j,k,r} \\
 Y_{j,p,k,r,t}^{ED} & \Rightarrow \bigwedge_{q \in [t, |T|]} \neg Y_{j,p,k,r,q}^{AV} \quad \forall (j, p, k, r) \in Pre, 1 < t \leq RT_{j,k,r}
 \end{aligned} \right\} \text{4) Generator can be available or early decommissioned until its lifetime expires} \\
 & \left. \begin{aligned}
 y_{j,p,k,r,t-1}^{AV} & = y_{j,p,k,r,t}^{RT} + y_{j,p,k,r,t}^{LE} \quad \forall (j, p, k, r) \in Pre, t = RT_{j,k,r} + 1 \\
 Y_{j,p,k,r,t}^{RT} & \Rightarrow \bigwedge_{q \in [t, |T|]} \neg Y_{j,p,k,r,q}^{AV} \quad \forall (j, p, k, r) \in Pre, t = RT_{j,k,r} + 1 \\
 Y_{j,p,k,r,t}^{LE} & \Rightarrow \bigwedge_{q=t} Y_{j,p,k,r,q}^{AV} \quad \forall (j, p, k, r) \in Pre, t = RT_{j,k,r} + 1
 \end{aligned} \right\} \text{5) Generator that reaches its lifetime can be retired or its lifetime can be extended} \\
 & IC_{j,k,r,t} = \sum_{p \in P_k} \delta_{k,p} y_{j,p,k,r,t}^{IN} \quad \forall j \in J_k, k \in K_r, r \in R, t \in T \quad \text{(6)} \quad AC_{j,k,r,t} = \sum_{p \in P_k} \delta_{k,p} y_{j,p,k,r,t}^{AV} \quad \forall j \in J_k, k \in K_r, r \in R, t \in T \quad \text{(7)} \\
 & TAC_{k,r,t} = \sum_{j \in J_k} AC_{j,k,r,t} \quad \forall k \in K_r, r \in R, t \in T \quad \text{(8)} \quad \sum_{p \in P_k} y_{j,p,k,r,t}^{AV} \leq 1 \quad \forall j \in J_k, k \in K_r, r \in R, t \in T \quad \text{(9)}
 \end{aligned}$$

3.2. Operation and system reliability constraints

There are two Boolean variables related to investment and operation decisions. $Z_{k,r,h,t}$ is true if design h is selected for power station k of region r in time t (Eqn. (a)). $W_{k,r,m,h,n,t}$ is true if power station k of region r is in operation mode m during sub-period n in time t for design h (Eqn. (b)). The binary variable $x_{j,p,k,r,n,t}$ indicates the operation of unit j with discrete capacity p in power station k of region r during sub-period n of time t . $OC_{j,k,r,n,t}$ represents an operating capacity, $P_{s,k,r,n,t}$ is the probability of station k of region r being in state s during sub-period n of time t , and $EP_{s,k,r,n,t}$ corresponds to the expected power output. $P_{k,r,n,t}^F$ and $P_{k,r,n,t}^P$ are the probabilities of station k being in successful and partial operations. $TEP_{k,r,n,t}$ is the summation of expected productions in all failure cases s (Eqn. (15)). $\lambda_{j,k}$ is the reliability of unit j in stage k and $\gamma_{j,k}$ defined in Eqn. (16), states that when the unit j belongs to set of backup units in failure state s under design h and operation mode m ($J_{s,m,h}^B$), the unit reliability ($\lambda_{j,k}$) will be 1 since the unit j does not

generate power in this failure state s . If the unit j belongs to set of failed units ($J_{s,m,h}^F$), the unit unreliability ($1 - \lambda_{j,k}$) will be used to calculate the system availability.

$$\begin{array}{l}
 \bigvee_{h \in H_k} \bigvee_{m \in M_{k,h}} \left[\begin{array}{l}
 \sum_{j \in J_k} \sum_{p \in P_k} Z_{k,r,h,t} y_{j,p,k,r,t}^{AV} = i \quad i = 0, 1, \dots, h \\
 \sum_{j \in J_k} \sum_{p \in P_k} W_{k,r,m,h,n,t} x_{j,p,k,r,n,t} = i \quad i = 1, \dots, m \\
 \sum_{p \in P_k} \varphi_j \delta_{k,p} x_{j,p,k,r,n,t} \leq OC_{j,k,r,n,t} \quad \forall j \in J_k \\
 OC_{j,k,r,n,t} \leq \sum_{p \in P_k} \chi_{j,k,r,n,t} \delta_{k,p} x_{j,p,k,r,n,t} \quad \forall j \in J_k \\
 P_{k,r,n,t}^F = \sum_{s \in S_{m,h}^F} \left\{ \prod_{j \in J_{s,m,h}^O} \lambda_{j,k} \prod_{j \in J_{s,m,h}^N} \gamma_{j,k} \right\} \\
 P_{k,r,n,t}^P = \sum_{s \in S_{m,h}^P} \left\{ \prod_{j \in J_{s,m,h}^O} \lambda_{j,k} \prod_{j \in J_{s,m,h}^N} \gamma_{j,k} \right\} \\
 EP_{s,k,r,n,t} = \sum_{j \in J_{s,m,h}^O} \rho_n OC_{j,k,r,n,t} P_{s,k,r,n,t} \quad s = 1, s \in S_{m,k,h} \\
 EP_{s,k,r,n,t} = \sum_{j \in J_{s,m,h}^N} \rho_n \chi_{j,k,r,n,t} AC_{j,k,r,n,t} P_{s,k,r,n,t} \quad s \neq 1, s \in S_{m,k,h}
 \end{array} \right] \quad \forall n \in N
 \end{array}
 \quad \left. \begin{array}{l}
 \text{a) Investment decision} \\
 \text{10) Number of available unit} \\
 \text{b) Operation decision} \\
 \text{11) Number of operating unit} \\
 \text{12) Operating capacity of available unit} \\
 \forall k \in K_r^{TH}, r \in R, t \in T \\
 \text{13) Successful and partial operation reliability} \\
 \text{14) Expected production by successful and partial operation}
 \end{array} \right\}
 \end{array}
 \quad (15)$$

$$TEP_{k,r,n,t} = \sum_{s \in S_{m,k,h}} EP_{s,k,r,n,t} \quad (15) \quad \left\{ \begin{array}{l}
 \gamma_{j,k} = 1 \quad \forall j \in J_{s,m,h}^B, k \in K_r^{TH} \quad J_{s,m,h}^B \cup J_{s,m,h}^F = J_{s,m,h}^N \quad \forall j \in J_k \\
 \gamma_{j,k} = (1 - \lambda_{j,k}) \quad \forall j \in J_{s,m,h}^F, k \in K_r^{TH} \quad J_{s,m,h}^O \cap J_{s,m,h}^N = \emptyset \quad \forall j \in J_k
 \end{array} \right. \quad (16)$$

The objective function is to minimize the total cost (Eqn. (17)), which includes the investment costs ($\theta_{j,p} y_{j,p,k,r,t}^{IN}$), the start-up costs ($\varphi_{j,p} x_{j,p,k,r,n,t}$), the expected fuel costs to purchase natural gas or coal ($\kappa_k FS_{k,r,n,t}$), the downtime penalty ($\varepsilon DT_{k,r,n,t}$), and the unmet demand penalty ($PN_{n,t}$). The unmet demand penalty is determined with the disjunction shown in Eqn. (18).

$$\min COST = \sum_{j \in J_k} \sum_{p \in P_k} \sum_{k \in K_r} \sum_{r \in R} \sum_{n \in N} \sum_{t \in T} (\theta_{j,p} y_{j,p,k,r,t}^{IN} + \varphi_{j,p} x_{j,p,k,r,n,t} + \kappa_k FS_{k,r,n,t} + \varepsilon DT_{k,r,n,t} + PN_{n,t}) \quad (17)$$

$$\left[\begin{array}{l}
 \sum_{k \in K_r} \sum_{r \in R} TEP_{k,r,n,t} < \sum_{r \in R} \rho_n D_{r,n,t} (1 + R_t^{min}) \\
 PN_{n,t} = (\sum_{r \in R} D_{r,n,t} - \sum_{r \in R} TEP_{r,n,t}) \alpha
 \end{array} \right] \vee \left[\begin{array}{l}
 \sum_{r \in R} TEP_{r,n,t} \geq \sum_{r \in R} \rho_n D_{r,n,t} (1 + R_t^{min}) \\
 PN_{n,t} = 0
 \end{array} \right] \quad \forall n \in N, t \in T \quad (18)$$

where ρ_n is a duration (hours) of sub-period n , $D_{r,n,t}$ is a power demand, and R_t^{min} is a reserve margin related to the peak demand of time t . The GDP given by (1)–(18) can be transformed into a Mixed-Integer Nonlinear Programming (MINLP) using Big-M (BM) and/or Hull Reformulation (HR) (Trespacios & Grossmann, 2014), and this paper uses Big-M method. Also, the MINLP model is reformulated as MILP model by using exact linearization of Eqn. (14) (Garcia-Herreros et al., 2015).

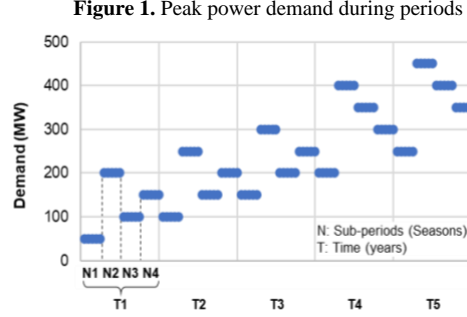
4. Illustrative example

To illustrate the application of the proposed model, we consider a simple case consisting of two regions, each with two power stations (i.e., coal power plants (1 existing, 1 potential), and natural gas power plants (1 existing, 1 potential)), and five time periods (i.e., 5 years). Note that renewable power technologies are not considered in this example.

Table 1 and Figure 1 display the major parameters and power demands during for each of the time periods.

Table 1. Parameters of the example

Parameters	Symbols	Values
Nameplate capacity (MW)		
Coal plants	$\delta_{k,p}$	100, 150, 200
Natural gas plants		100, 200, 300
Installation cost (k\$/unit)		
Coal plants	$\theta_{j,p}$	50, 64, 76
Natural gas plants		60, 91, 116
Start-up cost (\$/unit)		
Coal plants	$\varphi_{j,p}$	50, 80, 100
Natural gas plants		60, 100, 150
Unit reliability		
Coal plants	$\lambda_{j,k}$	0.95
Natural gas plants		0.96



To analyze the impacts of reliability in the expansion problem, three different cases are considered: Case A – typical expansion model without reliability, Case B – reliability evaluation for the given design of Case A, Case C – Proposed expansion model in this work. Table 2 shows the numerical results of the cases, and the results are obtained with CPLEX in GAMS 32.1.0 on an Intel Core i7-10510U CPU, 1.80GHz.

Table 2. Numerical results of the example

	# Constraints	# Cont. variables	# Binary variables	CPU (sec)	Cost (\$)
Case A	63,659	15,773	1,646	48.16	294,393
Case B	63,659	15,773	1,525	0.16	426,281
Case C	63,659	15,773	1,646	1,513.58	414,086

As shown in Figure 2(a), when reliability is not considered (Case A), the model newly installs one large generator (200MW) in region 1 and one large generator (300MW) in region 2 during period 1. Note that the design of Case B is the same to that of Case A. As shown in Figure 2(b), in Case C where the reliability is included as in the above GDP, the model installs two medium sizes (200MW) in region 2 in period 1. Figures 2(c) and 2(d) show the operation results of the two cases during sub-periods of period 5, respectively.

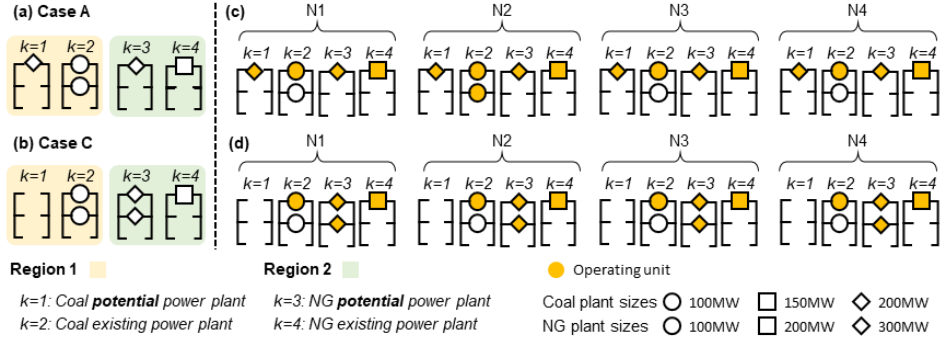


Figure 2. (a) and (b): investment results of Cases A and B; (c) and (d): operation results during period 5 (T5)

As shown in Figure 3(a), the total cost of Case A is \$294,390. When considering failure rates of generators for the given design of Case A (i.e., Case B), the total cost increases to \$426,281 due to unmet and low reliability penalties, and the system has availability of 0.94. On the other hand, the results of the proposed model (i.e., Case C) show that the system has a lower cost of \$414,086 and higher availability of 0.96 than the results of Case B. This example proves that the proposed model is more effective designing reliable power systems than typical expansion model that unit failures are not considered.

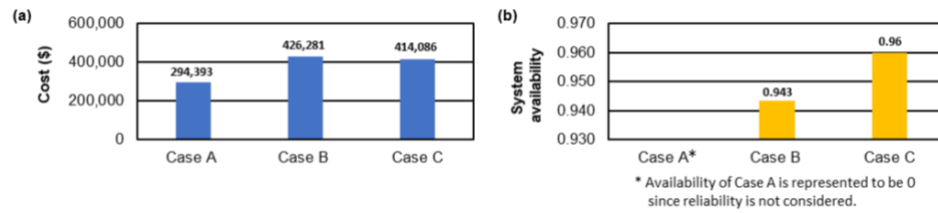


Figure 3. (a) Total cost and (b) System availability of the cases

5. Conclusions

This paper has presented an optimization model for expansion planning of reliable power generation systems. The proposed GDP model optimizes investment decisions (number, size, location, and time of generators to install, retire, and decommission) and operation decisions (number of operating/backup generators, operating capacity, and expected power production level). Future work will involve other operation problems in power systems such as economic dispatch and unit commitment to evaluate the reliability more precisely by using a more rigorous reliability models such as Markov chain theory. Moreover, renewable technologies such as wind turbine and solar panel will be included.

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