Stochastic Programming Models for Optimal Shale Well Development and Refracturing Planning under Uncertainty

Markus G. Drouven[1], Diego C. Cafaro[2] and Ignacio E. Grossmann[1]

[1] Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A.

[2] INTEC (UNL – CONICET), Güemes 3450, 3000 Santa Fe, ARGENTINA

In this work we present an optimization framework for shale gas well development and refracturing planning. This problem is concerned with if and when a new shale gas well should be drilled at a prospective location, and whether or not it should be refractured over its lifespan. We account for exogenous gas price uncertainty and endogenous well performance uncertainty. We propose a mixed-integer linear, two-stage stochastic programming model embedded in a moving horizon strategy to dynamically solve the planning problem. A generalized production estimate function is described that predicts the gas production over time depending on how often a well has been refractured, and when exactly it was restimulated last. From a detailed case study, we conclude that early in the life of an active shale well, refracturing makes economic sense even in low-price environments, whereas additional restimulations only appear to be justified if prices are high.

Correspondence concerning this article should be addressed to I. E. Grossmann at grossmann@cmu.edu

Introduction

Shale wells are well-known for their rapid production declines. Fig. 1 shows four different type curves for shale gas wells in different areas of the Marcellus Play in the U.S. Although the initial production rates vary, all wells are characterized by steep production declines. To upstream operators these characteristic declines are both a blessing and a curse. On the upside, the initial production peaks as seen in Fig. 1 imply that the majority of the hydrocarbons are produced early in the life of the well. As a result, most of the revenues that can be expected are obtained early on, with only minimal additional economic impact to the overall profitability the longer the well produces natural gas[1]. This allows operators to achieve quick returns on their investments. At the same time, the characteristic decline curves also put considerable emphasis on the precise timing of the initial production of a shale well. If natural gas prices happen to be low when the well is turned in-line, then operators could be wasting the "prime time" of their assets. Clearly, the timing of turning a well in-line is critical for profitability. However, it is notoriously difficult to predict natural gas prices accurately. Therefore, when upstream operators decide to drill, fracture and complete new shale wells, they oftentimes base their decisions on the prevalent price environment. By the time the well is actually turned in-line several months later, prices may already have deteriorated quickly and dramatically. For the longest time the industry consensus was that there is practically nothing upstream operators could do to manipulate shale decline curves favorably over time; the belief was that operators could not take any economically feasible recourse actions to reinvigorate their assets.

However, it turns out that shale wells can be fractured more than once. And there is increasing evidence suggesting that many mature shale wells still contain large volumes of oil and gas that can be recovered through the process of *refracturing* or *well restimulation*[2]. Fig. 2 shows a comparison of a horizontal well and its surrounding fracture network after the initial well stimulation (left) and after a refracture treatment (right). Kotov and Freitag (2015)[4] argue that the steep decline curves in unconventional reservoirs after the initial fracturing operation typically result in 10 percent or less recovery of the available reserves. This implies that refracturing theoretically has the potential to recover 90 percent of remaining hydrocarbons in the shale formation.

The oilfield service company Schlumberger estimates that roughly 10,000 horizontal shale oil and gas wells drilled in the past five years in North America are candidates for refracturing[5]. The belief is that a well restimulation can restore production to near-initial levels at far less cost than drilling and completing a new well[6]. Fig. 3 shows the production history of a Marcellus shale well refractured by Consol Energy after approximately four years. In this particular case the production data clearly reveals that the restimulation was very effective; production rates are restored and even the decline appears to be less drastic following the recompletion. Naturally, restimulation costs vary between operators or development areas and generally they depend on the selected refracturing technique. While Consol Energy estimates that a restimulation costs approximately two million USD, Encana is refracturing wells for less than one million USD[7]. In comparison, the process of drilling, fracturing and completing a new shale well in the Marcellus Play ranges in cost between three and six million USD.

However, not only does the cost of restimulations vary between operators. King (2015)[8] reports that both timing and frequency of recompletions oftentimes differ too. While some operators choose to refracture early into the lifespan of their wells, others wait several years to restimulate horizontal laterals. Moreover, there is increasing evidence suggesting that multiple restimulations of the same wellbore may make economic sense. In fact, Broderick et al. (2011)[9] claim that shale gas wells could be refractured up to five times over their expected lifespan of 20-25 years.

Despite its practical potential, the refracturing planning problem is not a well-studied problem in the literature. To this day, notably few researchers have addressed the challenges that shale gas producers face when scheduling and performing well restimulations. Among the few works that have been published in this field, Sharma (2013)[10] proposes guidelines and dimensionless type curves to accomplish two things: a) determine the ideal timing of a refracture treatment in the life of a well, and b) evaluate the potential increase in well production after the restimulation. Eshkalak et al. (2014)[11] study the refracturing planning problem with an emphasis on the economics of well restimulations. Through a comprehensive case study involving a total of 50 shale wells, the authors find that refracturing is profitable even in low-price environments, although the actual timing of the well restimulations is pre-determined. Tavassoli et al. (2013)[12] propose a comprehensive, numerical simulation model to evaluate the impact of well restimulations on the production performance of shale wells as a function of reservoir parameters, the recompletions design, and the timing of the refracture treatment. As a rule of thumb, the authors advise upstream operators to consider the restimulation of their shale wells whenever production decline rates are below 10-15%.

Lastly, Cafaro et al. (2016)[13] present an optimization framework to plan shale gas well refracture treatments. In their work, the authors assume that the decision to drill, fracture and complete a prospective shale well has already been made. In order to determine if, when and how often the well should be restimulated over its lifespan, Cafaro et al. (2016)[13] propose two optimization models: a continuous-time nonlinear programming (NLP) model and a discrete-time mixed-integer linear programming (MILP) model. Whereas the NLP model is primarily designed to identify the optimal time to refracture a well such that its expected ultimate recovery (EUR) is maximized, the MILP model can be used to schedule multiple refracture treatments over the life of a well. Both models, however, are purely deterministic in nature.

In this publication we present an important extension of the work by Cafaro et al. (2016)[13]. Our primary objective is to explicitly account for two major sources of uncertainty: price developments over time, and production performance before/after restimulations. In addition, we present a generalized production estimate function, and a moving horizon framework that enables upstream operators to schedule refracture treatments as true recourse actions to uncertainty realizations and/or potential disruptions.

General Problem Statement

We assume that an upstream operator has identified a prospective location to drill, fracture and complete a single shale gas well. The estimated production of the prospective well over time is characterized by a given type curve. Once completed, it is assumed that this well can be refractured multiple times over its expected lifespan. Every restimulation leads to a reinvigoration of the well's gas production rate.

However, in this work we recognize that a prospective well's production rate over time cannot always be forecasted accurately a priori, i.e. before the well has actually been drilled and completed. Particularly, the post-refracture response performance can be difficult to anticipate. Hence, we assume that – in addition to the default type curve – a discrete set of well performance scenarios is provided. These scenarios account for the possibility that the gas production prior to and after any number of well restimulations exceeds or fails to meet expectations.

In addition to well performance scenarios, we assume that the operator chooses to consider a set of natural gas price forecast scenarios. These scenarios reflect the fact that it is generally challenging to predict natural gas price developments reliably over time, especially over multiple years. By considering a number of unique, potential price development scenarios, the intent is to "robustify" the proposed well development strategy over a wide range of possible outcomes.

In light of uncertain well production performance and uncertain gas price forecast, the goal of this work is to determine: (a) if a well should be drilled at the prospective location at the present time, (b) whether or not the well should be refractured eventually, (c) how often the well should be refractured over its expected lifespan, and (d) when exactly the refracture treatments should be performed. The objective is to maximize the expected net present value of the well development project.

This article is organized as follows. Initially, we present a generalized production estimate function that explicitly considers the possibility of refracturing a shale well multiple times over its lifespan. Next, we briefly review general concepts of stochastic programming to address optimization problems under uncertainty and we discuss why two-stage stochastic programming is particularly suitable for addressing the well development and refracturing planning problem in light of exogenous price uncertainty and endogenous well performance uncertainty. The integrated planning problem is tackled with two distinct mixed-integer linear programming models: a) a two-stage stochastic programming model for well development planning, and b) a two-stage stochastic programming model for refracturing planning. Both models are embedded in a moving horizon strategy, which allows decision-makers to recognize refracturing as an opportunity to periodically respond to uncertainty realizations and/or potential disruptions. Moreover, we show that the moving horizon strategy can be used to effectively address the endogenous nature of the well performance uncertainty. Thereafter, we present a comprehensive case study to demonstrate how the proposed optimization framework can be used to solve the practical well development and refracturing planning problem under uncertainty. Lastly, we discuss qualitatively what role refracturing may have in field-wide shale development projects.

Generalized Production Estimate Function Considering Multiple Refracture Treatments

In this section we propose a generalized production estimate function that predicts how much gas a well is expected to produce over time as a function of when and how often it has been restimulated. For this purpose we introduce the parameter $Q_{i,t,\hat{t},p}$. This parameter captures the amount of gas to be produced by a well in time period t given that it has been refractured i times total, and the last stimulation was performed in time period $\hat{t} < t$. Since we wish to account for the uncertainty in predicting gas production over time, this parameter also includes the well-performance scenario index p. This index highlights the fact that the estimated gas production is scenario-dependent, which will be outlined in more detail below. As suggested by Cafaro et al. (2016)[13], the gas production of an unconventional well can be represented adequately by a decreasing power function. This power function is defined by an expected initial production peak parameter k_p and an expected initial production decline parameter a_p, both of which are assumed to be scenario-dependent. Yet, these two parameters by themselves can only represent the production of a shale well that has not been refractured. In order to account for restimulation measures we propose Eq. (1).

$$Q_{i,t,\hat{t},p} \approx \tilde{\gamma}_{i,p} \cdot k_p \cdot t^{-a_p} + r_{i,\hat{t},p} \cdot \left(t - \hat{t} - rt + 1\right)^{-a_{i,p} - b_{i,p} \cdot \hat{t}} \tag{1}$$

The function in Eq. (1) contains the previously introduced expected initial production peak parameter k_p and the expected initial production decline parameter a_p. However, this function also considers a number of additional factors that play an important role as soon as a well has been refractured once or more often. For instance, every time a shale well is refractured, the contribution of its initial fractures to the overall production changes. Some operators report an increase in production contribution, while others have experienced decreases. We introduce the parameter $\tilde{\gamma}_{i,p}$ to capture this aspect. Initially, after a well has been drilled and fractured for the first time, this parameter equals one. After every restimulation, however, the parameter may be set to a different expected value. This information can typically be provided by completions design engineers, geologists, or reservoir engineers. By default, we assume that $\tilde{\gamma}_{i,p} = \tilde{\gamma}_p^{\,i}$, meaning that every additional refracture treatment has the same impact on the original fractures.

More importantly, the second term of the gas production estimate function in Eq. (1) captures the characteristic peak in production following a well restimulation. We assume that every restimulation takes rt time periods (usually, one month). For this reason, we introduce the expected, supplemental production peak parameter $r_{i,\hat{t},p}$. The value of this parameter changes depending on how many times (i) the well has been refractured and when the last restimulation occurred, captured by the index $\hat{t}$. With every additional refracture treatment, this supplemental production peak becomes less pronounced. Also, field tests have revealed that the peak following a restimulation decreases the longer an upstream operator waits to refracture a well. We note that previous work by Cafaro et al. (2016)[13] does not consider the timing of a well restimulation to anticipate the supplemental production peak following the refracture treatment. For simplicity, the authors assume that every time a well is refractured, the supplemental production peak lessens by a peak reduction factor $\beta_{i,p}$, which explicitly considers the number of total restimulations, but does not account for the timing of these measures. Conceptually, the approximation $r_{i,\hat{t},p} \approx \beta_{i,p} \cdot r_p$ is valid and may be used to simplify the problem at hand.

Finally, we address the exponent $(-a_{i,p} - b_{i,p} \cdot \hat{t})$ in Eq. (1). Essentially, this exponent is an estimate of the post-refracture production decline after *i* restimulations in scenario *p*. The term is made up of three criticial factors that are believed to determine the production decline following a refracture treatment: a) the initially expected production decline of the well after *i* restimulations $a_{i,p}$ (which may vary depending on how many stages of the well are actually recompleted), b) the expected additional decline after *i* restimulations $b_{i,p}$, and c) the timing of the *i*-th refracture treatment $\hat{t}$. This composite decline exponent is motivated by the work of Tavassoli et al. (2013)[12] who show that the post-refracture production decline increases the longer an upstream operator waits to refracture a shale well. As before, all production decline parameters are scenario-dependent and can therefore be defined to account for different well-performance scenarios *p*. Fig. 4 shows an illustration of the generalized production estimate function with added "noise" (created via Monte Carlo simulation). The type curves are representative for a well having been fractured once, twice or three times considering different production performance parameter settings.

Altogether, the generalized production estimate function in Eq. (1) is more rigorous and comprehensive than the previously proposed correlation by Cafaro et al. (2016)[13], since it explicitly considers: a) how often a well has been refractured in total, b) when a well was last restimulated, and c) by how much production may deviate depending on the degree of uncertainty.

Concepts of Stochastic Programming

In this section, we briefly review concepts and premises of stochastic programming. The motivation for stochastic programming originates from the fact that decision-makers often face problems involving uncertain parameters. These parameters could include price forecasts, processing times, or cost assumptions. Stochastic programming allows decision-makers to solve problems involving uncertain parameters through rigorous mathematical optimization. The premise of stochastic programming is that a problem is essentially split into two broad categories of decisions: a) those that have to be made in light of the uncertainty, i.e., not knowing the actual realization of the uncertain parameters, and b) those decisions that can be taken as soon as the uncertainty has revealed itself. The former decisions are referred to as *here-and-now decisions*, whereas the latter can be classified as *wait-and-see, corrective* or *recourse actions*. The interpretation of this categorization is as follows: in light of uncertain parameters the goal is to identify a particular here-and-now solution strategy (e.g. a schedule, an assignment, or a particular design) that works best for a set of possible scenarios. This solution should be such that regardless of which of the scenarios is true, the selected strategy hedges against the risk of uncertainty and, in theory, it is prepared for any possible outcome. At the same time, the aforementioned recourse actions provide a decision-maker with the flexibility to respond to particular uncertainty realizations. The more flexibility a decision-maker has in terms of recourse actions, the less impactful the here-and-now decisions are. In stochastic programming the time horizon is generally discretized and all potential uncertainty realizations are obtained from discretized probability distributions. Therefore, a given set of discrete scenarios merely represents a finite set of different realizations for the uncertain parameters[14]. Given the probability of each scenario, we use mathematical programming to maximize the expected value of the objective function, subject to the constraints from all scenarios. We refer to the formulations of these optimization problems as the *deterministic equivalent* of the stochastic problem.

In this work we primarily focus on two-stage stochastic programming[15] where the entire set of decision variables is split into two subsets: here-and-now decision variables (stage one) and wait-and-see decision variables (stage two recourse actions). Alternatively, optimization problems under uncertainty can be addressed via multi-stage stochastic programming. In this case decision-makers have the opportunity to make here-and-now decisions at three or more stages throughout the time horizon. Clearly, multi-stage stochastic programming is a more rigorous and accurate representation of the decision-making process in practice. However, these formulations lead to significantly larger models that are oftentimes computationally intractable.

Within the realm of stochastic programming, we can distinguish between two types of uncertainty: exogenous and endogenous uncertainty. Exogenous uncertainty realizes regardless of what a decision-maker does. If we consider the future natural gas price as an uncertain parameter, for instance, we can presume that the uncertainty will realize eventually, i.e., the market will settle on a particular gas price – regardless of whether a shale gas producer drills or refractures a prospective well or not. The realization of endogenous uncertainty, on the other hand, depends on what a decision-maker ends up doing. For example, in this work we assume that the production performance of an unconventional well before and/or after a restimulation is uncertain prior to actually drilling the well. However, once an upstream operator has actually drilled the lateral section of the well, completions engineers gather reservoir data including permeability and porosity readings, which in turn can be used to predict the well's production performance much more accurately. Therefore, similar to Zeng and Cremaschi (2017)[16], we consider well performance to be an endogenous uncertainty. It is worth mentioning that in the past, optimization problems under exogenous and endogenous uncertainty have been addressed almost exclusively with multi-stage stochastic programming. For a detailed examination of multi-stage stochastic programming under endogenous and exogenous uncertainties we refer to the comprehensive work by Apap and Grossmann (2016)[14].

Two-Stage Stochastic Programming Model for Well Development Planning

In this work, we assume that the practical well development planning problem under exogenous price uncertainty and endogenous well performance uncertainty can be formulated as a two-stage stochastic programming approach. First, we consider the premise of the generic well development planning problem. Initially, an upstream operator has identified a prospective location to drill, fracture and complete a single shale gas well. For this well, a long-term forecast of its production over time can be estimated. However, since this production estimate is uncertain, the operator wants to consider alternative well performance scenarios. Moreover, even at this stage in the development process, the operator may want to consider the possibility of refracturing the well at some point over the course of its life, possibly even multiple times. Although the post-refracture production performance can also be estimated, it is likely that the operator's confidence in this estimate is limited. To this day, operators have drilled thousands of unconventional wells, but only restimulated a small fraction of them. Therefore, the post-refracture well performance is also assumed to be uncertain. In order to hedge against the risk of uncertainty, we consider a set of well performance scenarios $p \in P$, each with probability φ_p, as part of the well development planning problem. The planning problem is also challenging because commodity prices are known to fluctuate dramatically. Upstream operators need to know that their potential investment in a prospective shale well makes economic sense across a range of possible price developments. Hence, we explicitly factor price uncertainty into our analysis, and therefore consider a set of price forecast scenarios $f \in F$. Furthermore, our framework gives operators the opportunity to assign a likelihood π_f to the realization of each scenario. The purpose of the well development planning problem is then to determine, first and foremost, if an upstream operator should drill the prospective shale well at the present time. In the proposed optimization framework this key decision is captured by introducing the binary decision variable w^{DRILL}. In the context of two-stage stochastic programming, as outlined earlier, this variable is classified as a stage one, here-and-now decision. Without knowing what the ultimate production performance will be or how natural gas prices will develop, the optimization model sets this variable either to one or to zero in light of the considered spectrum of uncertainty scenarios. Beyond the actual well development, there are a number of decisions that can be made individually for every considered scenario; these are denoted as scenario-dependent, wait-and-see decisions. Among these decisions are: a) whether or not the well should be refractured eventually, b) how often the well should be restimulated over its expected lifetime, and c) when exactly the refracture treatments should be performed. All of the aforementioned aspects of the well development planning problem are captured by the binary decision variable $x_{i,t,f,p}$. This variable is equal to one if the well is scheduled to be restimulated for the *i*-th time in time period t under price scenario f and well performance scenario p. The following constraints are designed around these two key decision variables, w^{DRILL} and $x_{i,t,f,p}$.

For instance, the inequality in Eq. (2) is added to the proposed model to ensure that unless the well has actually been drilled, it cannot be refractured.

$$w^{DRILL} \geq x_{i,t,f,p} \qquad \forall i \in I, t \in T, f \in F, p \in P \tag{2}$$

We note that Eq. (2) is expressed as an inequality constraint to allow for the possibility of drilling the well but never actually refracturing it over its lifespan. In turn, Eq. (3) ensures that the prospective well cannot be restimulated for the *i*-th time more than once.

$$\sum_{t \in T} x_{i,t,f,p} \leq 1 \qquad \forall i \in I, f \in F, p \in P \tag{3}$$

Eq. (4) is a sequencing constraint ensuring that if in time period t the well is restimulated for the i-th time, then it has to have been refractured for the $(i-1)$-th time previously. We note that the parameter rt is introduced to represent the number of time periods it actually takes to recomplete the well.

$$x_{i,t,f,p} \leq \sum_{\tau < t-rt} x_{i-1,\tau,f,p} \qquad \forall i \in I, t \in T, f \in F, p \in P, i > 1 \tag{4}$$

At this point we rely on a simple but effective step to strengthen the quality of the proposed model formulation. We introduce an auxiliary binary variable $y_{i,t,f,p}$ to determine whether as of time period t the well has been refractured i times in scenarios f and p. Although the variables $x_{i,t,f,p}$ and $y_{i,t,f,p}$ are closely related, they serve different purposes. The variable $x_{i,t,f,p}$ marks the exact timing of a restimulation, whereas the variable $y_{i,t,f,p}$ keeps track of the "state of restimulation". For instance, it is entirely possible that in time period $t = 36$ (typically months) the well has been refractured twice for a particular scenario combination, hence $y_{i2,t36,f,p} = 1$. Yet, this does not necessarily imply that the restimulation actually occurred in this particular time period. Instead, the well could have been refractured in time period $t = 24$ for the second time, in which case $x_{i2,t24,f,p} = 1$. To establish the relationship between these two variables, we include Eqs. (5), (6) and (7) in the model.

$$x_{i,t,f,p} \le y_{i,t,f,p} \qquad \forall i \in I, t \in T, f \in F, p \in P \tag{5}$$

$$y_{i,t,f,p} \le \sum_{\tau \le t} x_{i,\tau,f,p} \qquad \forall i \in I, t \in T, f \in F, p \in P \tag{6}$$

$$y_{i,t,f,p} \ge y_{i,t-1,f,p} - x_{i+1,t,f,p} \qquad \forall i < |I_0|, f \in F, p \in P, t > 1 \tag{7}$$

All three equations above can easily be derived using propositional logic[17] and we refer to the work by Cafaro et al. (2016)[13] for the actual derivation. Similar to the previously introduced Eq. (3), we also add Eq. (8) to the model.

$$\sum_{i \in I_0} y_{i,t,f,p} = w^{DRILL} \qquad \forall t \in T, f \in F, p \in P \tag{8}$$

However, we note that unlike Eq. (3), the above constraint is actually expressed as an equality constraint. That is because at any point in time the well has to be in a particular "refracturing state" $i \in I_0$, if drilled. In fact, the set I_0 includes the element i_0 which represents the state "drilled and fractured, but not refractured". Next, we introduce an additional binary variable $z_{i,t,\hat{t},f,p}$. This variable can be derived from the previously defined decision variables $x_{i,t,f,p}$ and $y_{i,t,f,p}$ as follows:

$$[y_{i,t,f,p} = 1] \wedge [x_{i,\hat{t},f,p} = 1] \Leftrightarrow [z_{i,t,\hat{t},f,p} = 1] \qquad \forall i \in I, t \in T, \hat{t} \le t, f \in F, p \in P \tag{9}$$

Practically speaking, the variable $z_{i,t,\hat{t},f,p}$ indicates whether in time period t the well has been refractured i times in the past, and the last restimulation occurred in time period $\hat{t}$ for the scenario combination f and p. By applying propositional logic to the statement in Eq. (9), we derive Eqs. (10)-(12).

$$y_{i,t,f,p} + x_{i,\hat{t},f,p} \leq z_{i,t,\hat{t},f,p} + 1 \qquad \forall i \in I, t \in T, \hat{t} \leq t, f \in F, p \in P \tag{10}$$

$$z_{i,t,\hat{t},f,p} \leq y_{i,t,f,p} \qquad \forall i \in I, t \in T, \hat{t} \leq t, f \in F, p \in P \tag{11}$$

$$z_{i,t,\hat{t},f,p} \leq x_{i,\hat{t},f,p} \qquad \forall i \in I, t \in T, \hat{t} \leq t, f \in F, p \in P \tag{12}$$

For the particular element $i_o \in I_0$, Eqs. (10)-(12) take the following form:

$$y_{i,t,f,p} + w^{DRILL} \leq z_{i,t,\hat{t},f,p} + 1 \qquad \forall i = i_o, t \in T, \hat{t} = t_1, f \in F, p \in P \tag{13}$$

$$z_{i,t,\hat{t},f,p} \leq y_{i,t,f,p} \qquad \forall i = i_o, t \in T, \hat{t} = t_1, f \in F, p \in P \tag{14}$$

$$z_{i,t,\hat{t},f,p} \leq w^{DRILL} \qquad \forall i = i_o, t \in T, \hat{t} = t_1, f \in F, p \in P \tag{15}$$

The actual gas production of the prospective well $P_{t,f,p}$ is directly linked to the previously proposed production estimate function $Q_{i,t,\hat{t},p}$ in Eq. (16).

$$P_{t,f,p} = \sum_{i \in I_o} \sum_{\hat{t}=1}^{t} Q_{i,t,\hat{t},p} \cdot z_{i,t,\hat{t},f,p} \qquad \forall t \in T, f \in F, p \in P \tag{16}$$

Since the previously introduced decision variable $z_{i,t,\hat{t},f,p}$ captures the current time period (index t), how often the well has been refractured (index i), and when the last restimulation occurred (index $\hat{t}$) for every scenario combination (indices f and p), we link it directly to the production $Q_{i,t,\hat{t},p}$ predicted by Eq. (1). However, we note that the proposed optimization framework can be linked to any alternative production forecast function simply by replacing $Q_{i,t,\hat{t},p}$ by the preferred estimation.

Finally, Eq. (17) ensures that, if drilled, the well should be in one “refracturing state” at every point in time for every scenario combination.

$$\sum_{i \in I_0} \sum_{\hat{t}=1}^{t} z_{i,t,\hat{t},f,p} = w^{DRILL} \qquad \forall t \in T, f \in F, p \in P \tag{17}$$

The objective of the well development project is to maximize the expected net present value. This means that in light of the considered price forecast uncertainty and well performance uncertainty, revenues from gas sales have to be maximized, whereas expenses for well development and recompletions are to be minimized.

$$\max ENPV = \underbrace{-\left(DC + CC\right) \cdot w^{DRILL}}_{\substack{\text{Stage 1 decision (here-and-now):} \\ \text{Develop the well: yes or no?}}} + \underbrace{\sum_{f \in F} \pi_f \sum_{p \in P} \varphi_p \sum_{t \in T} \left(1+d\right)^{-t} \cdot \left\{ P_{t,f,p} \cdot gp_{t,f} - \sum_{i \in I} rc_i \cdot x_{i,t,f,p} \right\}}_{\substack{\text{Stage 2 decisions (wait-and-see):} \\ \text{Restimulate the well: yes or no? how often? when? (scenario-dependent)}}} \tag{18}$$

The objective function in Eq. (18) clearly exemplifies the two-stage nature of the proposed optimization model. The initial summation term captures the stage one, here-and-now decision concerned with whether or not the well should be developed at the present time. This is a yes-or-no design decision that involves a development expense for drilling and completions operations, as represented by the parameters DC and CC, respectively. The binary variable w^{DRILL} is clearly scenario-independent, accounting for the fact that this decision needs to be made in light of the uncertainty, i.e., not knowing which of the scenarios will turn out to be true.

The second summation term in Eq. (18) represents the stage two, wait-and-see decisions that reflect whether or not, how often and when the well needs to be refractured. These decisions represent scenario-dependent recourse actions that can be made individually and independently for every single scenario combination of price forecast and well performance. Since every unique scenario combination may result in a different production profile and/or restimulation strategy, revenues and expenses may vary scenario-by-scenario. In particular, this term of the objective function contains the scenario-dependent gas price parameter $gp_{t,f}$ as well as the refracture cost rc_i which may depend on the total number of recompletions. Moreover, every scenario combination is individually weighted based on specified scenario realization probabilities, π_f and φ_p, for price forecasts and well performance, respectively. These probability parameters allow decision-makers to specify their confidence in individual scenarios, which will then be reflected in the solution identified by the optimization. Altogether, Eqs. (2)-(8) and (10)-(18) define the formulation of the well development planning problem.

In the previous section we pointed out that the pre-and post-refracture well performance uncertainty is endogenous in nature, whereas the price forecast uncertainty can be categorized as exogenous. However, at this stage in the planning process both uncertainty sources are treated the same because the prospective well has not actually been drilled and completed yet. Once this has been done, completions and reservoir engineers can use collected subsurface data to refine their production estimates to the point where a stochastic analysis is no longer necessary. This leads us to the refracturing planning problem, which will be addressed in greater detail in the following section.

Two-Stage Stochastic Programming Model for Refracturing Planning

The setup of the refracturing planning problem is as follows. Unlike before, we assume that an upstream operator is dealing with an actively producing shale gas well. Similar to the premise of the well development planning problem, however, a long-term type-curve forecast for this well's gas production is available. Yet, at this point into the well's lifespan we assume that the gas production over time can be predicted fairly accurately. Right after turning the well in-line, operators record the initial production and get early readings on its decline rate. Also, subsurface data gathered during drilling and fracturing operations allows producers to anticipate a well's response behavior to one or many restimulations relatively precisely. Hence, it is no longer necessary to account for well performance uncertainty as part of the refracturing planning problem. Price uncertainty, on the other hand, continues to present a major challenge to the operator. Therefore, we still consider a set of natural gas price forecast scenarios $f \in F$ with probability π_f when scheduling refracture treatments for the given well.

The refracturing planning problem is meant to address a number of important decisions that an upstream operator needs to make in the situation described above. The primary purpose is to determine if the active shale well should be refractured at the present time. That is ultimately the question that motivates this section. Beyond this decision, however, we also wish to determine: a) whether or not the well should be refractured again, b) how often it should be restimulated over its expected lifespan, and c) when exactly subsequent refracture treatments should ideally be scheduled. As before, we propose a two-stage stochastic programming model to address the refracturing planning problem in light of uncertain price forecasts. In the spirit of two-stage stochastic programming, the here-and-now design decision is concerned with the possibility of restimulating the well at the present time, whereas all other decisions can be classified as scenario-dependent, wait-and-see recourse actions. Although the model formulation of the refracturing planning problem is clearly inspired by the well development planning problem, there are some distinct differences that are highlighted below.

Eqs. (19)-(34), introduced next, compare directly to Eqs. (3)-(8), (10)-(12) in the previous section. However, there are a few notable differences. First, and most importantly, we introduce a new binary decision variable x^{REFRAC} to capture whether or not the actively producing well should be refractured at the present time. In the context of two-stage stochastic programming, x^{REFRAC} represents the stage one, here-and-now decision variable. Unlike before, we now also link the index i to the "current refracture state" cr which tracks how often a well has been restimulated thus far. The introduction of the current refracture state is necessary since the model proposed in this section is intended to be used repeatedly over the course of a well's lifespan, even after multiple refracture treatments may already have occurred. For more details regarding this scheme we refer to the next section (Moving Horizon Framework for Well Development and Refracturing Planning). Lastly, we point out that unlike the previous model, the constraints below are not set up over the set of well performance scenarios $p \in P$ since the production uncertainty is assumed to have resolved itself at this point in the planning process.

The inequalities in Eqs. (19) and (20) are included in the refracturing planning model to make sure that the shale well cannot be refractured for the *i*-th time more than once. Here and in several other constraints below, the introduction of the stage-one decision variable x^{REFRAC} makes it necessary to distinguish between the first opportunity to recomplete the well (here-and-now), represented by the refracture state $i = cr + 1$ in time period t_1, and additional opportunities for future well restimulations $i > cr + 1$.

$$\sum_{t \in T} x_{i,t,f} + x^{REFRAC} \leq 1 \qquad \forall i = cr + 1, f \in F \tag{19}$$

$$\sum_{t \in T} x_{i,t,f} \leq 1 \qquad \forall i > cr + 1, f \in F \tag{20}$$

Eqs. (21) and (22) represent sequencing constraints ensuring that the optimizer cannot schedule the *i*- th well restimulation, unless the (*i*-1)-th recompletion has been performed. As before, the practical constraint is expressed via two inequalities due to the stage-one decision variable x^{REFRAC}.

$$x_{i,t,f} \leq \sum_{\tau < t - rt} x_{i-1,\tau,f} + x^{REFRAC} \qquad \forall i \in I, t \in T, f \in F, i = cr + 2 \tag{21}$$

$$x_{i,t,f} \leq \sum_{\tau < t - rt} x_{i-1,\tau,f} \qquad \forall i \in I, t \in T, f \in F, i > cr + 2 \tag{22}$$

Eqs. (23)-(28) are directly adapted from Eqs. (5)-(7) in section Two-Stage Stochastic Programming Model for Well Development Planning. They are added to the model to account for the auxiliary variable $y_{i,t,f}$, which captures the "state of restimulation". For more details we refer to the previous section.

$$x^{REFRAC} \leq y_{i,t,f} \qquad \forall i = cr + 1, t = t_1, f \in F \tag{23}$$

$$x_{i,t,f} \leq y_{i,t,f} \qquad \forall i > cr, t > 1, f \in F \tag{24}$$

$$y_{i,t,f} \leq \sum_{\tau \leq t} x_{i,\tau,f} + x^{REFRAC} \qquad \forall i = cr + 1, t \in T, f \in F \tag{25}$$

$$y_{i,t,f} \leq \sum_{\tau \leq t} x_{i,\tau,f} \qquad \forall i > cr + 1, t \in T, f \in F \tag{26}$$

$$y_{i,t,f} \geq 1 - x^{REFRAC} \qquad \forall i = cr, t = t_1, f \in F \tag{27}$$

$$y_{i,t,f} \geq y_{i,t-1,f} - x_{i+1,t,f} \qquad \forall i \geq cr, t > 1, f \in F \tag{28}$$

As before, we also include Eq. (29) in the proposed model to ensure that at any point in time the well can be categorized by its “refracturing state” $i \in I_0$.

$$\sum_{i \geq cr} y_{i,t,f} = 1 \qquad \forall t \in T, f \in F \tag{29}$$

Eqs. (30)-(34) can be traced back to constraints (10)-(12) in the previous section. These inequalities capture the relation between the decision variables $z_{i,t,\hat{t},f}$, $y_{i,t,f}$, $x_{i,\hat{t},f}$ and x^{REFRAC}.

$$y_{i,t,f} + x^{REFRAC} \leq z_{i,t,\hat{t},f} + 1 \qquad \forall i = cr+1, t \in T, \hat{t} = t_1, f \in F \tag{30}$$

$$y_{i,t,f} + x_{i,\hat{t},f} \leq z_{i,t,\hat{t},f} + 1 \qquad \forall i \geq cr+1, t \in T, 1 < \hat{t} \leq t, f \in F \tag{31}$$

$$z_{i,t,\hat{t},f} \leq y_{i,t,f} \qquad \forall i \geq cr+1, t \in T, \hat{t} \leq t, f \in F \tag{32}$$

$$z_{i,t,\hat{t},f} \leq x^{REFRAC} \qquad \forall i = cr+1, t \in T, \hat{t} = t_1, f \in F \tag{33}$$

$$z_{i,t,\hat{t},f} \leq x_{i,\hat{t},f} \qquad \forall i \geq cr+1, t \in T, 1 < \hat{t} \leq t, f \in F \tag{34}$$

If the optimization concludes that a well restimulation is not justified here-and-now, then $x^{REFRAC} = 0$ and by Eq. (27) $y_{cr,t1,f} = 1$. This means that the well’s “current refracturing state” cr does not change in time period $t1$. In fact, the well will remain in the refracturing state cr until an additional refracture treatment $cr+1$ is proposed as a recourse action in a future time period. To account for this particular case, we introduce the binary variable $z^N_{t,f}$. For as long as no refracture treatment is scheduled, this variable will be set to one according to Eq. (35).

$$\sum_{i \geq cr+1} \sum_{\hat{t}=1}^{t} z_{i,t,\hat{t},f} + z^N_{t,f} = 1 \qquad \forall t \in T, f \in F \tag{35}$$

We note that Eq. (35) ensures that $z_{t,f}^{N} = 0$ whenever a recompletion $i = cr + 1$ is performed, since the corresponding variable $z_{i,t,\hat{t},f}$ will automatically take value one. Finally, we determine the well's production in time period t for price forecast scenario f by Eq. (36).

$$P_{t,f} = \sum_{i>cr} \sum_{\hat{t}=1}^{t} Q_{i,t,\hat{t}} \cdot z_{i,t,\hat{t},f} + Q_t^N \cdot z_{t,f}^N \qquad \forall t \in T, f \in F \tag{36}$$

Similar to the well development planning model, the gas production in time period t depends on the "refracturing state" of the well at that time, which in general is determined by the variable $z_{i,t,\hat{t},f}$. This decision variable is multiplied by the parameter $Q_{i,t,\hat{t}}$, capturing the anticipated production of the well in time period t considering that it was last refractured for the *i*- th time in time period $\hat{t}$. However, in this refracturing planning model we introduce an additional term into this key constraint. If no further refracture treatment is scheduled during the first t time periods of the current planning horizon, and therefore $z_{i,t,\hat{t},f} = 0$, then then the production is given by the parameter Q_t^N. This parameter reflects the default production of the well without any restimulations. For the refracturing planning problem we also rely on a slightly modified objective function as seen in Eq. (37). The distinction between stage one, here-and-now and stage two, wait-and-see decisions is clearly highlighted.

$$\max ENPV = \underbrace{-RC \cdot x^{REFRAC}}_{\substack{\text{Stage 1 decision (here-and-now):} \\ \text{Refracture the well now: yes or no?}}} + \underbrace{\sum_{f \in F} \pi_f \sum_{t \in T} (1+d)^{-t} \cdot \left\{ P_{t,f} \cdot gp_{t,f} - rc_i \cdot \sum_{i>cr} x_{i,t,f} \right\}}_{\substack{\text{Stage 2 decisions (wait-and-see):} \\ \text{Restimulate the well later: yes or no? when? how often? (scenario-dependent)}}} \tag{37}$$

Altogether, Eqs. (19)-(37) complete the proposed model for addressing the refracturing planning problem.

Moving Horizon Framework for the Well Development and Refracturing Planning

For clarification purposes, we contrast moving and rolling horizon strategies since these expressions are sometimes used interchangeably by different authors. By moving horizon we mean that a fixed-length planning horizon is periodically moved forward in time, and the corresponding optimization problem is re-solved based on updated input data. We refer to Fig. 5 for a graphic illustration of the moving horizon concept. A rolling horizon approach, on the other hand, is often presented as a decomposition technique for planning and scheduling problems[18], in which the entire time horizon is divided into two blocks: a detailed time block and a subsequent aggregate time block.

In this work, we propose a moving horizon strategy as shown in Fig. 5 to address the integrated well development and refracturing planning problem. At each step of this moving horizon strategy, we solve one of the two stochastic programming models presented in the previous sections. The overall algorithm is illustrated in Fig. 6. Which of the two models is used depends on the current stage of the well development plan. At the beginning of the planning horizon (current time $ct = t_1$), the well is considered to be in a "ready-for-drilling" state. At this point in time, the well development planning problem is solved considering endogenous and exogenous uncertainties. The key here-and-now model decision is whether or not to drill the shale well (represented by the binary variable w^{DRILL}). If w^{DRILL} is zero, stating that the well is not to be drilled at the present time, then the planning horizon moves forward *fwd* periods (typically one year) and the model is solved again at $t = ct + fwd$, at which point revised uncertainty scenarios can be incorporated. If instead the decision is to drill the well ($w^{DRILL} = 1$), then the planning horizon also moves forward *fwd* periods, but the problem at hand changes conceptually. Now, the well has actually been drilled, fractured and completed. The drilling time (*dt*) is recorded and its current state (*cr*) changes to i_o. Thereafter, the first gas production peak is observed. Moreover, after some time of continuous production, we are able to determine which of the well performance scenarios $p \in P$ has actually realized; we say that the "production estimate scenario" is revealed. In the next step, given that the actual performance of the shale gas well is now known, and after updating the gas price forecast scenarios, the refracturing planning problem is solved (only considering exogenous price uncertainty).

The problem now enters the second phase: deciding on recourse actions. The new key here-and-now decision is whether or not to refracture the well (denoted by the binary x^{REFRAC}). Note that the well is currently producing, and a restimulation will temporarily reduce production flow to zero. If the model proposes not to refracture the well at this time, then the well continues to produce, the planning horizon is moved forward *fwd* periods, and the potential recourse action (a restimulation) is evaluated once again at $t = ct + fwd$, under revised price scenarios. If, however, the decision is made to refracture ($x^{REFRAC} = 1$) then the refracturing time (*lt*) is registered, and a new production peak is induced. The magnitude of the peak and the production decline that follows depend on the age and the performance of the well. We assume that both of them are known data for the recourse model. The refracture state of the well changes to $cr = i_1$, the planning horizon moves forward once again, and the refracturing planning model is re-solved to determine if a further restimulation would be economically attractive, considering continuously revised price forecast scenarios.

We note that the proposed algorithm compares to the work by Cui and Engell (2010)[19], who propose a moving horizon strategy based on a two-stage stochastic mixed-integer linear programming for multi-period, medium-term planning of a multiproduct batch plant considering uncertainty in terms of demand, plant capacity and product yields. However, in their work the authors do not consider endogenous uncertainty as part of the planning problem.

It is important to acknowledge some of the proposed framework's shortcomings, as well as some of its advantages. For instance, at every iterative step in the proposed algorithm in Fig. 6 the optimization assumes that upon the implementation of the here-and-now decisions, the considered price uncertainty realizes instantaneously over the entire remaining planning horizon. Practically speaking, this is obviously not the case. Hence, in the scheme in Fig. 6 we move the planning horizon up one time increment periodically and re-solve the problem in light of the uncertainty realization and under consideration of updated price forecasts (once again giving rise to a stochastic program). Yet, this sequential realization of uncertainty is not directly captured "a priori" at every step of the algorithm.

Other optimization frameworks, such as multi-stage stochastic programming, explicitly account for this sequence of here-and-now decisions, uncertainty realizations and recourse actions at specific, future points in time or stages[14]. Hence, multi-stage stochastic programming is clearly a more rigorous and accurate representation of the decision-making process over extended periods of time. At the same time, it is well-known that multi-stage stochastic programming leads to significantly larger models that quickly become computationally intractable. Therefore, we advocate the proposed strategy of embedding two-stage stochastic programs in moving horizon approaches as a practical optimization framework for problems involving exogenous and endogenous uncertainty to bridge the existing gap between deterministic programming and multi-stage stochastic programming.

Case Study: Well Development and Refracturing Planning under Uncertainty

In order to demonstrate how the proposed optimization framework can support upstream operators in deciding whether or not to drill and refracture shale wells, we present and discuss a comprehensive well development and refracturing planning case study. For this purpose, we assume that an operator has identified a prospective location to drill, fracture and complete a single shale gas well. The initial well development is assumed to cost $3,000,000. Every potential restimulation of the well can be performed for $700,000. The planning problem at hand is complicated by the fact that commodity prices are subject to significant fluctuations. For this reason, the operator wishes to consider a total of nine different, equally probable price development scenarios, all of which are defined by the operator's business development strategy. In considering all nine scenarios, the intention is to hedge against the risk of price uncertainty.

It is assumed that, once turned in-line, the prospective well could potentially be refractured up to five times over its expected lifespan of 20 years. However, we expect the operator's experience with shale well restimulations to be fairly limited – which is presently true for many oil and gas companies. As long as the prospective well has not actually been drilled yet, implying that access to subsurface geological data is very limited, it is difficult to anticipate the well's production. Hence, we explicitly consider well performance uncertainty in this case study to account for the possibility that the expected production performance of the well is under- or overestimated. In this case study a total of three different, equally probable production scenarios are considered ("low", "avg", "hgh"), determined by completions design engineers, geologists, or reservoir engineers based on production data of neighboring wells. However, it is assumed that once the well has been drilled and completed, the operator can refine the production forecast to the point where it becomes deterministic. Therefore, the case study at hand represents a well development and refracturing planning problem under exogenous price uncertainty and endogenous production uncertainty. Given a 10 year planning horizon, discretized by months, the operator wishes to maximize the expected net present value of the proposed project. In order to address the described problem, we rely on the proposed two-stage stochastic programming models and embed them in a moving horizon strategy based on annual re-evaluations. All input data used in this case study can be provided upon request.

Initially, in the first year of this case study, we assume that the market is operating in a low-to-moderate price environment as illustrated in Fig. 7. Natural gas is selling for \$ 3.3 /Mscf (\$ 0.1165 /m^3). Future price uncertainty is captured by the aforementioned nine price scenarios defining a "cone of uncertainty" based on positive, null and negative price trends with underlying cyclic fluctuations. At this point in time, the key question that the operator faces is whether or not to drill the prospective shale well. The proposed MILP model for the well development planning problem involves 19,441 binary variables, 131,059 continuous variables and a total of 413,317 constraints. Using CPLEX 24.7.3 on an Intel i7, 2.93 Ghz machine with 8 GB RAM, the problem solves in 59 seconds. The optimization converges to the "zero-solution" (NPV= \$0), indicating that it does not make economic sense to drill the prospective well at the present time despite the consideration of possible future refracture treatments. The assumed price environment does not allow for economic well development. As a result of this analysis, the operator would refrain from developing the prospective well at this time and pursue alternative, more promising investment opportunities.

In the spirit of the moving horizon strategy, we fast-forward into year 2 of the case study (*fwd* = 12 months in Fig. 6). The prospective well is once again considered for development. We assume that in the meantime the natural gas price has climbed to \$ 4.0 /Mscf (\$ 0.1413 /m^3) as shown in Fig. 7. The well development planning model is re-applied to the problem at hand. Although the problem size is identical, it now takes 269 seconds to solve the problem to zero relative optimality gap on the same machine as before. In light of the elevated price environment, the optimization concludes that it does make sense to drill, fracture and complete the prospective well here-and-now. The expected NPV for the well development project is $195,482. At this point in the case study, the decision-maker and the optimization are still unaware of the well's true production performance over time. Either of the three performance scenarios ("low", "avg", "hgh") could potentially realize. However, the optimization rigorously evaluates all three well performance scenarios (and all nine price performance scenarios), and proposes a well development strategy for each possible realization. Upon closer inspection the results reveal that refracturing of the well is proposed in nearly all scenarios, although the timing of the restimulations varies significantly. If the performance of the well is found to be average or high, the model suggests to refracture early into the well's lifespan – almost regardless of which price scenario realizes. If, however, the well's performance turns out to be low, then the restimulations tend to be scheduled later in life, and oftentimes selectively for elevated price development scenarios. The results also show that multiple recompletions of the well (up to five times) are proposed for some scenario combinations. In the context of this case study, we assume that the decision-maker indeed agrees to drill and complete the prospective well.

Once more we fast-forward a year; now into year 3 of the case study. The well is assumed to have actively produced for the past twelve months. In addition to early production readings, the operator has also gathered sufficient reservoir data to refine the well's performance forecast. For the purpose of this case study we therefore assume that the production performance can now be classified as "high", according to the respective scenario. This marks the realization of the endogenous well performance uncertainty. The following, detailed analysis is based on this particular realization. However, Table 1 summarizes here-and-now decisions based on the moving horizon strategy for all possible well performance realizations.

In year 3 of the case study, the natural gas price has decreased to \$ 3.2 /Mscf (\$ 0.1130 /m^3). Since the well is actively producing at this point, we now rely on the proposed refracturing planning model for decision-support. As outlined earlier, this model no longer considers well performance uncertainty, but it does account for price forecast uncertainty. For the problem at hand, this model involves 541 binary variables, 44,767 continuous variables and 45,308 constraints. Using the same machine and solver as before, it takes 44 seconds to solve the problem to optimality. Under the current conditions, the optimization proposes to refracture the well here-and-now for the first time. Moreover, the results suggest that given the set of considered price development scenarios, additional well restimulations will be justified over time as shown in Fig. 7. The expected NPV for the proposed refracturing strategy is $ 2,927,232. It should be noted that the expected NPV is significantly higher in year 3 than in year 2 due to the realization of the well production uncertainty according to the “high” performance scenario. As before, we assume that the operator chooses to implement the proposed here-and-now decision, which results in the well being restimulated in year 3.

By year 4 of the case study the natural gas price has decreased further, down to $ 2.1 /Mscf ($ 0.0742 /m^3) as shown in Fig. 8. Re-applying the refracturing planning model reveals that the well should not be refractured at the present time under these circumstances. In light of the depressed price environment, the expected NPV diminishes to $ 1,756,846. Yet, the solution indicates that future restimulations can improve the economics of the well development project for selected price development scenarios. Year 5 is characterized by an increase in gas price to $ 4.0 /Mscf ($ 0.1413 /m^3). Interestingly however, the optimization does not propose to refracture the well at this time despite the higher price environment. It appears that the increase in expected revenues after the well reinvigoration does not outweigh the restimulation costs at this time. The improved expected NPV of $ 2,829,758 does show though, that the well development project clearly benefits from the recent price increase.

One last time, we fast-forward into year 6 of the case study and assume that the natural gas price has spiked to $ 6.9 /Mscf ($ 0.2437 /m^3) as illustrated in Fig. 8. Under these circumstances the optimization proposes to refracture the well <u>a second time</u> here-and-now, and it recognizes scenario-dependent opportunities for additional restimulations. If implemented, the suggested restimulation strategy leads to an expected NPV of $ 4,759,961. Although the analysis could be continued for several more years, we conclude our case study at this point. By year 6 of this case study we find that refracturing increases gas recovery from 805 MMscf (22.8 10^6 m^3) without refracturing, to 1,243 MMscf (35.2 10^6 m^3) with two refractures, and the profitability of the well development project is improved from -$ 173,311 (without refracturing) to $ 1,366,314 (with refracturing) over the first six years. This clearly indicates the potential of well restimulations for unconventional wells. More importantly, the analysis demonstrates that the proposed optimization framework can be used effectively to address the well development and refracturing planning problem under exogenous price uncertainty and endogenous well performance uncertainty.

Discussion: General Recommendations for Refracturing Shale Wells

In this section we discuss general recommendations for refracturing shale wells motivated by the results of the case study presented in the previous section. First, if a recompletion is believed to be effective, then refracturing is promising early into the life of a shale well even when commodity prices are relatively low. The reasoning behind this is that early, effective well restimulations have a lasting impact on gas production over time. They alter the overall decline curve favorably, and thereby increase the expected ultimate recovery (EUR) significantly. Hence, economics greatly benefit from these early workovers even in low-price environments. In fact, King (2015)[8] also argues that refracturing within the first two years of production may provide significant economic benefits, especially during periods of downturns in oil and gas prices when drilling budgets are oftentimes reduced.

Secondly, as wells mature, refracture treatments should only be performed: a) in elevated price environments, or b) in direct response to projected price peaks. The reasoning here is as follows: the longer a shale well has already been producing, the less effective and impactful a recompletion is typically expected to be. This practical observation is also reflected in the previously introduced generalized production estimate function. What this implies is that, assuming the cost of refracturing a well remains the same, the potential “return on investment” of a well restimulation generally diminishes over time.

In order to substantiate the above claims, we analyze a particular solution of the previously presented case study in more detail. In year 2 of the case study the situation is as follows: given is a prospective shale well. The decision has not yet been made whether, at the present time, this well should be drilled or not. The decision-maker faces price uncertainty and well performance uncertainty. The respective optimization problem is solved and reveals that, at the present time, well development does make economic sense. At the same time, the optimization specifies scenario-dependent refracturing strategies for every scenario combination of price and performance uncertainty. Here we examine the solution for one particular price forecast scenario in detail. As Fig. 9 shows, the optimization proposes to drill the well here-and-now, despite the fact that its true production performance is uncertain; it could turn out to be either “low”, “average” or “high”. For each of these possibilities the optimization proposes a refracturing strategy that would maximize economics, given the particular price forecast scenario.

Interestingly, for the “average” and “high” performance scenarios, the optimization proposes to refracture the well just one year after turning it in-line, even though the price is expected to decrease significantly. However, if the well’s production performance turns out to be “low”, then it is suggested not to recomplete. This confirms the previously stated recommendation that early refracture treatments can be justified even in low-price environments.

The results in Fig. 9 also show a clear trend for late-life refracture treatments. As prices are forecasted to increase, the optimization proposes to exploit the projected price peak by scheduling multiple well restimulations. This trend holds true regardless of which well performance scenario realizes. It confirms that the timing of the late-life recompletions is very sensitive to the price environment at the time.

Discussion: Field-Wide Shale Development Planning Considering Refracturing Opportunities

The presented results raise the question how the proposed optimization framework could be used for field-wide shale development planning, rather than merely being applied to a single, prospective well. Given a set of prospective locations for developing new wells and a set of mature, actively producing wells, an upstream operator may have to decide how many new wells to drill, fracture and complete and/or whether existing wells should be restimulated instead. The proposed optimization framework can be embedded in field-wide development planning models such as those proposed by Drouven and Grossmann (2016)[20] or Cafaro and Grossmann (2015)[21]. Even though we do not address the field-wide development planning problem explicitly in this work, we attempt to discuss and evaluate refracturing opportunities within mature shale development areas qualitatively. The motivation for this discussion is that in mature development areas new wells and refracturing opportunities will compete against one another; especially in light of limited resources such as development capital, fracturing crews or drilling rigs.

Conceptually, refracturing provides operators with a number of promising opportunities. For instance, Drouven and Grossmann (2016)[20] find that the equipment utilization in shale gas gatherings systems is often poor due to the characteristically steep decline curves of unconventional wells. Operators tend to size pipelines and compressors such that they can handle the high initial production rates of shale wells. However, within months after these wells are turned in-line, production declines dramatically and operators are left with oversized and under-utilized pipelines and compressors. To offset volumes lost to decline and to maintain constant production, operators are forced to drill and complete new shale wells in quick succession[4]. As Fig. 10 illustrates, the impact of these development strategies on rural landscapes can be quite significant. Moreover, for every new well that is drilled an operator needs to install additional gathering equipment such as production units or well lines.

However, by reinvigorating existing wells through restimulations upstream operators can increase the utilization of gathering pipelines and compressor stations without constantly opening up new wells; in simple terms: refracturing can help operators keep their pipelines and compressors “full”. In this way, by drilling fewer new shale wells and “reusing” existing infrastructure, operators can decrease the well count in development areas, lay out fewer gathering pipelines, and thereby reduce the overall surface disruption. It is also worth mentioning that refracturing an existing well takes far less time than drilling and completing a new well. The process of developing a prospective shale well involves securing additional acreage, applying for permits, relocating and assembling a drilling rig, drilling the vertical and horizontal segments of the well, completing the well and installing production equipment as well as gathering pipelines. From start to finish the entire process may take several months to complete. Refracturing an existing well, on the other hand, can be done within weeks. Considering the recent, dramatic fluctuations in natural gas prices, refracturing could allow upstream operators to very quickly respond to projected price increases by ramping up field-wide production in a short period of time. From this perspective, refracturing conceptually compares to shut-in based production schemes such as those proposed by Knudsen and Foss (2013)[23] and Knudsen et al. (2014)[24].

Even from a water management perspective refracturing makes sense. It is well-known that hydraulic fracturing requires significant volumes of water of up to 20 million liters per well. However, over the lifespan of a shale well up to 50% of the injected water is eventually recovered at the surface again as flowback or produced water. The recovered water is generally contaminated and may not be released back into the environment unless it has undergone extensive (and therefore costly) treatment. Alternatively, operators have two options: a) dispose of the impaired water by injecting it into abandoned wells (which is strictly regulated, very expensive, and known to lead to undesirable seismic activity), or b) reuse the recovered water for future fracturing operations. Given these options, upstream operators have increasingly been reusing impaired water for hydraulic fracturing in an attempt to reduce disposal volumes and avoid costly treatment. For this purpose, however, the recovered water oftentimes needs to be transported from one well pad to another – depending on where the development activity is occurring. Transportation is usually performed with water hauling trucks, which leads to increased truck traffic, road deterioration, the potential for accidents, and added costs. These issues can be mitigated if upstream operators choose to restimulate more of their horizontal wells as part of field-wide development programs. Rather than transporting impaired water across and in between development areas, operators could temporarily store the recovered water on-site and re-use it to refracture other producing wells eventually. Finally, it is important to note that refracturing is significantly cheaper than drilling and completing new wells. This cost advantage can be of significant importance to smaller, capital-constrained upstream operators, who do not always have access to the financial markets and therefore fresh capital. Instead of being able to drill just one new shale well, refracturing may allow these companies to reinvigorate production at up to six of their assets. By embedding the proposed optimization framework in field-wide development models, these benefits could easily be quantified and may convince operators to increasingly exploit refracturing opportunities in mature development areas.

Conclusions

In this article, we have presented stochastic programming models for optimal shale well development and refracturing planning under exogenous price uncertainty and endogenous well performance uncertainty. The proposed optimization framework is intended to help upstream operators decide: a) if and when a prospective shale well should be drilled and fractured, and b) how often and when the well should be refractured. In our work, we accounted for uncertain price forecasts and uncertain well performance by proposing mixed-integer linear, two-stage stochastic programming models. The endogenous nature of the well performance uncertainty was addressed through a moving horizon strategy into which the proposed models were embedded. As part of a comprehensive case study, we demonstrated how the proposed optimization framework can be used to determine when to drill and/or refracture a shale well in light of price and performance uncertainty. The case study also revealed two interesting observations: a) even if commodity prices are low, it can make economic sense to refracture active shale wells early into their lifespan, and b) late-life refracture treatments only appear justified in elevated price environments or in direct response to projected price peaks. Finally, we concluded our analysis with a qualitative discussion on refracturing opportunities for field-wide shale development planning projects.

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Nomenclature

Sets

$i \in I$ Refracture treatments

$f \in F$ Price forecast scenarios

$p \in P$ Well performance scenarios

$t \in T$ Time periods

Binary variables

w^{DRILL} Active if the well is drilled here-and-now (stage one decision variable)

x^{REFRAC} Active if the well is refractured here-and-now (stage one decision variable)

$x_{i,t,f,p}$ Active if the well is refractured for the *i*-th time in time period t for price forecast scenario f and well performance scenario p

$y_{i,t,f,p}$ Active if in time period *t* the well has been refractured a total of *i* times for price forecast scenario f and well performance scenario p

$z_{i,t,\hat{t},f,p}$ Active if in time period *t* the well has been refractured a total of *i* times and the last refracturing occurred in time period $\hat{t}$ for price forecast scenario f and well performance scenario p

$z_{t,f}^{N}$ Active if in time period t the producing well has <u>not</u> been refractured for price forecast scenario f

Continuous variables

$P_{t,f,p}$ Gas production of the shale gas well in time period t for price forecast scenario f and well performance scenario p

Parameters

$a_{i,p}$ Production decline after i well restimulations for well performance scenario p

$b_{i,p}$ Post-refracturing decline after i well restimulations for well performance scenario p

$\beta_{i,p}$	Peak reduction factor after i well restimulations for well performance scenario p
cr	Current refracturing state of the shale well
ct	Current time period
d	Discount rate
dt	Drilling time period
DC	Drilling cost
CC	Completions cost
RC	Refracturing cost
φ_p	Realization probability for well performance scenario p
fwd	Number of periods the planning horizon moves forward at every iteration
$gp_{t,f}$	Gas price in time period t for price forecast scenario f
$\tilde{\gamma}_{i,p}$	Original fracture contribution after i restimulations for well performance scenario p
k_p	Initial production peak for well performance scenario p
lt	Last refracture time period
π_f	Realization probability for price forecast scenario f
$r_{i,\hat{t},p}$	Supplemental production peak after i well restimulations when the last one occurred in time period $\hat{t}$
rc_i	Cost of i-th well restimulation
rt	Duration of refracture treatment
$Q_{i,t,\hat{t},p}$	Shale gas well production in time period t given that it has been refractured i times, and the last restimulation was performed in time period $\hat{t} \le t$ for well performance scenario p

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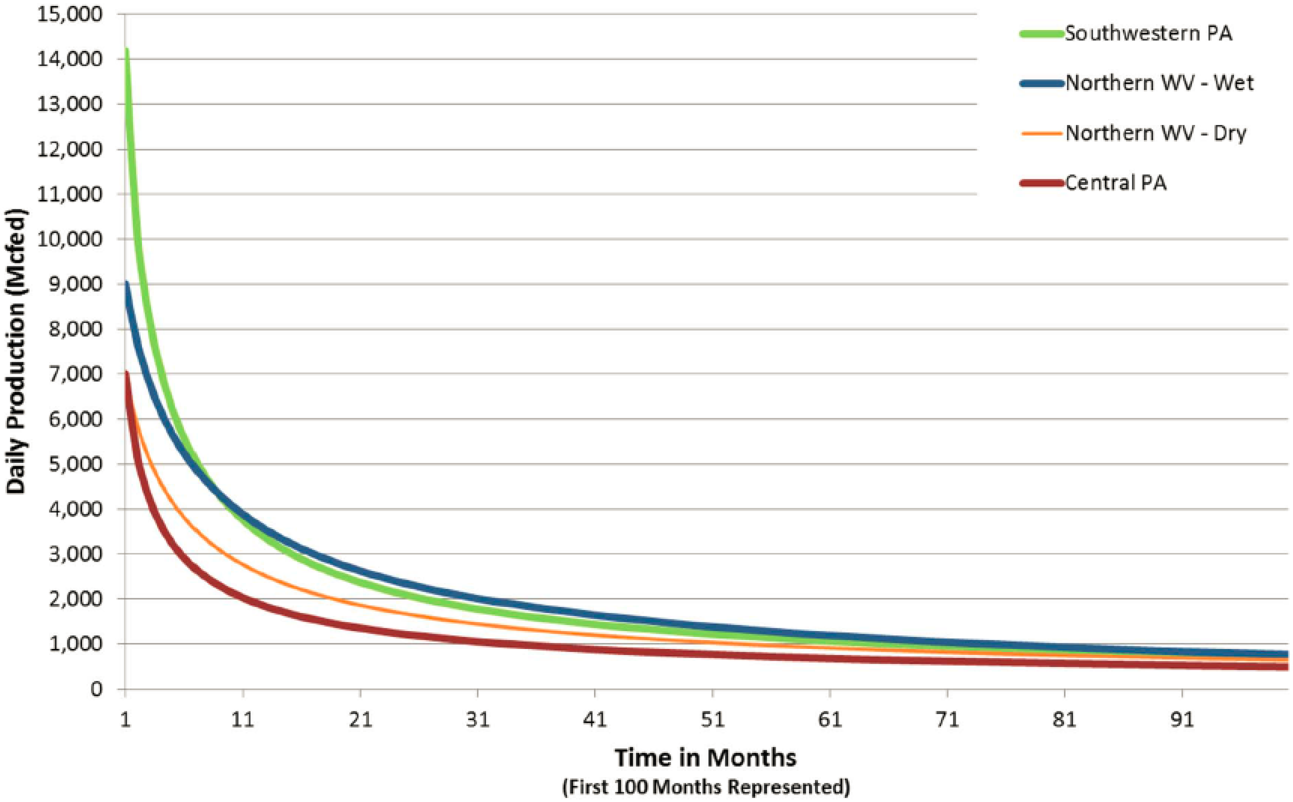

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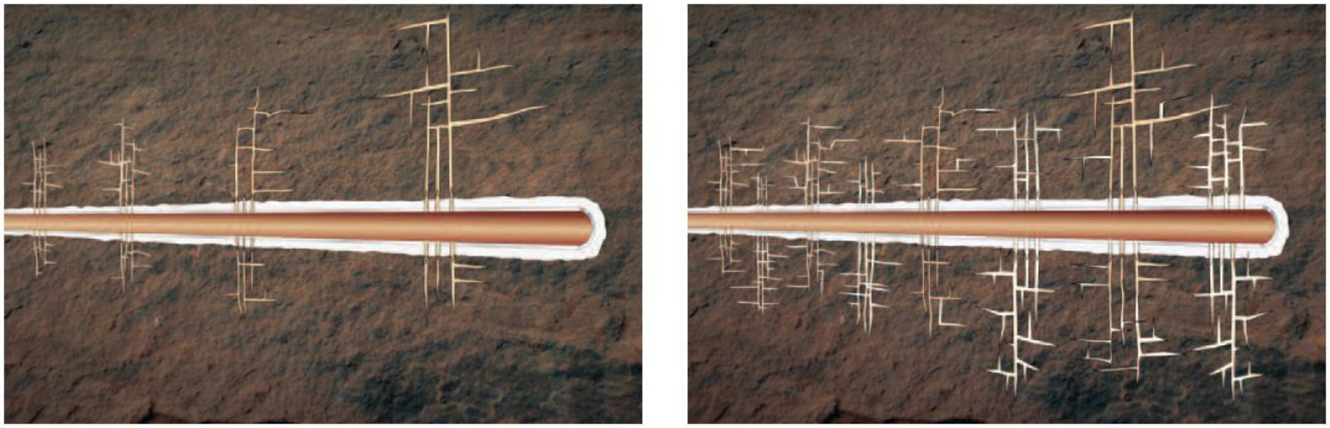

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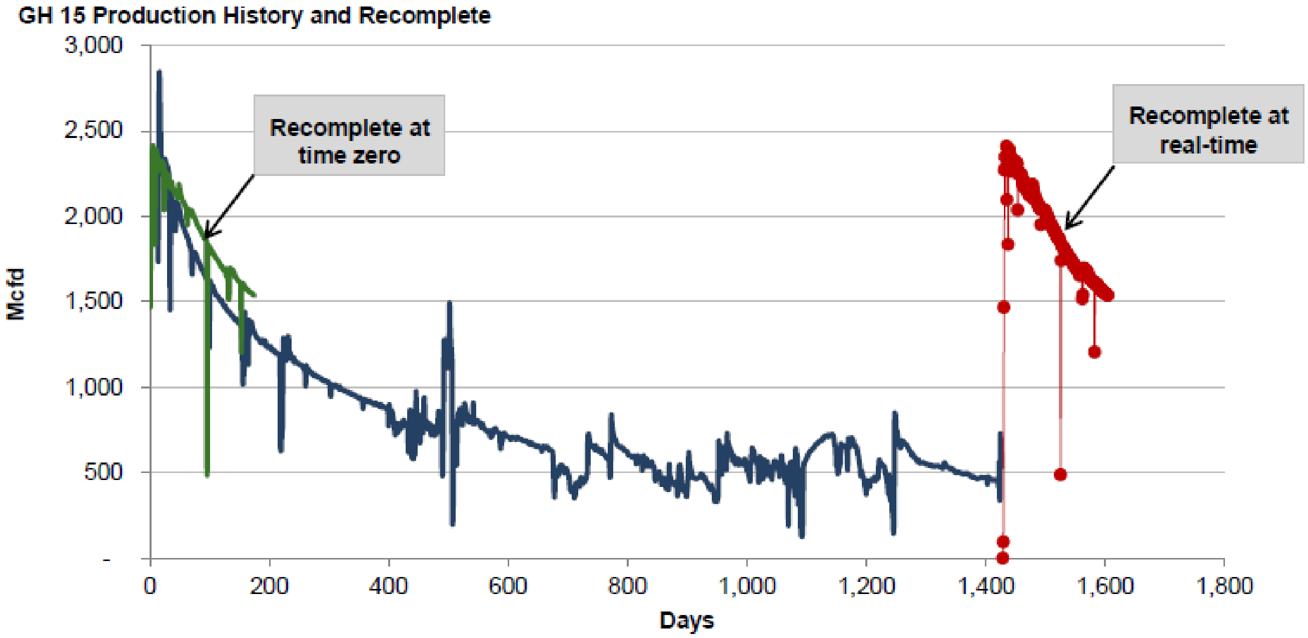

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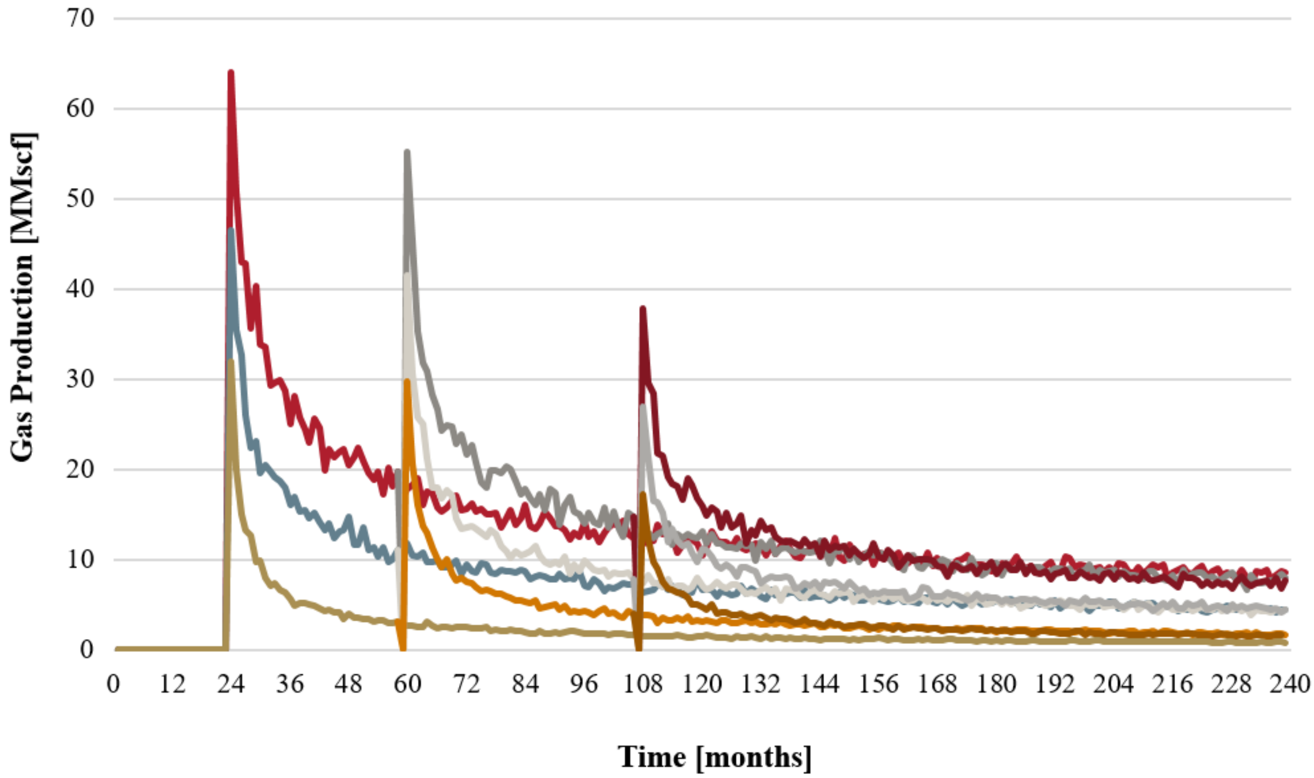

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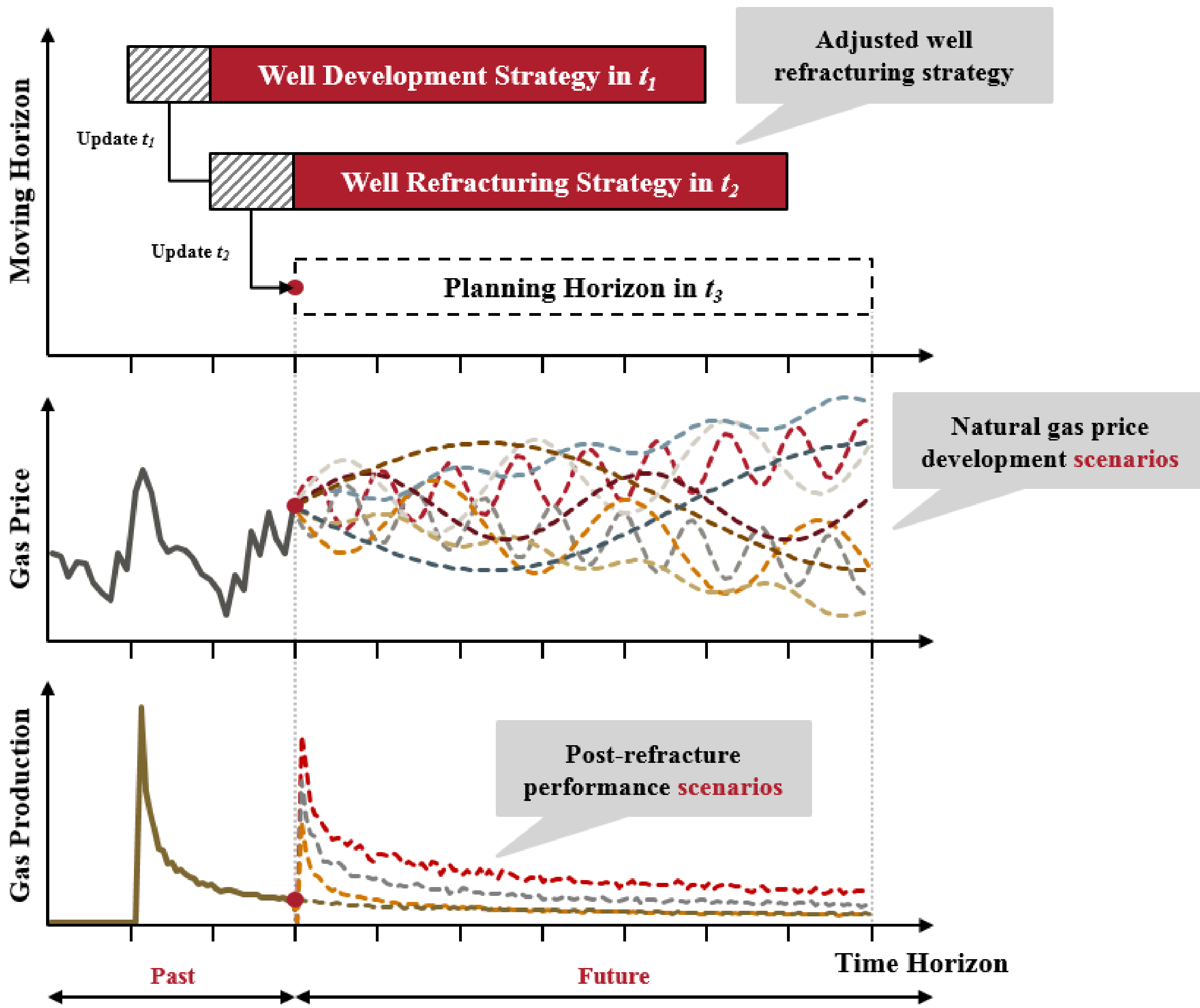

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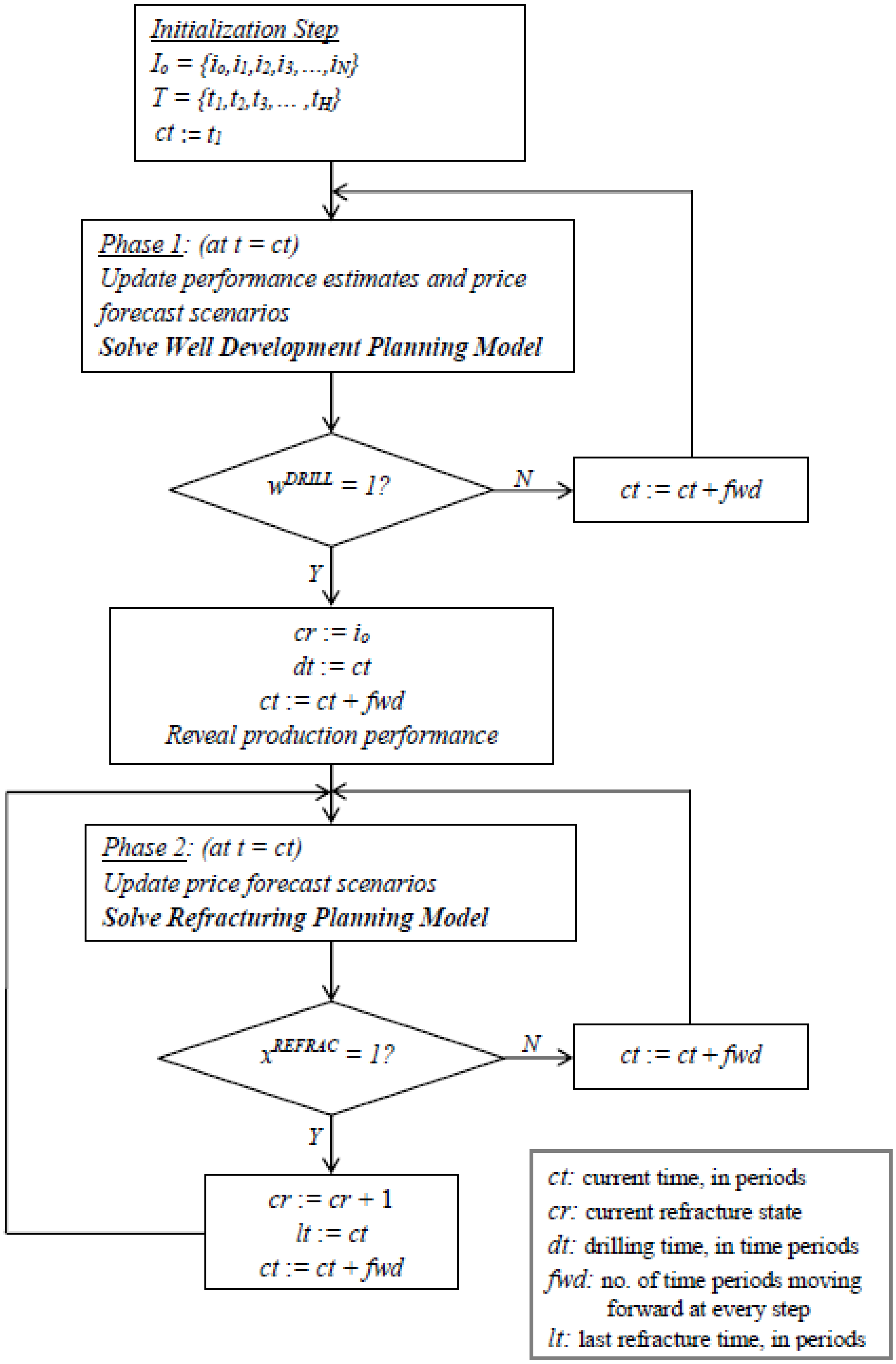

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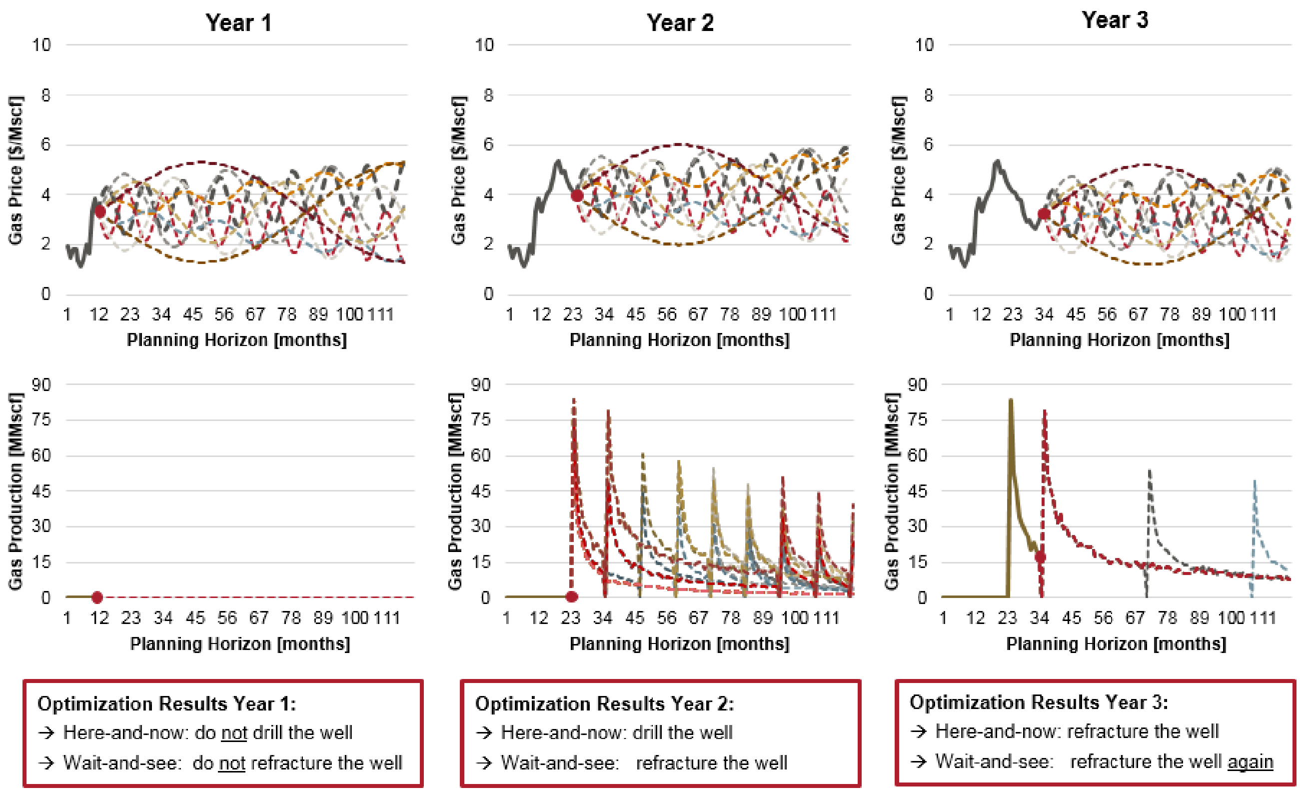

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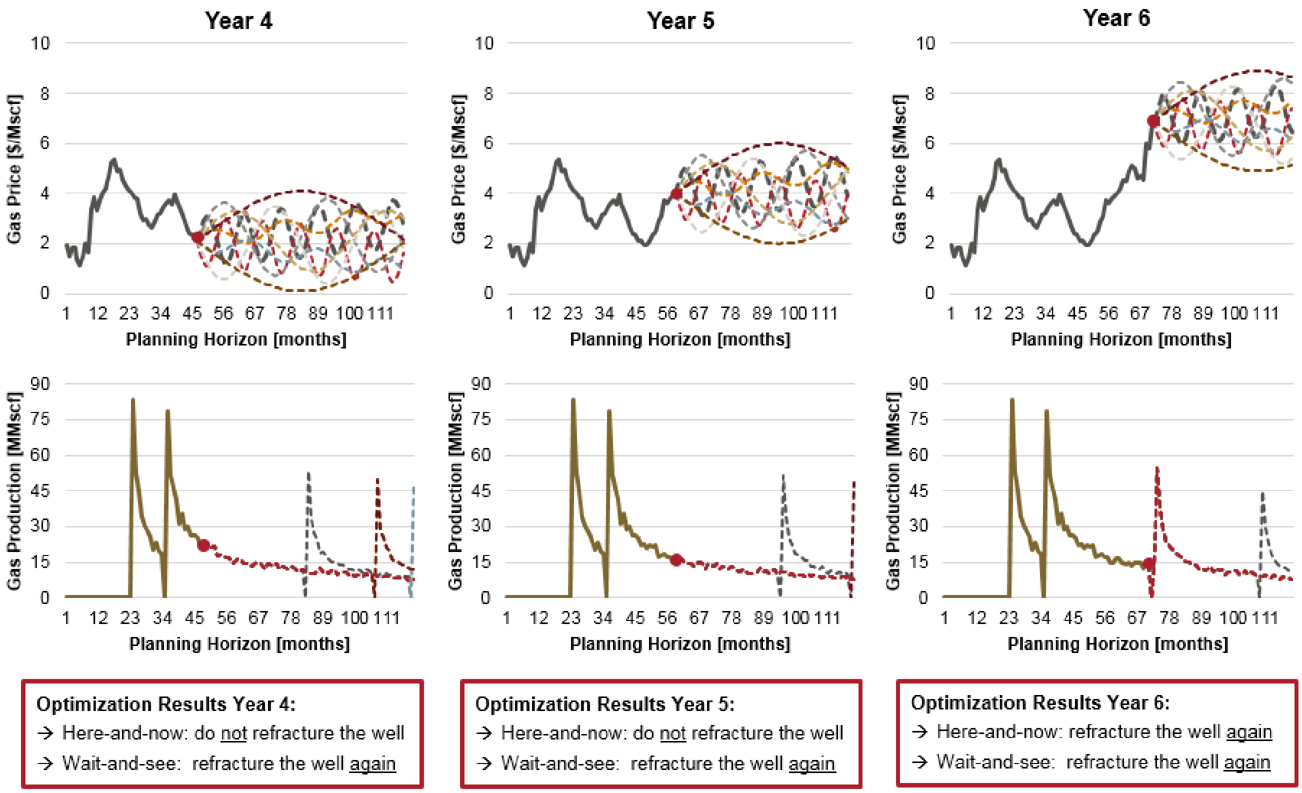

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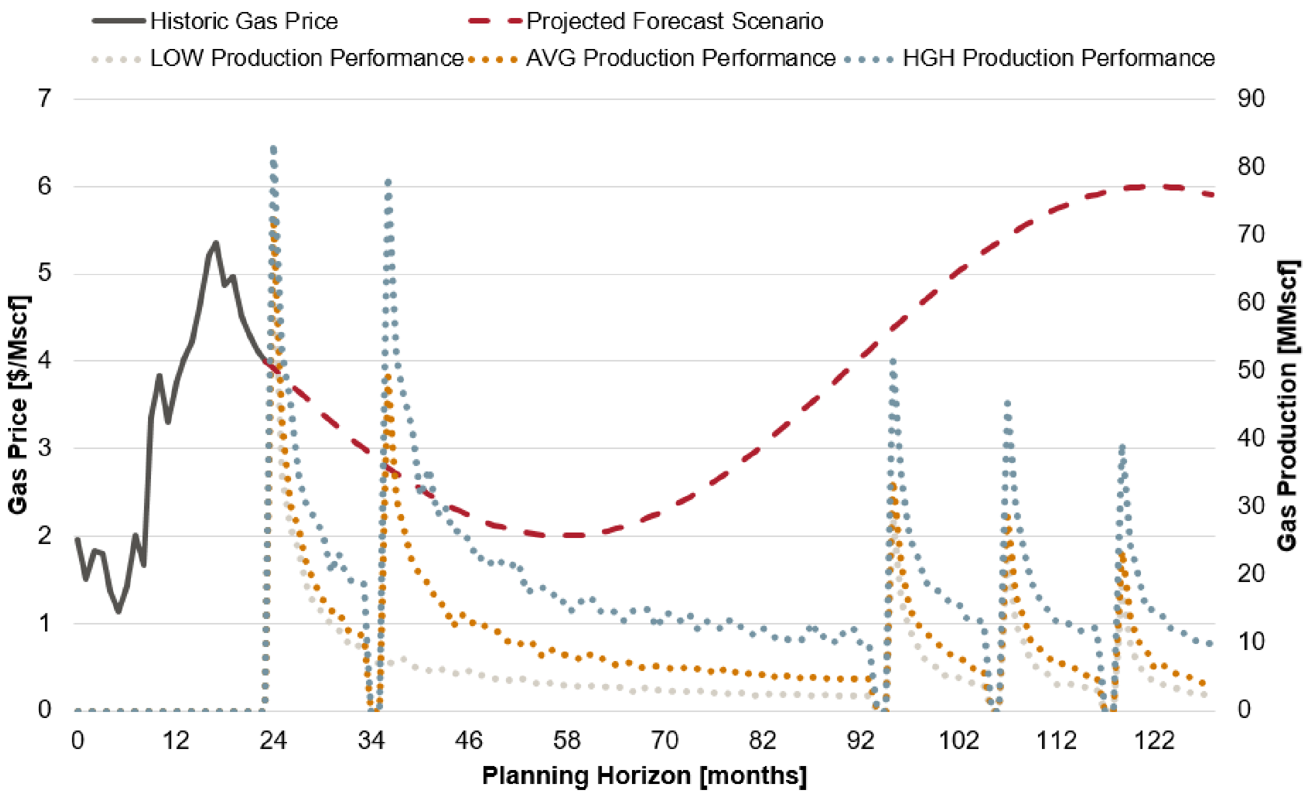

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Well performance realization	Year 3 (3.2 $/Mscf)	Year 4 (2.1 $/Mscf)	Year 5 (4.0 $/Mscf)	Year 6 (6.9 $/Mscf)
"LOW"	do not refracture	do not refracture	do not refracture	refracture (first time)
"AVG"	refracture (first time)	do not refracture	do not refracture	do not refracture
"HGH"	refracture (first time)	do not refracture	do not refracture	refracture (second time)

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