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An Optimization Model for the Design and Operation of Reliable Power Generation Systems

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Abstract

This paper aims to develop a new optimization model for the design and operation of reliable power generation systems. This work optimizes the selection of redundant or backup units and operating units to maximize the reliability and to minimize the cost. In particular, every possible failure state that the power generation systems can have is investigated to evaluate the system reliability. To achieve this goal, we develop an optimization model that minimizes the total cost using Generalized Disjunctive Programming (GDP). The GDP model includes two decision variables: the first is a selection of redundant units with different sizes to increase the reliability of systems, and the second is a selection of operating units to satisfy the power demand. Specifically, the model determines the system reliability and corresponding expected power production by considering the number of redundant and operating units, and possible failure states under each design and operation mode. The model imposes a penalty when the demand is not satisfied, and the system has a low reliability. We have applied the proposed model in a small power plant (one stage with up to three generators) and verified through a sensitivity analysis that the model installs larger and more units to improve the system reliability as penalty rates increase.

Keywords: Redundancy, Reliability, Design, Operation, Optimization

1. Introduction

As evidenced by the Texas power crisis in 2021[1], the failure of power generation systems can lead to extreme events. Therefore, power generation systems should be designed to have high reliability to withstand failures of one or multiple components, and supply near uninterrupted electric power to industries and households. Reliability indicates the probability that a system will perform its required function properly even if one or multiple units fail (Sherwin et al., 2020). Since the goal of power generation systems is to consistently provide electric power, securing high reliability in their design and operation is a highly desirable objective. Numerous studies on the design/planning of power generation systems and reliability evaluation have been reported. However, previous works have dealt with these problems separately. Lara et al. (2018) have focused on optimizing the generation capacity to satisfy the power demand, whereas Amusat et al. (2016) have evaluated the reliability after designing the power generation systems. Studies that integrate design and reliability have also been reported, but they only consider a couple of generators or transmission lines failures (Moreira et al., 2016). Given the

[1] In February 2021, massive electricity generation systems in Texas were failed due to severe winter storms. Due to this, more than 4.5 million homes and businesses were left without power for several days, and at least 210 people were killed.

recent crisis, there is a strong motivation for a more comprehensive method to consider reliability in the design and operation phases of the power generation systems.

One method to improve reliability of the power generation systems at the design phase is to add redundant or backup units, which allows the systems to operate even if one or multiple generators fail (Kim et al., 2016). This approach is known as 'reliability-based design optimization,' and various studies on this topic have been conducted. Ye et al. (2018) develop a mixed integer nonlinear programming model for the optimal design of chemical process. Ortiz-Espinoza et al. (2021) present a multi-objective reliability-based design optimization model by combining economics, reliability, and safety. Chen et al. (2021) propose a two-stage stochastic generalized disjunctive programming (GDP) model by considering reliability and endogenous/exogenous uncertainties. Since these works assume that their target systems operate at a steady state, the authors mainly focus on optimizing the number of redundant units for the reliable design. However, since power systems operate in unsteady state due to time-varying power demand, reliability is also influenced by the operational strategies that the systems use to satisfy the load demand. Specifically, backup units can have a dual role in power generation systems. They can remain as backup units in case of low power demand or change to operating units when the power demand is high. Such dual purpose of redundant units depending on the load demand should be considered in the design and operation of reliable power generation systems. To our knowledge, this issue has not been addressed before in the literature for the design optimization of power generation systems with considerations of reliability.

This paper aims to develop a new optimization model for the design and operation of reliable power generation systems. This work optimizes the number of redundant units and operating units to maximize the reliability and to minimize the cost. In the remainder of this paper, we develop an optimization model using Generalized Disjunctive Programming (GDP), which is a high-level model representation that involves equations, disjunctions, and logic propositions. We then verify the effectiveness of the proposed model by solving an illustrative example.

2. Problem statement

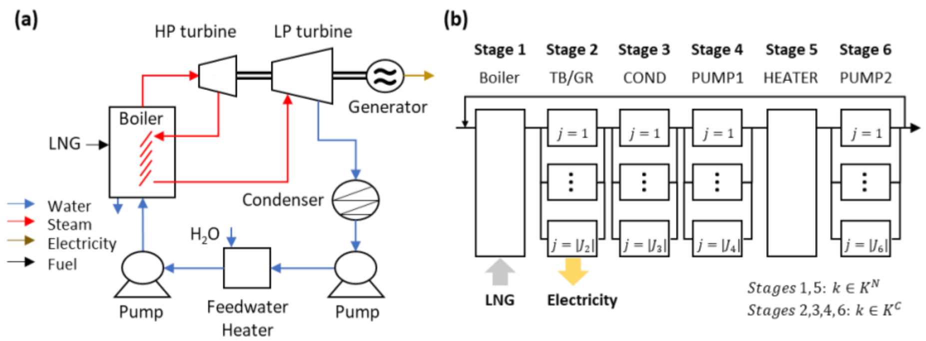

Figure 1. (a) Flow diagram of thermal power plant, (b) circular-parallel systems structure

Given is a natural gas power plant with multiple stages $k \in K$ including turbine, boiler, and pump, parallel identical units $j \in J_k$ for each stage k, and set of discrete capacities $c \in C_k$ of the units j in stages k. The stages can be classified into two groups: noncritical stage $k \in K^N$ that do not consider reliability, and critical stages $k \in K^C$ that do consider reliability. A set of stage designs $h \in H_k$ and corresponding operation modes $m \in M_{k,h}$, and time periods $t \in T$ are also given. Specifically, $h = 1$ means one unit installation and $h = H$ means all potential units are installed. Likewise, $m = 1$ represents one unit operation mode, $m = M$ refers to the mode in which all units are simultaneously operated. Each stage k has different failure states $s \in S_{m,k,h}$ depending on design h and operation mode m, which can be classified into (i) successful operation states ($S^F_{m,k,h}$) and (ii) partial operation states $\left(S^P_{m,k,h}\right)$. There are also corresponding operating reliabilities: successful operation reliability ($R^F_{k,t}$) and partial operation reliability ($R^P_{k,t}$). While 'successful operation states' indicate the operation states in which the power generation capacity is sufficient to satisfy the load demand, 'partial operation states' refer to the operation states in which the power generation capacity is insufficient to meet the load demand, but still can produce electric power at a limited level. The $R^F_{k,t}$ and $R^P_{k,t}$ are probabilities that such successful or partial operation states exist in stage k in time *t*, respectively. The major assumptions used in the model are: (i) There is one unit in noncritical stages, which has sufficiently large capacity; (ii) A redundant unit can be a backup of any operating unit; (iii) Repair and maintenance processes are not considered.

3. Model formulation

The model is developed using Generalized Disjunctive Programming (GDP) (Grossmann and Trespalacios, 2013), which can be expressed in terms of Boolean and continuous variables, algebraic constraints, disjunctions, and logic propositions.

$$
\bigvee_{h \in H_k} \left[\begin{array}{c}
Z_{k,h} \\
\sum_{c \in C_k} y_{k,j,c} = i, \qquad i = 1, \dots, h \\
\bigvee_{m \in M_{k,h}} \left[\begin{array}{c}
W_{m,k,h,t} \\
\sum_{j \in J_k} \sum_{c \in C_k} x_{k,j,c,t} = i, \qquad i = 1, \dots, m \\
\sum_{c \in C_k} \varepsilon_j \rho_{k,c} x_{k,j,c,t} \le AUC_{k,j,t} \quad \forall j \in J_k \\
AUC_{k,j,t} \le \sum_{c \in C_k} \rho_{k,c} x_{k,j,c,t} \quad \forall j \in J_k \\
R^F_{k,t} = \sum_{s \in S^F_{m,k,h}} \left\{ \prod_{j \in J^O_{s,m,h}} \lambda_{j,k} \prod_{j \in J^N_{s,m,h}} \sigma_{j,k} \right\} \\
R^P_{k,t} = \sum_{s \in S^P_{m,k,h}} \left\{ \prod_{j \in J^O_{s,m,h}} \lambda_{j,k} \prod_{j \in J^N_{s,m,h}} \sigma_{j,k} \right\} \\
EP^s_{k,t} \le \omega_t \sum_{j \in J^O_{s,m,h}} \sum_{c \in C_k} AUC_{k,j,t} R^s_{k,t} \quad s = 1 \\
EP^s_{k,t} = \omega_t \sum_{j \in J^N_{s,m,h}} IC_{k,j} R^s_{k,t} \quad \forall s \neq 1, s \in S_{m,k,h}
\end{array} \right] t \in T
\end{array} \right] k \in K^C
\qquad
\begin{array}{ll}
\text{(a)} & \text{Investment disjunction} \\
\text{(1)} & \text{Number of installed unit} \\
\text{(b)} & \text{Operation disjunction} \\
\text{(2)} & \text{Number of operating unit} \\
\text{(3)} & \text{Available operating capacity of unit in time} \\
\text{(4)} & \text{Successful and partial operation reliability} \\
\text{(5)} & \text{Expected production by successful operation} \\
\text{(6)} & \text{Expected production by partial operation}
\end{array}
$$

$$
\begin{cases}
\sigma_{j,k} = 1 & \forall j \in J^B_{s,m,h}, k \in K \\
\sigma_{j,k} = (1 - \lambda_{j,k}) & \forall j \in J^F_{s,m,h}, k \in K
\end{cases}
\qquad
\begin{array}{l}
J^B_{s,m,h} \cup J^F_{s,m,h} = J^N_{s,m,h} \quad \forall j \in J_k \\
J^O_{s,m,h} \cup J^N_{s,m,h} = \emptyset \quad \forall j \in J_k
\end{array}
\qquad \text{(7)}
$$

There are two Boolean variables related to investment and operation decisions. $Z_{k,h}$ is true if design *h* is selected for stage *k*; false otherwise (Equation (a)). $W_{m,k,h,t}$ is true if stage *k* is in operation mode *m* in time *t* for design *h*; false otherwise (Equation (b)). The binary variable $y_{k,j,c}$ indicates the installation of unit *j* with specified capacity *c* in stage *k* and $x_{k,j,c,t}$ indicates the operation of unit *j* with specificed capacity *c* in stage *k* and time *t*. $\lambda_{j,k}$ is a reliability of unit *j* in stage k and $\sigma_{j,k}$ defined in Eqn (7), states that when the unit *j* belongs to set of backup units in failure state *s* under design *h* and operation mode *m* ($J^B_{s,m,h}$), the unit reliability ($\lambda_{j,k}$) will be 1. If the unit *j* belongs to set of failed unit ($J^F_{s,m,h}$), the unit unreliability ($1 - \lambda_{j,k}$) will be used to calculate system reliability.

$$\sum_{c \in C_k} y_{k,j,c} \leq 1 \quad \forall k \in K^C, j \in J_k \tag{8}$$

$$\sum_{c \in C_k} x_{k,j,c,t} \leq 1 \quad \forall k \in K^C, j \in J_k, t \in T \tag{9}$$

$$IC_{k,j} = \sum_{c \in C_k} \rho_{k,c} y_{k,j,c} \quad \forall k \in K^C, j \in J_k \tag{10}$$

$$TEP_{k,t} = \sum_{s \in S} EP^s_{k,t} \quad \forall k \in K^C, t \in T \tag{11}$$

$$ASC_{k,t} = \sum_{j \in J_k} AUC_{k,j,t} \quad \forall k \in K^C, t \in T \tag{12}$$

$$DC_{k,t} = \xi R^P_{k,t} \quad \forall k \in K^C, t \in T \tag{13}$$

$$\left.\begin{array}{ll} \veebar_{h \in H_k} Z_{k,h} & \forall k \in K^C \\ Z_{k,h} \Leftrightarrow \veebar_{m \in M_{k,h,t}} W_{m,k,h,t} & \forall h \in H_k, k \in K^C, t \in T \end{array}\right\} \tag{14}$$

$$\left.\begin{array}{ll} y_{k,j+1,c} \leq y_{k,j,c} & \forall k \in K^C, j \in J_k, c \in C_k \\ x_{k,j+1,c,t} \leq x_{k,j,c,t} & \forall k \in K^C, j \in J_k, c \in C_k, t \in T \end{array}\right\} \tag{15}$$

$$y_{k,j,c} \geq x_{k,j,c,t} \quad \forall k \in K^C, j \in J_k, c \in C_k, t \in T \tag{16}$$

$$\left.\begin{array}{ll} ASC_{k,t} \leq \psi_k & \forall k \in K^N, t \in T \\ TEP_{k,t} \leq ASC_{k,t} & \forall k \in K^N, t \in T \end{array}\right\} \tag{17}$$

$$TEP_{k+1,t} = \eta_k TEP_{k,t} \quad \forall k \in K, t \in T \tag{18}$$

$$Z_{k,h}, W_{m,k,h,t} \in \{True, False\};\; x_{k,j,c,t}, y_{k,j,c} \in \{0,1\};\; IC_{k,j}, AUC_{k,j,t}, ASC_{k,t}, R^s_{k,t}, EP^s_{k,t}, TEP_{k,t}, DC_{k,t}, F_t \geq 0$$

Eqns. (8) and (9) state that only one capacity can be installed and operated. Eqns. (10) – (12) indicate capacity of unit installed in stage *k*, total expected power production, and available capacity of stage *k*, respectively. Eqn. (13) is a downtime penalty and Eqn. (14) is a logic constraint for disjunction. Eqn. (15) are symmetry breaking constraints, meaning that a unit can only be selected if the one with higher priority is selected. Eqn. (16) indicates that installed units only can be used. Eqn. (17) constrains the expected production and operating capacity of noncritical stage $k \in K^N$. Eqn. (18) states that the expected production of stage *k+1* is estimated by using expected production of stage *k* and conversion rate of stage *k*.

The objective function in (19) is to minimize the total cost, which includes the investment cost ($\delta_{k,c} y_{k,j,c}$), start-up cost ($\theta_{k,c} x_{k,j,c,t}$), expected fuel cost to purchase natural gas ($\pi_t F_t$), expected operating cost ($\gamma_k TEP_{k,t}$), downtime penalty ($DC_{k,t}$), and unmet demand penalty (PN_t). The system is charged penalties for the unmet demand, as shown by Equation (20). Rather than considering a bi-criterion optimization problem, we assume that shortfalls in power demand and low reliability are penalized so as to formulate the optimization problem as a single objective problem for cost minimization.

$$\min Z = \sum_{k \in K} \sum_{j \in J_k} \sum_{c \in C_k} \delta_{k,c} y_{k,j,c} + \sum_{k \in K} \sum_{j \in J_k} \sum_{c \in C_k} \sum_{t \in T} \theta_{k,c} x_{k,j,c,t} + \sum_{t \in T} \pi_t F_t + \sum_{k \in K} \sum_{t \in T} \gamma_k TEP_{k,t} + \sum_{k \in K} \sum_{t \in T} DC_{k,t} + \sum_{t \in T} PN_t \tag{19}$$

$$\left.\begin{array}{c} \begin{bmatrix} V_{1,t} \\ TEP_{k,t} < \omega_t D_t \\ PN_t = (\omega_t D_t - TEP_{k,t})\alpha \end{bmatrix} \vee \begin{bmatrix} V_{2,t} \\ TEP_{k,t} \geq \omega_t D_t \\ PN_t = 0 \end{bmatrix} \quad \forall k = GN, t \in T \\ V_{1,t} \veebar V_{2,t} \quad \forall t \in T \end{array}\right\} \tag{20}$$

The GDP given by (1)−(20) can be transformed into a Mixed-Integer Nonlinear Programming (MINLP) using Big-M (BM) and/or Hull Reformulation (HR) (Grossmann and Trespalacios, 2013). This paper uses both methods, and Eqns. (5) and (6) are transformed into MILP constraints by using an exact linearization (Avraamidou and Pistikopoulos, 2019, and Garcia-Herreros et al., 2015).

4. Illustrative example

To verify the proposed model, the power system that has one stage (k = 1) involving up to three generators ($|J_k|$ = 3) and three different sizes ($|C_k|$ = 3) are analysed. Here the one stage stands for the generator stage. The total time horizon is 10 months, which is divided into 10 periods (i.e., 1 month). Table 1 shows the parameter values for the example.

Table 1. Parameter for illustrative example

Parameter	Symbol	Value	Parameters	Symbol	Value
Nameplate capacity (MW)	$\rho_{k,c}$	50,80, 100	Purchase cost of natural gas ($/MMBtu)	π_t	5
Minimum operating capacity (ratio of $\rho_{k,c}$, %)	ε_j	10	Production cost ($/MWh)	γ_k	5
Unit reliability	$\lambda_{j,k}$	0.97	Downtime penalty rate ($/hr)	ξ	1000
Conversion rate	η_k	0.4278	Installation cost (k$/unit)	$\delta_{k,c}$	10, 13, 15
Unmet demand penalty rate ($/MWh)	α	100	Start-up cost ($/unit)	$\theta_{k,c}$	100, 160, 200

Table 2. Numerical results of illustrative example

Solution method	MINLP		MILP	
Solver	BARON		CPLEX	
Cost (k$)	164.1		164.1	
Average reliability	**0.9758**		**0.9758**	
Reformulation	BM	HR	BM	HR
Equations	1,110	1,212	2,190	4,452
Cont. variables	267	1,083	627	2,883
Binary variables	108	360	108	360
CPU (sec.)	4.040	11.110	0.360	0.687

Figure 2. Optimal design and operation

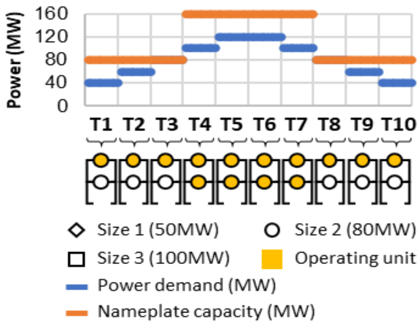

Table 2 shows the numerical results obtained with BARON and CPLEX in GAMS 32.1.0 on an Intel Core i7-10510U CPU, 1.80GHz. Although the sizes of the MILP reformulations are larger, their CPU times are significantly shorter than the MINLP. The proposed model predicts the total cost of $164,192 including unmet demand penalty of $144 and downtime penalty of $2,328. As shown in Figure 2, the model installs two medium size generators of 80 MW each yielding a total of 160MW. While the second generator remains as a backup when the demand is relatively low (from T1 – T3 and T8 – T10), both generators are used to meet the demand during T4 – T7.

5. Sensitivity analysis

To analyze the impact of unmet demand and downtime penalty rates on design and operation of reliable power systems, two alternative cases that have different penalty rates are suggested (Case 1: α = $500/MWh, ξ = $5,000/hr, Case 2: α = $1,000/MWh, ξ = $10,000/hr). As shown in Figure 3(a), the system with higher penalty rates than base case (c.f., Base case: α = $100/MWh, ξ = $1,000/hr) tends to install two larger units (each 100 MW) so as to improve reliability, and the cost is also increased to 171.9 k$. When the unmet and downtime penalties are significantly higher than other two cases (Base case and Case 1) (Figure 3(b)), the system decides to install three medium size generators (each 80 MW) and have one unit as a backup during all the periods, which results in the highest reliability (0.9989) and cost (175.1 k$).

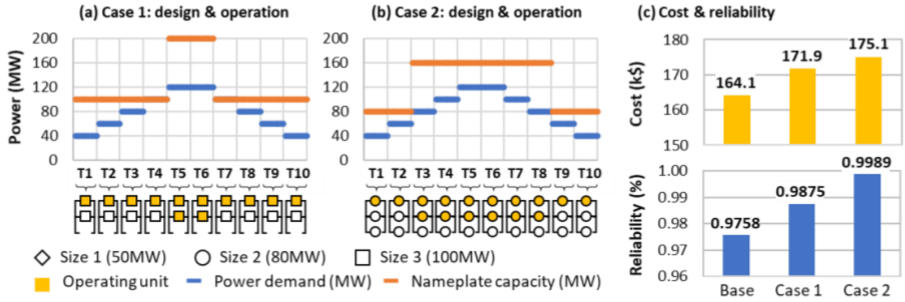

Figure 3. (a) and (b): Optimal design and operation of alternative cases, (c): total cost and average reliability of all cases

6. Conclusions

This paper has presented a mathematical optimization model for the design and operation of reliable power generation system. This work optimizes the number of redundant units and operating units to maximize the reliability and to minimize the cost. We propose a GDP formulation to represent the reliability and expected production, which are essential factors to determine the design and operation of power generation systems. Through a small example and sensitivity analysis, we found that the optimal system involves more and larger units to improve the system reliability as the penalty rates increase. Future work will involve other operation problems in power systems such as economic dispatch and unit commitment to evaluate the reliability more precisely by using a more rigorous reliability model such as Markov chain theory.

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