# Recent Advances and Challenges in Optimization Models for Expansion Planning of Power Systems and Reliability Optimization

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# Abstract

Optimization models for expansion planning of power systems aim to determine capacities, investment timing, and location of power systems to satisfy the power demands while minimizing the total cost. The models have become complex in recent years to reflect both regulations on conventional energy sources and the increasing penetration of renewable energy sources (RES). This paper reviews the basic concepts and optimization models for expansion planning of power systems. We first explain the definition and features of generation expansion planning (GEP), transmission expansion planning (TEP), and generation and transmission expansion planning (GTEP). To address the computational challenges, we review several simplifications including temporal and spatial aggregation, and decomposition methods for large-scale problems. This paper also addresses power system reliability defined as the probability of satisfying the demand while withstanding failures of components. The goal of this paper is to provide a research overview, discuss trends on expansion planning of power systems, and suggest directions for future research. *Keywords:* Power Systems, Optimization, Expansion Planning, Mixed-integer

Programming, Reliability

# 1. Introduction

In a power grid, as shown in Figure 1, electricity is generated by power plants, transmitted through high-voltage transmission lines (approximately from 138kV to 765kV) to reduce electricity losses during transmission, and then distributed to different customers after lowering voltages in substations, from 120V to 69kV depending on the purpose (Zhang and Grossmann, 2016). The main goal of power systems is to supply uninterrupted power to customers, thereby matching power supply and demand through reliable system

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designs and flexible operations. However, due to high variations and uncertainties in power demands, unexpected disruption of power supply by extreme weather, and inherent failure rates of generators, transformers, and transmission lines, it is challenging to balance the supply and demand without interruption.

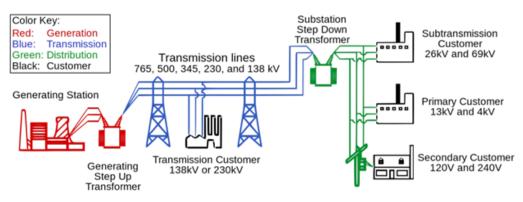


Figure 1: Structure of power systems (Zhang and Grossmann, 2016)

In 2020, 4.01 billion MWh of electricity were generated in the United States. 80% of the total electricity was generated from conventional sources such as natural gas, petroleum, coal, and nuclear, whereas 20% was produced by renewable power generators, such as wind turbines and solar panels (EIA, 2021). The International Renewable Energy Agency (IRENA) reported the 'REmap 2030' for the US, which aims to increase the renewable energy share in total energy generation up to 50% by 2030 (IRENA, 2015). However, unlike the conventional technologies in which power outputs are dispatchable by operators, power outputs of renewable generators are not controlled by operators, and are susceptible to weather conditions of the regions where the renewable generators are installed. Therefore, advanced methods to effectively integrate conventional and renewable technologies must be developed for sustainable and reliable power systems.

Studies on power system optimization can be categorized into short-term operations and long-term investment planning from a temporal perspective (see Figure 2). While short-term problems have mainly dealt with operation and scheduling problems, such as real-time economic dispatch or day-ahead unit commitment, long-term problems have focused on design and planning of power systems.

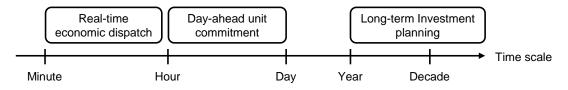


Figure 2: Major decision-making problems in power systems by time scale (modified from Sun (2016))

Real-time economic dispatch (ED) aims to find the optimal power output from available power generators to meet the power demand while minimizing the operating cost (Xia and Elaiw, 2010; Al Farsi et al., 2015; Reddy and Bijwe, 2015). In practice, ED problems are conducted every 5-15 minutes, depending on the electricity market structures. The optimization model for ED problems is subject to the bounds on power output of generating units and the power balance constraints. A number of studies on ED problems have been reported. Some researchers have solved ED problems by using optimization techniques such as linear programming (Conejo and Baringo, 2018), dynamic programming (Lee et al., 1984), and nonlinear programming (Liang and Glover, 1992). Others have applied heuristic methods such as Genetic Algorithms (Walters and Sheble, 1993) and Artificial Neural Networks (Yalcinoz and Short, 1998). Due to both an increasing interest in renewable energy sources, which are intermittent and non-dispatchable, and time-sensitive load demand, researchers also have taken into account uncertainty in economic dispatch by using different approaches: such as sub-hourly coupling models (Gangammanavar et al., 2016), probabilistic envelopes approach (Nosair and Bouffard, 2017), and real-time economic dispatch (Surender Reddy et al., 2015).

The goal of unit commitment (UC) is to determine the optimal scheduling of generators (i.e., ON and OFF status) and corresponding power output to meet the power demand while: a) satisfying technical and security constraints, and b) minimizing the cost. UC problems are typically solved daily for the day-ahead market. The objective function is to minimize the total cost, which can consist of operating costs,  $CO_2$  emission costs, and security costs. Various constraints regarding start-up, shut-down, and ramping up and down are included. Readers interested in power system operations can refer to the paper by Conejo and Baringo (2018). Many studies on UC problems have been reported. Padhy (2004) reviews various types of optimization models and algorithms for UC problems, including dynamic programming, integer programming, linear programming, Lagrangean relaxation, and artificial neural networks. Zondervan and Grossmann (2010) develop a mixed-integer quadratically constrained programming (MIQCP) model to solve UC problems while minimizing the total cost. Moreover, a stochastic optimization model for demand uncertainty is developed in the following paper (Zondervan and Grossmann, 2016). Bhardwaj et al. (2012) address evolutionary algorithms such as particle swarm optimization. Mallipeddi and Suganthan (2014) apply deterministic and stochastic approaches to UC problems, and provide a brief discussion and comparison on those methods. Zheng et al. (2015) describe two-stage and multistage stochastic models and algorithms, and compare their performances.

An expansion planning (EP) of power systems aims to determine capacities, investment timing, and location of generation, transmission, and distribution to satisfy the projected power demand within a set of technical and economic constraints. The model encompasses a time horizon of more than 10 or 20 years and evaluates investment cost, operating cost, and expected power output over the selected time horizon (Gacitua et al., 2018). A more detailed explanation of EP problems can be found in section 2. This paper focuses on reviewing expansion planning models and pointing out challenges that still need to be addressed. The contributions of this paper are as follows:

- This paper provides basic description of generation expansion planning (GEP), transmission expansion planning (TEP), and generation and transmission planning (GTEP).
- Both optimization models and simplification methods used to improve computational tractability of expansion planning models, such as spatial and temporal aggregations, are investigated.
- Reliability, resilience, and flexibility of power systems are identified as areas, which should be further addressed in the future.

# 2. Review of expansion planning of power systems

Expansion planning (EP) of power systems aims to determine the optimal investment strategy related to number, capacity, location, and investment timing of generators and transmission lines to satisfy power demands, while following technical, economic, and environmental constraints (Conejo et al., 2016; Hemmati et al., 2013). Initial EP research was not able to consider operation problems together due to a lack of appropriate methods. Therefore, it simplified investment decisions with continuous variables by applying linear programming (LP) models (Beglari and Laughton, 1975; Garver, 1970). However, recent EP studies have integrated investment and operation problems because of both the development of advanced techniques and the increased need for variable generation technologies such as wind turbines and solar panels (Gacitua et al., 2018). The structure of optimization models has also changed from LP to mixed-integer linear programming (MILP) models so as to take into account both continuous and discrete natures of power systems (such as power flows or on/off status of power plants). However, the introduction of discrete variables has significantly increased computational times, leading researchers to develop solution methods such as simplification or decomposition. Meanwhile, advanced approach (e.g., game-theoretical models) has also been used to reflect characteristics of modern power systems such as regulated or deregulated electricity markets. For instance, Akbari et al. (2017) propose a new structure for decentralized generation and transmission expansion planning using Stackelberg bilevel game theory with multi-leaders and multi-followers. Chuang et al. (2001) apply non-cooperative game theory (i.e., the Cournot model of oligopoly behavior) to formulate an expansion planning model for a competitive electricity industry.

In general, the basic MILP model for EP problems can be represented as follows, min  $Z = c^{\top}x + d^{\top}y$  (1a)

s.t. 
$$Ax + By \le f$$
 (1b)

$$Ey \le g$$
 (1c)

$$x \in \mathbb{R}^m, \quad y \in \mathbb{Z}^n$$
 (1d)

where x represents the continuous variables and y represents the discrete variables. For example, power flows between regions and power output from each generator correspond to continuous variables, whereas the numbers of installed or retired generator are discrete variables. c and d are operating and investment cost coefficients, respectively. A, B, E are coefficients matrices for the constraints, while f and g are vectors of the right-hand sides. The size of coefficient matrices is dependent on the size of the problems. The objective function is to minimize the total cost consisting of investment and operating costs, as shown in equation (1a). It should be noted that other cost terms such as carbon emission cost, social cost, and risk penalty also can be included in the objective function. The constraints involving both investment and operating decisions are represented by equation (1b), and the constraints for investment decisions are expressed by equation (1c). For instance, energy balances, demand satisfaction, and ramping up/down constraints belong to equation (1b), whereas installation and retirement schedule of generators belong to equation (1c).

EP problems can be classified into generation expansion planning (GEP), transmission expansion planning (TEP), generation and transmission expansion planning (GTEP), and distribution expansion planning (DEP) (Koltsaklis and Dagoumas, 2018; Gonzalez-Romero et al., 2020; Muñoz-Delgado et al., 2020). Note that since investment costs for generation and transmission are more expensive than those for distribution, this paper does not address DEP problems.

# 2.1. Generation expansion planning (GEP)

Generation expansion planning (GEP) is the basic model in a long-term planning of electric power systems. The main goal of GEP is to determine the number, size, location, and investment timing of power generators/power stations so as to satisfy the projected power demand over time horizon. GEP models are generally subjected to constraints regarding power demand satisfaction, power output limit of generators, and reserve margin for reliability (Gacitua et al., 2018; Lara et al., 2018). Recently, researchers have included operational problems such as ED and UC in expansion planning to capture the hourly features of power generators. Lara et al. (2018) propose a MILP model that includes both long-term investment decisions and hourly unit commitment decisions, which minimize the overall costs including investment, operating, and environmental costs. Palmintier and Webster (2016) present a MILP model that integrates capacity planning, unit commitment, maintenance, and critical operating constraints. Moreover, Abdin et al. (2022) propose a novel multistage robust optimization model for multi-period and multi-region capacity planning. Short-term operation constraints such as unit commitment and ramping up/down, as well as uncertainty of load demand and renewable generation are taken into account. Scott et al. (2021) develop a stochastic MILP model to account for uncertainties of four parameters: electricity demand, fuel cost (i.e., natural gas price), carbon tax, and technology cost. A summary of GEP studies can be found in Table 1.

#### 2.2. Transmission expansion planning (TEP)

Transmission expansion planning (TEP) aims to install new transmission lines and/or expands the capacity of existing transmission lines. TEP models are in general subjected to constraints regarding power balances, power limits on transmission lines, voltage requirement, capacity limit of transmission line, and security (Gacitua et al., 2018). As in GEP, a typical objective function of TEP is to minimize cost while maintaining reliability and security. Different optimization techniques have been used for TEP problems. Bahiense et al. (2001) propose a mixed-integer disjunctive model for transmission network expansion and suggest three different formulations (i.e., big-M formulation, hull formulation, and alternative big-M formulation). Freitas et al. (2019) adopt a linear disjunctive model so as to evaluate expansion scenarios with multiple generation technologies. Moreover, Escobar et al. (2008) propose a new TEP model suitable for a competitive electricity market, and employ a tailored genetic algorithm to solve the problem. Sousa and Asada (2015) propose a multi-objective evolutionary algorithm to obtain promising solutions from the alternative pool. Haghighat and Zeng (2018) present a two-stage stochastic mixed-integer second-order conic programming (MISOCP) model for expansion planning of nonlinear AC power flows. Moreira et al. (2015) develop a robust adjustable optimization model to improve the tractability of the contingency-constrained TEP models. Jabr (2013) propose a robust MILP model to capture the impact of uncertainties of renewable power generations and power load, and employ Benders decomposition to solve the robust MILP model. A summary of TEP studies can be found in Table 1.

#### 2.3. Generation and transmission expansion planning (GTEP)

GEP and TEP have seldom been combined due to the difference in market agents: GEP problems have been dealt with by generators companies (known as GENCOs), whereas TEP problems have been controlled by transmission companies (known as TRANSCOs). However, the significant penetration of renewables into power systems has accelerated the development of co-optimization methods. Unlike the conventional energy sources such as coal and natural gas, which are collectible and can be processed in centralized power plants, it is challenging to aggregate renewable sources since they are distributed and intermittent (Won et al., 2017). Moreover, the regions rich in renewable energy sources

might not be connected to the power grid; for example, offshore wind farms can generate a large amount of electricity but might not be well connected to inland areas (Koltsaklis and Dagoumas, 2018). Therefore, in modern power systems with a high share of renewable power technologies, GEP and TEP should be combined (Krishnan et al., 2016).

The main goal of GTEP is to determine the optimal investment decisions such as number, size, location, and time of power generators/power stations and transmission lines, along with operation decisions, so as to satisfy the projected power demand over the time horizon. GTEP models are also subjected to constraints regarding power balances, unit commitment, operating reserve, capacity limits, and capacity factors for renewable generation technologies (Gacitua et al., 2018; Li et al., 2021a). A large number of GTEP works have been reported. Li et al. (2021a) extend the GEP work of Lara et al. (2018) to GTEP by considering expansion planning of transmission lines. Shu et al. (2015) propose a MILP model for network expansion planning. In particular, two methods (dynamic programming and simplified MILP) are applied to address the computational complexity. Pozo et al. (2013) present a three-level expansion planning model. The first and second levels determine the optimal investment decisions for transmission and generation, respectively. At the last level, operation strategies are determined under the fixed generation and transmission designs.

Stochastic programming or robust optimization are also implemented to take into account the impact of uncertainties in GTEP. Moreira et al. (2017) present a two-stage min-max-min model to consider high security standards and uncertainty of renewable generation. The proposed model also determines the optimal reserve systems by employing n - K security criteria. Aghaei et al. (2014) propose a new probabilistic expansion planning model for GTEP problems by considering reliability criteria. Alizadeh and Jadid (2015) propose a mixed-integer nonlinear programming (MINLP) for GTEP, as well as an algorithm for reliability evaluation. A summary of GTEP studies can be found in Table 2.

# 2.4. Extension of the expansion planning framework

Expansion planning studies of power systems have focused mainly on electric power systems. However, as modern energy systems have become more complex and uncertain due to the high penetration of renewable sources and regulations on conventional sources, the need of integrated energy systems research has been augmenting in recent years. Integrated energy systems (IES), also known as *energy hubs*, produce multiple products (e.g., electricity, heat, chemicals including  $H_2$  and  $CO_2$ ) from multiple energy sources such as fossil fuel, uranium, wind, and solar through various technologies such as generators, chemical plants, and storages (Maroufmashat et al., 2019; Walker et al., 2017; Geidl et al., 2007). A number of studies on IES systems have been reported. Shao et al. (2017) propose a robust optimization model and algorithm for integrated power systems with natural gas so as to enhance resilience of power systems under extreme conditions. He et al. (2021)

Generation	Generation expansion planning						
Refs.	<b>Optimization model</b>	Solution methods	Uncertainty	Reliability	Flexibility	$CO_2$ emission	Power flow
Lara et al. $(2018)$	MILP	Nested-BD	I	ı	ı	Y	TM
Palmintier and Webster (2016)	MILP	I	ı	ı	Υ	Υ	ı
Abdin et al. $(2022)$	MILP, RO	RIB	RES, demand	ı	Υ	I	·
Nguyen and Felder (2020)	MINLP	BLD	·	ı	I	ı	ı
Aghaei et al. (2013)	MINLP, MOO	CNBI	ı	Υ	I	Υ	ı
Chuang et al. $(2001)$		$\mathrm{GT}$	ı	Υ	ı	I	ı
Contt of al (9091)	MIT D SD		Technology cost, load demand,		>	>	
17070 CA CT. (7071)			$CO_2$ price, feedstocks price		ч	ч	
			RES availability,				
Lara et al. $(2020)$	MILP, SP	SDDiP	load demand, $CO_2$ tax, fuel price	I	Y	Y	MT
Transmission	Transmission expansion planning						
Refs.	<b>Optimization model</b>	Solution methods	Uncertainty	Reliability	Flexibility	$CO_2$ emission	Power flow
Bahiense et al. (2001)	DP	HR	1	1	ı	1	DC
Freitas et al. (2019)	MILP	I	ı	ı	I	I	DC
Escobar et al. $(2008)$	CO	GA	ı	ı	ı	I	DC
Sousa and Asada $(2015)$	MINLP	HR	ı	ı	Y	I	DC
Moreira et al. $(2015)$	MILP, RO	BD	ı	Υ	I	I	DC
Jabr (2013)	MILP, RO	BD	RES, load demand	ı	Υ	ı	DC
Tor et al. $(2008)$	MILP	BD	ı	Υ	I	I	DC
Haghighat and Zeng (2018)	MISOCP, SP	BD	Price, load demand	I	Υ	I	AC
Y: yes, MILP: Mixed-integer linear programming, MINLP: Mixed-integer nonlinear programming, CO: Combinatorial optimization, DP: disjunctive programming, MOO: Multi-objective optimization, SP: Stochastic programming, RO: Robust optimization, SDDiP: Stochastic dual dynamic integer programming, MISOCP: Mixed-integer second-order conic programming, BD: Benders decomposition, BLD/TLD: Bilevel (or Tri-level) decomposition, GA: Genetic algorithm, HR: Heuristics, GT: Game theory approach, RIB: Reduction of the information basis, CNBI: Corrected normal boundary intersection, TM: transport model, DC: DC flows, AC: AC flows	Mixed-integer linear programming, MINLP: Mixed-integer nonlinear programming, CO: Combinatorial op ramming, MOO: Multi-objective optimization, SP: Stochastic programming, RO: Robust optimization, SI integer programming, MISOCP: Mixed-integer second-order conic programming, BD: Benders decompositi vel) decomposition, GA: Genetic algorithm, HR: Heuristics, GT: Game theory approach, RIB: Reduction o basis, CNBI: Corrected normal boundary intersection, TM: transport model, DC: DC flows, AC: AC flows	<pre>ILP: Mixed-integer n ization, SP: Stochast; integer second-order hm, HR: Heuristics, v v intersection, TM: t</pre>	ionlinear program ic programming, F conic programmin GT: Game theory transport model, I	ming, CO: Co RO: Robust or ag, BD: Bendé approach, RI OC: DC flows	mbinatorial ptimization, ers decompos B: Reductior AC: AC flo	optimization, DJ SDDiP: Stochas ition, BLD/TLI t of the informat vs	D: D: ion

	Table 2: A summary of generation and transmission expansion planning (GTEP) models	generation and transm	nission expansion	planning (GT	'EP) models		
Refs.	Optimization model	Solution methods	Uncertainty	Reliability	Flexibility	$CO_2$ emission	Power flow
Guerra et al. (2016)	MILP	I	I	ı	ı	Υ	DC
Shu et al. $(2015)$	MILP, DNP	BLD	ı	ı	ı	ı	I
Pozo et al. $(2013)$	MILP	TLD	ı	ı	ı	ı	DC
Li et al. $(2021a)$	MILP	BD	ı	ı	ı	Υ	DC
Moreira et al. $(2017)$	MILP, MMM	TLD	ı	Υ	ı	ı	I
Aghaei et al. (2014)	MINLP	LF	ı	Υ	ı	Υ	DC
Alizadeh and Jadid (2015)	MINLP	BD	ı	Υ	ı	ı	DC
Roh et al. $(2009)$	MILP, SP	BD	Unit failures	Υ	·	Υ	DC
Munoz and Watson (2015)	MILP, SP	НЧ	RES, load demand	I	Υ	ı	DC
Praveen et al. (2018)	MILP, MOO	MOGWO	Wind power, load demand		Υ	ı	DC
Ahmadi et al. $(2020)$	MILP, RO	FDM	load demand	ı	Υ	Υ	DC
Kim et al. $(2015)$	MINLP	GBD	I	ı	I	I	AC
Y: yes, MILP: Mixed-integer linear programming, MINLP: Mixed-integer nonlinear programming, DNP: dynamic programming, MOO: Multi-objective optimization, SP: Stochastic programming, RO: Robust optimization, BD: Benders decomposition, GBD: Generalized Benders decomposition, MMM: min-max-min, BLD/TLD: Bilevel (or Tri-level) decomposition, PH: Progressive hedging, LF: linear formulation, MOGWO: Multi-objective grey wolf optimization, FDM: Fuzzy decision making, TM: transport model, DC: DC flows, AC: AC flows	ger linear programming, , SP: Stochastic program n-max-min, BLD/TLD: e grey wolf optimization	<ul> <li>ng, MINLP: Mixed-integer nonlinear programming, DNP: dynamic programming, MOO: amming, RO: Robust optimization, BD: Benders decomposition, GBD: Generalized BendeD: Bilevel (or Tri-level) decomposition, PH: Progressive hedging, LF: linear formulation, ion, FDM: Fuzzy decision making, TM: transport model, DC: DC flows, AC: AC flows</li> </ul>	eger nonlinear   pptimization, B ) decompositio sion making, T1	programming D: Benders da n, PH: Progr M: transport	, DNP: dyna ecomposition essive hedgin model, DC: J	mic programmir , GBD: Generaliz g, LF: linear for DC flows, AC: A	ış, MOO: zed Benders mulation, .C flows

develop a capacity expansion planning model for a combined electricity and  $H_2$  system, so called Decision Optimization of Low-Carbon Power-Hydorgen Network (DOLPHYN), and evaluate sector-coupling effects of  $H_2$  in power systems. Brown et al. (2018) present an LP optimization model for multi-region (or multi-country) and multi-period planning of integrated energy systems including electricity, heat, hydrogen, and methane. The coupling effects of multiple energy carriers enable the systems to have 95% reduction in  $CO_2$  emission. Han and Kim (2019) propose an expansion planning model for complex renewable energy supply systems with multiple products such as electricity, hydrogen, and fuel from renewable sources. Allen et al. (2019) present a two-stage stochastic MILP model for planning and scheduling of power systems exploiting the Energy-Water Nexus framework.

#### 3. Large-scale example of generation and transmission expansion planning

Expansion planning, especially multi-region and multi-period expansion planning, is regarded as a complex problem due to its large number of decision variables and constraints. The example from the paper by Li et al. (2021a) is used in this section to explain how the model works and what decisions the model can provide for large-scale problems. In the paper by Li et al. (2021a), five different types of generators are considered: natural gas plants, coal plants, nuclear plants, wind turbines, and solar panels. Given are the corresponding nameplate (maximum) capacity of the generators, capital costs, fixed and variable operating costs, ramp-up/ramp-down rates for conventional generators, capacity factors for renewable generators, phase angle, and susceptance of transmission lines. The optimization model is formulated as an MILP to determine: a) the optimal location and timing to install or retire generators, storage, and transmission lines, b) the optimal scheduling of thermal generators from the unit commitment, and c) the power output of generators and power flows through transmission lines. The objective function is to minimize the total cost, including investment, operating, and environmental costs. The model is applied to the ERCOT electricity market in Texas, consisting of five subregions such as Panhandle, Northeast, West, South, and Coast, over a 20 year horizon from 2019 to 2038.

Figures 3 and 4 show generation and transmission expansion results, respectively. As shown in Figure 3, the total capacity of natural gas plants increases and reaches a peak in 2024 (approximately 70GW). However, the capacity decreases afterward due to both the retirement of existing generators and strengthened regulation on  $CO_2$  emissions, preventing the systems from installing new fossil-fuel power generators. The total capacity of nuclear and coal power plants decreases due to the same reasons, and finally accounts for 8% of the total capacity in 2038. On the other hand, the total capacity of wind turbines and solar panels, which accounts for 20% of the total capacity in 2019, greatly increases and becomes the largest contributor in 2038 by accounting for 52% of the total capacity. As shown in Figure 4, most of the transmission lines are built from Panhandle to others so as to transmit the power, it is because Panhandle can produce more renewable electricity than others due to its higher wind speed and solar radiations.

The corresponding MILP model includes 2,800 binary variables, 1,024,680 integer variables, 4,120,606 continuous variables, and 7,787,266 constraints. The example was solved in 33,207 seconds (i.e., 9.224 hours) using a tailored Benders decomposition algorithm in CPLEX within an optimality gap of 0.4%. This example shows that solution strategies for handling large-scale problems must be developed, as will also be discussed in the next section.

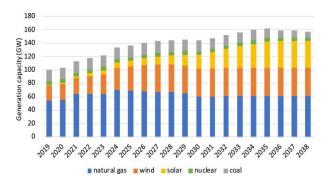


Figure 3: Generation expansion results (Li et al., 2021a)

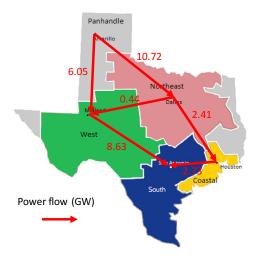


Figure 4: Transmission expansion results (Li et al., 2021a)

#### 4. Computational challenges in expansion planning

One of the main issues of multi-region and multi-period expansion planning models is the computational intractability caused by investment and operation decisions over multiple decades. In order to improve the computational tractability of the optimization models, temporal and spatial aggregation methods have been proposed in the literature, as will be discussed in sections 4.1 and 4.2. In addition, decomposition algorithms for large-scale problems will also be described in section 4.3.

#### 4.1. Temporal aggregation

While operation problems such as ramping up/down and unit commitment are solved on an hourly basis, investment decisions such as installing new generators are based on a yearly basis. It is possible to solve hourly operation problems every hour; however, it is intractable in expansion planning as it focuses on system design over several decades. To tackle such a problem, previous researchers have developed temporal aggregation approaches. According to the paper by Helistö et al. (2019), there are three temporal aggregation methods: (a) screening curve method, (b) time slice method, and (c) representative period method.

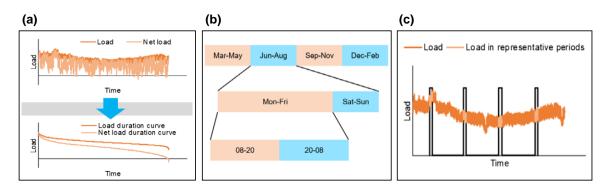


Figure 5: Temporal aggregation methods: (a) Screening curve, (b) Time slice, (c) Representative period (modified from Helistö et al. (2019))

A screening curve uses a load duration curve or a net load duration curve to simplify the high variation in demand. This method plots power demand over a specified period in the order of decreasing magnitude, as shown in Figure 5(a). The screening curve method can intuitively provide minimum and maximum power demands over the period; however, it is not used to solve operational problems such as hourly ramping up/down an unit commitment due to the absence of chronological representation. As shown in Figure 5(b), a time-slice method splits the whole time horizon into multiple periods based on seasonal or day-night variations. The time slice method represents the entire data of interest (such as annual power demand or capacity factor for renewable generation) by averaging the data set that belongs to each time slice. Therefore, the accuracy of optimization models is improved as the number of time slices increases (Mallapragada et al., 2018; Poncelet et al., 2016).

A representative period method in Figure 5(c) selects a subset of days or weeks from a fullspace data set and solves problems based on the representative periods. In the literature, both representative days (Mallapragada et al., 2018; Teichgraeber and Brandt, 2019; Scott et al., 2019; Tso et al., 2020) and representative weeks (Bahl et al., 2018) have been used as representative periods. Lara et al. (2018) use the representative days method to effectively simplify the time set, while keeping the chronology of hourly data such as loads and capacity factors. Figure 6 shows the temporal aggregation method used in the paper by Lara et al. (2018). Each year  $t \in \mathcal{T}$  can be represented by the summation of days  $d \in \mathcal{D}$ , and each day d is composed of sub-period  $s \in \mathcal{S}$  (i.e. 24 hours). While investment decisions are made in year t, operation decisions are determined on each representative days d. The most important decision in this method is to select representative days d from the fullspace data set. Clustering methods such as k-means or k-medoids can be used to find the optimal number of representative days depending on the input data (Li et al., 2021b). Note that the accuracy of the optimization models can improve as the number of representative days increases. By multiplying the weight factor ( $W_d$ ) by the sub-period sof representative days d, the model indirectly solves the operation problems for all given times.

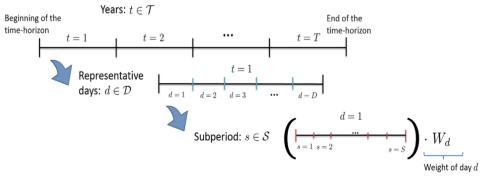


Figure 6: Representative days approach for temporal aggregation (Lara et al., 2018)

#### 4.2. Spatial aggregation

Expansion planning is typically performed on large-scale power systems consisting of thousands of buses. Note that a bus stands for a node connected by multiple lines in a power grid, and contains components such as loads and generators. It is possible for small-scale planning problems focusing on a few regions to consider all buses. However, it is intractable to explicitly model all buses in large-scale expansion planning such as national-level or state-level planning. Therefore, various spatial aggregation methods have been proposed to improve computational tractability of large-scale problems.

The method to reduce spatial complexity is to aggregate some buses using clustering analysis. According to the paper by Frysztacki et al. (2022), there are three clustering methods for spatial reduction: K-means, Ward's method, and Modularity maximization. K-means is one of the common algorithms that aggregates some buses based on geographical information such as coordinates. One drawback of K-means is that it might distort the connectivity of regions. For example, two regions that are not physically connected can be aggregated as one cluster if they are adjacent (Horsch and Brown, 2017). While Ward's method uses hierarchical clustering that combines buses based on capacity factors or resolved capacity factors, Modularity maximization clusters the buses based on electrical distances between nodes. Lara et al. (2018) and Li et al. (2021a) use the Ward's method with capacity factor aggregation. In particular, the area of interest is divided into several regions that have similar load profiles and climate conditions (e.g., wind speed and solar radiation). Since each region is treated as one bus in the power flow model of those papers, only the tielines between two neighboring regions are considered. The spatial aggregation is close to the original full-space model if the generators using the same technology have similar parameters and the transmission lines within each aggregated regions.

#### 4.3. Decomposition algorithms for large-scale problems

Despite the aggregation methods explained in previous sections, the MILP models for GEP, TEP, and GTEP can involve millions of variables that cause the model to be computationally intractable. Computational experiments in Lara et al. (2018) and Li et al. (2021a) have shown that the commercial solvers like CPLEX and Gurobi fail to solve the large-scale problems involving millions of variables and constraints. To address this computational intractability, researchers have proposed different decomposition algorithms such as column generation (Flores-Quiroz et al., 2016), Dantzig-Wolfe decomposition (Singh et al., 2009), nested-Benders decomposition (Lara et al., 2018), and tailored Benders decomposition (Li et al., 2021a), as well as heuristics (Palmintier and Webster, 2016, 2014) and advanced approach such as stochastic dual dynamic programming (Pereira and Pinto, 1991) and stochastic dual dynamic integer programming (Zou et al., 2019). Tables 1 and 2 also provide solution methods used in each paper. Since Benders decomposition has been widely used (See Tables 1 and 2), we explain here two Benders decomposition methods, nested Benders and tailored Benders decompositions proposed by Lara et al. (2018) and Li et al. (2021a), respectively.

The nested Benders decomposition has two main phases: forward pass and backward pass. A high-level description is shown in Figure 7. In the forward pass, the model is solved sequentially year by year in a myopic fashion. At a given year t, only the variables and constraints associated to year t are included. Benders cuts from the backward pass are also added to provide a relaxation of the cost-to-go function at year t. The forward pass provides a feasible solution when it reaches the end of the planning horizon, which is an upper bound of the cost minimization problem. The backward pass solves a relaxation of the MILP model from the end of the planning horizon to the first year sequentially. After the LP relaxation at year  $t \in \mathcal{T}$  is solved, a Benders cut can be generated to update the cost-to-go function at year  $t - 1 \in \mathcal{T}$ . The problem at year t = 1 provides a lower bound of the problem. The nested Benders algorithm iterates between forward pass and backward pass until the specified convergence tolerance is reached.

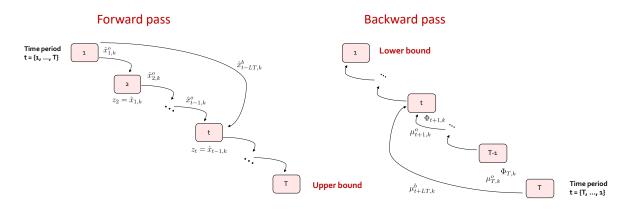


Figure 7: Nested Benders decomposition algorithm (Lara et al., 2018)

The tailored Benders decomposition decomposes the fullspace model into a Benders master problem that only involves the investment variables y, and into  $|\mathcal{T}|$  Benders subproblems. A high level description is given in Figure 8. At each iteration, the algorithm first solves the Benders master problem after which the investment decisions are fixed, and each Benders subproblem can then be solved independently. Note that there are some integer variables in the operating decisions, such as the number of generators that are on/off. In order to generate valid Benders cuts for the Benders master problem, integer variables regarding operating decisions are relaxed. The algorithm iterates between the master problem and the subproblems until the predicted lower and upper bounds lie within a specified tolerance.

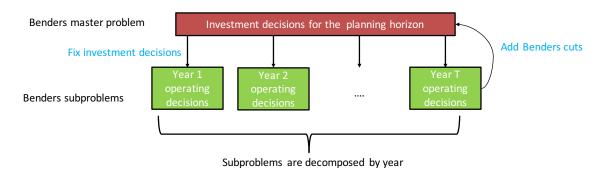


Figure 8: Tailored Benders decomposition algorithm (Li et al., 2021a)

#### 5. Reliability in power systems

Reliability is the probability of a device or system to perform its function adequately under the given operating conditions (Endrenyi, 1979). Since the main goal of power systems is to supply customers with uninterrupted power so as to satisfy the load demand, power system reliability indicates the probability that power demand is satisfied even if some units fail (Billinton and Allan, 1990). However, meeting the time-varying demand is not always possible due to the stochastic nature affecting power systems, such as random failures of equipment, and supply disruption by extreme weather conditions. Moreover, it is challenging to exactly predict power output generated from variable generation technologies, such as wind turbines and solar panels, due to the intermittency of renewable sources.

In power generation systems, reliability can be evaluated based on *security* and *adequacy* (Billinton and Allan, 1990). *Security* indicates the ability of power systems to operate stably under unplanned outages, which is associated with how the systems respond to the perturbations during operation. On the other hand, *adequacy* focuses on securing sufficient generation, transmission, and distribution facilities to satisfy the load demand and operational constraints under normal operation conditions. High-level studies such as design and expansion planning have mainly analyzed system adequacy to evaluate power system reliability. As shown in Figure 9, the system adequacy can be evaluated using deterministic and probabilistic approaches.

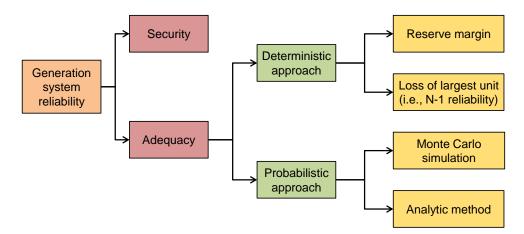


Figure 9: Reliability assessment for power generation systems (modified from Ballireddy and Modi (2019))

There are two different methods in the deterministic approach for adequacy: reserve margin and loss of the largest unit. The reserve margin method installs a larger capacity than required to prevent power shortages caused by the contingency (also known as overdesign of the systems). Note that in power systems, contingency is the same term as a disturbance resulting from the component outages (Abul'Wafa et al., 2019). This reserve margin capacity is usually equivalent to a fixed percentage of the peak demand and operating margins. The loss of the largest unit method, also known as N-1 reliability, means that the power systems can withstand an unexpected failure or outage of the single and largest component (Ovaere, 2016). Although there are N-2, N-3,..., N-k reliability methods stating the power systems can stably operate under multiple unit failures, N-1 reliability method has been commonly used in expansion planning due to its simplicity. A shortcoming of these methods is that they do not account for the stochastic nature such as failures of the generators. Securing sufficient capacity does not guarantee that the systems will not fail because each power generating unit has an inherent failure rate.

A probabilistic approach is more rigorous and accurate than the deterministic approach in the way that multiple failure scenarios are explicitly included to predict the system reliability. As shown in Figure 9, there are two methods in the probabilistic approach: Monte Carlo simulation and an analytic method (Singh and Mitra, 1995; Singh et al., 2019). Monte Carlo simulation generates different failure scenarios and evaluates the system reliability based on the probability distribution function of components composing each failure scenario (Singh and Mitra, 1995). The analytical method evaluates the system reliability by using mathematical models, which include the state space method using Markov processes, network reduction method, conditional probability method, and cut-set or tie-set method (Singh et al., 2019; Prada, 1999). Although the analytical method is rigorous and useful in reliability evaluation, it is challenging to integrate this method into expansion planning due to the complexity of the resulting optimization models.

# 5.1. Reliability indices in a probabilistic approach

There are three reliability indices used in the probabilistic approach for expansion planning of power generation systems: loss of load probability (LOLP), loss of load expectation (LOLE), and loss of energy expectation (LOEE) (Gbadamosi and Nwulu, 2020). LOLP is the probability that the power system fails to satisfy the power demand due to outages of available generating capacities. LOLE indicates the expected number of hours or days where the power demands are not satisfied (Husain Saleh et al., 2019). As the name implies, LOLP should be defined as a probability, but it is measured in numbers of hours or days for convenience, which states LOLP and LOLE have been interchangeably used in power systems. Both indices have commonly been used to determine the installed capacity of power systems based on peak load demand. However, a shortcoming of LOLE and LOLP is that they do not provide information regarding the duration and frequency of failures and incidence of power loss. On the other hand, LOEE stands for the power demand not supplied due to inadequate capacity of power generation systems (Hakimi et al., 2022). It is also known as expected energy not served (EENS). LOEE or EENS is also frequently used in expansion planning; in general, LOLE and LOEE have been used together so as to provide information about duration and amounts of power loss.

Three important steps should be conducted to use these indices for reliability assessment. The first is to enumerate all capacity failure states composed of combinations of operating and failed generators. For instance, there is a power generation system consisting of two 50MW generators, each with failure rate of 0.1. This illustrative example can have four capacity failure states as shown in Table 3. It should be noted that the number of capacity failure states increases exponentially as the number of generators increases. The power system with three 50MW generators has eight failure states. All failure states can explicitly enumerate using Markov properties, and readers interested in Markov process for reliability assessment can refer to the paper by Rausand and Hoyland (2003).

1	able 5: Capa	acity failure	states of the mustrative ex	ampie
Failure states	Generate	or status	Availability capacity	State probability
(k)	Gen #1	Gen $#2$	(MW)	$(p_k)$
1	Active	Active	100	0.81
2	Failed	Active	50	0.09
3	Active	Failed	50	0.09
4	Failed	Failed	0	0.01

Table 3: Capacity failure states of the illustrative example

The second is to predict the probability of each capacity failure state k based on failure rates of generators, which are known as forced outage rates (FOR). The state probability  $(p_k)$  can be calculated as follows:

$$p_k = \prod_{j \in J_k^F} (1 - \lambda_j) \prod_{j \in J_k^O} \lambda_j \tag{2}$$

where  $\lambda_j$  is the forced outage rate (or simply failure rate) of units j, such as generators or transmission lines, it is assumed as 0.1 in the above example.  $J_k^F$  and  $J_k^O$  are the sets of failed and active units under capacity outage state k, respectively. As depicted in Table 3, the probability of the failure state 1 becomes 0.81, whereas the probability of the failure state 4 becomes 0.01. Finally, *outage time* ( $t_k$ ) and corresponding *unmet demand* ( $E_k$ ) under the specific capacity failure state (k) are projected based on the state probability ( $p_k$ ) and the load profile. For the illustrative example composing of two 50MW generators, the load profile is assumed to be given as depicted in Figure 10(a).

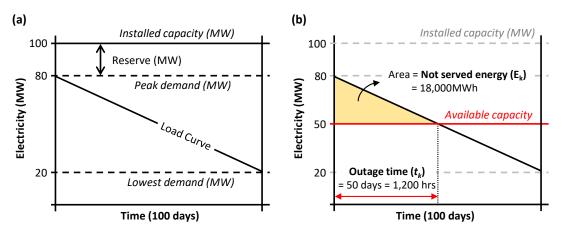


Figure 10: (a) load curve and installed capacity, (b) outage time and power energy not served of the illustrative example

As shown in 10(a), the load demand gradually decreases from 80MW to 20MW for 100 days, with its peak demand at 80MW. The system is designed to have a 25% larger capacity than the peak demand, resulting in a total installed capacity of 100MW. The difference between the installed capacity and the peak demand is known as *reserve*. If one

of the generator fails, the power system only has available capacity of 50MW as shown in 10(b). Such power system is not able to fully satisfy the load demand for the first 50 days (i.e., 1200 hours), and the unmet demand is calculated as 18,000 MWh for the given period. 50 days and 18,000 MWh stand for *outage time* ( $t_k$ ) and *unmet demand* ( $E_k$ ) at the failure state k, respectively. Since this power system has four different failure states, four different outage times and unmet demands are calculated. LOLE and LOEE of the power systems are calculated by aggregating the products of outage time and state probability, and unmet demand and state probability for all states, as shown in equations (3) and (4). As a result, the illustrative example expects to have 240 hours of LOLE and 3600 MWh of LOEE (see Table 4). In practice, power systems aim to design and operate to have lower LOLE and LOEE.

$$LOLE = \sum_{k=1}^{n_k} p_k t_k \quad (unit:hours) \tag{3}$$

$$LOEE = \sum_{k=1}^{n_k} p_k E_k \quad (unit: MWh) \tag{4}$$

For instance, the goal of PJM electricity systems that coordinates the movement of electricity of 13 eastern states in the US is to maintain their LOLE level at 0.1 days/year (Garrido, 2020). As mentioned earlier, the main challenge of the probabilistic approach is that the number of capacity failure states grows exponentially with the number of generators installed. Therefore, solution strategies to handle the large number of failure states should be developed.

Failure states $(k)$	Out capacity (MW)	State probability $(p_k)$	Outage time $(\text{hours}, t_k)$	Unmet demand (MWh, $E_k$ )		
1	0	0.81	0	0		
2	50	0.09	$1,\!200$	18,000		
3	50	0.09	$1,\!200$	18,000		
4	100	0.01	$2,\!400$	36,000		
LOLE	(hours)	240				
LOEE (MWh)			3600			

Table 4: LOLE and LOEE results of the illustrative example

#### 5.2. Reviews of probabilistic model for reliable expansion planning

Several expansion planning works using a probabilistic method to evaluate power system reliability have been reported. Slipac et al. (2019) and Choi et al. (2005) propose a new optimization model for expansion planning of power systems based on probabilistic reliability indices. Aghaei et al. (2013) present a multi-objective generation expansion planning model that minimizes the cost and environmental impact, and maximizes reliability. The model is subject to a reliability constraint expressed by LOLP criterion. Aghaei et al. (2014) incorporate EENS criterion into GTEP optimization models to account for the reliability of power systems. The non-linearity of the model caused by reliability constraints is eliminated by reformulating the model. The authors also compare the results of typical GTEP and reliability-constrained GTEP, and evaluate the effect of non-linearity of the models on finding feasible solutions in a reasonable time. Jooshaki et al. (2019) propose a reliability-constrained MILP model for the distribution expansion planning. Power loss is penalized as a reliability-related cost in the model, and three reliability indices (EENS, SAIFI, and SAIDI) are applied. Note that system average interruption frequency index (SAIFI) and system average interruption duration index (SAIDI) are used to measure the reliability of distribution system. Gbadamosi and Nwulu (2020) propose a multi-objective optimization model for the expansion planning, which minimize the cost, power outage costs, power losses, and  $CO_2$  emission. Markov processes and three indices (i.e., LOLP, LOLE, and EENS) are applied in the model to evaluate the power system reliability. Cho and Grossmann (2022) propose an optimization model for expansion planning of reliable power generation systems. The model determines both investment decisions (such as number, size, location, and investment timing) and operation decisions (number of operating/backup generators, operating capacity, and expected power output) while minimizing total costs consisting of investment and operation costs, and LOLE and LOEE penalties. Interesting points of this paper are that: a) backup generators are considered to improve power systems reliability, and b) all possible capacity failure states are explicitly considered in the model. Moreover, the backup generators have dual roles in the paper. They remain as backups in case of low power demand, or change to operating generators when the power demand is large.

# 6. Future directions

Previous studies have not only developed optimization models for expansion planning of power systems but also extended the scope of the models by taking into account reliability and flexibility constraints. However, there are several challenges that researchers should address for expansion planning research.

#### 6.1. Power systems resilience

Resilience is the ability of a system to quickly recover its normal conditions after the occurrences of disruptions that are typically due to extreme weather conditions (Hosseini et al., 2016). Reliability has been widely analyzed in expansion planning, whereas resilience is an emerging concept in power systems. Note that resilience of chemical process such as heat exchanger network had been actively studied (Lenhoff and Morari, 1982; Marselle et al., 1982; Morari, 1983; Saboo et al., 1987a,b). In power systems, reliability is concerned with *high probability and low impact (HPLI) events*, such as single unit failure, whereas

resilience is related to *low probability and high impact (LPHI) events* such as natural disasters and extreme weather conditions. Power system reliability mainly aims to ensure stable design and operation of power systems by: a) installing sufficient capacities and/or b) preventing events that can frequently occur through reliable operation strategies. Power systems with high reliability can consistently satisfy the load demand. On the other hand, power system resilience analyzes the recovery process after a power outage. The resilience research not only takes preventive measures to reduce the probability of power outage but it also suggests ways to restore the damaged power systems. Power systems with high resilience can withstand unexpected disruption and quickly recover from the damaged operating conditions (Bhusal et al., 2020). Further explanation of power system resilience can be found in the paper (Gacitua et al., 2018).

Previously, resilience was not considered in expansion planning because the probability of extreme weather events was assumed to be significantly low, and because of the computational intractability of combining dynamic features and long-term capacity planning. However, recently, researchers have integrated resilience and expansion planning as the power outages caused by extreme weather events have increased worldwide, such as the 2021 Texas power crisis (Aldhous et al., 2021). It should be noted that unlike the reliability research which relies on the probability of equipment failure, power system resilience is significantly affected by highly uncertain and unpredictable information such as frequency or magnitude of extreme weather (Gacitua et al., 2018). Therefore, advanced methods and approach that can account for such uncertainty should be developed. Although some studies on resiliency-constrained expansion planning have been reported (Zakernezhad et al., 2021; Hamidpour et al., 2021; Anderson et al., 2021; Qorbani and Amraee, 2021), follow-up studies should propose advanced expansion planning models that combine reliability and resilience to rigorously account for all possible failures of components and disruption.

# 6.2. Integration of reliability, resilience, and flexibility

In chemical engineering, flexibility is the capability of a chemical process to achieve feasible operation over a given range of uncertain conditions (Grossmann and Floudas, 1987). However, power system flexibility has a different meaning. Power system flexibility is the ability to balance supply and demand without disruption, even under uncertain operation conditions (EPRI, 2016). However, it is challenging to satisfy the demand of modern power systems consisting of many variable generators due to intermittency of renewable sources. In order to improve flexibility of power systems, alternatives such as large-scale electricity storage, sector-coupling with hydrogen or heat, and demand response have been proposed. As discussed in section 2, expansion planning models accounting for either reliability or flexible operation under uncertainty have been areas of active research (See Tables 1 and 2). However, because of the complexity of the optimization models, there are not many studies that have simultaneously taken into account reliability, resilience, and flexibility in expansion planning. Future expansion planning studies should integrate these three topics together through development of advanced optimization models.

# 7. Conclusions

Expansion planning, one of the areas in power system research, aims to determine adequate capacities for generation, transmission, and distribution to satisfy the projected power demand within a set of technical and economic constraints. The objective function of optimization models is to minimize the cost, including investment and operating costs. Constraints related to power balance, capacity limits, and security are usually applied. The optimization model usually considers a time horizon of more than 20 years, and determines investment decisions such as the number, type, capacity, investment timing, and locations of the units. However, the models have become more complex in recent years so as to reflect both regulations on conventional energy sources and the increasing penetration of renewable energy sources.

This paper has reviewed the basic concepts and optimization models of expansion planning. We also have investigated computational challenges of expansion planning models, such as temporal and spatial aggregation, and decomposition methods for large-scale problems. In addition, this paper has reviewed the definition of power system reliability and the methods frequently used to evaluate power system reliability, such as reserve margin, N-1 reliability, Monte Carlo simulation, and an analytic method. Finally, this paper has briefly discussed future directions including integration of reliability, resilience, and flexibility.

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