Disjunctive optimization model and algorithm for long-term capacity expansion planning of reliable power generation systems

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Abstract

This paper proposes a new optimization model and algorithm for long-term capacity expansion planning of reliable power generation systems. The model optimizes both investment decisions (e.g., size, location, and time to install, retire and decommission facilities) and operation decisions (e.g., on/off status, operating capacity, and expected power output). It is also able to optimize reserve systems (or backup systems), as well as the main systems, to improve power systems reliability. An impact of operational strategies of generators (i.e., participating in electricity production vs. remaining as idle units during operation) on power systems reliability is considered. Probability of equipment failures and capacity failure states are used to rigorously estimate the power systems reliability depending on design and operation strategies. The optimization model is formulated with Generalized Disjunctive Programming (GDP), which is reformulated as a mixed-integer linear programming (MILP) model using the Hull relaxation. Two reliability-related penalties, such as downtime penalty and unmet demand penalty, are included in the objective function to maximize reliability while minimizing the total net present cost. Furthermore, a bilevel decomposition with tailored cuts is developed to reduce computational times of the multi-scale optimization model. The effectiveness of the proposed model is shown by comparing the results with the results obtained from the expansion planning models that do not explicitly consider reliability. We also show that the proposed bilevel decomposition is computationally efficient for solving large scale problems through 5-years and 10-years planning case studies.

Keywords: Power Systems, Optimization, Expansion Planning, Reliability

1. Introduction

Recently, the National Renewable Energy Laboratory (NREL) reported that electricity consumption in the US would increase more than previously expected due to an increasing interest in electrification. Specifically, the electricity demand projection by 2050 is revised to be 5,656 - 6,504 TWh from 4,722 TWh (Mai et al., 2018). Along with the increase in electricity demand, many countries in the world aim to establish decarbonized energy systems (Davis et al., 2018). The solution for decarbonizing power systems is to invest

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in renewable generators such as wind turbines and solar panels, that is zero-emission technologies (Jafari et al., 2020). However, due to their intermittent nature, it is difficult for renewable power systems to satisfy a power demand without interruption.

Expansion planning of power generation systems optimizes investment decisions such as size, location, generation technology, and investment timing of generation facilities to satisfy electricity demand. The decisions on the installation and/or decommissioning of power plants are complex due to technical and environmental issues such as operation limits of dispatchable generators (i.e., thermoelectric and combined cycle power plants), level of penetration of renewable energy sources, and addition of energy storage systems (Conejo et al., 2016; Hemmati et al., 2013).

A number of studies regarding power systems planning have been reported. Lara et al. (2018) propose a mixed-integer linear programming (MILP) model that optimizes long-term investment decisions and hourly operation decisions such as unit commitment. Bahiense et al. (2001) propose a mixed-integer disjunctive model for transmission network expansion and compare three different formulations (i.e., big-M formulation, hull formulation, and alternative big-M formulation) to reformulate the disjunctive model as an MILP model. Haghighat and Zeng (2018) present a two-stage stochastic mixed-integer second-order conic programming (MISOCP) model to capture the non-linearity of the power flows. Li et al. (2021) extend the work done by Lara et al. (2018) by adding expansion planning of transmission lines, and applying Benders decomposition rather than the nested Benders decomposition proposed by Lara et al. (2018). Shu et al. (2015) combine dynamic programming and MILP techniques to resolve the computational intractability, proving its effectiveness for solving large-scale problems. Pozo et al. (2013) present a three-level expansion planning model that can simultaneously determine the optimal investment decisions and operation strategies.

As indicated above, electricity supply is becoming critical in most production processes and its importance is expected to increase. Therefore, it becomes necessary for power systems to have a reliable infrastructure that can guarantee a stable electricity supply. Power systems reliability refers to the ability of power systems to supply uninterrupted electricity to fulfill the load demand. A detailed explanation on power systems reliability is provided in section 2.

Several reliability-constrained expansion planning works have been reported. Slipac et al. (2019) and Choi et al. (2005) propose new optimization models for expansion planning of power systems based on probabilistic reliability indices. Aghaei et al. (2013) present a multi-objective generation expansion planning model that minimizes the cost and environmental impact, and maximizes reliability. Aghaei et al. (2014) incorporate a reliability evaluation criterion (i.e. expected energy not served (EENS)) into the optimization model. The authors also compare the results of typical expansion planning models and the reliability-constrained expansion planning model, and evaluate the effect of non-linearity for finding feasible solutions in a reasonable time. Jooshaki et al. (2019) propose a reliability-constrained MILP expansion planning model for distribution systems in which power loss is penalized in the model. Gbadamosi and Nwulu (2020) propose a multi-objective optimization model for expansion planning, which minimizes the cost, power losses, and CO_2 emissions. Markov processes and three reliability evaluation indices (i.e., loss of load probability (LOLP), loss of load expectation (LOLE), and EENS) are applied in the model to evaluate power systems reliability.

Although a number of studies on expansion planning of reliable power systems have been reported, in this work, we explicitly consider the impact of failures of equipment on power systems reliability by enumerating all the possible capacity failures. This paper proposes a novel Generalized Disjunctive Programming (GDP) model for expansion planning of reliable power generation systems by considering the failure probability of power plants. The model determines both long-term investment decisions, such as size, location, and installation time, and short-term (or hourly) operation decisions, such as operating/idle status, operating capacity, and expected power production level, while minimizing the total net present cost including reliability penalties.

2. Background: availability and reliability

2.1. Power plant availability and power systems reliability

Power plant availability indicates the ability of a power plant to generate electricity over a certain period of time (Cho et al., 2022). It is the desired goal to improve the availability of power plants, as higher availability implies that the power plants can produce electricity over longer times compared to plants with lower availability. Power plant availability is affected by multiple factors such as inherent failure rates of equipment, maintenance and inspection, fuel supply, and weather conditions (for renewable power generations). Among the factors, the failure rates of equipment can substantially impact the availability of dispatchable power plants (e.g., coal, natural gas, and nuclear power plants) as these have relatively higher failure rates than other power plants (e.g., coal plants: 10%, wind turbines: $< 2\%$ (Fakhry, 2019)). On the other hand, for renewable power generators such as wind turbines and solar panels, weather conditions such as wind speed or solar irradiation are more important than failure rates in determining the plant availability (Fakhry, 2019). Therefore, a method that can complement the impact of failure rates and intermittence of resources should be developed so as to enhance the availability of power plants.

power systems reliability represents the probability of power systems (or a power grid) consisting of a number of individual power plants to supply uninterrupted electricity so as to satisfy customers' power demands. In order to enhance power systems reliability, it is important to have sufficient capacity of generation systems as the reliability can be enhanced by improving the availability of individual generation facilities. It should be noted that 'reliability' has multiple definitions in different research areas. In reliability engineering and chemical engineering, reliability is the probability of a single plant successfully performing its required task without failures (Endrenyi, 1979). The reliability is measured by failure rates of plants, and the plants with low failure rates are known to be highly reliable (Singh et al., 2019; Prada, 1999). While the definition of reliability in these two disciplines is closely related to the performance of individual plants, the meaning in power system engineering is related to the ability of the power grid (or network) to meet demands all the time. To avoid confusion, in this paper, 'power systems reliability' only indicates the ability of power systems to satisfy power demands.

2.2. Parallel units for improvement of power systems reliability

Power systems reliability can be enhanced by improving the availability of each power plant. Specifically, highly available power plants can reduce the probability of not satisfying power demands of power generation systems by producing more electricity. One method that can be used to improve the availability of power plants is to add more units in parallel, allowing the plants to operate when failures occur (Kim and Kim, 2017). This approach is known as 'reliability-based design optimization (RBDO)' in reliability engineering (Kuo and Prasad, 2000), and estimates the plant availability by using all possible failure scenarios and the probability of failures of each unit. Since the number of failure scenarios to be explored greatly increases as the number of components increases, this approach has been used to a lesser extent. Ye et al. (2018) propose an MINLP model that can optimize the number of parallel units while minimizing cost. The proposed MINLP model is applied to air separation units and a methanol synthesis process. Chen et al. (2022) propose a two-stage stochastic generalized disjunctive programming (GDP) model in order to account for both exogenous (e.g., power demand) and endogenous (e.g., probability of failures) uncertainties. The proposed GDP model is reformulated as an MINLP model, and a logic-based outer approximation algorithm is applied to solve the problem. Ortiz-Espinoza et al. (2021) present a multi-objective optimization model that maximizes process safety and reliability, and minimizes total costs. The proposed model is used to optimize the design of a distillation column and corresponding operation conditions. Massim et al. (2005) present a new method that can evaluate power systems reliability indices by employing an ant colony method. Contrary to classical reliability theories that are based on binary failure states, these authors consider a multi-state case and use the universal moment generating function (UMGF) approach to evaluate reliability.

In previous RBDO research, it is assumed that parallel units can only be used when operating units fail. This assumption is valid for plants that operate in steady-state such as air separation units. However, parallel units of power plants (or parallel generators) might be required to increase power production together with the main plants so as to satisfy a time-sensitive load demand. That is, parallel units of power plants have a dual role based on their operational status: they can either remain as backups, or else operate to produce additional electricity (Cho and Grossmann, 2022a,b). Although the amount of electricity produced by the power plants increases as operating both the main and parallel generators, the power plant under this operation mode is less likely to constantly satisfy the required power output, because there are no available backup generators. It should be noted that the case where all main and parallel generators operate to produce electricity is equivalent to the case without any backups from the plant availability point of view. In other words, the power systems reliability can be changed not only by the number of parallel units but also operational strategies of these units. Therefore, instead of using a traditional reliability estimation method that relies only on the design of parallel units, a new approach that considers both the design and the impact of operation strategies, including the dual role of parallel units, is proposed in this paper.

3. Problem statement and assumptions

The proposed planning problem involves optimizing both design of generation infrastructure and operation schedules, while considering reliability constraints. The general structure of the optimization model is explained in section 3.1, a temporal representation that is used to combine yearly-basis planning and hourly-basis operation problems is described in section 3.2, and an explanation on reliability estimation is presented in section 3.3.

3.1. Generation representation

As shown in Figure 1, given is a set of regions $r \in R$ and corresponding power plants $k \in K_r$ of each region. Six different types of power plants are taken into account, such as coal plants, natural gas plants, nuclear plants, biomass-fired plants, wind turbines, and solar panels. There are two types of power plants: existing power plants, $k \in K_r^{EX}$, and potential power plants, $k \in K_r^{PN}$. While the capacity of the existing power plants is known, that of the potential power plants will be optimized by choosing one size from among a set of available sizes $c \in C$. Coal, natural gas, and biomass-fired plants belong to the power plants that consider redundancy, $k \in K_r^{RD}$, meaning parallel generators $j \in J_k$ can be added to these power plants for both capacity expansion and power systems reliability. The size of parallel generators is also optimized from different available sizes. Parallel generators are not considered for nuclear power plants, wind turbines, and solar panels, $k \in K_r^{ND}$. The reason that parallel generators are not considered especially for nuclear power plants is that nuclear power plants are highly reliable themselves (Ballard et al., 1989). To secure a stable electricity supply from wind turbines and solar panels, Li-ion batteries, $i \in I_k$, are considered as backups rather than parallel generators. Unlike power plants that can improve their availability by adding parallel generators, batteries added to wind turbines and solar panels do not reduce the failure rates of power plants, but they can supply electricity during periods of low availability of renewable energy or periods of high demand. It should also be noted that in this work, transmission lines are not considered.

The power plants can also be classified into two groups based on their operational properties: dispatchable power plants, $k \in K_r^D$, and renewable power plants, $k \in K_r^N$. The dispatchable power plants indicate the power plants whose the power output can be adjusted depending on the power demands, and include coal, natural gas, nuclear, and biomass-fired power plants. Since power output from the dispatchable power plants can be controlled by operators, detailed operational constraints such as ramping up and down, unit commitment must be included. In contrast, the power output of renewable power plants, such as wind turbines and solar panels, is mostly determined not by operators but by weather conditions.

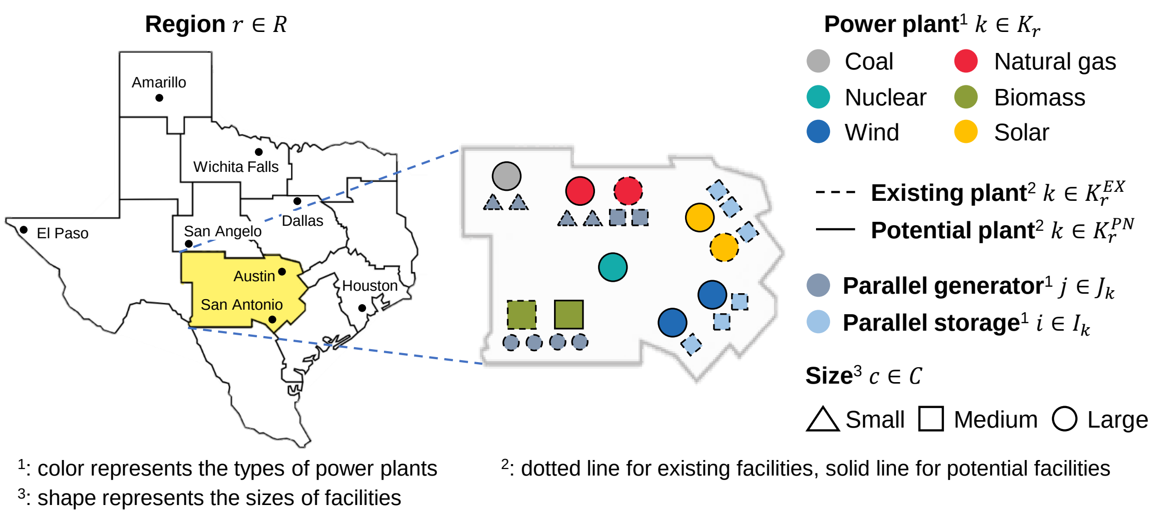

Figure 1: Graphical representation of the model structure

3.2. Temporal representation

As shown in Figure 2, different sets of times are considered: years $t \in T$, representative days $n \in N$, and subperiods (hours) $b \in B$. A year comprises 365 days and each day consists of 24 hours. To address the computational complexity caused by the very large number of time periods, each year is modeled using a subset of representative days (Li et al., 2022). The number of representative days can be selected by the modeller, but in this paper for simplicity, we use 4 representative days method. It states that 4 days are selected from every year, and hourly operational problems are solved for those representative days. The investment decisions are made at the beginning of each year. For potential power plants, the optimal location, size, and installation timing of the main and parallel generators are determined. On the other hand, the existing power plants can either be decommissioned before their remaining lifetimes expire, or else their lifetimes can be extended to operate longer than expected. Operation decisions such as power output are made on a hourly basis for the representative days by considering operational properties of power plants, failure rates, and operation costs. The way to select representative days can be changed by modellers depending on the problems they desire to solve.

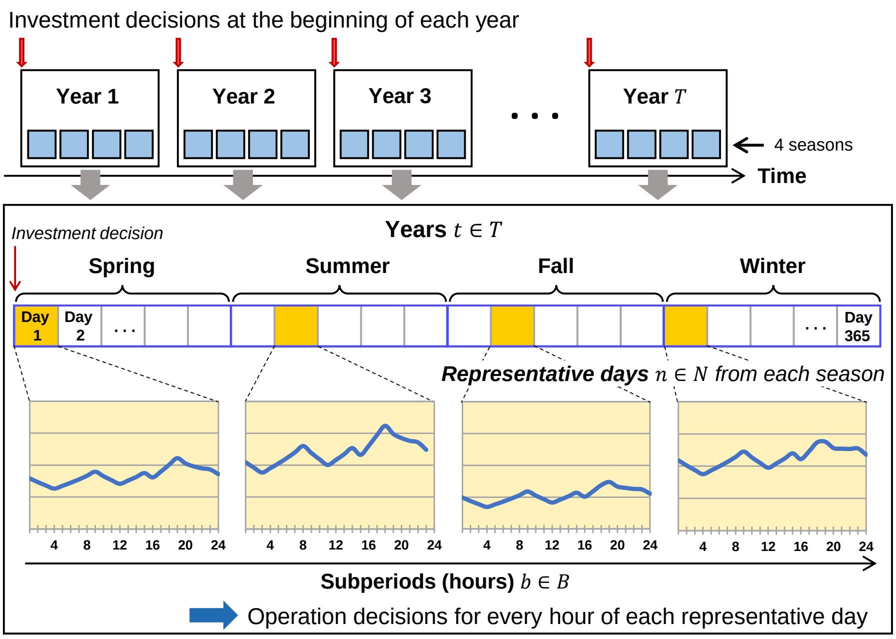

Figure 2: Temporal resolution for co-optimization of investment planning and operation

3.3. Power plant operational reliability estimation

As explained previously, adding units in parallel enables a power plant to improve its availability. However, the method that has been used to calculate the power plant availability only analyzes the impact of the number of parallel units, meaning that the impact of dual role of parallel units is not taken into account. In this work, we define a new term called 'power plant operational reliability', representing the ability of a power plant to perform its required function under the specific designs and operations.

We assume that a set of power plant designs, $h \in H_k$, and operation modes, $m \in M_{k,h}$ is given. For the power plants that consider redundancy $k \in K_r^{RD}$, $h = 1$ indicates that only the main generator is available, while $h = H$ means that all main generator and parallel generators are available for the power plant. Similarly, $m = 1$ represents the main generator operates, $m = M$ means that all generators including the main and parallel operate. Each power plant $k \in K_r^{RD}$ has different capacity failure states, $s \in S_{k,h}$, depending on the designs h. The failure states can be classified into successful operational states ($S^F_{k,h,m}$) and partial operational states ($S^P_{k,h,m}$) by taking into account an operational status. 'Successful operational states' refer to cases where the remaining capacity after one or multiple operating units fail is larger than or equal to the operating capacity of no failures cases. On the other hand, 'Partial operational states' refer to the cases where the remaining capacity after one or multiple operating units fail is less than the operating capacity of no failures cases. The detailed method to calculate successful and partial operational states can be found in Appendix A.

There are two criteria used to evaluate power systems reliability in this work: *unmet demand penalty* and *downtime penalty*. An unmet demand penalty indicates a cost penalty that occurs when the power demand is not satisfied. A downtime penalty indicates a cost penalty that occurs when the individual power plant is expected to fail. Note that in power systems engineering, there are two terms used to evaluate power systems reliability: loss of load expectation (LOLE) and loss of energy expectation (LOEE) (Allan and Billinton, 2000). While LOLE refers to the expected number of hours or days where the power demand is not satisfied, LOEE stands for the power demand not supplied due to insufficient capacity of generation systems. Downtime penalty corresponds to LOLE, and unmet demand penalty corresponds to LOEE. However, we here use the terms, a downtime penalty and an unmet demand penalty instead of LOLE and LOEE to help readers of this journal understand this work better. The unmet demand penalty is decided by comparing the actual power demand and the expected power output at each subperiod, and the downtime penalty is evaluated based on the partial operational reliability of each power plant. Details for calculating the penalties are explained in section 4. Given all the above specifications and assumptions, the proposed model is also given the following data:

- Electricity demand over the time horizon of all regions
- Economic data such as capital and fixed/variable operating costs of facilities, start-up/shut-down costs, fuel costs, CO_2 tax
- Available nameplate capacities, lifetimes, conversion efficiencies, capacity factors for renewable generator, CO_2 emission rates
- Probability of failures of facilities, unmet demand penalty rate, downtime penalty rate

The goal is to determine the following decisions:

- The configuration such as location, type, time, and sizes of the main and parallel generators and batteries
- Operation schedules of all facilities such as on/off schedules
- Expected power output and successful/partial operational reliability along with the total system cost

The objective is to minimize a net present cost including capital and operating (e.g., fixed and variables) costs, reliability penalties, CO_2 emission costs, and fuel cost.

4. Model formulation

Previous RBDO studies have mainly focused on determining the optimal design of parallel units to maximize the availability of power plants. Since satisfying the power demand as well as improving the availability of individual power plants are important in power systems, the impact of operation strategies, including the dual role of parallel units (i.e., remaining as backups vs. participating in electricity production), should be considered in reliability-constrained expansion planning. However, there are two computational challenges that must be addressed: i) an advanced RBDO model that can effectively include the impact of operation strategies (including the dual role of parallel units) should be proposed, and ii) the advanced RBDO model should be combined with an expansion planning model. Detailed explanation of the advanced RBDO model can be found in Appendix A.

In this section, we combine the reliability estimation model explained in Appendix A and an expansion planning model so as to develop a reliability-constrained expansion planning (RP) model. The RP model is formulated using *Generalized Disjunctive Programming (GDP)* involving Boolean variables, continuous variables, binary variables, algebraic equations, and logic propositions (Trespalacios and Grossmann, 2014; Grossmann and Trespalacios, 2013). Two reformulation strategies (the Big-M reformulation (Trespalacios and Grossmann, 2015) and the Hull relaxation (Lee and Grossmann, 2000)) can be used to reformulate the GDP model into MILP/MINLP models. We use here the Hull relaxation that yields a tighter LP relaxation than the Big-M reformulation.

4.1. Outline of the model

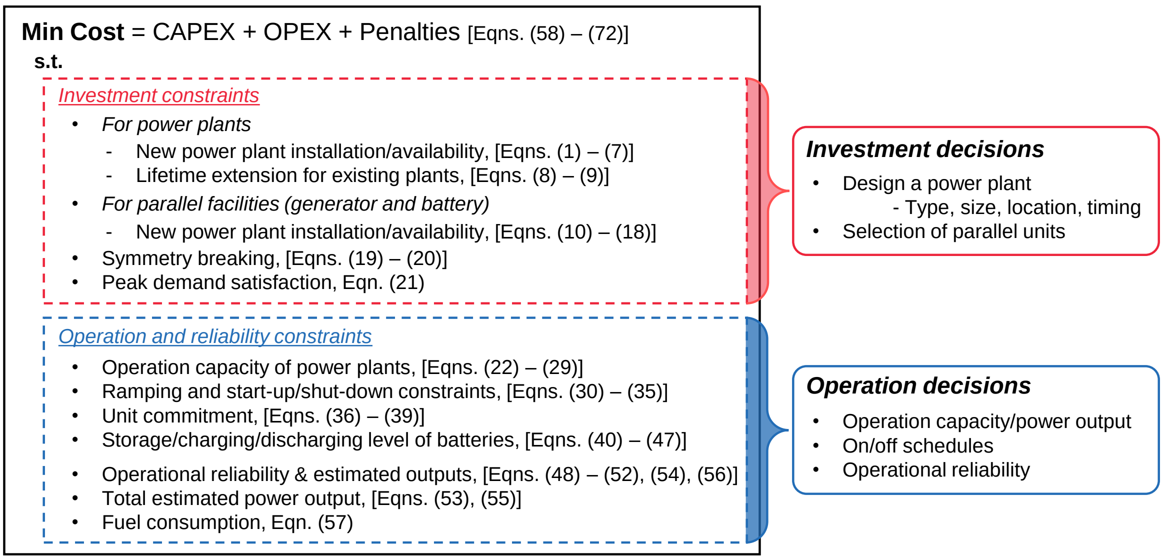

Figure 3: Overview of the optimization model (RP model)

Figure 3 provides an overview of the original full-space model (RP model). The superstructure of the optimization model including major Boolean and continuous variables is shown in Figure B.1 of Appendix B. All constraints can be classified into investment constraints and operation/reliability constraints. While design decisions about installation and lifetime extension are subjected to Eqns. (1) - (21), operation and reliability decisions such as operation capacity, on/off schedules, and operational reliability, are subjected to Eqns. (22) - (57). The objective function is to minimize a net present cost consisting of capital expenditure (CAPEX), operating expenses (OPEX), and reliability-related penalties. The corresponding equations used to calculate the cost are shown in Eqns. (58) - (72).

4.2. Investment constraints

4.2.1. Installation and availability of power plants

Eqns. (1) - (2) ensure that when a potential power plant $k \in K_r^{PN}$ is installed in region $r \in R$ at year t ($Y_{r,k,t}^{PI}$), only one size $c \in C$ is chosen. The adjusted investment cost of the potential power plant ($AIC_{r,k,t}^{P}$) and the installed capacity of the potential power plant ($C_{r,k,t}^{PI}$) are also calculated when the power plant is installed. The original investment cost is adjusted by considering both lifetime of the potential power plant and a planning horizon of this work,

$$\left[\begin{array}{c} Y_{r,k,t}^{PI} \\ \bigvee_{c\in C}\left[\begin{array}{c} Y_{r,k,c,t}^{UPI} \\ AIC_{r,k,t}^{P} = \alpha_{k,c}^{P} \\ C_{r,k,t}^{PI} = \varphi_{k,c}^{P} \end{array}\right] \end{array}\right] \vee \left[\begin{array}{c} \neg Y_{r,k,t}^{PI} \\ AIC_{r,k,t}^{P} = 0 \\ C_{r,k,t}^{PI} = 0 \end{array}\right] \quad \forall r \in R, k \in K_r^{PN}, t \in T \tag{1}$$

$$Y_{r,k,t}^{PI} \Longleftrightarrow \bigvee_{c\in C} Y_{r,k,c,t}^{UPI} \quad \forall r \in R, k \in K_r^{PN}, t \in T \tag{2}$$

where the Boolean variable, $Y_{r,k,t}^{PI}$, is true if the potential power plant $k \in K_r^{PN}$ is installed in region $r \in R$ at year $t \in T$. $Y_{r,k,c,t}^{UPI}$, is true if the potential power plant $k \in K_r^{PN}$ with discrete size $c \in C$ is installed in region r at year t. $\alpha_{k,c}^{P}$ is the coefficient of the investment cost of the power plant k with size c, $\varphi_{k,c}^{P}$ is the nameplate capacity of the power plants k with size c.

Eqn. (3) states that the potential power plant k of region r can be installed once over the planning period t if it is installed, where $y_{r,k,t}^{PI}$ is the binary variable representing the Boolean variable $Y_{r,k,t}^{PI}$,

$$\sum_{t\in T} y_{r,k,t}^{PI} \leq 1 \quad \forall r \in R, k \in K_r^{PN} \tag{3}$$

To indicate whether the power plant is available at year t or not, a new Boolean variable ($Y_{r,k,t}^{PA}$) is introduced. If the potential power plant $k \in K_r^{PN}$ is available in region r at year t, it should be either available at the previous year $t-1$ or installed at current year t as shown in Eqn. (4). For the existing power plant $k \in K_r^{EX}$, the plant should be available at year $t-1$ so as to be available at year t as shown in Eqn. (5).

$$Y_{r,k,t}^{PA} \Longleftrightarrow Y_{r,k,t-1}^{PA} \vee Y_{r,k,t}^{PI} \quad \forall r \in R, k \in K_r^{PN}, t > 1 \tag{4}$$

$$Y_{r,k,t}^{PA} \Longrightarrow Y_{r,k,t-1}^{PA} \quad \forall r \in R, k \in K_r^{EX}, t > 1 \tag{5}$$

The disjunction in Eqn. (6) states that the fixed operating cost ($FOC_{r,k,t}^{P}$) occurs for the available power plant, and is calculated by the available capacity, $C_{r,k,t}^{PA}$, and the coefficient of the fixed operating cost, β_k^P. As shown in Eqn. (7), while the available capacity of the potential power plant $k \in K_r^{PN}$ is the same as the installed capacity ($C_{r,k,t}^{PI}$), that of the existing power plant $k \in K_r^{EX}$ is the same as the preinstalled capacity (ω_k), which is a parameter.

$$\begin{bmatrix} Y_{r,k,t}^{PA} \\ FOC_{r,k,t}^{P} = \beta_k^P C_{r,k,t}^{PA} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{r,k,t}^{PA} \\ FOC_{r,k,t}^{P} = 0 \end{bmatrix} \quad \forall r \in R, k \in K_r, t \in T \tag{6}$$

$$C_{r,k,t}^{PA} = \begin{cases} C_{r,k,tq}^{PI} & \forall r \in R, k \in K_r^{PN}, t \in T, tq \le t \\ \omega_k & \forall r \in R, k \in K_r^{EX}, t \in T \end{cases} \tag{7}$$

Eqn. (8) states that if the lifetime of an existing power plant reaches its limit, the lifetime can be extended by paying the extension cost ($EC_{r,k,t}$). The Boolean variable, $Y_{r,k,t}^{PL}$, is true if the existing power plant $k \in K_r^{EX}$ of region r extends its lifetime ($\sigma_{r,k}$) at year t. δ_k is the coefficient of the lifetime extension cost. The power plant can operate afterward once the lifetime is extended (Eqn. (9)).

$$\begin{bmatrix} Y_{r,k,t}^{PL} \\ EC_{r,k,t} = \delta_k \end{bmatrix} \vee \begin{bmatrix} \neg Y_{r,k,t}^{PL} \\ EC_{r,k,t} = 0 \end{bmatrix} \quad \forall r \in R, k \in K_r^{EX}, t \in T \tag{8}$$

$$Y_{r,k,tq}^{PA} \Longrightarrow Y_{r,k,t}^{PL} \quad \forall r \in R, k \in K_r^{EX}, t = \sigma_{r,k}, t+1 \le tq \le |T| \tag{9}$$

4.2.2. Addition of parallel generators and batteries

As explained in section 3, while the dispatchable power plants consider redundancy $k \in K_r^{RD}$ can add parallel generators $j \in J_k$, the renewable power plants $k \in K_r^N$ can add parallel batteries $i \in I_k$. Eqns. (10) - (11) state that only one size c is chosen if the parallel generator is installed. Eqn. (12) indicates the installation of the battery i. The installed capacities of the parallel generator and battery ($C_{r,k,j,t}^{BI}$ and $C_{r,k,i,t}^{SI}$) are also calculated as well as the adjusted investment costs ($AIC_{r,k,j,t}^{B}$ and $AIC_{r,k,i,t}^{S}$). Boolean variables, $Y_{r,k,j,t}^{BI}$ and/or $Y_{r,k,i,t}^{SI}$, are true if the parallel generator j and/or the parallel battery i are added to power plant k of region r at year t, respectively. $Y_{r,k,j,c,t}^{UBI}$, the Boolean variable, is true if the parallel generator j with discrete size $c \in C$ is installed in the power plant $k \in K_r^{RD}$ of region r at year t.

$$\begin{bmatrix} Y_{r,k,j,t}^{BI} \\ \bigvee_{c \in C} \begin{bmatrix} Y_{r,k,j,c,t}^{UBI} \\ AIC_{r,k,j,t}^{B} = \alpha_{k,j,c}^{B} \\ C_{r,k,j,t}^{BI} = \varphi_{k,j,c}^{B} \end{bmatrix} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{r,k,j,t}^{BI} \\ AIC_{r,k,j,t}^{B} = 0 \\ C_{r,k,j,t}^{BI} = 0 \end{bmatrix} \quad \forall r \in R, k \in K_r^{RD}, j \in J_k, t \in T \tag{10}$$

$$Y_{r,k,j,t}^{BI} \Longleftrightarrow \bigvee_{c \in C} Y_{r,k,j,c,t}^{UBI} \quad \forall r \in R, k \in K_r^{RD}, j \in J_k, t \in T \tag{11}$$

$$\begin{bmatrix} Y_{r,k,i,t}^{SI} \\ AIC_{r,k,i,t}^{S} = \alpha_{k,i}^{S} \\ C_{r,k,i,t}^{SI} = \varphi_{k,i}^{S} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{r,k,i,t}^{SI} \\ AIC_{r,k,i,t}^{S} = 0 \\ C_{r,k,i,t}^{SI} = 0 \end{bmatrix} \quad \forall r \in R, k \in K_r^{N}, i \in I_k, t \in T \tag{12}$$

Similarly to the power plants, the parallel generator j and battery i should be either available at the previous year $t-1$ or installed at current year t to be able to operate in region r at year t, as shown in Eqns. (13) - (14).

$$Y_{r,k,j,t}^{BA} \Longleftrightarrow Y_{r,k,j,t-1}^{BA} \vee Y_{r,k,j,t}^{BI} \quad \forall r \in R, k \in K_r^{RD}, j \in J_k, t > 1 \tag{13}$$

$$Y^{SA}_{r,k,i,t} \Longleftrightarrow Y^{SA}_{r,k,i,t-1} \vee Y^{SI}_{r,k,i,t} \quad \forall r \in R, k \in K^N_r, i \in I_k, t > 1 \tag{14}$$

Eqns. (15) - (16) state that the fixed operating costs for parallel units ($FOC^B_{r,k,j,t}$ and $FOC^S_{r,k,i,t}$) are incurred when they are available at year t, respectively. Since the parallel generators are auxiliary facilities to the main power plant, Eqns. (17) - (18) ensure that the parallel facilities are available if and only if the power plants k are available.

$$\begin{bmatrix} Y^{BA}_{r,k,j,t} \\ FOC^B_{r,k,j,t} = \beta^B_{k,j} C^{BA}_{r,k,j,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y^{BA}_{r,k,j,t} \\ FOC^B_{r,k,j,t} = 0 \end{bmatrix} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{15}$$

$$\begin{bmatrix} Y^{SA}_{r,k,i,t} \\ FOC^S_{r,k,i,t} = \beta^S_{k,i} C^{SA}_{r,k,i,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y^{SA}_{r,k,i,t} \\ FOC^S_{r,k,i,t} = 0 \end{bmatrix} \quad \forall r \in R, k \in K^N_r, i \in I_k, t \in T \tag{16}$$

$$Y^{BA}_{r,k,j,t} \Longrightarrow Y^{PA}_{r,k,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{17}$$

$$Y^{SA}_{r,k,i,t} \Longrightarrow Y^{PA}_{r,k,t} \quad \forall r \in R, k \in K^N_r, i \in I_k, t \in T \tag{18}$$

Eqns. (19) - (20) are symmetry breaking constraints that enforce the parallel generator j and battery i with higher priority (i.e., lower j and i values) should be selected first before the facilities with lower priority.

$$Y^{BI}_{r,k,j+1,t} \Longrightarrow Y^{BI}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{19}$$

$$Y^{SI}_{r,k,i+1,t} \Longrightarrow Y^{SI}_{r,k,i,t} \quad \forall r \in R, k \in K^N_r, i \in I_k, t \in T \tag{20}$$

Eqn. (21) states that the total capacity of the main and parallel generators that are available in year t should be greater than or equal to the peak demand of year t (Θ_t).

$$\sum_{r \in R} \sum_{k \in K_r} C^{PA}_{r,k,t} + \sum_{r \in R} \sum_{k \in K^{RD}_r} \sum_{j \in J_k} C^{BA}_{r,k,j,t} \geq \Theta_t \quad \forall t \in T \tag{21}$$

4.3. Operational constraints

4.3.1. Operation of renewable power plants

A capacity factor ($\Phi_{r,k,t,n,b}$) is used to estimate the power output of the renewable power plant as shown in Eqn. (22). The variable operating cost ($VOC^P_{r,k,t,n,b}$) is calculated as stated in Eqn. (23).

$$C^{PO}_{r,k,t,n,b} = \Phi_{r,k,t,n,b} C^{PA}_{r,k,t} \quad \forall r \in R, k \in K^N_r, t \in T, n \in N, b \in B \tag{22}$$

$$VOC^P_{r,k,t,n,b} = \varrho_{n,b} \varepsilon^P_k C^{PO}_{r,k,t,n,b} \quad \forall r \in R, k \in K^N_r, t \in T, n \in N, b \in B \tag{23}$$

where $\varrho_{n,b}$ is the operation time during subperiod b of representative day n, ε^P_k is the coefficient of the variable operating cost of power plant k.

The renewable power plant will operate during subperiod b of representative day n if they are available in year t (Eqn. (24)).

$$Y^{PA}_{r,k,t} \Longleftrightarrow X^{PO}_{r,k,t,n,b} \quad \forall r \in R, k \in K^N_r, t \in T, n \in N, b \in B \tag{24}$$

4.3.2. Operation of dispatchable power plants

Eqns. (25) - (26) ensure that the dispatchable power plant $k \in K_r^D$ and parallel generator $j \in J_k$ added to the power plant considering redundancy $k \in K_r^{RD}$ should be available to be operated during subperiod b of representative day n in year t.

$$X_{r,k,t,n,b}^{PO} \Longrightarrow Y_{r,k,t}^{PA} \quad \forall r \in R, k \in K_r^D, t \in T, n \in N, b \in B \tag{25}$$

$$X_{r,k,j,t,n,b}^{BO} \Longrightarrow Y_{r,k,j,t}^{BA} \quad \forall r \in R, k \in K_r^{RD}, j \in J_k, t \in T, n \in N, b \in B \tag{26}$$

where $X_{r,k,t,n,b}^{PO}$ and $X_{r,k,j,t,n,b}^{BO}$ are true if the dispatchable power plant $k \in K_r^D$ and parallel generator $j \in J_k$ added to the power plant considering redundancy $k \in K_r^{RD}$ operate during subperiod b of representative day n in year t, respectively.

As shown in Eqns. (27) - (28), if the power plant and parallel generator operate, the operating capacities of power plant and parallel generator ($C_{r,k,t,n,b}^{PO}$ and $C_{r,k,j,t,n,b}^{BO}$, respectively) and the variable operating costs ($VOC_{r,k,t,n,b}^{P}$ and $VOC_{r,k,j,t,n,b}^{B}$, respectively) are calculated.

$$\begin{bmatrix} X_{r,k,t,n,b}^{PO} \\ VOC_{r,k,t,n,b}^{P} = \varrho_{n,b}\varepsilon_k^P C_{r,k,t,n,b}^{PO} \\ \gamma_k^{min} C_{r,k,t}^{PA} \le C_{r,k,t,n,b}^{PO} \le \gamma_k^{max} C_{r,k,t}^{PA} \end{bmatrix} \vee \begin{bmatrix} \neg X_{r,k,t,n,b}^{PO} \\ VOC_{r,k,t,n,b}^{P} = 0 \\ C_{r,k,t,n,b}^{PO} = 0 \end{bmatrix} \tag{27}$$
$$\forall r \in R, k \in K_r^D, t \in T, n \in N, b \in B$$

$$\begin{bmatrix} X_{r,k,j,t,n,b}^{BO} \\ VOC_{r,k,j,t,n,b}^{B} = \varrho_{n,b}\varepsilon_{k,j}^B C_{r,k,j,t,n,b}^{BO} \\ \gamma_{k,j}^{min} C_{r,k,t}^{BA} \le C_{r,k,j,t,n,b}^{BO} \le \gamma_{k,j}^{max} C_{r,k,j,t}^{BA} \end{bmatrix} \vee \begin{bmatrix} \neg X_{r,k,j,t,n,b}^{BO} \\ VOC_{r,k,j,t,n,b}^{B} = 0 \\ C_{r,k,j,t,n,b}^{BO} = 0 \end{bmatrix} \tag{28}$$
$$\forall r \in R, k \in K_r^{RD}, t \in T, n \in N, b \in B$$

The parallel generator j can operate if and only if the main power plant k operates (Eqn. (29)).

$$X_{r,k,j,t,n,b}^{BO} \Longrightarrow X_{r,k,t,n,b}^{PO} \quad \forall r \in R, k \in K_r^{RD}, j \in J_k, t \in T, n \in N, b \in B \tag{29}$$

Operational constraints such as unit commitment and ramping up/down of the dispatchable power plant k and parallel generator j are considered to take into account hourly-operations. The operational constraints for the dispatchable power plants k are included in this section, and the detailed constraints for parallel generators j are included in Appendix C.

Ramping up/down constraints. Ramping up/down constraints capture the limitation on how fast the dispatchable power plant can adjust to its power output. The ramping up constraint denotes that the dispatchable power plant cannot increase the power output above a maximum level (so called ramping-up limit) during operation. The ramping down constraint states that the dispatchable power plant cannot decrease its power output below the minimum level (so call ramping-down limit) during operation. Eqns. (30) - (31) are ramping up and down constraints for the dispatchable power plant, respectively.

$$\begin{bmatrix} X_{r,k,t,n,b-1}^{PO} \vee U_{r,k,t,n,b}^{PU} \\ C_{r,k,t,n,b}^{PO} - C_{r,k,t,n,b-1}^{PO} \le \kappa_k^U C_{r,k,t}^{PA} \end{bmatrix} \vee \begin{bmatrix} \neg(X_{r,k,t,n,b-1}^{PO} \vee U_{r,k,t,n,b}^{PU}) \\ 0 \le C_{r,k,t,n,b}^{PO}, C_{r,k,t,n,b-1}^{PO} \le \gamma_k^{max} C_{r,k,t}^{PA} \end{bmatrix} \tag{30}$$
$$\forall r \in R, k \in K_r^D, t \in T, n \in N, b \in B$$

$$\begin{bmatrix} X^{PO}_{r,k,t,n,b} \lor U^{PD}_{r,k,t,n,b} \\ C^{PO}_{r,k,t,n,b-1} - C^{PO}_{r,k,t,n,b} \leq \kappa^D_k C^{PA}_{r,k,t} \end{bmatrix} \lor \begin{bmatrix} \neg(X^{PO}_{r,k,t,n,b} \lor U^{PD}_{r,k,t,n,b}) \\ 0 \leq C^{PO}_{r,k,t,n,b}, C^{PO}_{r,k,t,n,b-1} \leq \gamma^{max}_k C^{PA}_{r,k,t} \end{bmatrix} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{31}$$

where $U^{PU}_{r,k,t,n,b}$ and $U^{PD}_{r,k,t,n,b}$ are true if the dispatchable power plant $k \in K^D_r$ of region r starts up or shuts down during subperiod b of representative day n in year t, respectively. κ^U_k and κ^D_k are the start-up and shut-down limits of the dispatchable power plant k, respectively.

Start-up/Shut down constraints. When the dispatchable power plant starts up and/or shuts down, the start-up and shut-down costs are incurred, as shown in Eqns. (32) - (33).

$$\begin{bmatrix} U^{PU}_{r,k,t,n,b} \\ SUC^{P}_{r,k,t,n,b} = \zeta^{PU}_k \end{bmatrix} \lor \begin{bmatrix} \neg U^{PU}_{r,k,t,n,b} \\ SUC^{P}_{r,k,t,n,b} = 0 \end{bmatrix} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{32}$$

$$\begin{bmatrix} U^{PD}_{r,k,t,n,b} \\ SDC^{P}_{r,k,t,n,b} = \zeta^{PD}_k \end{bmatrix} \lor \begin{bmatrix} \neg U^{PD}_{r,k,t,n,b} \\ SDC^{P}_{r,k,t,n,b} = 0 \end{bmatrix} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{33}$$

where $SUC^{P}_{r,k,t,n,b}$ and $SDC^{P}_{r,k,t,n,b}$ are the start-up and shut-down costs of the dispatchable power plant k, respectively.

As shown in Eqns. (34) - (35), the start-up and shut-down constraints are applied only to the available power plant.

$$U^{PU}_{r,k,t,n,b} \Longrightarrow Y^{PA}_{r,k,t} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{34}$$

$$U^{PD}_{r,k,t,n,b} \Longrightarrow Y^{PA}_{r,k,t} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{35}$$

Unit commitment. Unit commitment constraints are used to determine operation schedules of the dispatchable power plant during subperiod b of representative day n in year t. The dispatchable power plant can start up during subperiod b if and only if it does not operate during the previous subperiod $b-1$ (Eqn. (36)). Also, it can shut down during subperiod b if and only if it operates during the previous subperiod $b-1$ (Eqn. (37)).

$$\neg X^{PO}_{r,k,t,n,b-1} \land X^{PO}_{r,k,t,n,b} \Longleftrightarrow U^{PU}_{r,k,t,n,b} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{36}$$

$$X^{PO}_{r,k,t,n,b-1} \land \neg X^{PO}_{r,k,t,n,b} \Longleftrightarrow U^{PD}_{r,k,t,n,b} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{37}$$

Start-up and shut-down of the dispatchable power plant cannot occur at the same time (Eqns. (38) - (39)).

$$U^{PU}_{r,k,t,n,b} \Longrightarrow \neg U^{PD}_{r,k,t,n,b} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{38}$$

$$U^{PD}_{r,k,t,n,b} \Longrightarrow \neg U^{PU}_{r,k,t,n,b} \quad \forall r \in R, k \in K^D_r, t \in T, n \in N, b \in B \tag{39}$$

It should be mentioned that each representative day n is assumed to be connected to other representative days. Therefore, operational results such as start-up and shut-down schedules and ramping up and down ratio of the last subperiod ($b = |B|$) of the previous representative day $n-1$ should be transferred to the first subperiod ($b = 1$) of the following representative day n. Likewise, operational results of the last subperiod ($b = |B|$) of the last representative day ($n = |N|$) of the previous planning year $t-1$ should be transferred to the first subperiod ($b = 1$) of the first representative day ($n = 1$) of the following planning year t. However, those constraints are not shown here as they have the same structure to the constraints explained in this section.

4.3.3. Operation of batteries

Eqn. (40) ensures that the battery i should be available in year t to be operated during subperiod b of representative day n. The Boolean variable, $X^{SO}_{r,k,i,t,n,b}$, is true if the battery i of power plant k of region r operates during subperiod b of representative day n in year t.

$$X^{SO}_{r,k,i,t,n,b} \Longrightarrow Y^{SA}_{r,k,i,t} \quad \forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B \tag{40}$$

As shown in Eqn. (41), the storage level of the battery ($SL_{r,k,i,t,n,b}$) and the variable operating cost ($VOC^S_{r,k,i,t,n,b}$) are calculated if the battery i operates. The storage level cannot exceed the available capacity.

$$\begin{bmatrix} X^{SO}_{r,k,i,t,n,b} \\ VOC^S_{r,k,i,t,n,b} = \varrho_{n,b}\varepsilon^S_{k,i} SL_{r,k,i,t,n,b} \\ \gamma^{min}_{k,i} C^{SA}_{r,k,i,t} \leq SL_{r,k,i,t,n,b} \leq \gamma^{max}_{k,i} C^{SA}_{r,k,i,t} \end{bmatrix} \vee \begin{bmatrix} \neg X^{SO}_{r,k,i,t,n,b} \\ VOC^S_{r,k,i,t,n,b} = 0 \\ SL_{r,k,i,t,n,b} = 0 \end{bmatrix} \tag{41}$$
$$\forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B$$

Eqn. (42) enforces that when the battery i operates, it can be either discharged or charged. The Boolean variables, $V^{DC}_{r,k,i,t,n,b}$ and $V^{CH}_{r,k,i,t,n,b}$, are true if the battery i is discharged or charged during subperiod b, respectively.

$$X^{SO}_{r,k,i,t,n,b} \Longleftrightarrow V^{DC}_{r,k,i,t,n,b} \vee V^{CH}_{r,k,i,t,n,b} \quad \forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B \tag{42}$$

The charging and discharging level cannot exceed the installed capacity (Eqns. (43) - (44)).

$$\begin{bmatrix} V^{CH}_{r,k,i,t,n,b} \\ \xi^{Cmin} C^{SA}_{r,k,i,t} \leq L^{CH}_{r,k,i,t,n,b} \leq \xi^{Cmax} C^{SA}_{r,k,i,t} \end{bmatrix} \vee \begin{bmatrix} \neg V^{CH}_{r,k,i,t,n,b} \\ L^{CH}_{r,k,i,t,n,b} = 0 \end{bmatrix} \tag{43}$$
$$\forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B$$

$$\begin{bmatrix} V^{DC}_{r,k,i,t,n,b} \\ \xi^{Dmin} C^{SA}_{r,k,i,t} \leq L^{DC}_{r,k,i,t,n,b} \leq \xi^{Dmax} C^{SA}_{r,k,i,t} \end{bmatrix} \vee \begin{bmatrix} \neg V^{DC}_{r,k,i,t,n,b} \\ L^{DC}_{r,k,i,t,n,b} = 0 \end{bmatrix} \tag{44}$$
$$\forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B$$

where ξ^{Cmin} and ξ^{Cmax} are the minimum and maximum charging ratios, and ξ^{Dmin} and ξ^{Dmax} are the minimum and maximum discharging ratios, respectively.

The total charging level of all batteries added to power plant k is calculated by Eqn. (45), whereas the total discharging level pf battery is determined by Eqn. (46).

$$\varrho_{n,b} \sum_{i \in I_k} L^{CH}_{r,k,i,t,n,b} = TCH_{r,k,t,n,b} \quad \forall r \in R, k \in K^N_r, t \in T, n \in N, b \in B \tag{45}$$

$$\varrho_{n,b} \sum_{i \in I_k} L^{DC}_{r,k,i,t,n,b} = TDC_{r,k,t,n,b} \quad \forall r \in R, k \in K^N_r, t \in T, n \in N, b \in B \tag{46}$$

Eqn. (47) determines the storage level of the battery ($SL_{r,k,i,t,n,b}$) during subperiod b. The storage level during subperiod b is affected by the storage level during previous period $b-1$, the charging and the discharging level during current subperiod b.

$$SL_{r,k,i,t,n,b} = (1-\iota) SL_{r,k,i,t,n,b-1} + \pi^{CH} L^{CH}_{r,k,i,t,n,b} - L^{DC}_{r,k,i,t,n,b}/\pi^{DC} \tag{47}$$
$$\forall r \in R, k \in K^N_r, i \in I_k, t \in T, n \in N, b \in B$$

where ι is the energy loss in storage, π^{CH} and π^{DC} are the charging and discharging ratios of the battery i, respectively.

4.4. Power plant operational reliability

As discussed in the earlier section, a power plant operational reliability is estimated based on the design and operation of the power plant. The nested disjunction proposed in Appendix A is modified and applied to the power plant that considers redundancy $k \in K_r^{RD}$ so as to estimate the operational reliability using Eqns. (48) - (50). Since we explain in detail how we estimate the operational reliability with a simple example in Appendix A, readers are encouraged to read it first to understand this section better.

$$\bigvee_{h \in H_k} \left[\begin{array}{c} Z_{r,k,h,t} \\ \bigvee_{m \in M_{k,h}} \left[\begin{array}{c} W_{r,k,h,m,t,n,b} \\ P^F_{r,k,t,n,b} = \sum_{s \in S^F_{k,h,m}} P_{s,k} \\ EP_{s,r,k,t,n,b} = P_{s,k}(C^{PO}_{r,k,t,n,b} + \sum_{j \in J^O_{h,m,s}} C^{BO}_{r,k,j,t,n,b}) \quad \forall s \in S^F_{k,h,m} \\ EP_{s,r,k,t,n,b} = P_{s,k}(C^{PA}_{r,k,t} + \sum_{j \in J^O_{h,m,s}} C^{BA}_{r,k,j,t}) \quad \forall s \in S^P_{k,h,m} \end{array} \right] \forall n \in N, b \in B \end{array} \right]$$

$$\forall r \in R, k \in K_r^{RD}, t \in T \tag{48}$$

$$\underline{\bigvee_{h \in H_k}} Z_{r,k,h,t} \quad \forall r \in R, k \in K_r^{RD}, t \in T \tag{49}$$

$$Z_{r,k,h,t} \Longleftrightarrow \bigvee_{m \in M_{k,h}} W_{r,k,h,m,t,n,b} \quad \forall r \in R, k \in K_r^{RD}, h \in H_k, t \in T, n \in N, b \in B \tag{50}$$

where $Z_{r,k,h,t}$ is true if the power plant k of region r select the design h in year t, and $W_{r,k,h,m,t,n,b}$ is true if the power plant k with the design h of region r has the operation mode m during subperiod b of representative day n in year t. $S^F_{k,h,m}$ and $S^P_{k,h,m}$ are the new sets of successful and partial operation states s. $P^F_{r,k,t,n,b}$ is the successful operational reliability of the power plant k. $EP_{s,r,k,t,n,b}$ is the expected power output of the power plant k of region r under the failure state s during subperiod b of representative day n in year t. While the expected power output of successful operation state $EP_{s,r,k,t,n,b}$ is calculated based on the probability of a power plant being in state s and operating capacity of the power plant k when there is no failure, that of partial operation state is calculated by using the probability of state s and installed capacity.

Eqns. (51) - (52) provide logic propositions between the available facilities and the design, and operating facilities and the operation mode, respectively.

$$Z_{r,k,h,t} \Longrightarrow \Big(\bigwedge_{(k,h) \in D^P} Y^{PA}_{r,k,t} \Big) \Big(\bigwedge_{(k,h) \notin D^P} \neg Y^{PA}_{r,k,t} \Big) \Big(\bigwedge_{(k,j,h) \in D^B} Y^{BA}_{r,k,j,t} \Big) \Big(\bigwedge_{(k,j,h) \notin D^B} \neg Y^{BA}_{r,k,j,t} \Big)$$
$$\forall r \in R, k \in K_r^{RD}, h \in H_k, t \in T \tag{51}$$

$$W_{r,k,h,m,t,n,b} \Longrightarrow \Big(\bigwedge_{(k,h,m) \in O^P} X^{PO}_{r,k,t,n,b} \Big) \Big(\bigwedge_{(k,h,m) \notin O^P} \neg X^{PO}_{r,k,t,n,b} \Big)$$
$$\Big(\bigwedge_{(k,j,h,m) \in O^B} X^{BO}_{r,k,j,t,n,b} \Big) \Big(\bigwedge_{(k,j,h,m) \notin O^B} \neg X^{BO}_{r,k,j,t,n,b} \Big) \tag{52}$$
$$\forall r \in R, k \in K_r^{RD}, h \in H_k, m \in M_{k,h}, t \in T, n \in N, b \in B$$

where D^P and D^B are the new sets connecting available power plant k and corresponding design h, and available parallel generator j of power plant k and corresponding design h, respectively. O^P and O^B are the new sets connecting operating power plant k and corresponding operation mode m under design h, and operating parallel generator j of power plant k and corresponding operation mode m under design h, respectively.

The total expected power output ($TEP_{r,k,t,n,b}$) is calculated as shown in Eqn. (53).

$$TEP_{r,k,t,n,b} = \varrho_{n,b} \sum_{s \in S_{k,h}} EP_{s,r,k,t,n,b} \quad \forall r \in R, k \in K_r^{RD}, t \in T, n \in N, b \in B \tag{53}$$

Since parallel units are not considered for the power plants without redundancy $k \in K_r^{ND}$, a new method calculating successful operational reliability without parallel units is proposed. Successful operational reliability and the total expected power output of the power plants without redundancy are calculated by using Eqns. (54) - (55). λ_k^P is the probability of failures of power plant k.

$$P^F_{r,k,t,n,b} = \lambda_k^P + (1-\lambda_k^P)(1-X^{PO}_{r,k,t,n,b}) \quad \forall r \in R, k \in K_r^{ND}, t \in T, n \in N, b \in B \tag{54}$$

$$TEP_{r,k,t,n,b} = \varrho_{n,b} \lambda_k^P C^{PO}_{r,k,t,n,b} \quad \forall r \in R, k \in K_r^{ND}, t \in T, n \in N, b \in B \tag{55}$$

Since the summation of successful and partial operational reliability should be 1, partial operational reliability ($P^P_{r,k,t,n,b}$) is simply calculated as shown in Eqn. (56).

$$P^P_{r,k,t,n,b} = 1 - P^F_{r,k,t,n,b} \quad \forall r \in R, k \in K_r, t \in T, n \in N, b \in B \tag{56}$$

The amount of fuel such as natural gas and coal consumed by the dispatchable power plant $k \in K_r^D$ is estimated based on the operating capacity of dispatchable facilities (Eqn. (57)).

$$FS_{r,k,t,n,b} = \eta_k (C^{PO}_{r,k,t,n,b} + \sum_{j \in J_k} C^{BO}_{r,k,j,t,n,b}) \quad \forall r \in R, k \in K_r^D, t \in T, n \in N, b \in B \tag{57}$$

where η_k is the conversion ratio of the power plants k.

4.5. Objective function

The objective function is to minimize the net present cost (Ψ) consisting of the capital expenditure (Ψ^{CAPEX}), operating expenses (Ψ^{OPEX}), downtime penalty (Ψ^{DTP}), and unmet demand penalty (Ψ^{UMP}), as shown in Eqn. (58). Ψ^{CAPEX} is represented by Eqn. (59) and τ_t is the discount factor over time t.

$$min \quad \Psi = \Psi^{CAPEX} + \Psi^{OPEX} + \Psi^{DTP} + \Psi^{UMP} \tag{58}$$

$$\begin{aligned} \Psi^{CAPEX} = \sum_{r \in R} \sum_{t \in T} \tau_t (&\sum_{k \in K_r^{PN}} AIC^P_{r,k,t} + \sum_{k \in K_r^{RD}} \sum_{j \in J_k} AIC^B_{r,k,j,t} + \\ &\sum_{k \in K_r^N} \sum_{i \in I_k} AIC^S_{r,k,i,t} + \sum_{k \in K_r^{EX}} EC_{r,k,t}) \end{aligned} \tag{59}$$

Ψ^{OPEX} consists of the fixed operating cost (Ψ^{FOC}), variable operating cost (Ψ^{VOC}), start-up cost (Ψ^{SUC}), shut-down cost (Ψ^{SDC}), fuel cost (Ψ^{FUC}), and CO_2 emission cost (Ψ^{CEM}) as shown in Eqn. (60). Detailed equations for the cost terms are shown in Eqns. (61) - (66),

$$\Psi^{OPEX} = \Psi^{FOC} + \Psi^{VOC} + \Psi^{SUC} + \Psi^{SDC} + \Psi^{FUC} + \Psi^{CEM} \tag{60}$$

$$\Psi^{FOC} = \sum_{r \in R} \sum_{t \in T} \tau_t \Big(\sum_{k \in K_r} FOC^P_{r,k,t} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} FOC^B_{r,k,j,t} + \sum_{k \in K^N_r} \sum_{i \in I_k} FOC^S_{r,k,i,t} \Big) \tag{61}$$

$$\begin{aligned} \Psi^{VOC} = \sum_{r \in R} \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \tau_t \Big(\sum_{k \in K_r} VOC^P_{r,k,t,n,b} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} VOC^B_{r,k,j,t,n,b} + \\ \sum_{k \in K^N_r} \sum_{i \in I_k} VOC^S_{r,k,i,t,n,b} \Big) \end{aligned} \tag{62}$$

$$\Psi^{SUC} = \sum_{r \in R} \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \tau_t \Big(\sum_{k \in K^D_r} SUC^P_{r,k,t,n,b} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} SUC^B_{r,k,j,t,n,b} \Big) \tag{63}$$

$$\Psi^{SDC} = \sum_{r \in R} \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \tau_t \Big(\sum_{k \in K^D_r} SDC^P_{r,k,t,n,b} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} SDC^B_{r,k,j,t,n,b} \Big) \tag{64}$$

$$\Psi^{FUC} = \sum_{r \in R} \sum_{k \in K^D_r} \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \tau_t \chi_k FS_{r,k,t,n,b} \tag{65}$$

$$\Psi^{CEM} = \sum_{r \in R} \sum_{k \in K^D_r} \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \tau_t \theta_k \phi_t FS_{r,k,t,n,b} \tag{66}$$

where χ_k is the price of the fuels consumed in the power plants k, θ_k is the CO_2 emission rate of the power plants k, and ϕ_t is the CO_2 tax over time t.

As discussed earlier, two reliability-related cost penalties (i.e., unmet demand penalty and downtime penalty) are taken into account in this paper. The unmet demand is calculated as shown in Eqns. (67) - (69).

$$TDM_{t,n,b} = \varrho_{n,b} \sum_{r \in R} \Big(\Omega_{r,t,n,b} + \sum_{k \in K^N_r} TCH_{r,k,t,n,b} \Big) \quad \forall t \in T, n \in N, b \in B \tag{67}$$

$$TSP_{t,n,b} = \sum_{r \in R} \Big(\sum_{k \in K_r} TEP_{r,k,t,n,b} + \sum_{k \in K^N_r} TDC_{r,k,t,n,b} \Big) \quad \forall t \in T, n \in N, b \in B \tag{68}$$

where $\Omega_{r,t,n,b}$ is the demand profile of region r during subperiod b of representative day n in year t. $TDM_{t,n,b}$ is the total amount of electricity required, and $TSP_{t,n,b}$ is the total amount of electricity produced during subperiod b of representative day n in year t. While the excess is curtailed if the amount of electricity produced is larger than the amount of electricity required, the demand is not fully satisfied in the opposite case.

$$\begin{bmatrix} T_{t,n,b} \\ TDM_{t,n,b} \leq TSP_{t,n,b} \\ UMD_{t,n,b} = 0 \\ CT_{t,n,b} = TSP_{t,n,b} - TDM_{t,n,b} \end{bmatrix} \vee \begin{bmatrix} \neg T_{t,n,b} \\ TDM_{t,n,b} \geq TSP_{t,n,b} \\ UMD_{t,n,b} = TDM_{t,n,b} - TSP_{t,n,b} \\ CT_{t,n,b} = 0 \end{bmatrix} \quad \forall t \in T, n \in N, b \in B \tag{69}$$

where $UMD_{t,n,b}$ is the unmet demand and $CT_{t,n,b}$ is the curtailment during subperiod b of representative day n in year t.

The expected downtime of the power plant k ($DT_{r,k,t}$) is calculated based on the partial operational reliability as shown in Eqn. (70).

$$DT_{r,k,t} = \sum_{n \in N} \sum_{b \in B} P^{P}_{r,k,t,n,b} / (|N| \cdot |B|) \quad \forall r \in R, k \in K_r, t \in T \tag{70}$$

The unmet demand penalty (Ψ^{UMP}) and downtime penalty (Ψ^{DTP}) are calculated as stated in Eqns. (71) - (72),

$$\Psi^{UMP} = \sum_{t \in T} \sum_{n \in N} \sum_{b \in B} \mu \tau_t UMD_{t,n,b} \tag{71}$$

$$\Psi^{DTP} = \sum_{r \in R} \sum_{k \in K_r} \sum_{t \in T} \psi \tau_t DT_{r,k,t} \tag{72}$$

where μ is the unmet demand penalty rate and ψ is the downtime penalty rate.

5. Solution strategy

The proposed GDP model (Eqns. (1) - (72)) is reformulated as a multi-period MILP model using the Hull relaxation (Lee and Grossmann, 2000). This model generally entails millions of constraints and variables that make the model computationally very expensive. Li et al. (2021) prove that commercial solvers such as CPLEX and Gurobi fail to solve large-scale problems with millions of constraints and variables. Since the model proposed in this work not only takes into account multi-period plannings and hourly operations, but also estimates operational reliability depending on the designs and operations, it is necessary to develop a solution method that can reduce the computational expense. To address this challenge, we apply a bilevel decomposition that decomposes the original full-space model into a master problem and a subproblem. Figure 4 shows the bilevel decomposition algorithm.

The deterministic planning master problem (DPM) determines the capacity of the generation systems required to satisfy the load demand. Hourly operation constraints such as unit commitment and ramping up/down constraints are neglected at this level. It is also assumed that operational reliability of power plants is only dependent on the number of parallel units as operational problems are not taken into account in the master problem. Since the master problem is a relaxation of the original model, it yields a lower bound on the cost. The reliability-constrained operation subproblem (ROS) finds the optimal hourly operational results for the design predicted by the master problem. Unlike other bilevel decomposition algorithms where the subproblem only solves operational problems for the design selected by the master problem, the algorithm of this work allows the subproblem to change the configuration obtained from the master problem by adding more parallel generators and/or batteries. The parallel units not selected at the master problem can be added to the subproblem to improve power systems reliability. The impact of dual role of parallel generators are explicitly taken into account depending on the operational strategies. An upper bound is obtained from the solution of the subproblem, because the solution corresponds to a feasible solution of the original problem. The master problem and subproblem are solved iteratively, and the master problem is updated at every iteration by adding tailored cuts. Three heuristic cuts (i.e., capacity pruning cut, timing pruning cut, and capacity & timing fixing cut) that can expedite the convergence are also proposed along with integer cuts. Detailed explanations regarding the cuts can be found in section 5.3. Convergence is achieved when the gap between the lower and upper bounds lies within a specified tolerance, or the lower bound is greater than the upper bound due to the inclusion of integer cuts. At a certain iteration, if the lower bound is greater than the upper bound, it implies the search can be stopped as no better solution can be found in the following iterations (You et al., 2011).

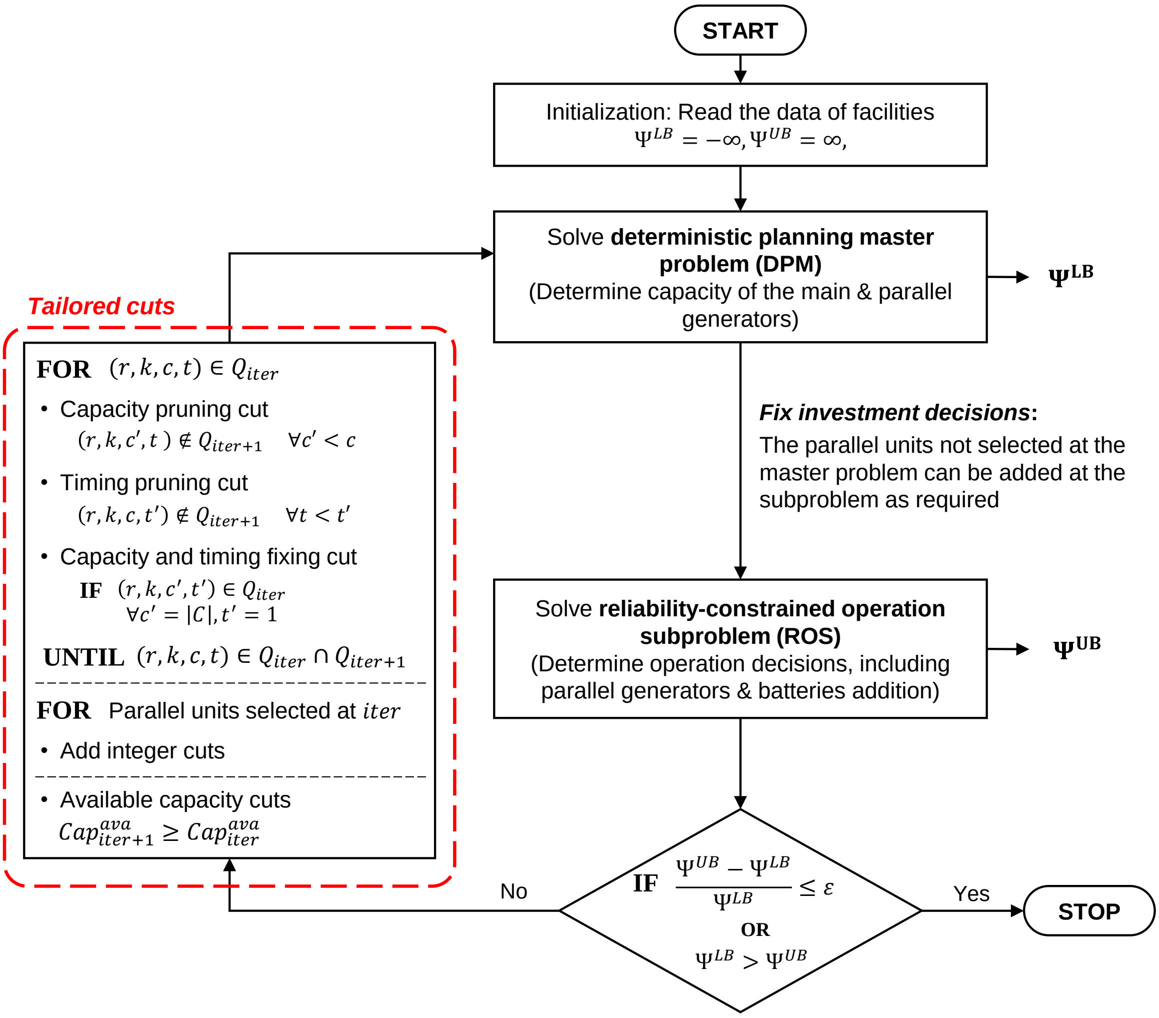

Figure 4: Bilevel decomposition algorithm

5.1. Master problem

The DPM model determines the optimal configuration of power generation systems while ignoring the detailed hourly operation and reliability estimation constraints. Installation of power plants, Eqns. (1) - (9), and installation of parallel generators, Eqns. (73) - (78), and peak demand constraint (Eqn. (21)) are included in the master problem.

$$\left[\begin{array}{c} YU^{BI}_{r,k,j,t} \\ \bigvee_{c\in C}\left[\begin{array}{c} YU^{BI}_{r,k,j,c,t} \\ AIC^{B}_{r,k,j,t} = \alpha^{B}_{k,j,c} \\ C^{BI}_{r,k,j,t} = \varphi^{B}_{k,j,c} \end{array}\right] \end{array}\right] \vee \left[\begin{array}{c} \neg YU^{BI}_{r,k,j,t} \\ AIC^{B}_{r,k,j,t} = 0 \\ C^{BI}_{r,k,j,t} = 0 \end{array}\right] \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{73}$$

$$\left[\begin{array}{c} YU^{BA}_{r,k,j,t} \\ FOC^{B}_{r,k,j,t} = \beta^{B}_{k,j} C^{BA}_{r,k,j,t} \end{array}\right] \vee \left[\begin{array}{c} \neg YU^{BA}_{r,k,j,t} \\ FOC^{B}_{r,k,j,t} = 0 \end{array}\right] \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{74}$$

$$YU^{BI}_{r,k,j,t} \iff \bigvee_{c \in C} YU^{UBI}_{r,k,j,c,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{75}$$

$$YU^{BA}_{r,k,j,t} \iff YU^{BA}_{r,k,j,t-1} \vee YU^{BI}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t > 1 \tag{76}$$

$$YU^{BA}_{r,k,j,t} \implies YU^{PA}_{r,k,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{77}$$

$$YU^{BI}_{r,k,j+1,t} \implies YU^{BI}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{78}$$

where $YU^{BI}_{r,k,j,t}$ and $YU^{UBI}_{r,k,j,c,t}$ are the Boolean variables for installation of parallel generators used in the master problem, and $YU^{BA}_{r,k,j,t}$ is the Boolean variable for availability of parallel generators used in the master problem.

The objective function of the master problem is defined by Eqns. (79) - (80). While the exact Ψ^{CAPEX} and Ψ^{FOC} can be calculated as power plants and parallel generators are installed (Eqns. (81) - (82)), the variable operating cost (Ψ^{VOC}), start-up/shut-down costs (Ψ^{SUC} and Ψ^{SDC}), fuel cost (Ψ^{FUC}), CO_2 emission cost (Ψ^{CEM}), and reliability-related penalties (Ψ^{DTP} and Ψ^{UMP}) cannot exactly be computed since the detailed operational constraints and reliability estimation are excluded in the master problem:

$$min \quad \Psi^{LB} = \Psi^{CAPEX} + \Psi^{OPEX} + \Psi^{DTP} + \Psi^{UMP} \tag{79}$$

$$\Psi^{OPEX} = \Psi^{FOC} + \Psi^{VOC} + \Psi^{SUC} + \Psi^{FUC} + \Psi^{CEM} \tag{80}$$

$$\Psi^{CAPEX} = \sum_{r \in R} \sum_{t \in T} \tau_t \Big(\sum_{k \in K^{PN}_r} AIC^{P}_{r,k,t} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} AIC^{B}_{r,k,j,t} + \sum_{k \in K^{EX}_r} EC_{r,k,t} \Big) \tag{81}$$

$$\Psi^{FOC} = \sum_{r \in R} \sum_{t \in T} \tau_t \Big(\sum_{k \in K_r} FOC^{P}_{r,k,t} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} FOC^{B}_{r,k,j,t} \Big) \tag{82}$$

$$\Psi^{VOC} = \sum_{r \in R} \sum_{t \in T} \tau_t \Big(\sum_{k \in K_r} \varepsilon^{P}_{k} \overline{C^{PO}_{r,k,t}} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} \varepsilon^{B}_{k,j} \overline{C^{BO}_{r,k,j,t}} \Big) \tag{83}$$

$$\Psi^{SUC} = \sum_{r \in R} \sum_{t \in T} \tau_t \Big(\sum_{k \in K^{RD}_r} \zeta^{PU}_{k} y^{PA}_{r,k,t} + \sum_{k \in K^{RD}_r} \sum_{j \in J_k} \zeta^{BU}_{k,j} y^{BA}_{r,k,j,t} \Big) + \sum_{r \in R} \sum_{k \in K^{ND}_r} \tau_{t=1} \zeta^{PU}_{k} y^{PA}_{r,k,t=1} \tag{84}$$

$$\Psi^{FUC} = \sum_{r \in R} \sum_{k \in K^{D}_r} \sum_{t \in T} \tau_t \chi_k \overline{FS_{r,k,t}} \tag{85}$$

$$\Psi^{CEM} = \sum_{r \in R} \sum_{k \in K^{D}_r} \sum_{t \in T} \tau_t \theta_k \phi_t \overline{FS_{r,k,t}} \tag{86}$$

To obtain a valid and tight lower bound, Ψ^{VOC} is estimated using the operating capacity of the main and parallel generators that are expected to be operated in year t (Eqn. (83)). While shut-down of facilities is excluded in the master problem, it is assumed that all facilities start up every year t if they are available. Since the nuclear power plants produce electricity constantly once they operate, start-up of nuclear power plants is considered only one time at the beginning of the planning year (Eqn. (84)). As stated in Eqns. (85) - (86), Ψ^{FUC} and Ψ^{CEM} are calculated after estimating the amount of fuel required.

The estimated operating capacities of the main plant ($\overline{C^{PO}_{r,k,t}}$) and parallel generators ($\overline{C^{BO}_{r,k,j,t}}$) are calculated as shown in Eqns. (87) - (89), and the estimated amount of fuel ($\overline{FS_{r,k,t}}$) is calculated based on the operating capacity (Eqn. (90)).

$$\overline{C^{PO}_{r,k,t}} = \overline{\Phi}_{r,k,t} \upsilon_t C^{PA}_{r,k,t} \quad \forall r \in R, k \in K^N_r, t \in T \tag{87}$$

$$\gamma^{min}_k \upsilon_t C^{PA}_{r,k,t} \leq \overline{C^{PO}_{r,k,t}} \leq \gamma^{max}_k \upsilon_t C^{PA}_{r,k,t} \quad \forall r \in R, k \in K^{RD}_r, t \in T \tag{88}$$

$$\gamma^{min}_{k,j} \upsilon_t C^{BA}_{r,k,j,t} \leq \overline{C^{BO}_{r,k,j,t}} \leq \gamma^{max}_{k,j} \upsilon_t C^{BA}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{89}$$

$$\overline{FS_{r,k,t}} = \eta_k (\overline{C^{PO}_{r,k,t}} + \sum_{j \in J_k} \overline{C^{BO}_{r,k,j,t}}) \quad \forall r \in R, k \in K^D_r, t \in T \tag{90}$$

where $\overline{\Phi}_{r,k,t}$ is the average capacity factor for renewable power plant, υ_t is the operation time in year t (i.e., 8,760 hours).

Since the operation constraints are excluded in the master problem, successful operational reliability ($\overline{P^F_{r,k,t}}$) is only dependent on the number of parallel generators as stated in Eqn. (91) (Ye et al., 2018). Downtime penalty (Ψ^{DTP}) related to the partial operational reliability is formulated as shown in Eqn. (92). $\lambda^B_{k,j}$ is probability of failure of parallel generator j in power plant k.

$$\overline{P^F_{r,k,t}} = 1 - (1 - \lambda^P_k)\Big(y^{PA}_{r,k,t} + \sum_{j \in J_k} (1 - \lambda^B_{k,j})^{j-1} y^{BA}_{r,k,j,t}\Big) \quad \forall r \in R, k \in K_r, t \in T \tag{91}$$

$$\Psi^{DTP} = \sum_{r \in R} \sum_{k \in K_r} \sum_{t \in T} \psi \tau_t \upsilon_t (1 - \overline{P^F_{r,k,t}}) \tag{92}$$

The disjunction for unmet demand calculation (Eqn. (69)) is substituted by Eqns. (93) - (94) in the master problem.

$$\sum_{r \in R} \sum_{k \in K_r} \overline{C^{PO}_{r,k,t}} + \sum_{r \in R} \sum_{k \in K_r} \sum_{j \in J_k} \overline{C^{BO}_{r,k,j,t}} + \overline{UMD_t} = \overline{CT_t} + \upsilon_t \Lambda_t \quad \forall t \in T \tag{93}$$

$$\Psi^{UMP} = \sum_{t \in T} \mu \tau_t \overline{UMD_t} \tag{94}$$

where $\overline{UMD_t}$ and $\overline{CT_t}$ are the estimated unmet demand and curtailment in year t, respectively. Λ_t is the average demand in year t.

The master problem that can provide a tight lower bound is formulated with Eqns. (1) - (9), Eqn. (21), and Eqns. (73) - (94).

5.2. Subproblem

Since the reliability penalties are underestimated by ignoring the dual role of parallel units and operational constraints, the master problem is unlikely to add parallel units unless necessary to satisfy the load demand. To consider the impact of dual role of generators on power systems reliability, we assume that additional parallel units can be further built in the subproblem. The backup generators that decide to be installed at the master problem ($YU^{BI}_{r,k,j,t}$) should be fixed at the subproblem. For backup generators that do not install at the master problem ($\neg YU^{BI}_{r,k,j,t}$), the variables should be relaxed to allow the subproblem to further install them as required. These constraints are formulated by Eqns. (95) - (96).

$$YU^{BI}_{r,k,j,t} \Longrightarrow Y^{BI}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{95}$$

$$YU^{BA}_{r,k,j,t} \Longrightarrow Y^{BA}_{r,k,j,t} \quad \forall r \in R, k \in K^{RD}_r, j \in J_k, t \in T \tag{96}$$

The subproblem determines operation decisions and evaluate operational reliability, as well as determines whether more parallel units are required for improving power systems reliability. The model is formulated with Eqns. (1) - (72), and Eqns. (95) - (96). The subproblem yields a valid upper bound since its solution is a feasible solution of the full-space model. The master problem and subproblem are solved iteratively until the gap between the lower and upper bounds lie within a specified tolerance.

5.3. Tailored cuts generation

A new solution is obtained in the following iteration if the lower bound obtained from the master problem and the upper bound obtained from the subproblem do not lie within the specified tolerance in the previous iteration. We add *cuts* into the master problem at the following iteration so as to eliminate the incumbent solution and to tighten the lower bound. Different cuts such as design cuts, superset cuts, and subset cuts have been proposed in bilevel decomposition works (Iyer and Grossmann, 1998; Dogan and Grossmann, 2006). However, those cuts are not applicable in this work because the subproblem of this work can change the design obtained from the master problem by adding parallel units.

The available capacity of power systems in the master problem will be equal to or slightly larger than the load demand as reliability penalties are underestimated. Therefore, higher reliability penalties are expected in the subproblem, resulting in a large gap between lower and upper bounds. In order to reduce this gap, the power systems reliability is improved as iterations proceed by expanding the available capacity. Eqn. (97) states that the total available capacity of iteration $iter+1$ (Cap^{ava}_{iter+1}), which includes both the main and parallel generators, cannot be less than that of iteration $iter$ (Cap^{ava}_{iter}).

$$Cap^{ava}_{iter+1} \geq Cap^{ava}_{iter} \quad \forall iter \in IT \tag{97}$$

Even if Eqn. (97) enforces the model to increase the available capacity in the master problem as iterations proceed, it can still take many iterations and long computational times. After some tests, we find that the discrete sizes of facilities and discrete periods hinder the model from solving in reasonable computational time. Therefore, heuristic cuts to accelerate the iteration processes are developed, such as the capacity pruning cut, timing pruning cut, and capacity and timing fixing cut, as described below.

a. Capacity pruning cuts. Let Q_{iter} be a set of power plants k with sizes c installed in regions r in year t at iteration $iter$, i.e., $(r,k,c,t) \in Q_{iter}$. If a size c belongs to this set at iteration $iter$, smaller sizes c', $c' < c$, cannot be chosen for the power plant k at the following iteration $iter+1$, i.e., $(r,k,c',t) \notin Q_{iter+1}$, $c' < c$, if $(r,k,c,t) \in Q_{iter}$.

Let Ψ^{LB}_{iter} be the lower bound obtained from the master problem at iteration $iter$. For the algorithm to converge in a finite number of iterations, the lower bound should increase as iteration proceeds, i.e., $\Psi^{LB}_{iter} \leq \Psi^{LB}_{iter+1}$. Assume a size c is selected for a power plant k at iteration $iter$. Since the total available capacity should be increased by constraint (97), it is not recommended to choose smaller sizes c', $c' < c$, for the power plant k at the following iteration $iter + 1$. It should be noted that it is not impossible for the large-sized power plant selected at the previous iteration to have smaller size at the following iterations. However, installing multiple small-sized units cannot give the optimal solution in cost minimization model by the economy of scale. If a specific power plant is not chosen at the previous iteration, this cut cannot be applied to the power plant. It is because this cut removes the design combination of the power plant selected in the previous iteration for the following iteration.

b. Timing pruning cuts. If a power plant k is installed in year t at iteration $iter$, the power plant k cannot be installed after year t at the following iteration $iter + 1$, i.e., $(r, k, c, t') \notin Q_{iter+1}$, $t < t'$, if $(r, k, c, t) \in Q_{iter}$.

Assume a power plant k is installed in year t at the initial iteration $iter = 1$. Since the capacity constraint (Eqn. (97)) is not considered at the first iteration, the power plant k is installed when the capacity needs to be expanded to satisfy the load demand. Also assume the power plant k installed in year t at the first iteration $iter = 1$ is installed in year t', $t' < t$, at the second iteration $iter = 2$. Since the power plant is installed earlier than required, it can act as a backup in year t', resulting in improving the reliability of year t'. As a result, by enforcing the power plant k selected in year t at the previous iteration $iter$ not to be installed in a later time t', $t < t'$ at the following iterations, the power systems reliability will be maintained or improved in each iteration.

c. Capacity and timing fixing cuts. If the largest size $c = |C|$ is selected for a power plant k at the initial year $t = 1$, the size and installation time for the power plant k are fixed from the following iteration $iter + 1$.

By cuts (a) and (b), it is changed for the power plant k installed at the previous iteration $iter$ to choose larger size and install it earlier than required as iterations continue. Therefore, the size and investment timing should be fixed if the largest size (i.e., $c = |C|$) and the earliest time ($t = 1$) are selected since there is no further way to change either size and installation time.

Cuts (a), (b), and (c) are added until there are no plant design changes in consecutive iterations. It should be noted that the three cuts are only used to eliminate the combinations of the main power plants. Since the parallel units can only be added to the available power plants, integer cuts effectively remove the previous combination while expanding the total available capacity. Moreover, parallel units may not follow economies of scale for improving power systems reliability. Therefore, only integer cuts are added for the parallel units at every iteration.

6. Comparison of the planning models with and without reliability constraints

To show the advantage of the proposed model with reliability constraints (RP) (Eqns. (1) - (72)), it is designed a small power generation system that consists of two regions with three power plants (i.e., coal, natural gas, and biomass-fired). It is assumed that, at most, two parallel generators can be added to the main generator, and three sizes are available for potential facilities. The planning horizon is 5 years, and 4 representative days from each year are selected. Parameters used in this example, such as load demand, available sizes, probability of failures, and capital/operating costs, can be found in Appendix D.1.

Additionally, the model without reliability constraints (DP) is applied to the same example, and the expected reliability penalties (i.e., unmet demand penalty and downtime penalty) are calculated for the design obtained from the DP model so as to compare the impact of reliability in design stage.

The two models are coded in GAMS 32.1.0, and solved using Gurobi 32.1.0 as the MIP solver on an Intel Core i7-10510U CPU, 1.80GHz. Table 1 presents the computational results of the two models. Since the probability of failures is considered in the RP model, it has more constraints and continuous variables than the DP model, resulting in a longer computational time (e.g., RP: 10.84 sec, DP: 0.66 sec). However, the RP model has lower total cost and higher system average availability than the DP model due to the sufficient capacity. Note that the system average availability is calculated by taking the average availability of all power plants over the planning horizon.

Table 1: Computational and economic results of the illustrative example

Model	Constraints	Cont. Vars	Bin. Vars	CPU (sec)	Cost (M$)	Sys. Avail (%)
RP	23,503	12,435	1,644	10.84	530.1	0.984
DP	13,419	7,391	1,644	0.66	649.0	0.948

Figure 5 shows the optimal configuration of both cases in the last year (T5). Since the probability of failure is taken into account in the RP model, the total available capacity of the RP model (2900MW) is larger than that of the DP model (2200MW). Two parallel generators are installed together with the main generator in the natural gas power plants of regions 1 and 2, because of low capital/operating costs and CO_2 emission rate. Contrary to the design obtained by the DP model, the RP model adds more parallel generators into the coal and biomass-fired power plants (i.e., 1 unit of 100MW to coal power plant, and 2 units of 200MW to biomass-fired plant) with a relatively higher probability of failure to prevent the plants from entirely failing.

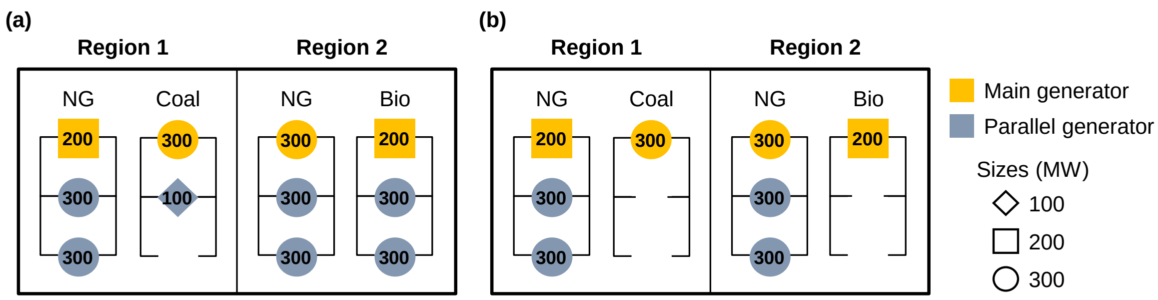

Figure 5: The configuration in the last year: (a) proposed model (RP) and (b) the model without reliability (DP)

Figure 6 displays on/off snapshots of all power plants during subperiods of the last year (T5). As shown in Figure 6(a), some parallel generators are operated to produce electricity than remaining as idle generators in the RP model. Interestingly, the power plants with two parallel generators (i.e., the natural gas plant in regions 1 and 2 and the biomass-fired plant in region 2) avoid using all three generators simultaneously except in exceptional cases because operating all generators is likely to decrease the successful operational reliability. On the other hand, as shown in Figure 6(b), the main and parallel generators are used to produce electricity and to satisfy the load demand in the DP model as the probability of failure and the dual role of parallel generators are not taken into account.

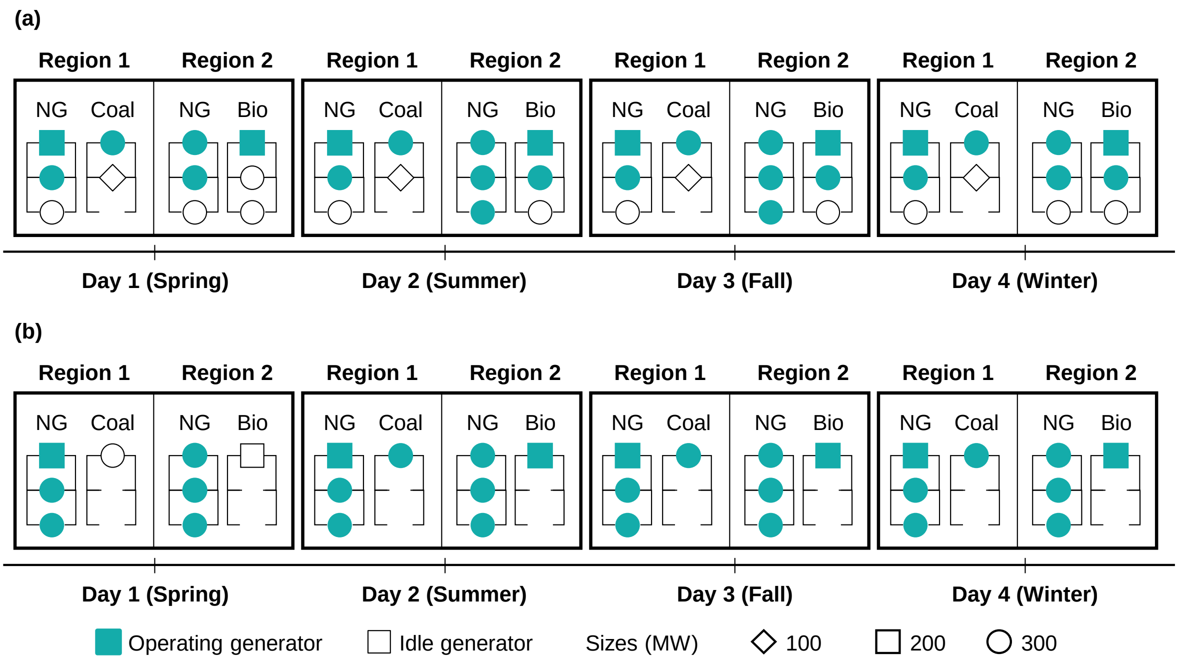

Figure 6: On/off snapshots during subperiods in the last year: (a) proposed model (RP) and (b) the model without reliability (DP)

As shown in Figure 7, the RP model requires higher CAPEX ($108M) and OPEX ($373M) over 5 years due to having more parallel generators. However, since the RP model considers slack capacity to reallocate the load demand when the generators fail, lower reliability penalties are occurred (unmet demand penalty: 0, and downtime penalty: $49M). In contrast, the DP model has lower CAPEX ($64M) and OPEX ($366M) but incurs in higher reliability penalties (unmet demand penalty: $57M, and downtime penalty: $163M) due to its insufficient capacity. Consequently, the more reliable design obtained by the RP model enables the power generators systems to have a better economic performance than the DP model (Total cost: RP - $530M, DP - $649M). This example shows that the proposed model is more effective for designing reliable power systems than the expansion model in which the probability of failures is not considered.

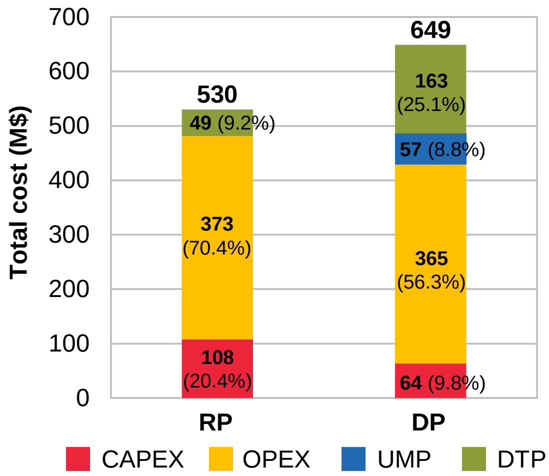

Figure 7: Cost contribution: (a) proposed model (RP) and (b) the model without reliability (DP)

7. Case studies

The proposed model and bilevel algorithm are applied to two cases: i) Case 1: 5-year planning problem and ii) Case 2: 10-year planning problem. 1 representative day is selected from each season of a year, meaning a total of 4 representative days are chosen for a year, and 24-hours operations are solved for each of the representative days. As shown in Figure 8, each case consists of two regions with six types of power plants: coal, natural gas, nuclear, biomass-fired, wind turbines, and solar panels. Each region is assumed to have a maximum of 9 power plants (region 1: 5 existing and 4 potential; region 2: 4 existing and 5 potential). Among the power plants, parallel generators can be added to coal, natural gas, and biomass-fired power plants, whereas batteries can be installed in wind turbines and solar panels. The power demands of the two regions over 5 and 10 years are depicted in Figure D.2 in Appendix D.2. Technical and economic parameters used in the two cases, such as available sizes, probability of failures, and capital/operating costs, can also be found in Table D.2 in Appendix D.2.

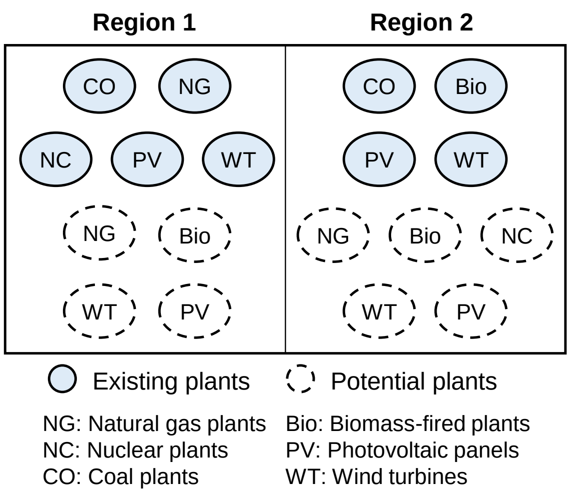

Figure 8: Configuration of the case studies

The two cases are modeled in GAMS 32.1.0, and solved with the proposed bilevel

Table 2: Computational results of the two cases

Cases	Models	Constraints	Cont. Vars.	Bin. Vars.	CPU (sec)	Cost (M$)	Gap (%)
Case 1	Full-space	2,036,308	869,869	218,770	3,377	7,470	0.90
	Bilevel	-	-	-	407	7,485	1.1
	DPM	2,394	2,659	1,390	0.95	7,403	-
	ROS	2,043,990	1,098,239	218,770	406	7,485	-
Case 2	Full-space	5,173,241	2,211,134	565,580	$36,000^a$	*	-
	Bilevel	-	-	-	2,469	15,108	0.13
	DPM	6,106	3,314	3,820	2.02	14,995	-
	ROS	5,192,464	2,234,174	565,580	2,467	15,108	-

[a]: Computational time limit = 36,000 sec

*: No feasible solution found

decomposition and full-space model using Gurobi 32.1.0 on an Intel Core i7-10510U CPU, 1.80GHz. Table 2 presents the computational results of the two cases. Case 1 is solved using the full-space model in an hour within a 1% optimality gap. The proposed bilevel decomposition dramatically reduces the computational time, solving Case 1 in 407 seconds, compared to 3,377 seconds for the full-space model. Since the operational and reliability evaluation constraints are not explicitly considered but roughly estimated in the master problem (DPM), the number of constraints, continuous variables, and binary variables of the DPM problem is smaller than those of the ROS subproblem by at least two orders of magnitude. The solution gap between the full-space model and the bilevel decomposition is also less than 1% (i.e. 0.02%). The full-space model does not solve Case 2 within the 10 hours (36,000 seconds) due to its very large computational size. However, the bilevel decomposition solves the same case in 2,469 seconds within a 0.13% optimality gap. From these two cases, it is shown that the decomposition algorithm is effective in solving very large-scale expansion planning problems with millions of constraints and variables.

Figure 9 shows the accumulated capacity and the ratio between the main and parallel generation systems in Case 1. As shown in Figure 9(a), the nuclear power plants are the largest electric power suppliers, accounting for 56% of the total available capacity (3,128MW) at the last planning period (T5), followed by natural gas power plants (2,270MW, 41% of the total available capacity). Preinstalled coal power plants are decommissioned due to their lower efficiency, higher probability of failures, and significant CO_2 emission rate. Renewable power plants such as solar panels and wind turbines are not installed as they are less reliable than the nuclear power plants, and are highly intermittent. Figure 9(b) depicts that approximately 25% of the total available capacity is the capacity of parallel generators that are installed to prevent the cases where the operating natural gas power plants fail. It should be noted that since this paper aims to propose a new optimization model considering the probability of equipment failures, additional constraints to promote the installation of renewable generation technologies are not taken into account, such as CO_2 emission regulation and renewable penetration targets. Therefore, the results can be changed when these constraints are considered in the planning horizon.

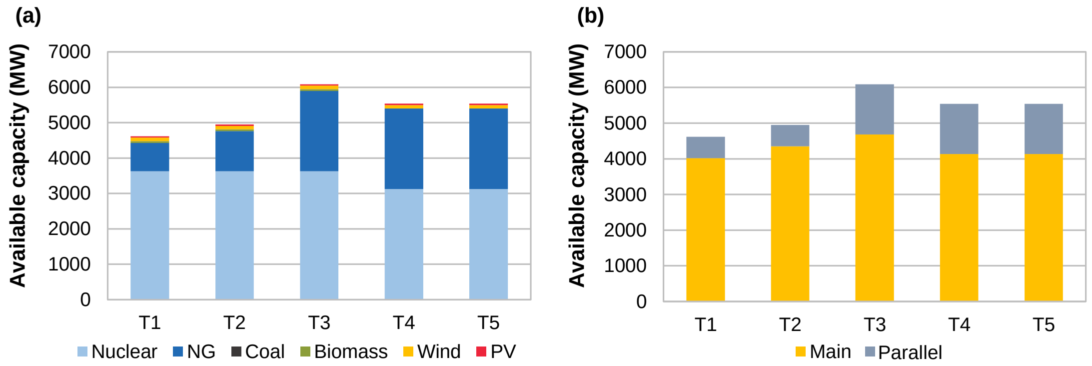

Figure 9: (a) Available capacity over the planning horizon, (b) ratio of the main and parallel systems of Case 1

Figure 10 provides the design results of Case 2. As shown in Figure 10(a), the main generation technology is changed from nuclear power plant (from T1 to T6) to natural gas power plant (from T7 to T10) due to a limit of installation of nuclear power plants, and relatively cheaper costs of natural gas power plants than other types of power plants. At the last planning period (T10), the natural gas power plants account for 64% of the total available capacity (5,206MW), and the nuclear power plants, the second largest power plant, accounts for 34% of the total available capacity (2,735MW). The capacity of parallel generators also increases as the natural gas power plants are installed so as to prevent the failures of equipment in natural gas power plants (Figure 10(b)).

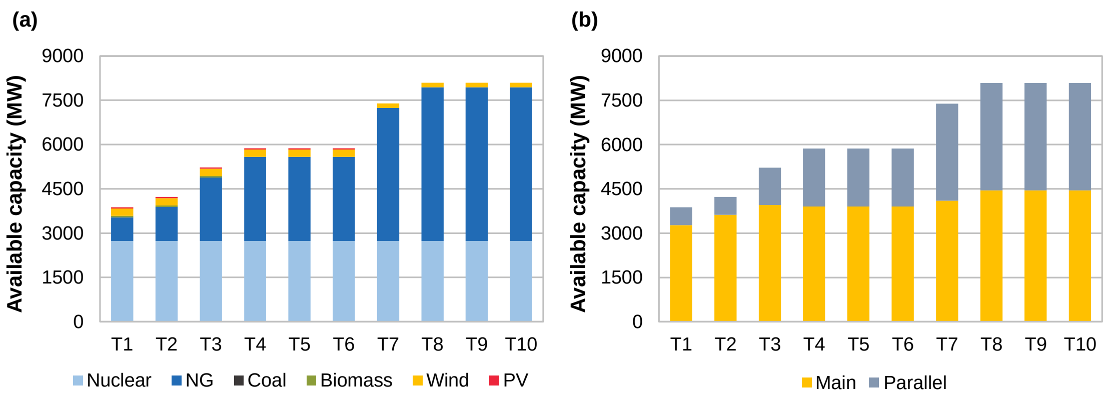

Figure 10: (a) Available capacity over the planning horizon, (b) ratio of the main and parallel systems of Case 2

Figure 11 depicts a 24-hours operational schedule at the last representative day $|N|$, $n = 4$, of the last planning year $|T|$, $T = 10$, of Case 1. Nuclear power plants constantly operate during 24 hours due to their slower ramping up and down capacity and expensive start-up & shut-down costs. Natural gas power plants buffer the unmet demand that nuclear power plants do not cover as natural gas power plants can quickly change their power outputs and have relatively cheap operation costs than nuclear power plants. It should be noted that 2.7% of the hourly demand during subperiods 18 and 19, i.e., 18:00PM and 19:00PM, (i.e., 11.37 GWh) is not satisfied. The model decides that paying the penalty for the small amount of the unmet demand is cheaper than starting up a new facility or ramping up the operating facility. It also should be noted that such operation results are dependent on the parameter used, therefore, the penalty of unmet demand can be increased to avoid this results.

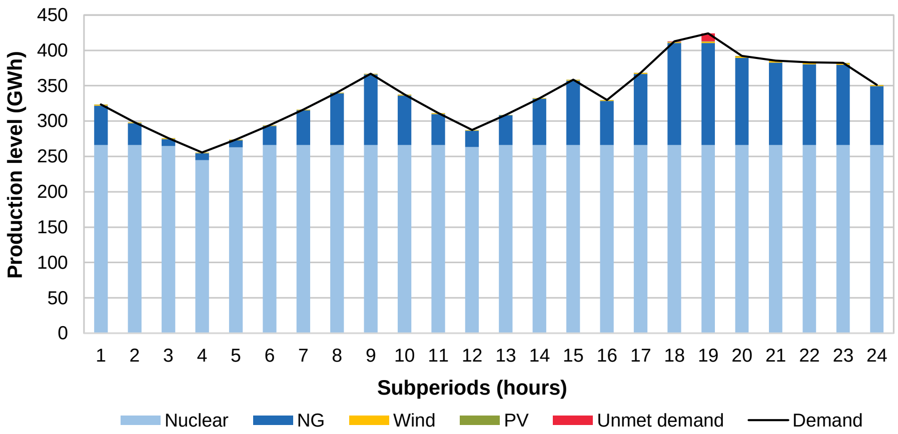

Figure 11: Operation strategies of Case 1 at the last representative day $|N|$ of the last planning year $|T|$

8. Conclusions

In this work, we have proposed a Generalized Disjunctive Programming (GDP) model and bilevel decomposition algorithm to optimize the expansion planning of reliable power generation systems. The model determines both long-term investment decisions (e.g., size, location, and time of facilities to install, retire, and decommission) and short-term operation decisions (e.g., on/off status, operating capacity, and expected power output).

The original contributions of the proposed model are as follows: i) it optimizes the size and operation of the reserve systems (or backup systems), as well as the main systems, to improve power systems reliability, and ii) the impact of dual role of parallel generators (i.e., participating in electricity production vs. remaining as backup units during operation) on power systems reliability is explicitly considered. Two penalties, downtime and unmet demand penalties, are included in the objective function to maximize reliability while minimizing the net present cost. Furthermore, a bilevel decomposition with tailored cuts has been proposed to reduce the computational expenses of the multi-scale expansion planning model. We compared two planning models with/without reliability constraints and shown that the proposed model provides more reliable design and operations at lower total cost when accounting for penalties. The effectiveness of the bilevel decomposition algorithm is also verified by solving two large-scale case studies, 5-years planning and 10-years planning.

As for future work, a generalized reliability-constrained expansion planning model will be developed, which includes alternative ways for improving reliability such as maintenance or inspection, as well as redundancy. The proposed reliability-constrained expansion planning model will include resilience, the ability of a system to quickly recover its normal conditions after the occurrences of disruptions, so as to establish reliable and resilient power systems.

Acknowledgements

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Nomenclature

Indices and Sets

$r \in R$	set of regions
$k \in K$	set of power plants (main generators)
$k \in K_r$	set of power plants in region r
$k \in K_r^{PN}$	set of potential power plants in region r, $K_r^{PN} \subseteq K_r$
$k \in K_r^{EX}$	set of existing power plants in region r, $K_r^{EX} \subseteq K_r$
$k \in K_r^{RD}$	set of power plants that consider redundancy in region r, $K_r^{RD} \subseteq K_r$
$k \in K_r^{ND}$	set of power plants that do not consider redundancy in region r, $K_r^{ND} \subseteq K_r$
$k \in K_r^{D}$	set of dispatchable power plants in region r, $K_r^{D} \subseteq K_r$
$k \in K_r^{N}$	set of renewable power plants in region r, $K_r^{N} \subseteq K_r$
$i \in I_k$	set of batteries for renewable plants $k \in K_r^N$
$j \in J_k$	set of parallel generators of power plants that consider redundancy $k \in K_r^{RD}$
$c \in C$	set of discrete sizes
$h \in H_k$	set of designs of power plants that consider redundancy $k \in K_r^{RD}$
$m \in M_{k,h}$	set of operation modes of power plants that consider redundancy $k \in K_r^{RD}$ under designs $h \in H_k$
$s \in S_{k,h}$	set of failure states of power plants that consider redundancy $k \in K_r^{RD}$ under designs $h \in H_k$
$s \in S_{k,h,m}^F$	set of successful operational states of power plants that consider redundancy $k \in K_r^{RD}$ under designs $h \in H_k$ and operation modes $m \in M_{k,h}$
$s \in S_{k,h,m}^P$	set of partial operational states of power plants that consider redundancy $k \in K_r^{RD}$ under designs $h \in H_k$ and operation modes $m \in M_{k,h}$
$t \in T$	set of planning years
$n \in N$	set of representative days
$b \in B$	set of subperiods (hours)
$iter \in IT$	set of iterations in bilevel decomposition algorithm

Parameters

$\varphi_{k,c}^P$	Nameplate capacity of potential power plant k with size c (MW)
$\varphi_{k,j,c}^B$	Nameplate capacity of parallel generator j of power plant k with size c (MW)
$\varphi_{k,i}^S$	Nameplate capacity of parallel battery i of power plant k (MW)
ω_k	Preinstalled capacity of existing power plant k (MW)
$\sigma_{r,k}$	Remaining lifetime of existing power plant k in region r (years)
Θ_t	Peak power demand in year t (MW)
$\Phi_{r,k,t,n,b}$	Capacity factor of power plant k of region r during subperiod b of representative day n in year t (%)
$\varrho_{n,b}$	Operation time during subperiod b of representative day n (hours)
γ_k^{min},γ_k^{max}	Minimum/maximum operating capacity of power plant k (%)
$\gamma_{k,j}^{min}$,$\gamma_{k,j}^{max}$	Minimum/maximum operating capacity of parallel generator j of power plant k (%)

$\gamma_{k,i}^{min}$,$\gamma_{k,i}^{max}$	Minimum/maximum operating capacity of parallel battery i of power plant k (%)
κ_k^U,κ_k^D	Start-up/shut-down limit of power plant k (%)
$\kappa_{k,j}^U$,$\kappa_{k,j}^D$	Start-up/shut-down limit of parallel generator j of power plant k (%)
π^{CH}	Charging ratio of battery (%)
π^{DC}	Discharging ratio of battery (%)
ι	Loss of energy in battery (%)
λ_k^P	Probability of failure of power plant k (%)
$\lambda_{k,j}^B$	Probability of failure of parallel generator j in power plant k (%)
η_k	Conversion ratio of power plant k (MMBtu/GWh)
θ_k	CO_2 emission rate of power plant k (kg/MMBtu)
$\Omega_{r,t,n,b}$	Power demand of region r during subperiod b of representative day n in year t (MW)
ξ^{Cmin},ξ^{Cmax}	Minimum/maximum charging ratio (%)
ξ^{Dmin},ξ^{Dmax}	Minimum/maximum discharging ratio (%)
$\alpha_{k,c}^P$	Investment cost coefficient of potential power plant k with size c ($)
$\alpha_{k,j,c}^B$	Investment cost coefficient of parallel generator j of power plant k with size c ($)
$\alpha_{k,i}^S$	Investment cost coefficient of parallel battery i of power plant k ($)
β_k^P	Fixed operating cost coefficient of potential power plant k ($)
$\beta_{k,j}^B$	Fixed operating cost coefficient of parallel generator j of power plant k ($)
$\beta_{k,i}^S$	Fixed operating cost coefficient of parallel battery i of power plant k ($)
δ_k	Lifetime extension cost coefficient of existing power plant k ($)
ε_k^P	Variable operating cost coefficient of power plant k ($/GWh)
$\varepsilon_{k,j}^B$	Variable operating cost coefficient of parallel generator j of power plant k ($/GWh)
$\varepsilon_{k,i}^S$	Variable operating cost coefficient of parallel battery i of power plant k ($/GWh)
ζ_k^{PU},ζ_k^{PD}	Start-up/shut-down cost coefficient of power plant k ($)
$\zeta_{k,j}^{BU}$,$\zeta_{k,j}^{BD}$	Start-up/shut-down cost coefficient of parallel generator j of power plant k ($)
τ_t	Discount factor over time t (%)
χ_k	Fuel price consumed by power plant k ($/MMBtu)
ϕ_t	CO_2 tax rate over time t ($/kg)
μ	Unmet demand penalty rate ($)
ψ	downtime penalty rate ($)
υ_t	Operation time of year t (hours) used in master problem
$\overline{\Phi}_{r,k,t}$	Average capacity factor for power plant k of region r in year t used in master problem
Λ_t	Average demand in year t (MW) used in master problem

Continuous variables

$C_{r,k,t}^{PI}$	Installed capacity of potential power plant k of region r at year t (MW)
$C_{r,k,j,t}^{BI}$	Installed capacity of parallel generator j of power plant k in region r at year t (MW)

$C^{SI}_{r,k,i,t}$	Installed capacity of parallel battery i of power plant k in region r at year t (MW)
$C^{PA}_{r,k,t}$	Available capacity of potential power plant k of region r at year t (MW)
$C^{BA}_{r,k,j,t}$	Available capacity of parallel generator j of power plant k in region r at year t (MW)
$C^{SA}_{r,k,i,t}$	Available capacity of parallel battery i of power plant k in region r at year t (MW)
$C^{PO}_{r,k,t,n,b}$	Operating capacity of power plant k of region r during subperiod b of representative day n in year t (MW)
$C^{BO}_{r,k,j,t,n,b}$	Operating capacity of parallel generator j of power plant k of region r during subperiod b of representative day n in year t (MW)
$SL_{r,k,i,t,n,b}$	Storage level of parallel battery i of power plant k of region r during subperiod b of representative day n in year t (MW)
$L^{CH}_{r,k,i,t,n,b}$	Charging level of parallel battery i of power plant k of region r during subperiod b of representative day n in year t (MW)
$L^{DC}_{r,k,i,t,n,b}$	Discharging level of parallel battery i of power plant k of region r during subperiod b of representative day n in year t (MW)
$TCH_{r,k,t,n,b}$	Total charging level of parallel battery installed in power plant k of region r during subperiod b of representative day n in year t (GWh)
$TDC_{r,k,t,n,b}$	Total discharging level of parallel battery installed in power plant k of region r during subperiod b of representative day n in year t (GWh)
$TDM_{t,n,b}$	Total amount of power required during subperiod b of representative day n in year t (GWh)
$TSP_{t,n,b}$	Total amount of power produced during subperiod b of representative day n in year t (GWh)
$P_{s,k}$	Probability of failure state s of power plant k (%)
$P^{F}_{r,k,t,n,b}$	Successful operational reliability of power plant k in region r during subperiod b of representative day n in year t (%)
$P^{P}_{r,k,t,n,b}$	Partial operational reliability of power plant k in region r during subperiod b of representative day n in year t (%)
$EP_{s,r,k,t,n,b}$	Expected power output in failure state s of power plant k in region r during subperiod b of representative day n in year t (GWh)
$TEP_{r,k,t,n,b}$	Total expected power output of power plant k in region r during subperiod b of representative day n in year t (GWh)
$FS_{r,k,t,n,b}$	Amount of fuel consumed by power plant k in region r during subperiod b of representative day n in year t (MMBtu)
$UMD_{t,n,b}$	Unmet demand during subperiod b of representative day n in year t (GWh)
$CT_{t,n,b}$	Curtailment during subperiod b of representative day n in year t (GWh)
$DT_{r,k,t}$	Expected downtime of power plant k of region r in year t (hours)
$\overline{C^{PO}_{r,k,t}}$	Estimated operating capacity of power plant k of region r in year t at master problem (MW)
$\overline{C^{BO}_{r,k,j,t}}$	Estimated operating capacity of parallel generator j of power plant k of region r in year t at master problem (MW)
$\overline{FS_{r,k,t}}$	Estimated amount of fuel consumed by power plant k of region r in year t at master problem (GWh)

$\overline{P^F_{r,k,t}}$	Estimated successful operational reliability of power plant k of region r in year t at master problem (%)
$\overline{UMD_t}$	Estimated unmet demand in year t at master problem (GWh)
$\overline{CT_t}$	Estimated curtailment in year t at master problem (GWh)
Cap^{ava}_{iter}	Total available capacity at iteration $iter$ (MW)
$AIC^P_{r,k,t}$	Adjusted investment cost for potential power plant k in region r at year t ($)
$AIC^B_{r,k,j,t}$	Adjusted investment cost for parallel generator j of power plant k in region r at year t ($)
$AIC^S_{r,k,i,t}$	Adjusted investment cost for parallel battery i of power plant k in region r at year t ($)
$FOC^P_{r,k,t}$	Fixed operating cost for potential power plant k in region r at year t ($)
$FOC^B_{r,k,j,t}$	Fixed operating cost for parallel generator j of power plant k in region r at year t ($)
$FOC^S_{r,k,i,t}$	Fixed operating cost for parallel battery i of power plant k in region r at year t ($)
$EC_{r,k,t}$	Lifetime extension cost of existing power plant k in region r at year t ($)
$VOC^P_{r,k,t,n,b}$	Variable operating cost of power plant k in region r during subperiod b of representative day n in year t ($)
$VOC^B_{r,k,j,t,n,b}$	Variable operating cost of parallel generator j of power plant k in region r during subperiod b of representative day n in year t ($)
$VOC^S_{r,k,i,t,n,b}$	Variable operating cost of parallel generator j of power plant k in region r during subperiod b of representative day n in year t ($)
$SUC^P_{r,k,t,n,b}$	Start up cost of power plant k in region r during subperiod b of representative day n in year t ($)
$SUC^B_{r,k,j,t,n,b}$	Start up cost of parallel generator j of power plant k in region r during subperiod b of representative day n in year t ($)
$SDC^P_{r,k,t,n,b}$	Shut down cost of power plant k in region r during subperiod b of representative day n in year t ($)
$SDC^B_{r,k,j,t,n,b}$	Shut down cost of parallel generator j of power plant k in region r during subperiod b of representative day n in year t ($)
Ψ	Net present cost throughout the planning horizon ($)
Ψ^{LB}	Net present cost (lower bound) obtained from the master problem ($)
Ψ^{UB}	Net present cost (upper bound) obtained from the subproblem ($)
Ψ^{CAPEX}	Capital expenditure ($)
Ψ^{OPEX}	Operating expenses ($)
Ψ^{DTP}	Total downtime penalties ($)
Ψ^{UMP}	Total unmet demand penalties ($)
Ψ^{FOC}	Total fixed operating costs ($)
Ψ^{VOC}	Total variable operating costs ($)
Ψ^{SUC}	Total start-up costs ($)
Ψ^{SDC}	Total shut-down costs ($)
Ψ^{FUC}	Total fuel costs ($)
Ψ^{CEM}	Total CO_2 emission costs ($)

Boolean variables

$Y^{PI}_{r,k,t}$	True if the potential plant k is installed in region r at year t
$Y^{BI}_{r,k,j,t}$	True if the parallel generator j is installed in power plant k of region r at year t
$Y^{SI}_{r,k,i,t}$	True if the parallel battery i is installed in power plant k of region r at year t
$Y^{UPI}_{r,k,c,t}$	True if the potential plant k with size c is installed in region r at year t
$Y^{UBI}_{r,k,j,c,t}$	True if the parallel generator j with size c is installed in power plant k of region r at year t
$Y^{PA}_{r,k,t}$	True if the potential plant k of region r is available at year t
$Y^{BA}_{r,k,j,t}$	True if the parallel generator j of power plant k of region r is available at year t
$Y^{SA}_{r,k,i,t}$	True if the parallel battery i of power plant k of region r is available at year t
$Y^{PL}_{r,k,t}$	True if the existing plant k of region r extends the lifetime at year t
$X^{PO}_{r,k,t,n,b}$	True if the power plant k of region r operates during subperiod b of representative day n in year t
$X^{BO}_{r,k,j,t,n,b}$	True if the parallel generator j of power plant k of region r operates during subperiod b of representative day n in year t
$X^{SO}_{r,k,i,t,n,b}$	True if the parallel battery i of power plant k of region r operates during subperiod b of representative day n in year t
$U^{PU}_{r,k,t,n,b}$	True if the power plant k of region r starts up during subperiod b of representative day n in year t
$U^{BU}_{r,k,j,t,n,b}$	True if the parallel generator j of power plant k of region r starts up during subperiod b of representative day n in year t
$U^{PD}_{r,k,t,n,b}$	True if the power plant k of region r shuts down during subperiod b of representative day n in year t
$U^{BD}_{r,k,j,t,n,b}$	True if the parallel generator j of power plant k of region r shuts down during subperiod b of representative day n in year t
$V^{DC}_{r,k,i,t,n,b}$	True if the parallel battery i of power plant k of region r is discharged during subperiod b of representative day n in year t
$V^{CH}_{r,k,i,t,n,b}$	True if the parallel battery i of power plant k of region r is charged during subperiod b of representative day n in year t
$Z_{r,k,h,t}$	True if the design h is selected for power plant k of region r in year t
$W_{r,k,h,m,t,n,b}$	True if the operation mode m of design h is selected for power plant k of region r during subperiod b of representative day n in year t
$T_{t,n,b}$	True if the amount of power supplied is larger than the amount of power required during subperiod b of representative day n in year t
$YU^{BI}_{r,k,j,t}$	True if the parallel generator j is installed in power plant k of region r in year t at master problem
$YU^{UBI}_{r,k,j,c,t}$	True if the parallel generator j with size c is installed in power plant k of region r in year t at master problem
$YU^{BA}_{r,k,j,t}$	True if the parallel generator j of power plant k of region r is available in year t at master problem

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