Mixed-Integer Programming Models for Line Pressure Optimization in Shale Gas Gathering Systems

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In this work we propose a mixed-integer nonlinear programming model to address the line pressure optimization problem for shale gas gathering systems. This model is designed to determine: a) the optimal timing for turning prospective wells in-line, b) the optimal pressure profile within a gathering network, and c) the necessary compression power for delivering produced gas to long-distance transmission lines. We rely on a pressure-normalized decline curve model to quantify how line pressure variations impact the gas production of individual wells. The reservoir model itself is incorporated in a transmission optimization framework which rigorously evaluates pressure drops along pipeline segments. Moreover, we explicitly consider compression requirements to lift line pressure from gas gathering levels to setpoints dictated by transmission pipeline companies. Since the resulting optimization models are large-scale, nonlinear and nonconvex, we propose a solution procedure based on an efficient initialization strategy. Finally, we present a detailed case study, and show that the proposed optimization framework can be used effectively to manage line pressures in shale gas gathering systems by properly scheduling when, and how many, new wells are brought online.

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Introduction

Fig. 1 shows a simplified illustration of a typical shale gas gathering system. The gathering network is characterized by a few key elements, namely, existing and prospective well pads, existing and prospective gathering pipelines of varying sizes, one or several compressors and a coupling link to a large-diameter, long-distance transmission pipeline. On every existent pad one or more shale wells actively produce natural gas, which is then fed into gathering lines at varying rates over time. The shale gas gathering system itself is typically operated at relatively low pressures, ranging from 50 to 200 psi. Low line pressures allow producers to extract the most gas from their shale wells, since they create a large differential between the reservoir pressure and the wellhead pressure. In other words, as the line pressure in a gathering system increases, overall production typically decreases (Lee & Wattenberger 1996, Boyan & Ghalambor 2014). Since the reverse statement is true as well, upstream producers generally prefer to operate their gathering systems at the lowest possible line pressure.

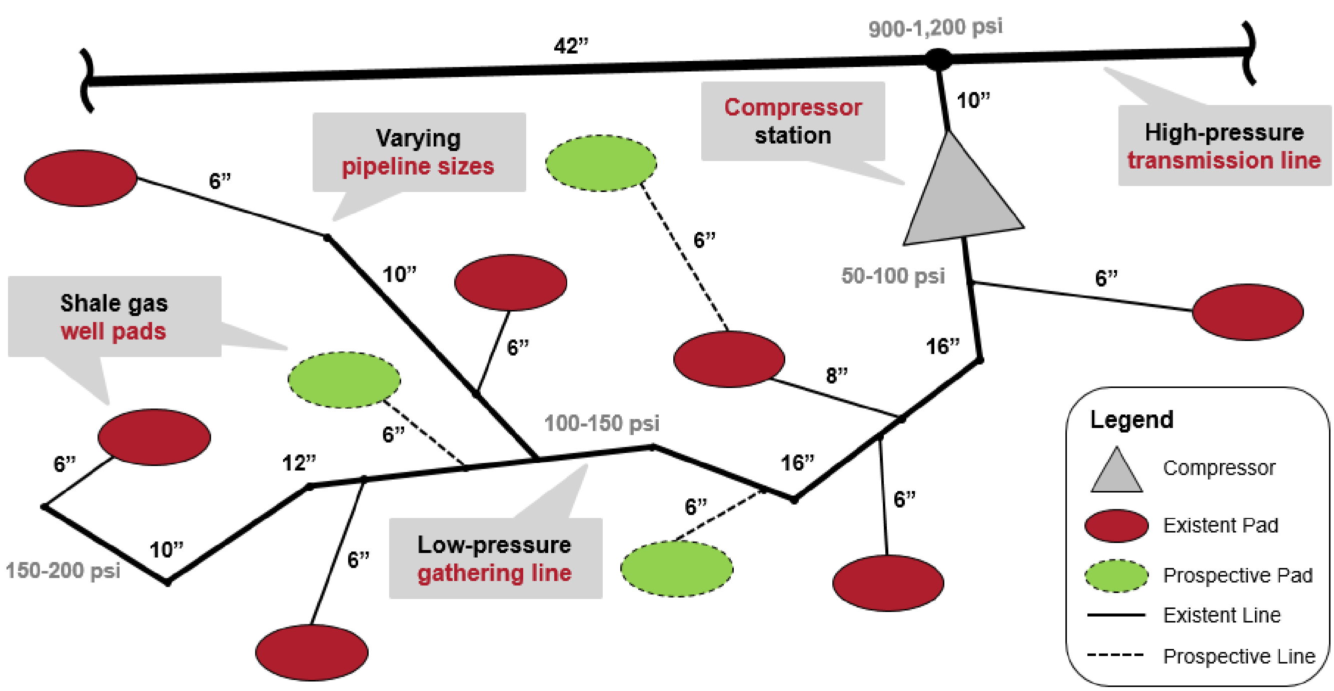

Fig. 1: Illustration of a typical shale gas gathering system with key network elements

Eventually the produced gas needs to be delivered to a transmission pipeline that will move the gas to major demand hubs. These transmission lines are operated by midstream companies at very high pressures between 900-1,200 psi in order to transport large quantities of natural gas over long distances. Therefore, it is up to the shale gas producer to overcome the pressure differential between the low-pressure gathering system and the high-pressure transmission line. This is typically accomplished through one or several compressors. Compressor stations allow upstream operators to produce the gas at low pressures, on the one hand, but still meet the transmission line's pressure delivery requirements on the other hand. Due to the significant pressure differential that needs to be overcome by the compressor, and the considerable volumes of gas that are processed, compression expenses can be a major cost factor in the operation of shale gas gathering systems. Consequently, upstream producers struggle to balance two conflicting objectives: a) operating their gathering systems at low pressures and thereby increasing gas production, and b) raising line pressures so as to minimize compression expenses.

In addition to this inherent line pressure management challenge, shale gas producing companies also struggle with the so-called *backoff effect*. Fig. 2 shows the historic gas production rate of a shale well in the Appalachian Basin over time, along with its wellhead pressure. The data shows that sudden line pressure variations have a direct and pronounced impact on the shale well's production. For instance, in mid-February 2014 the respective gathering system experienced an abrupt pressure increase (possibly due to an unscheduled compressor shut-down). The well's response is almost instantaneous, and it is characterized by a striking production backoff. The other way around, when the wellhead pressure dropped in mid-June 2014, this particular well saw a clear and immediate increase in production. The pressure spike is almost mirrored by the production rate.

It should also be pointed out that the pressure and production data in Fig. 2 shows dynamic effects and measurement variances. The purpose of this work is not to capture these dynamic effects that occur on a daily basis, or to account for measurement errors. Instead, we focus on the dominant backoff effects which are directly tied to strategic development decisions, such as deliberate line pressure variations or bringing new wells online. The problems we wish to address are characterized by extended planning horizons (3 to 12 months) in which a weekly discretization of time is assumed to be sufficient. Hence, we believe that the use of steady-state models with semi-dynamic extensions is applicable and justified.

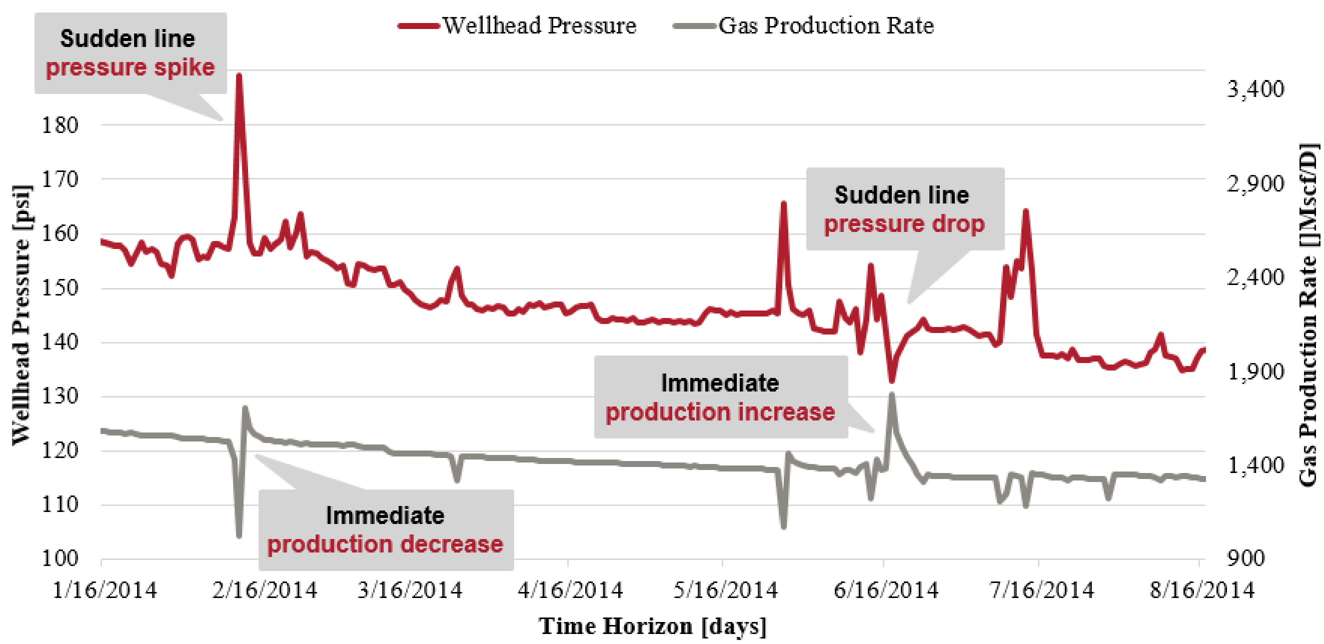

Fig. 2: Historic wellhead pressure and gas production rate of a single shale well over time

The backoff-effect itself is particularly prominent when new shale wells are turned in-line. Due to the characteristically high initial production rates of shale gas wells, gathering systems experience pronounced line pressure increases. These sudden pressure spikes result in immediate production cutbacks by all wells, but especially existing wells experience serious recovery declines. To demonstrate this effect, we turn to Fig. 3. This figure shows the overall gas production rate and average line pressure for a gas gathering system over time. The red dotted line shows the forecasted production of all existent wells – assuming that no new wells are opened up. Clearly, these wells are expected to experience a gradual decline with age.

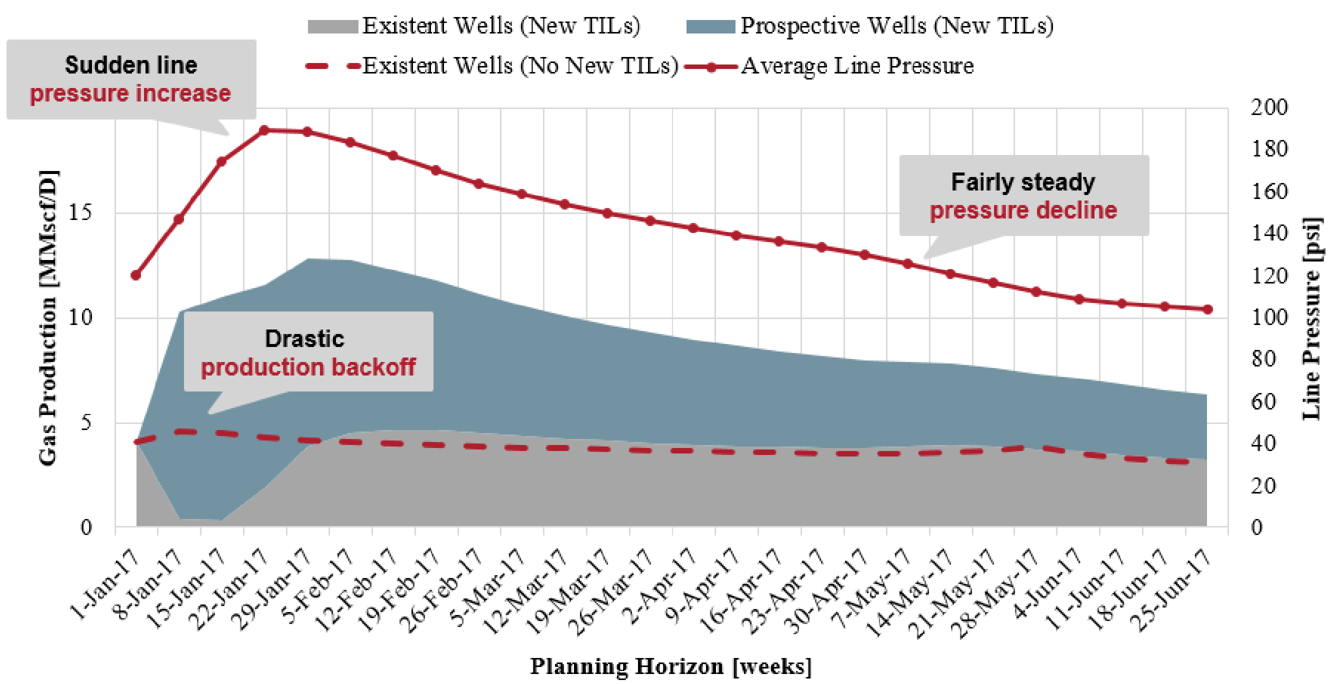

Fig. 3: Demonstration of the "backoff effect" in a shale gas gathering system when too many prospective wells are turned in-line at the same time

For demonstration purposes let us assume that an upstream operator decides to turn seven prospective shale wells in-line on January 1, 2017. For this scenario the stacked charts in grey and blue show the expected production of the existent and prospective wells respectively. In addition, the solid red line indicates how this development decision would affect the average line pressure in the gathering system. As expected, the line pressure increases by nearly 70 psi once the new wells are brought online. However, this sudden pressure spike results in an almost immediate, drastic production backoff by the existing wells collectively. These wells nearly cease to produce at all. What this implies is that a majority of the incremental production volumes added by turning new wells in-line is "lost" to making up for the reduced production of the existent wells. In other words, the system does not exhibit the desired production increase that one would expect from bringing seven additional wells online. Shale gas producers regularly struggle with this phenomenon since it seriously complicates development decisions regarding the right timing for turning wells in-line. For this precise reason the "backoff effect" lies at the heart of the line pressure optimization problem in shale gas gathering systems.

General Problem Statement

In this work we present a multiperiod mixed-integer nonlinear programming model to address the line pressure optimization problem in shale gas gathering systems. The problem at hand can be stated as follows. Within an active development area, an upstream operator is actively producing natural gas from a set of existing shale wells into an existing gas gathering system. This pipeline system delivers the produced gas to a compressor station which feeds into a long-distance, wide-diameter, high-pressure transmission line. Within the foreseeable future the producer wishes to open up additional prospective wells to maximize the utilization of the available gas gathering capacity.

Our work is concerned with: a) determining the optimal schedule to turn prospective wells in-line, also referred to as the "turn-in-line (TIL) schedule", b) identifying the optimal pressure profile within the gas gathering network, and c) calculating the required compression power to deliver the gas into the interstate transmission network. The problem is complicated by the fact that as new wells are brought online, the production of previously producing wells is negatively affected. In other words, the increase in line pressure due to additional gas production curtails gas recovery from mature wells. This effect is particularly prominent due to the characteristically steep decline curves of new shale gas wells. Hence, the objective of this work is to determine the optimal "TIL" schedule, line pressure profile and compressor operation such that the net present value of the field development project is maximized.

Literature Review

Over the years many researchers have proposed mathematical programming models for pressure optimization in natural gas transmission systems. Recently, Ríos-Mercado and Borraz-Sánchez (2015) published a comprehensive review of previous efforts in this domain to-date. The authors distinguish between three different topics in this general research area: a) line-packing problems focused on short-term natural gas storage in pipelines (Carter & Rachford 2003, Krishnaswami et al. 2004, Zavala 2014), b) pressure drop models capturing pressure losses due to frictional resistance along pipeline segments (Duran & Grossmann 1986, De Wolf & Smeers 2000, Martin et al. 2006), and c) fuel cost minimization problems that focus on compressor station modeling (Wu et al. 2000, Ríos-Mercado et al. 2006, Misra et al. 2015). All three topics have been addressed extensively – both individually and collectively. We only highlight selected papers focused on the use of mathematical programming techniques. For a comprehensive summary of related publications we refer to the work by Ríos-Mercado and Borraz-Sánchez (2015).

To the best of our knowledge, we are the first to address the rigorous line pressure optimization problem in the context of shale gas development. Previous optimization frameworks addressing the shale gas development problem do not rigorously capture pressure variations within gas gathering networks (Cafaro & Grossmann 2014, Guerra et al. 2016, Drouven & Grossmann 2016). In this work we explicitly consider three important aspects of the development problem: a) the incorporation of reduced-order, nonlinear shale well reservoir models, b) the rigorous consideration of pressure drops along gas gathering pipelines based on nonlinear and nonconvex gas flow equations, and c) the inclusion of nonlinear and nonconvex compressor models to determine necessary compression power. Specifically, in this work we rely on a pressure normalized decline model proposed by Anderson et al. (2012) to capture and quantify how line pressure variations affect individual well production rates. Anderson et al. (2012) observe that shale wells display a harmonic decline of pressure normalized production rate over time. In fact, the authors demonstrate that a linear relationship between pressure normalized production rate and cumulative gas production can be established. Fig 4. shows this linear relationship for selected wells from the Haynesville play. For more details regarding the proposed reservoir function we refer to section Model Description.

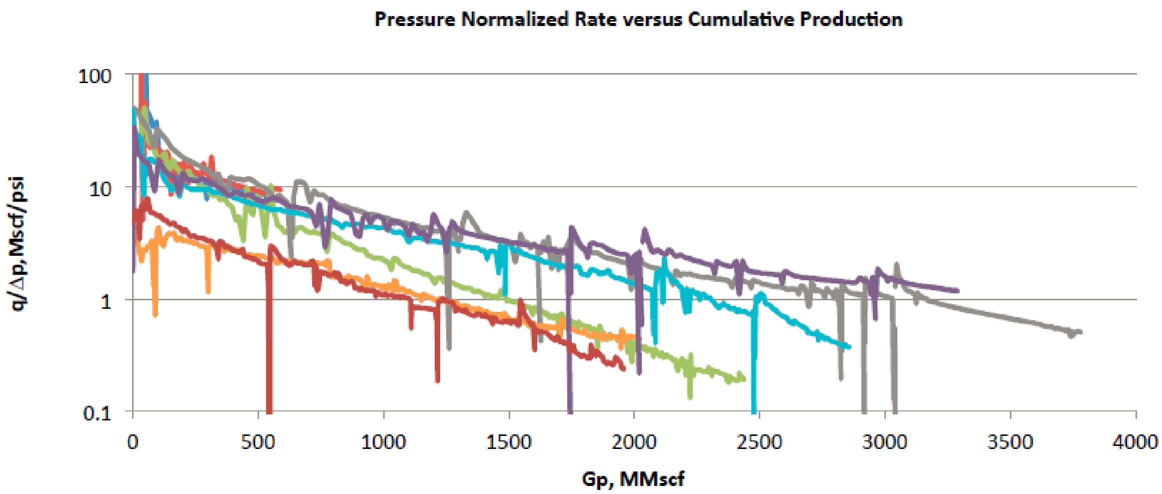

Fig. 4: Pressure normalized production rate over cumulative gas production for selected shale wells (Source: Anderson et al. 2012)

We also note that to date several alternative shale well reservoir models have been proposed. In particular, we highlight those proposed by Knudsen & Foss (2013), (2015) who derive reduced-order shale well and reservoir proxy models using first-principles physics of the subsurface storage and transport mechanisms. These models are particularly suitable for capturing rapid reservoir dynamics, such as those occurring during shut-in operations. Empirically derived reservoir models, on the other hand, generally assume steady-state operations and therefore these models are not suitable for the explicit consideration of short-term line pressure manipulations.

Model Assumptions

- The planning horizon is discretized by weeks. This discretization is motivated by the fact that it typically takes one week to turn a prospective well in-line. Moreover, by splitting the planning horizon into weekly time intervals we can track the dynamics of the gas gathering system (in terms of pressure variations and compression requirements) with sufficient accuracy.
- The reservoir model proposed by Anderson et al. (2012) captures the production of shale gas wells based on their cumulative production and the respective wellhead pressure.

- The focus of this work is on turning prospective wells in-line. Therefore, we assume that either, a) all prospective wells have already been drilled and completed, or b) any outstanding drilling and completions operations can be scheduled according to the preferred TIL schedule.

Model Description

In this section we describe the proposed mixed-integer nonlinear programming model to address the line pressure optimization problem in shale gas gathering systems.

Reservoir Model Constraints

The reservoir model proposed by Anderson et al. (2012) is one of the fundamental building blocks of the line pressure optimization model we present. Its purpose is to tie the default gas production rate of any shale well $F_{w,t}^{W,S}$ to its wellhead pressure $P_{p,t}^{W}$ and cumulative production $Q_{w,t}^{S}$. As outlined by Anderson et al. (2012), this can be accomplished by considering four parameters: a) the initial bottomhole pressure of the well $p_{w,t}^{0}$, b) the cumulative production of the well at the beginning of the planning horizon $q_{w}^{0,S}$, c) a slope parameter $m_{w,p}$, and d) an intercept parameter $a_{w,p}$. The latter two parameters need to be fit to historic production data for existent wells and estimated for prospective wells based on forecasted type curves. By including this reservoir model in our framework, we can quantify how pressure variations will affect gas production rates.

Eq. (1) represents the default reservoir model for existing wells. It turns out that this model is nonlinear and nonconvex within the domain of interest. It is also important to note that the default reservoir model inherently assumes that any line pressure variations occur gradually, over extended periods of time. As we established earlier, shale gas gatherings systems oftentimes experience abrupt and pronounced line pressure changes. These are not captured adequately by the default reservoir model.

$$\log\left(\frac{F_{w,t}^{W,S}}{p_{w,t}^{0}-P_{p,t}^{W}}+\epsilon\right)\leq m_{w,p}\cdot\left(q_{w}^{0,S}+Q_{w,t}^{S}\right)+a_{w,p}\qquad\forall w\in\mathcal{EW},(w,p)\in\mathcal{WPA},t\in\mathcal{T}\qquad(1)$$

Therefore, we propose a "backoff extension" to the default reservoir model. The extension in Eq. (2) considers wellhead pressures at pads in consecutive time periods along with a well-specific backoff parameter Δ_w^B in order to predict the backoff $F_{w,t}^{B,S}$. If the wellhead pressure increases from one time period to another, then this backoff extension in Eq. (2) will result in a production backoff "penalty", i.e., the well's production is less than forecasted by the default reservoir model. The extension also holds in the reverse direction, i.e., when the wellhead pressure drops, then the production will increase accordingly. This backoff parameter itself can be fit to historic data for existing wells but for prospective wells it has to be estimated.

$$F_{w,t}^{B,S} = -\left(P_{p,t-1}^W - P_{p,t}^W\right) \cdot \Delta_w^B \qquad \forall w \in \mathcal{EW}, (w,p) \in \mathcal{WPA}, t \geq 1 \tag{2}$$

Ultimately, the default reservoir model and the backoff extension are combined for all wells (existent and prospective) in Eq. (3) to capture the shale well's final production $F_{w,t}^{F,S}$. We note that typical gas production units (GPUs) are rate-restricted. Hence, we impose Eq. (4) using the maximum production rate parameter γ_w to ensure that the well cannot produce at infinitely high rates early in its lifespan.

$$F_{w,t}^{F,S} = F_{w,t}^{W,S} - F_{w,t}^{B,S} \qquad \forall w \in \mathcal{W}, t \in \mathcal{T} \tag{3}$$

$$F_{w,t}^{F,S} \leq \gamma_w \qquad \forall w \in \mathcal{W}, t \in \mathcal{T} \tag{4}$$

Although the modified reservoir model transfers over directly from existing to prospective wells, we examine the model formulation of the latter in more detail. We rely on a disjunctive programming formulation for prospective wells to link the corresponding reservoir model to the turn in-line decision. For this reason we introduce the Boolean variable $Y_{w,t}^{PROD}$ which is true if the prospective well $w \in \mathcal{PW}$ on pad $p \in \mathcal{P}$ is actively producing in time period $t \in \mathcal{T}$. This Boolean variable is involved in the disjunction in Eq. (5) to model the fact that as long as the well is not actively producing, its default production $F_{w,t}^{W,S}$ is less or equal than zero and the production backoff $F_{w,t}^{B,S}$ is greater or equal than zero.

Due to the formulation of Eq. (3) the optimization will generally aim for "negative" backoff (resulting in a production increase), hence this inequality ensures that the inactivity of a prospective well does not lead to any actual production.

$$\begin{bmatrix} Y_{w,t}^{PROD} \\ \log\left(\dfrac{F_{w,t}^{W,S}}{p_{w,t}^{0}-P_{p,t}^{W}}+\epsilon\right) \le m_{w,p}\cdot\left(q_{w}^{0,S}+Q_{w,t}^{S}\right)+a_{w,p} \\ F_{w,t}^{B,S}=-\left(P_{p,t-1}^{W}-P_{p,t}^{W}\right)\cdot\Delta_{w}^{B} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{w,t}^{PROD} \\ F_{w,t}^{W,S}\le 0 \\ F_{w,t}^{B,S}\ge 0 \end{bmatrix} \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\in\mathcal{T} \quad (5)$$

On the other hand, if the Boolean variable $Y_{w,t}^{PROD}$ is true, the well is active, and its production will be governed by the default reservoir model and the backoff extension as outlined previously. We should also note that the initial cumulative production of prospective wells $q_{w}^{0,S}$ in Eq. (5) is typically zero.

The disjunction in Eq. (5) is transformed into the mixed-integer constraints Eq. (6)-(10) using the big-M reformulation (Grossmann & Trespalacios 2013). For this purpose we introduce a corresponding binary variable $y_{w,t}^{Prod}$ for the Boolean variable $Y_{w,t}^{PROD}$. We note that the big-M parameters $M_1,\ldots,M_5$ need to be defined independently.

$$\log\left(\frac{F_{w,t}^{W,S}}{p_{w,t}^{0}-P_{p,t}^{W}}+\epsilon\right)\le m_{w,p}\cdot\left(q_{w}^{0,S}+Q_{w,t}^{S}\right)+a_{w,p}+M_1\cdot\left(1-y_{w,t}^{PROD}\right) \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\in\mathcal{T} \quad (6)$$

$$F_{w,t}^{B,S}\le-\left(P_{p,t-1}^{W}-P_{p,t}^{W}\right)\cdot\Delta_{w}^{B}+M_2\cdot\left(1-y_{w,t}^{PROD}\right) \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\ge 1 \quad (7)$$

$$F_{w,t}^{B,S}\ge-\left(P_{p,t-1}^{W}-P_{p,t}^{W}\right)\cdot\Delta_{w}^{B}-M_3\cdot\left(1-y_{w,t}^{PROD}\right) \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\ge 1 \quad (8)$$

$$F_{w,t}^{W,S}\le M_4\cdot y_{w,t}^{PROD} \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\in\mathcal{T} \quad (9)$$

$$F_{w,t}^{B,S}\ge -M_5\cdot y_{w,t}^{PROD} \qquad \forall w\in\mathcal{PW},(w,p)\in\mathcal{WPA},t\ge 1 \quad (10)$$

At this point, we establish a direct link between the decision to bring a well online and its "active production status". In order to accomplish this we introduce an additional binary variable $y_{w,t}^{TIL}$ which marks the time period in which the well is turned in-line ("TIL" for short). Using this binary variable we first impose Eq. (11) which ensures that any prospective well may be opened at most once over the planning horizon. We note that the formulation explicitly allows for rejecting the development of a candidate well.

$$\sum_{t \in \mathcal{T}} y_{w,t}^{TIL} \leq 1 \qquad \forall w \in \mathcal{PW} \tag{11}$$

Next, we add Eqs. (12) and (13) to the model formulation. These inequalities impose additional constraints on the timing of TIL operations. Eq. (12) limits the number of wells that can be brought online simultaneously to $n^{TIL,max}$. Eq. (13) on the other hand restricts in which time periods prospective wells may be turned in-line through the so-called land-cleared parameter $lc_{w,t}$.

$$\sum_{w \in \mathcal{PW}} y_{w,t}^{TIL} \leq n^{TIL,max} \qquad \forall t \in \mathcal{T} \tag{12}$$

$$\sum_{t \in \mathcal{T}} y_{w,t}^{TIL} \leq lc_{w,t} \qquad \forall t \in \mathcal{T}, w \in \mathcal{PW} \tag{13}$$

Finally, the logic constraint Eq. (14) states that any prospective well that has been turned in-line in one of the previous time periods has to be actively producing. In other words, unless the TIL operation has already occurred, a prospective well cannot be producing.

$$\sum_{\tilde{t}=1}^{t} y_{w,\tilde{t}}^{TIL} = y_{w,t}^{PROD} \qquad \forall w \in \mathcal{PW}, t \in \mathcal{T} \tag{14}$$

Pressure Drop Constraints

Next, we focus on how to capture pressure drops along gathering pipeline segments due to frictional resistance. In this work we rely on the Weymouth Equation to link gas flowrates with up- and downstream pressures (Weymouth 1912). This equation is commonly used to estimate pressure drops in small-diameter, short-distance gathering pipelines. The Weymouth Equation considers the diameter of the pipeline segment d, its length l, the specific gravity of the gas S^g, its compressibility Z and the inlet temperature T^L. We apply the Weymouth equation to all network arcs within the gathering system, which includes: a) Eq. (15) for pipelines connecting well pads and network nodes $(p,n) \in \mathcal{PNA}$, b) Eq. (16) for segments connecting network nodes leading up to the compressor station $(n,\tilde{n}) \in \mathcal{NNA}$, and c) Eq. (17) for delivery pipelines connecting the compressor station and the transmission line $(n,\tilde{n}) \in \mathcal{DNA}$. In terms of pressures we explicitly distinguish between wellhead pressures $P_{p,t}^L$ at the pads and line pressures $P_{n,t}^L$ at the network nodes. It should be noted that all inequality constraints below are nonlinear and nonconvex.

$$F_{p,n,t}^{PN,S} \leq 1.1 \cdot \left(d_{p,n}\right)^{2.67} \cdot \left[\frac{\left(P_{p,t}^P\right)^2 - \left(P_{n,t}^L\right)^2}{l_{p,n} \cdot S^g \cdot Z \cdot T^L}\right]^{1/2} \qquad \forall (p,n) \in \mathcal{PNA}, t \in \mathcal{T} \tag{15}$$

$$F_{n,\tilde{n},t}^{NN,S} \leq 1.1 \cdot \left(d_{n,\tilde{n}}\right)^{2.67} \cdot \left[\frac{\left(P_{n,t}^L\right)^2 - \left(P_{\tilde{n},t}^L\right)^2}{l_{n,\tilde{n}} \cdot S^g \cdot Z \cdot T^L}\right]^{1/2} \qquad \forall (n,\tilde{n}) \in \mathcal{NNA}, t \in \mathcal{T} \tag{16}$$

$$F_{n,\tilde{n},t}^{NN,S} \leq 1.1 \cdot \left(d_{n,\tilde{n}}\right)^{2.67} \cdot \left[\frac{\left(P_{n,t}^L\right)^2 - \left(P_{\tilde{n},t}^L\right)^2}{l_{n,\tilde{n}} \cdot S^g \cdot Z \cdot T^L}\right]^{1/2} \qquad \forall (n,\tilde{n}) \in \mathcal{DNA}, t \in \mathcal{T} \tag{17}$$

Along with Eqs. (15)-(17) we impose the inequalities (18)-(20) to ensure that – due to the pressure drops – upstream and downstream pressures of pipeline segments are not identical. In this case ϵ is a sufficiently small, non-negative parameter.

$$P_{p,t}^P \geq P_{n,t}^L + \epsilon \qquad \forall (p,n) \in \mathcal{PNA}, t \in \mathcal{T} \tag{18}$$

$$P_{n,t}^L \geq P_{\tilde{n},t}^L + \epsilon \qquad \forall (n,\tilde{n}) \in \mathcal{NNA}, t \in \mathcal{T} \tag{19}$$

$$P_{n,t}^L \geq P_{\tilde{n},t}^L + \epsilon \qquad \forall (n,\tilde{n}) \in \mathcal{DNA}, t \in \mathcal{T} \tag{20}$$

Flow Balances

We include flow balances in the proposed line pressure optimization model to ensure the conservation of mass. In particular, Eq. (21) has to hold for every network node $n \in \mathcal{N}$ within the gathering system in every time period $t \in \mathcal{T}$. We distinguish between the following flows: a) gas flows $F_{p,n,t}^{PN,S}$ from well pads to network nodes $(p,n) \in \mathcal{PNA}$, b) gas flows $F_{n,\tilde{n},t}^{NN,S}$ between regular network nodes $(n,\tilde{n}) \in \mathcal{NNA}$, c) gas flows $F_{n,\tilde{n},t}^{NN,S}$ through the compressor station $(n,\tilde{n}) \in \mathcal{CNA}$, and d) gas flows $F_{n,\tilde{n},t}^{NN,S}$ along the delivery arc $(n,\tilde{n}) \in \mathcal{DNA}$. All flows are measured in volume units at standard conditions (15° C and 101.325 kPa).

$$\begin{aligned}&\sum_{(p,n)\in\mathcal{PNA}} F_{p,n,t}^{PN,S} + \sum_{(\bar{n},n)\in\mathcal{NNA}} F_{\bar{n},n,t}^{NN,S} + \sum_{(\bar{n},n)\in\mathcal{CNA}} F_{\bar{n},n,t}^{NN,S} = \\ &\sum_{(n,\tilde{n})\in\mathcal{NNA}} F_{n,\tilde{n},t}^{NN,S} + \sum_{(n,\tilde{n})\in\mathcal{CNA}} F_{n,\tilde{n},t}^{NN,S} + \sum_{(n,\tilde{n})\in\mathcal{DNA}} F_{n,\tilde{n},t}^{NN,S} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T}\end{aligned} \tag{21}$$

In addition, Eq. (22) allows the gas produced at a well pad to be delivered to multiple network nodes (only applies if the respective pad is actually connected to multiple network nodes).

$$\sum_{(p,n)\in\mathcal{PNA}} F_{p,n,t}^{PN,S} = F_{p,t}^{P,S} \qquad \forall (p,n) \in \mathcal{PNA}, t \in \mathcal{T} \tag{22}$$

Eq. (23) determines how much gas every pad produces based on the final production rates of individual wells located at the respective pad.

$$F_{p,t}^{P,S} = \sum_{w\in\mathcal{W}} F_{w,t}^{F,S} \qquad \forall (w,p) \in \mathcal{WPA}, t \in \mathcal{T} \tag{23}$$

Finally, Eq. (24) calculates the cumulative gas production $Q_{w,t}^{S}$ of every well in every time period based on its weekly production rates $F_{w,t}^{F,S}$.

$$Q_{w,t}^{S} = \sum_{\tilde{t}=1}^{t} F_{w,\tilde{t}}^{F,S} \qquad \forall w \in \mathcal{W}, t \in \mathcal{T} \tag{24}$$

Compression Power

The compression model is included in the line pressure optimization model to capture the tradeoff between low line pressures and high operating costs. This model explicitly accounts for the pressure differential that needs to be overcome between the low-pressure gathering system and the high-pressure transmission line. In this case we rely on a straightforward compression model reported in Biegler et al. (1997), which is nonlinear and nonconvex. It calculates the necessary compression power W_t to process the gas flowrate $F^{NN,S}_{n,\tilde{n}}$ from suction pressure $P^L_{n,t}$ to discharge pressure $P^L_{\tilde{n},t}$. The parameters that need to be specified include: a) the heat capacity ratio k, b) the compressibility z and c) the compressor efficiency η^{CE}. It should be noted that we consider multi-stage compressors in this work.

$$W_t \geq F^{NN,S}_{n,\tilde{n}} \cdot 43.6 \cdot \left(\frac{k}{k-1}\right) \cdot \left[\left(\frac{P^L_{\tilde{n},t}}{P^L_{n,t}}\right)^{\frac{k-1}{k}} - 1\right] \cdot z \cdot 1/\eta^{CE} \qquad \forall (n,\tilde{n}) \in \mathcal{CNA}, t \in \mathcal{T} \tag{25}$$

For practical purposes we impose lower and upper bounds on the compression power since many compressors should not or cannot be operated below certain engine speeds.

$$W^{MIN} \leq W_t \leq W^{MAX} \qquad \forall t \in \mathcal{T} \tag{26}$$

Objective Function

The objective function for the line pressure optimization problem considers three line items to maximize the net present value of the field development project: a) revenues from natural gas sales, b) expenses for turning wells in-line, and c) expenses for compressor operation. In this work we assume that the compressor is powered by natural gas. Hence, the compression expenses translate to lost revenues from reduced gas sales. All revenues and expenses are discounted back to the present time. The parameters included in the objective function in Eq. (27) are: a) the natural gas price forecast π_t, b) the cost of turning any prospective well in-line $\delta^{TIL}_{w,p}$, and c) the compressor fuel consumption coefficient φ^C.

$$\max NPV = \sum_{t \in \mathcal{T}} (1+dr)^{-t/52} \left[\sum_{(n,\tilde{n}) \in \mathcal{DNA}} F_{n,\tilde{n},t}^{NN,S} \cdot \pi_t - \sum_{w \in \mathcal{PW}} \sum_{(w,p) \in \mathcal{WPA}} y_{w,p,t}^{TIL} \cdot \delta_{w,p}^{TIL} - \sum_{(n,\tilde{n}) \in \mathcal{CNA}} W_t \cdot \varphi^C \cdot \pi_t \right] \quad (27)$$

Altogether, the proposed line pressure optimization MINLP model is given by equations (1)-(4) and (6)-(27). In the following section we describe a tailored solution strategy for addressing this problem.

Solution Strategy

As outlined in the previous section, the line pressure optimization problem in shale gas gathering systems gives rise to large-scale, nonconvex MINLPs. Solving these problems to optimality with commercial solvers can be very challenging, and even finding good feasible solutions is not trivial. Due to the nonconvexities involved in the pressure drop constraints and the compression model, the proposed optimization model can exhibit multiple local optima. Hence, we present a tailored solution strategy for this particular problem type. Fig. 5 shows an overview of the proposed solution strategy.

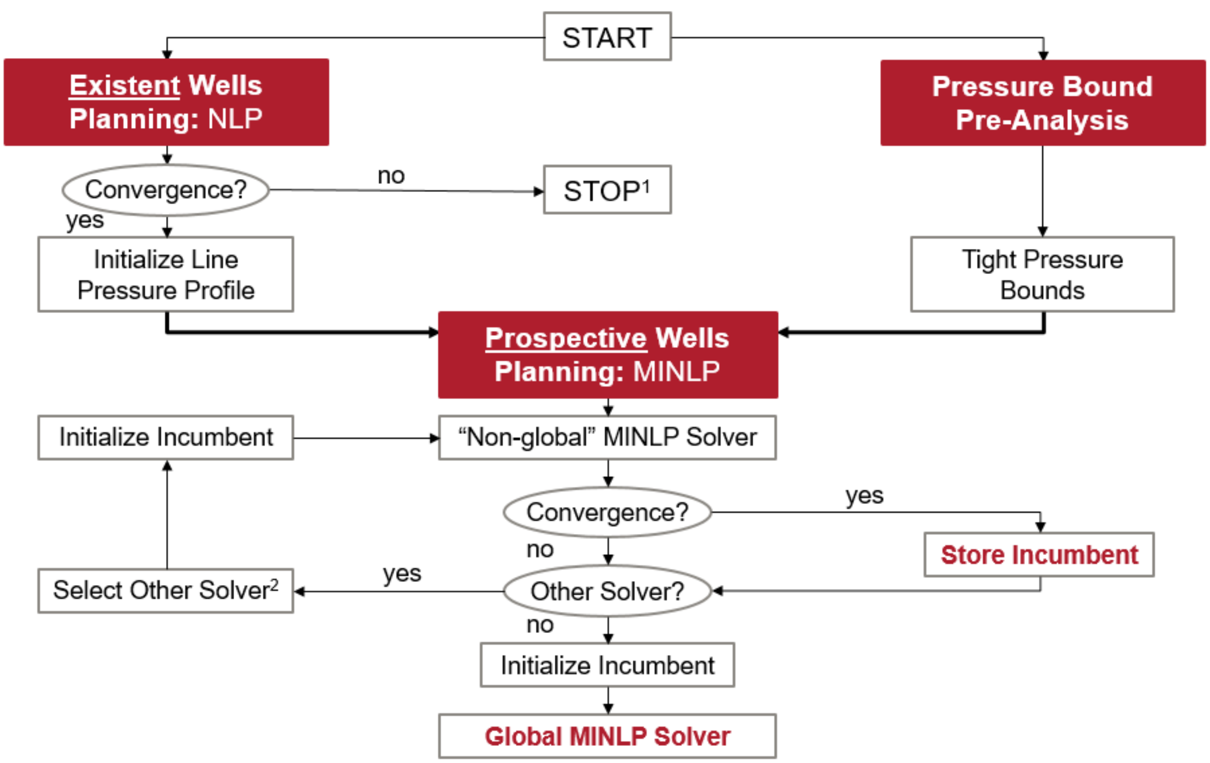

Fig. 5: Proposed solution strategy for addressing the line pressure optimization problem in shale gas gathering systems

The solution strategy illustrated in Fig. 5 begins by addressing a simplified version of the line pressure optimization problem, namely the *existent wells planning problem*. Initially, we only consider existing wells and do not account for any prospective wells that may be turned in-line. In particular, this version of the problem does not include Eqs. (6)-(14), and the objective function does not account for costs associated with TIL operations. Since the remaining model constraints do not include any binary variables, the existent wells planning problem reduces to a nonlinear programming problem. In general, the simplified NLP should be much easier to solve than the full-scale MINLP. Yet, the solution to the NLP yields a valid feasible solution to the actual MINLP problem. After all, one possible solution to the scheduling problem at hand is not to open up any new wells. More importantly though, any solution to the existent wells planning problem provides an initial line pressure profile within the respective shale gas gathering network. In other words, the solution specifies wellhead pressures, line pressures and gas flowrates throughout the network. This information can be used to effectively initialize the line pressure problem including any available prospective wells. We should note that the existent wells planning problem yields a nonconvex NLP. Hence, it may be necessary to use global NLP solvers such as BARON (Tawarmalani & Sahinidis 2005) or SCIP (Achterberg 2009) to obtain the global optimum.

Parallel to addressing the existent wells planning problem, we propose to perform a rigorous pressure bound pre-analysis throughout the gathering network. Oftentimes tight bounds can be specified on pressure variables by considering maximum allowable operating pressures (individually by pipeline segments) or low/high suction/discharge pressures at compressor inlets/outlets, respectively. These bounds are essential for strengthening the upper bounds of any (mixed-integer) nonlinear programming solver to increase the likelihood of convergence.

Finally, we address the full-scale line pressure optimization problem including any prospective wells. Despite all initialization and bound tightening attempts, this nonconvex MINLP can still be quite challenging to solve. In our experience, even minor problem data variations can change which MINLP solver performs best. In the spirit of multi-start approaches, we propose a strategy of alternating between different (non-global) MINLP solvers while initializing any solver run with the incumbent solution. For instance, the MINLP could first be solved using alpha-ECP (Westerlund & Pettersson 1995). If alpha-ECP provides a feasible solution that yields a better objective function value than the solution of the existent wells planning problem, then this solution is stored as the incumbent. Next, the initialized problem could be solved with DICOPT (Viswanathan & Grossmann 1990) by using CPLEX as the MIP solver and CONOPT as the NLP solver. Assuming that this solver combination does not yield a better solution, DICOPT could be run again – with the same incumbent as a starting solution – but now with IPOPT as the NLP solver. Our experimental analysis does not reveal a universally preferable sequence of MINLP solvers. However, considering the relatively short time it takes for most non-global MINLP solvers to converge, it can make sense to evaluate different possibilities depending on the problem data at hand. It should also be noted that this procedure does not guarantee convergence to the global optimum. In fact, all non-global solvers might fail at identifying the global optimum. Yet, in our experience, the proposed strategy increases the likelihood of converging to near-global solutions in a reasonable amount of time. Once all non-global solvers (or solver combinations) have been explored, the incumbent can be passed on to a global MINLP solver such as BARON or SCIP.

Case Study

For our case study we consider the mid-size shale gas gathering system shown in Fig. 6. This system itself consists of four existing well pads, a fully established pipeline network with segments of varying sizes, a single multi-stage compressor station providing up to 5,000 HP of compression power, and an interconnect to a long-distance transmission pipeline. Altogether, 21 wells are actively producing natural gas within the respective development area. In addition, there is the possibility to turn 14 additional wells in-line on the existent pads (five wells on PAD1, three wells on PAD2 and six wells on PAD4). We assume that all prospective wells have already been drilled, and that completions operations for these wells can be coordinated according to any feasible TIL schedule. Regardless of which schedule is ultimately selected, all produced gas must be delivered to the transmission line at 940 psi. Furthermore, the following pressure constraints need to be considered: the maximum allowable operating pressure throughout the gathering system is 1,440 psi, the suction pressure at the compressor inlet is constrained from above and below (70 psi and 250 psi respectively), and the discharge pressure at the compressor outlet may not exceed 1,400 psi. All of this information is used to impose tight bounds on line and wellhead pressures within the gathering network. Lastly, we assume that no more than two TIL operations can be performed per week. For a planning horizon of 26 weeks, we wish to determine the optimal TIL schedule and pressure profile within the gathering network that maximizes the net present value of the development project. We note that the data for this case study is provided by one of the largest upstream operators in the Appalachian Basin. For confidentiality reasons we cannot disclose the company's identity, nor the focus area of the case study.

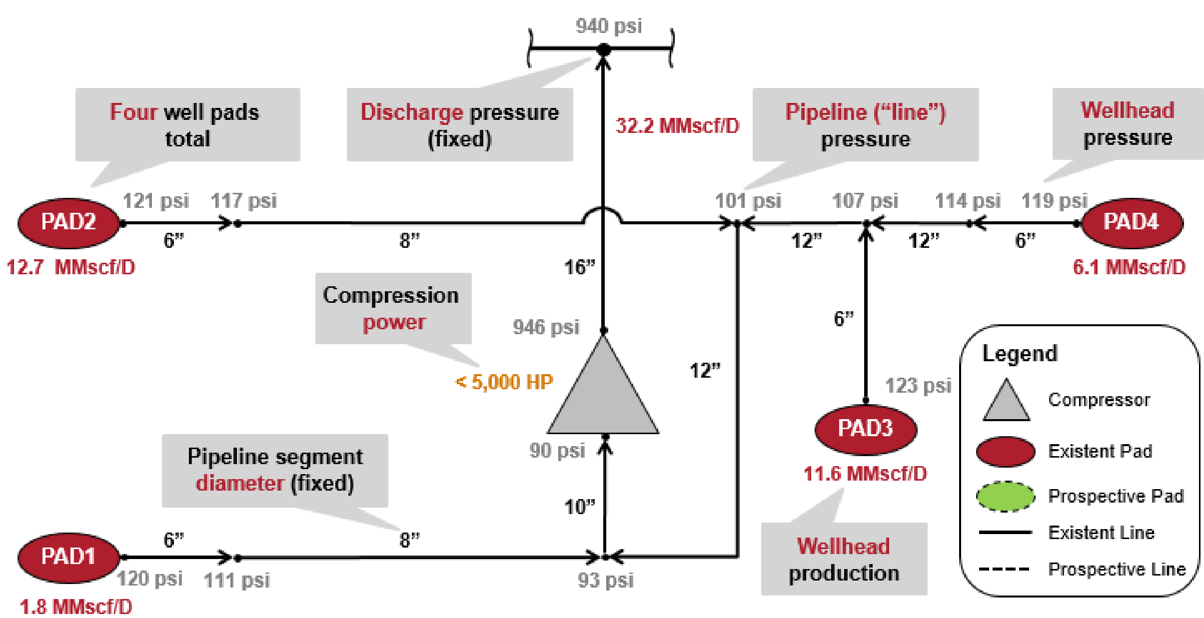

Fig. 6: Schematic of the mid-size gathering system considered for the case study

This particular line pressure optimization problem yields a nonconvex MINLP with 728 binary variables, 4,499 continuous variables and 6,530 constraints. We apply the solution strategy proposed in the previous section to this problem and terminate it after 10,000 s. All sub-problems and solvers are run on an Intel i7 with 2.93 Ghz and 12 GB RAM using GAMS 24.7.3. The reported NPV is 13.3 MM$.

Fig. 7 shows the improvement of the objective function value over time based on the proposed solution strategy. Initially, the simplified version of the problem is solved by only considering existing wells in the gathering system. This problem yields a nonconvex NLP which is initially solved using CONOPT 3.17. After approximately 20 seconds CONOPT converges to a solution with an NPV of 11.8 MM$. Due to the nonconvex nature of the existent wells planning problem, we initialize the global solver SCIP 3.2 with this solution and terminate its run after reaching an optimality gap of less than 10%.

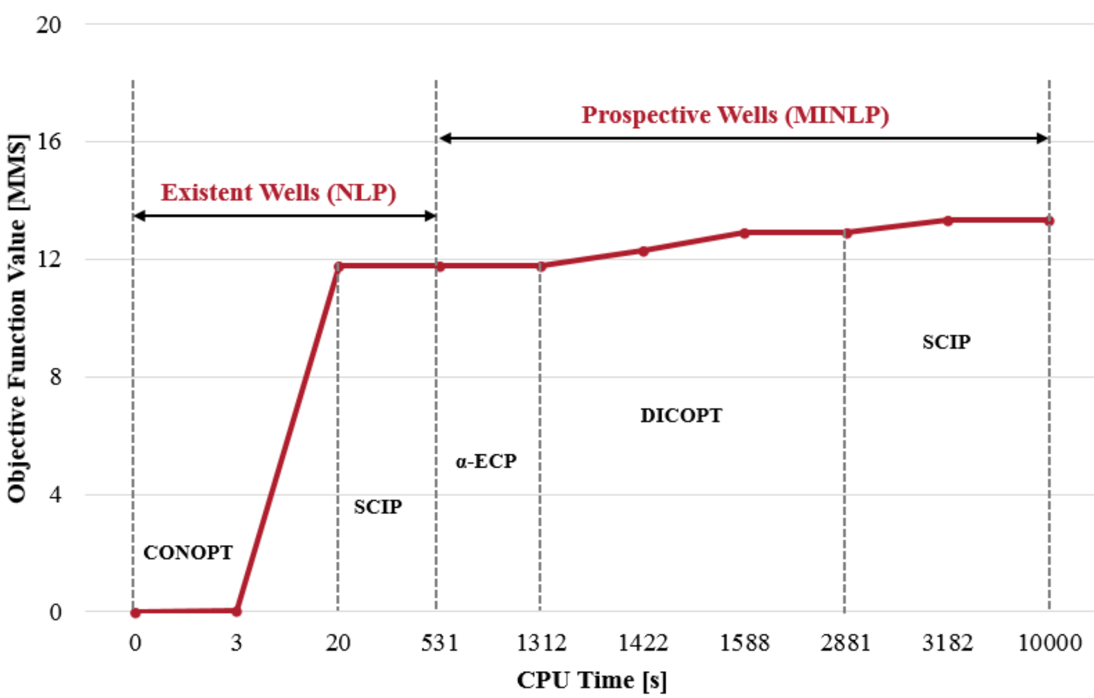

Fig. 7: Objective function value improvement over time based on the proposed solution strategy applied to the case study

The solution obtained from solving the existent wells planning problem provides an excellent starting point for addressing the full-scale line pressure optimization problem including all available prospective wells. The corresponding MINLP is first optimized using the non-global MINLP solver AlphaECP 2.20.06. This solver terminates after 780 s but does not manage to improve the objective function value beyond the solution of the existent wells planning problem. Next, we initialize the non-global MINLP solver DICOPT 24.7.3 using CPLEX 12.6.3.0 and CONOPT 3.17 with the incumbent. Fortunately, DICOPT converges to a solution with an improved objective function value. After 1,569s including 6 major iterations the reported NPV is 12.9 MM$ (up from 11.8 MM$) – and the solution suggests to turn selected prospective wells in-line. Finally, we pass this solution on to the global MINLP solver SCIP 3.2. Interestingly, SCIP identifies a solution with a slightly better objective function value of 13.3 MM$ within 300 s. Thereafter, SCIP spends 6,800 s attempting to close the optimality gap. Yet, the final gap after more than 7,100 s remains high, since the upper bound SCIP reports is 19.5 MM$. Nevertheless, it is important to note that SCIP provides a valid and rigorous upper bound to the problem that non-global MINLP solvers do not. Instead of SCIP, we also tested the global MINLP solver BARON 16.5.16. It turns out that BARON provides a slightly tighter upper bound (19.0 MM$). However, BARON converges to a marginally lower objective function value than SCIP within the time limit of 10,000 s.

First and foremost, the optimization yields the proposed TIL schedule as shown in Fig. 8. Altogether, a total of 9 out of 14 prospective wells are brought online: four on PAD1, three on PAD2 and two on PAD4. In the following paragraphs we analyze the implications of this schedule in more detail – and we attempt to outline why the timing and coordination of these particular TIL operations makes economic and practical sense.

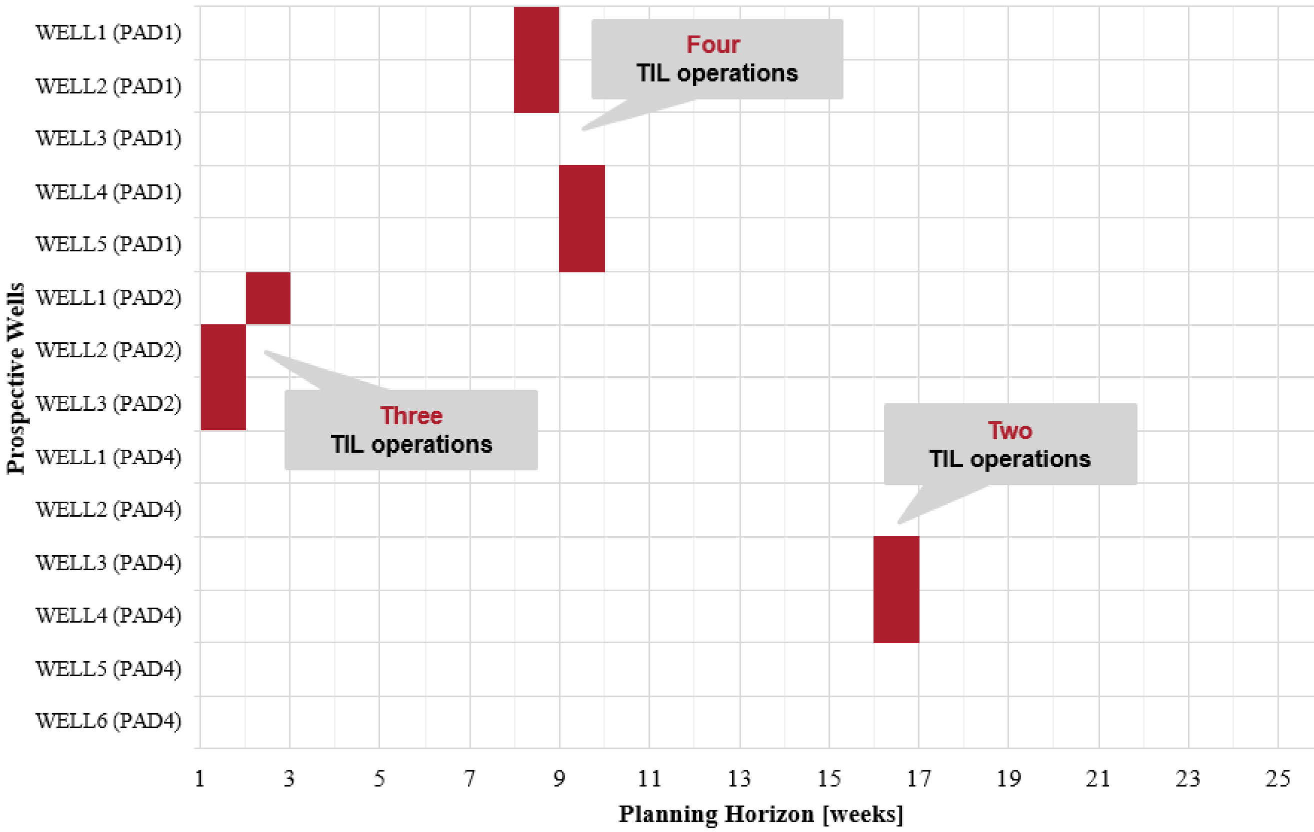

Fig. 8: Proposed TIL schedule for the shale gas gathering system considered in the case study

Fig. 9 shows production volumes by pads over time based on the proposed TIL schedule. Clearly, the entire system experiences volume growth over the full planning horizon, driven by new wells being brought online. At the same time, Fig. 9 reveals that the scheduled TIL operations are having a pronounced effect on the pressure profile within the gathering system. Every time a set of prospective wells are turned in-line, the line pressure increases noticeably, but then decreases again eventually. Unexpectedly though, the suction pressure does not appear to be abating between weeks 2 and 7 – after the TIL operations on PAD2 and before prospective wells are being brought online on PAD1.

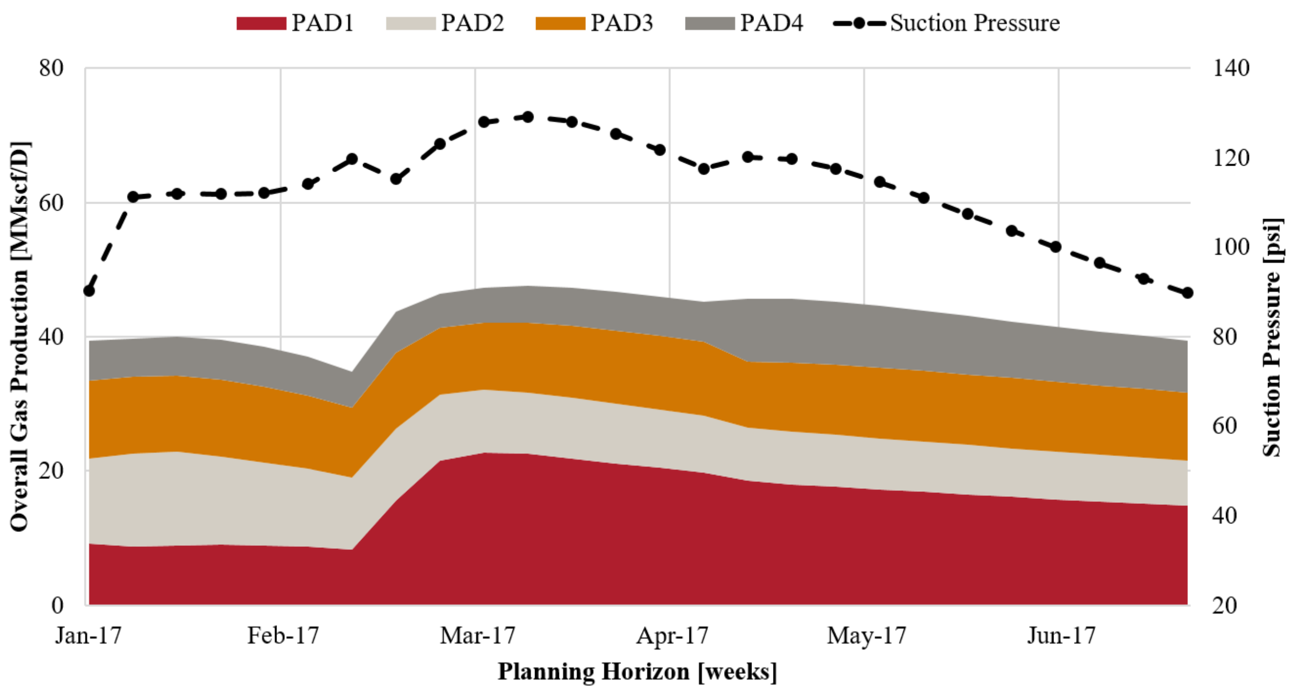

Fig. 9: Production volumes by pads over time compared to suction pressure at the compressor inlet

In fact, a detailed analysis of the solution in week 7 (towards the end of February 2017) allows for a number of interesting observations. As seen in Fig. 10, line pressure throughout the gathering system is fairly elevated even though the system is far from reaching its maximum capacity. This is unusual since upstream operators typically try to lower line pressure as quickly as possible to increase output of their shale wells. Moreover, the elevated line pressure implies that the pressure differential that needs to be overcome by the compressor is reduced. This explains why the compressor is running far below its maximum power of 5,000 HP.

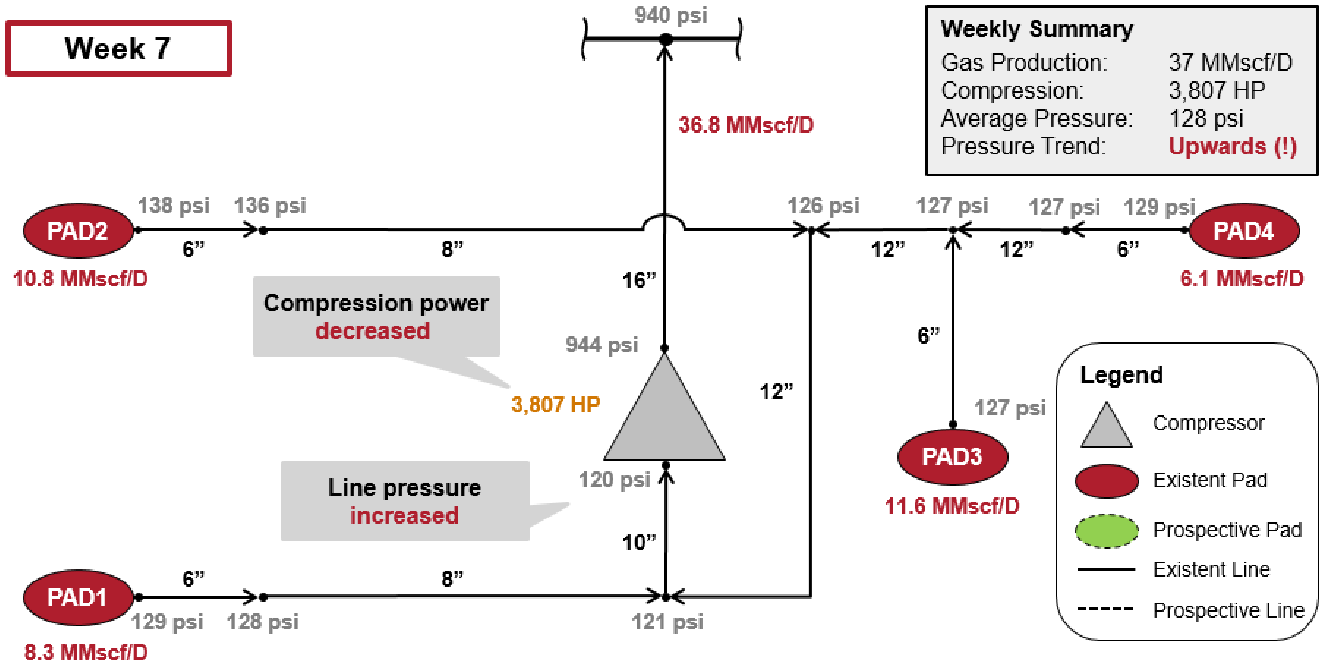

Fig. 10: Specific analysis of the proposed solution for week 7 of the case study

We turn to Fig. 11 to provide an explanation for the counter-intuitive solution suggested by the optimization. This figure shows overall gas production over time for two different cases. The lower dotted, red line marks the expected production of all existing wells assuming that no new wells are brought online (w/o development). The grey and blue stacked charts, on the other hand, show how much gas existent and prospective wells, respectively, contribute towards overall production based on the proposed TIL schedule (w/ development). The direct comparison of these two cases reveals that the backoff effect is having a prominent impact on gas production. Every time new wells are turned in-line, the existing wells produce significantly less than they would have by default. Yet, Fig. 11 allows for an intriguing observation. Although the pressure profile within the gathering system tracks the volume decline towards the end of the planning horizon, it does not do so early on. In fact, the clearly visible pressure increase between weeks 2 and 7 appears to be directly linked to the four TIL operations scheduled on PAD1. In other words, we have reason to believe that the optimization proactively <u>raises</u> line pressure throughout the system prior to bringing these four new wells online. In doing so, the optimization is actively mitigating the backoff effects associated with the upcoming TIL operations. Otherwise, if the pressure had subsided along with the production profile after week 2, the system would have experienced a pronounced pressure spike in week 8 (when the new wells are scheduled to come online). This pressure spike likely would have resulted in a substantial production loss – to the point where the existent wells might not have produced at all temporarily. Instead, the optimization proposes to "ready" and prepare the system for the upcoming pressure increase by maintaining the elevated pressure profile, and thereby effectively minimizing production backoff.

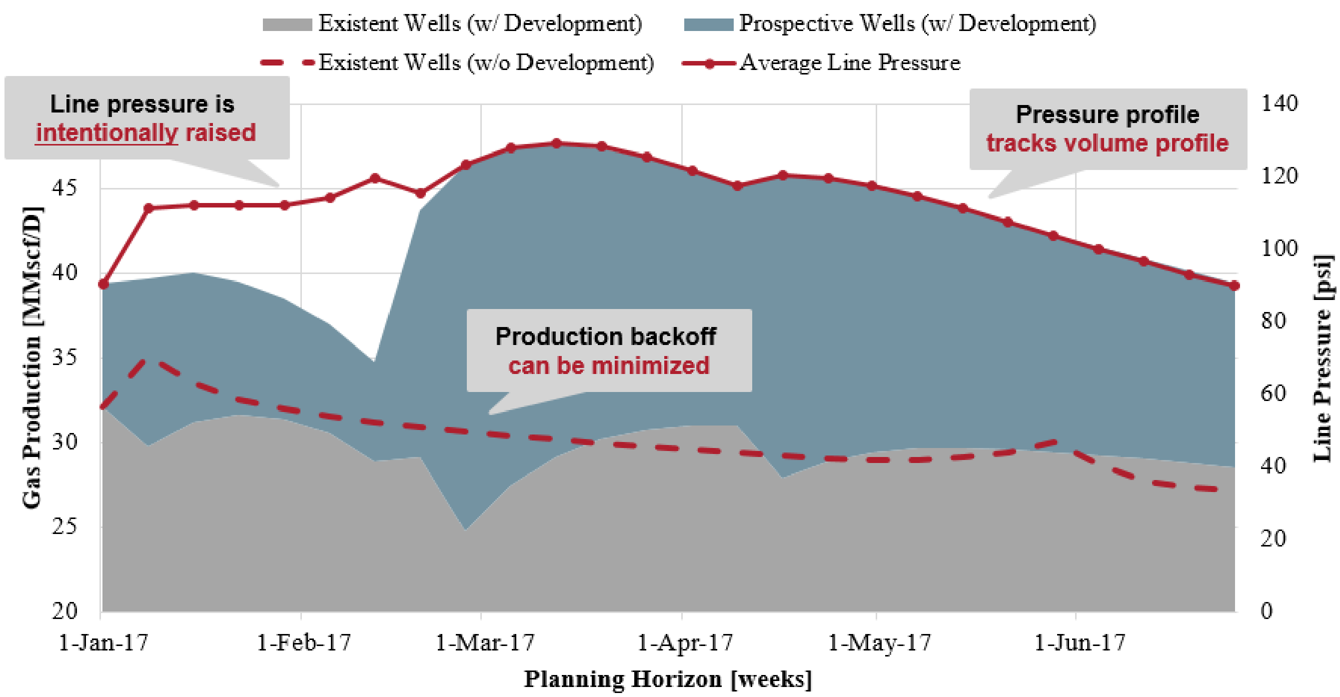

Fig. 11: Production volumes and line pressure over time distinguished by a) existent wells without development and b) existent and prospective wells with development

The findings described above are significant because they suggest that upstream operators can take a much more active role in terms of line pressure management when timing TIL operations. Rather than "wasting" the production potential of existing wells, the results suggest that producers need to evaluate carefully when new shale wells are turned in-line, and how their gathering systems should be operated prior to these events.

Conclusions

In this paper we have proposed a nonconvex mixed-integer nonlinear programming model for line pressure optimization in shale gas gathering systems. This model is designed to support shale gas producers in deciding when and how many prospective wells should be turned in-line, as well as how to manage line pressures and compressor stations throughout the gathering network. The model itself is based on three fundamental building blocks: a) a nonlinear and nonconvex reduced-order shale reservoir model, b) a nonlinear and nonconvex pressure drop model, and c) a nonlinear and nonconvex compression model. We modified the reservoir model specifically to account for production backoff effects that are prominent in shale gas gathering systems whenever new wells are brought online. Due to the nonconvex nature of the proposed model, we developed a tailored solution strategy that aims to provide valid and good initial solutions. Lastly, we applied the proposed optimization framework to a real-world case study using data from one of the largest upstream operators in the Appalachian Basin. Our results demonstrate that shale gas producers can proactively manage line pressures in their gathering systems to reduce undesirable production backoff as new wells are brought online. Finally, we note that the model proposed in this paper can be extended to other types of natural and associated gas gathering systems.

Acknowledgments

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Nomenclature

Sets

$t \in \mathcal{T}$	Time periods
$n \in \mathcal{N}$	Network nodes
$w \in \mathcal{W}$	Wells
$p \in \mathcal{P}$	Pads
$w \in \mathcal{EW}$	Existent wells
$p \in \mathcal{PW}$	Prospective wells
$(w, p) \in \mathcal{WPA}$	Well-to-pad assignments
$(p, n) \in \mathcal{PNA}$	Pad-to-node arcs
$(n, \tilde{n}) \in \mathcal{NNA}$	Node-to-node arcs
$(n, \tilde{n}) \in \mathcal{CNA}$	Compression node arcs
$(n, \tilde{n}) \in \mathcal{DNA}$	Delivery node arc

Binary Decision Variables

$y_{w,p,t}^{TIL}$	Active if well w on pad p turned in-line in time period t
$y_{w,p,t}^{PROD}$	Active if well w on pad p actively producing in time period t

Continuous, Non-Negative Decision Variables

$P_{n,t}^{L}$	Line pressure at network node n in time period t [psi]
$P_{p,t}^{P}$	Wellhead pressure at pad p in time period t [psi]
$F_{w,t}^{W,S}$	Default gas flow at well w in time period t (at standard conditions)
$F_{w,t}^{B,S}$	Backoff gas flow at well w in time period t (at standard conditions)
$F_{w,t}^{F,S}$	Final gas flow at well w in time period t (at standard conditions)
$F_{p,t}^{P,S}$	Gas flow at pad p in time period t (at standard conditions)
$F_{p,n,t}^{PN,S}$	Gas flow from pad p to network node n in time period t (at standard conditions)
$F_{n,\tilde{n},t}^{NN,S}$	Gas flow from network node n to network node $\tilde{n}$ in time period t

(at standard conditions)

$F_{n,\tilde{n},t}^{NN,A}$ Gas flow from network node n to network node $\tilde{n}$ in time period t (at actual conditions)

W_t Compressor power in time period t

$Q_{w,t}^{S}$ Cumulative gas production well w in time period t (at standard conditions)

$U_{n,t}^{L}$ Substitute line pressure at network node n in time period t

$U_{p,t}^{P}$ Substitute wellhead pressure at pad p in time period t

Parameters

$d_{p,n}$ Pipeline diameter for segment from pad p to network node n

$d_{\tilde{n},n}$ Pipeline diameter for segment from network node $\tilde{n}$ to network node n

$l_{p,n}$ Pipeline length for segment from pad p to network node n

$l_{\tilde{n},n}$ Pipeline length for segment from network node $\tilde{n}$ to network node n

$a_{w,p}$ Slope parameter pressure normalized decline curve for well w on pad p

$m_{w,p}$ Intercept parameter pressure normalized decline curve for well w on pad p

$lc_{w,t}$ Land-cleared date for well w in time period t

M Big-M parameter

$p_p^{L,MIN}$ Minimum wellhead pressure at pad p

$p_{w,p}^{0}$ Initial bottomhole pressure well w on pad p [psi]

$p_p^{L,MIN}$ Minimum wellhead pressure at pad p

$p_n^{L,MIN}$ Minimum line pressure at network node n

$p_p^{L,MAX}$ Maximum wellhead pressure at pad p

$p_n^{L,MAX}$ Maximum line pressure at network node n

q_w^{0} Cumulative gas production existent wells at begin of planning horizon

S^{g} Specific gravity of gas

W^{MAX} Maximum compression power

z Compressibility factor for gas

T^L	Gas temperature along pipelines
$\delta_{w,p}^{TIL}$	Well development cost at well w on pad p
π_t	Gas price in time period t
φ^C	Compressor fuel coefficient
γ_w	Rate restriction well w
N	Number of compressor stages
k	Heat capacity ratio
η^{CE}	Compression efficiency
R	Gas constant
dr	Annual discount rate
Δ_w^B	Backoff parameter well w

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