

OPTIMAL DEVELOPMENT PLANNING OF OFFSHORE OIL AND GAS FIELD INFRASTRUCTURE UNDER COMPLEX FISCAL RULES

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Abstract

The optimal development planning of offshore oil and gas fields has received significant attention in the recent years. In this paper, we present an efficient investment and operational planning model for this problem which is fairly generic and it is extended to include fiscal considerations. With the objective of maximizing total NPV for long-term planning horizon, the proposed non-convex multiperiod MINLP model involves decisions regarding facility installation and expansion, field-facility connections, well drilling schedule and production profiles of oil, water and gas in each time period. The model can be solved effectively with DICOPT for realistic instances and gives good quality solutions. Furthermore, it can be reformulated into an MILP after piecewise linearization and exact linearization techniques that can be solved globally in an efficient way. Solutions of the realistic instances are reported for the proposed models as well as the computational impact with consideration of the complex fiscal rules within development planning.

Keywords

Multiperiod Optimization, Planning, Offshore Oil and Gas, MINLP, MILP, FPSO, PSA, Fiscal Rules.

Introduction

The planning of offshore oil and gas field development represents a very complex problem and involves multi-billion dollar investments and profits. The complexity comes from the fact that usually there are many alternatives available for installation of the platforms and their sizes, for deciding which fields to develop and what should be the order to develop them, and which and how many wells are to be drilled in those fields and in what order, which field to be connected to which facility, and how much oil and gas to produce from each field. The other difficulties are the consideration of nonlinear profiles of the reservoir that are critical to predict the actual flowrates of oil, water and gas from each field as there can be significant variations in these flowrates over time, limitation on the number of wells that can be drilled each year due to availability of the drilling rigs, and long-term planning horizon that is the characteristics of the these projects. Moreover, installation and operation

decisions in these projects involve very large investments that can lead to large profits, or losses in the worst case if these decisions are not made carefully. Therefore, there is a clear motivation to optimize the investment and operations decisions for this problem to ensure reasonable return on the investments.

In this paper, we propose a new generic non-convex multiperiod MINLP model for the strategic/tactical planning of offshore oil and gas fields that on the one hand captures the realistic reservoir profiles, interaction among various fields and facilities, wells drilling limitations and other practical trade-offs involved in the offshore development projects, and on the other hand can be used as the basis for extensions that include fiscal considerations (e.g. Production Sharing Agreements) and/or for a stochastic programming model to handle uncertainties (e.g. uncertain reservoir size). As compared to the previous work (Iyer et al. (1998), Van den Heever

and Grossmann (2000), Goel and Grossmann (2004), Goel et al. (2006), Carvalho and Pinto (2006), Tarhan et al. (2009)), there are following major extensions and/or differences that are addressed in the paper:

1. Three components (oil, water and gas) are explicitly considered in the model for a multi-field offshore site to ensure realistic facility installation and capacity decisions.
2. Nonlinear reservoir behavior is approximated by high order polynomials to incorporate sufficient accuracy for the predicted reservoir profiles.
3. Reservoir profiles are expressed in terms of cumulative water and cumulative gas produced that are derived from water-to-oil (WOR) and gas-to-oil (GOR) ratio expressions using proposed properties.
4. The number of wells is a decision variable for each field to capture the realistic drill rig limitations and the resulting trade-offs among various fields.
5. The possibility of expanding the facility capacities in the future and lead times for construction and expansions of each facility are also considered.

We first present a multiperiod MINLP model for offshore oilfield development problem which is reformulated as an MILP problem. Furthermore, the models are reformulated with reduced number of binary variables. The models are then extended to include complex fiscal terms. The effectiveness of the proposed models and computational impact of using fiscal terms within investment and operational planning are demonstrated with the numerical results on realistic instances of the oilfield development problem.

Problem Statement

Given is a typical offshore oilfield infrastructure consisting of a set $F=\{1,2,\dots,f\}$ of oil fields available for producing oil using a set of FPSO (Floating, Production, Storage and Offloading) facilities, $FPSO = \{1,2,\dots,fpso\}$, (see Fig. 1) that can process the produced oil, store and offload it to the other tankers. Each oilfield consists of a number of potential wells to be drilled using drilling rigs, which are then connected to these FPSO facilities through pipelines to produce oil. We assume that the location of each FPSO facility and its possible connections to the given fields are known. Notice that each FPSO facility can be connected to more than one field to produce oil while a field can only be connected to a single FPSO facility. There is two-phase flow in the connecting pipelines due to the presence of gas and liquid that comprises oil and water. We assume here that there is no re-injection of water or gas in the fields for simplicity.

To develop and operate such a complex and capital intensive offshore oilfield infrastructure, we have to make the optimum investment and operation decisions to maximize the NPV considering a long-term planning horizon. The planning horizon is discretized into a number of time periods t , typically each with 1 year of duration. Investment decisions in each time period t include which FPSO facilities should be installed or expanded, and their respective installation or expansion

capacities for oil, liquid and gas, which fields should be connected to which FPSO facility, and the number of wells that should be drilled in a particular field f given the restrictions on the total number of wells that can be drilled in each time period t over all the given fields. Operating decisions include the oil/gas production rates from each field f in each time period t . It is assumed that all the installation and expansion decisions occur at the beginning of each time period t , while operation takes place throughout the time period. There is a limit on the number of expansions for each FPSO facility and lead time for its initial installation and expansion decision.

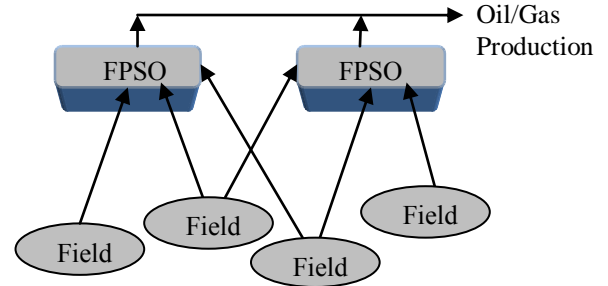


Figure 1. Typical Offshore Oilfield Infrastructure

When oil is extracted from a reservoir oil deliverability, eq. (1), water-to-oil ratio (WOR), eq. (2), and gas-to-oil ratio (GOR), eq. (3), change nonlinearly as a function of the fractional oil recovered, (fc), from the reservoir. The initial oil and gas reserves in the reservoirs, as well as the relationships for WOR and GOR in terms of fractional recovery are estimated from geologic studies. WOR and GOR values are further used in eqs. (4) and (5) to calculate the respective water and gas flow rates.

$$x_t^f \leq g(fc_{f,t-1}) \quad \forall t, f \quad (1)$$

$$wor_t^f = h(fc_{f,t-1}) \quad \forall t, f \quad (2)$$

$$gor_t^f = h(fc_{f,t-1}) \quad \forall t, f \quad (3)$$

$$water_t^f = wor_t^f oil_t^f \quad \forall t, f \quad (4)$$

$$gas_t^f = gor_t^f oil_t^f \quad \forall t, f \quad (5)$$

In this paper, we approximate the field deliverability, i.e. maximum oil flowrate from a field, eq. (1), cumulative water produced, eq. (6), and cumulative gas produced, eq. (7), from a field by high order polynomials in terms of the fractional oil recovered from that field.

$$wc_{f,t} = h(fc_{f,t}) \quad \forall t, f \quad (6)$$

$$gc_{f,t} = h(fc_{f,t}) \quad \forall t, f \quad (7)$$

Notice that eqs. (6) and (7) are derived from the corresponding equations for WOR, eq. (2), and GOR, eq. (3), using following two proposed properties:

1. The area under the curve WOR vs. cumulative oil produced for a field yields the cumulative amount of water produced.
2. The area under the curve GOR vs. cumulative oil produced for a field yields the cumulative amount of gas produced.

The motivation for using polynomials for cumulative water produced and cumulative gas produced as compared to WOR and GOR is to avoid bilinear terms in the formulation and to allow converting the resulting model into an MILP formulation. A generic MINLP model for oilfield development planning is presented next.

MINLP Model

We present here a new multiperiod MINLP model **(P1)** for the offshore oil and gas field investment and operations planning. The objective function (8) is to maximize the NPV of the project which is the difference between total revenue and total cost in each time period t taking the discount factors into account.

$$\begin{aligned}
 \text{(P1)} \quad & \text{Objective: Maximize NPV} & (8) \\
 & \text{s.t.} \\
 & \text{Economic Constraints} & (9) \\
 & \text{Reservoir Constraints} & (10) \\
 & \text{Field-FPSO Flow constraints} & (11) \\
 & \text{FPSO Capacity Constraints} & (12) \\
 & \text{Well drilling limitations} & (13) \\
 & \text{Logic Constraints} & (14)
 \end{aligned}$$

The gross revenues based on the total amount of oil and gas produced whereas total cost as the sum of capital and operating expenses in each time period t are calculated in economic constraints (9). Capital costs consist of the fixed FPSO installation cost, variable installation and expansion costs, field-FPSO connection costs and well drilling costs in each time period t while total operating expenses depend on the total amount of liquid and gas produced.

Constraints (10) predict the reservoir behavior for each field f in each time period t . In particular, constraint (1) are used to restrict the oil flow rate from each well for a particular FPSO-field connection to be less than the deliverability (maximum oil flow rate) of that field. The cumulative water and cumulative gas produced by the end of time period t from a field are represented by polynomial eqs. (6) and (7), respectively, in terms of fractional oil recovery by the end of time period t that are further used to calculate individual water and gas flow rates. The cumulative oil produced is also restricted by the recoverable amount of oil from the field.

Equations (11) represent the material balance constraints for the flow between fields and FPSOs. In particular, the total oil flow rate from field f in time period t is the sum of the oil flow rates over all FPSO facilities from this field, which depends on the oil flow rate per well and number of wells available for production. Total oil, water and gas flow rates into each FPSO facility, respectively, in time period t from all the given fields is calculated as the sum of the flow rates of each component over all the connected fields.

Equations (12) restrict the total oil, liquid and gas flow rates into each FPSO facility to be less than its corresponding capacity in each time period t . FPSO facility capacities in time period t are computed as the sum of the corresponding installation and expansion

capacities taking lead time into considerations. Furthermore, there are restrictions on the maximum installation and expansion capacities for each FPSO facility.

The number of wells available in a field is calculated as the sum of the wells available at the end of previous time period and the number of wells drilled at the beginning of time period t . The maximum number of wells that can be drilled over all the fields during each time period t and in each field f during complete planning horizon T are restricted by respective upper bounds in (13).

Logic constraints (14) include the restrictions on the number of installation and expansion of a FPSO facility, possible FPSO-field connections during the planning horizon T . Other logic constraints are also included to ensure that FPSO facility can be expanded and the connection between a field and that facility and corresponding flow can occur only if that facility has already been installed by that time period.

The proposed non-convex MINLP model **(P1)** for offshore oilfield planning involves nonlinear and non-convex constraints that can lead to suboptimal solutions when solved with a method that assumes convexity. In particular, constraints (1), (6) and (7) are univariate polynomials that represent reservoir profiles while constraints (15) involves bilinear terms with integer variables, $N_{f,t}^{well}$, that calculates the total oil flow rate from a field as the multiplication of the number of available wells in the field and oil flow rate per well.

$$x_{f,t} = N_{f,t}^{well} \cdot x_{f,t}^{well} \quad \forall t, f \quad (15)$$

In the following section, we reformulate this MINLP model **(P1)** into an MILP problem that can be solved to global optimality in an effective way.

MILP Reformulation

To approximate the univariate polynomials (1), (6) and (7) SOS1 variables $b_{f,t}^l$ are introduced to select the adjacent points $l-1$ and l for interpolation over an interval l . Constraints (16)-(19) represent the piecewise linear approximation for the fractional oil recovery and corresponding oil deliverability, cumulative water and cumulative gas produced for a field in each time period t , respectively, based on the reservoir profiles.

$$fc_{f,t} = \sum_{l=1}^n \lambda_{f,t}^l \tilde{f}c^l \quad \forall f, t \quad (16)$$

$$Q_{f,fpso,t+1}^{d,well} = \sum_{l=1}^n \lambda_{f,t}^l \tilde{Q}_{f,fpso}^{d,well,l} \quad \forall f, fpso, t \quad (17)$$

$$Q_{f,fpso,t}^{wc} = \sum_{l=1}^n \lambda_{f,t}^l \tilde{Q}_{f,fpso}^{wc,l} \quad \forall f, fpso, t \quad (18)$$

$$Q_{f,fpso,t}^{gc} = \sum_{l=1}^n \lambda_{f,t}^l \tilde{Q}_{f,fpso}^{gc,l} \quad \forall f, fpso, t \quad (19)$$

Equation (20) allows only one of the point l to be selected for which $b_{f,t}^l$ equals 1 while eq. (21) states that $\lambda_{f,t}^l$ can be non-zero for only two consecutive points l and $l-1$ that are used for convex combination during interpolation, eq. (22). Thus, the corresponding l th piece is used for linear interpolation as all other $\lambda_{f,t}^l$ are zero for a field in time period t and determines the value of the interpolated variable as a convex combination of their values at both the end of this piece l in eqs. (16)-(19).

$$\sum_{l=1}^{n-1} b_{f,t}^l = 1 \quad \forall f, t \quad (20)$$

$$\lambda_{f,t}^l \leq b_{f,t}^{l-1} + b_{f,t}^l \quad \forall f, t, l \quad (21)$$

$$\sum_{l=1}^n \lambda_{f,t}^l = 1 \quad \forall f, t \quad (22)$$

The other nonlinear constraints (15) in Model (P1) contain bilinear terms with integer variables that can be linearized using exact linearization techniques. Therefore, to linearize this constraint we first express the integer variable, $N_{f,t}^{well}$, for the number of wells in terms of the binary variables $Z_{f,k,t}^{well}$ using eq. (23) where $Z_{f,k,t}^{well}$ determines the value of the k th term of the binary expansion. The bilinear term in constraint (15) can then be rewritten as equation (24). Constraint (24) can be reformulated as a linear constraint (25) by introducing a nonnegative continuous variable $ZX_{f,fpso,k,t}^{well}$ which is further defined by the linear constraints (26)-(29) using an auxiliary variable $ZX1_{f,fpso,k,t}^{well}$.

$$N_{f,t}^{well} = \sum_k 2^{|k|-1} \cdot Z_{f,k,t}^{well} \quad \forall f, t \quad (23)$$

$$x_{f,fpso,t} = \sum_k 2^{|k|-1} \cdot Z_{f,k,t}^{well} \cdot x_{f,fpso,t}^{well} \quad \forall f, fpso, t \quad (24)$$

$$x_{f,fpso,t} = \sum_k 2^{|k|-1} \cdot ZX_{f,fpso,k,t}^{well} \quad \forall f, fpso, t \quad (25)$$

$$ZX_{f,fpso,k,t}^{well} + ZX1_{f,fpso,k,t}^{well} = x_{f,fpso,t}^{well} \quad \forall f, fpso, k, t \quad (26)$$

$$ZX_{f,fpso,k,t}^{well} \leq U_{f,fpso}^{well} Z_{f,k,t}^{well} \quad \forall f, fpso, k, t \quad (27)$$

$$ZX1_{f,fpso,k,t}^{well} \leq U_{f,fpso}^{well} (1 - Z_{f,k,t}^{well}) \quad \forall f, fpso, k, t \quad (28)$$

$$ZX_{f,fpso,k,t}^{well} \geq 0, ZX1_{f,fpso,k,t}^{well} \geq 0 \quad \forall f, fpso, k, t \quad (29)$$

The reformulated MILP Model (P2) involves all the constraints as in model (P1) except nonlinear constraints (1), (6), (7) and (15) which are replaced with linear and mixed-integer linear constraints (16)-(23) and (25)-(29).

Remark: Due to the potential computational expense of solving the large scale MINLP and MILP models presented in the previous sections, we further reformulate them by replacing the binary variables that represent the timing of the connections between fields and FPSOs without time index along with corresponding change in the logic constraints. The motivation for binary reduction comes from the fact that in the solution of these models the connection cost is only ~2-3% of the total cost, and hence, its exact discounting does not has a significant impact on the optimal solution. This results in a significant reduction in the number of binary variables (~33% reduction) and the solution time can be improved significantly for both the MINLP and MILP formulations. Hence, the proposed reduced models (P1-R) and (P2-R) correspond to the MINLP (P1) and MILP (P2) models, respectively, after binary reduction as explained above.

Example 1

In this example we consider 3 oil fields (see Fig. 2) that can be connected to 3 FPSOs with 7 possible connections among these fields and FPSOs. There are a total of 25 wells that can be drilled, and the planning horizon considered is 10 years, which is discretized into 10 periods of each 1 year of duration. We need to determine the optimum investment and operations decisions while maximizing total NPV over the given planning horizon.

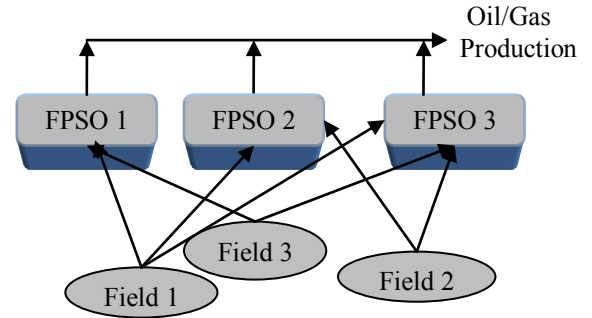


Figure 2. Oilfield Planning Example 1

The problem is solved using DICOPT 2x-C solver for MINLP (P1), and CPLEX 12.2 for MILP (P2). These models were implemented in GAMS 23.6.3 and run on Intel Core i7 machine with 4GB of RAM. The optimal solution of this problem that corresponds to (P1), suggests installing only FPSO 3 with a capacity of 300 kstb/d, 420.01 kstb/d and 212.09 MMSCF/d for oil, liquid and gas, respectively, at the beginning of year 1. All the three fields are connected to this FPSO facility at the beginning of year 4 when installation of the FPSO facility is completed and a total of 20 wells are drilled in these 3 fields to start production. One additional well is also drilled in field 3 in year 5 and there are no expansions in the capacity of FPSO facility. The total NPV of this project is \$6912.04 M. Table 1 compares the computational time of DICOPT (3.7s) for this instance with BARON which takes more than 36,000s to be within ~10% of optimality. Note that we use the DICOPT solution to initialize in this case, but BARON could only

provide a slightly better solution (6919.28 vs. 6912.04) than DICOPT in more than 10 hours.

Table 1. Comparison of the solvers for Example 1 (P1)

Solver	Const.	Cont. var.	Dis. var.	NPV Millions	Time (s)
DICOPT	1,997	1,271	151	6,912.04	3.07
BARON	1,997	1,271	151	6,919.28	>36,000

The MILP (P2) and its binary reduction (P2-R), are solved with CPLEX 12.2 and results in Table 2 show the significant reduction in the solution time after binary reduction (6.55s vs. 37.03s) while both the models give same optimal NPV i.e. \$7030.90M. Notice that these approximate MILP models are solved upto global optimality in few seconds while global solution of the original MINLP formulation is much expensive to obtain. We use 5 point estimates for piecewise linearization to formulate (P2) and (P2-R) as beyond that limit the change in optimal solution was very small as compared to large increase in the computational time.

Table 2. Model comparison for Example 1 (P2 vs. P2-R)

Model	Const.	Cont. var.	Dis. var.	SOS1 Var.	NPV Millions	Time (s)
P2	3,094	2,228	219	120	7,030.90	37.03
P2-R	3,057	2,165	177	120	7,030.90	6.55

The global solution from the MILP approximation (P2-R) gives a higher NPV for this example as compared to solving (P1) directly (7030.90 vs. 6912.04). Therefore, this model can potentially be used for finding global or near optimal solution to the original MINLP formulation. We fix the discrete variables coming from the model (P2-R) in the MINLP model (P1) and solve the resulting NLP that significantly improve the local solutions (7004.08 vs. 6912.04). Notice also that no other MINLP solver could find the better solution than this in reasonable computational time.

Incorporating Complex Fiscal Rules

Including fiscal considerations, Van den Heever and Grossmann (2001), Lin and Floudas (2003), as part of the investment and operation decisions for the oilfield development problem can significantly impact the optimal NPV and required computational time. Therefore, in this section we extend the proposed models to include the generic complex fiscal rules within planning.

There are a variety of contracts (e.g. Production Sharing Agreements or PSAs, Concessionary system) that are used in the offshore Oil and Gas industry. The specific rules defined in such a contract between operating Oil Company and host Government determine the profit that the oil company can keep as well as the royalties, profit share that are paid to the government. These profit oil splits, royalty rates etc. are usually based on the profitability of the project (progressive fiscal terms), where cumulative oil produced, rate of return, R-factor etc. are the typical profitability measures that determine

the tier structure for these contract terms. Given that the resulting royalties and/or Government profit oil share can be significant amount of the gross revenues, it is critical to consider these contracts terms explicitly during oilfield planning to access the actual economic potential of such a project. For instance, a very promising oilfield or block can turn out to be a big loss or less profitable than projected in the long-term if significant royalties are attached to that field which was not considered during the development planning phase involving large investments. On contrary, there could be possibility of missing an opportunity to invest in a field which has very difficult conditions for production and looks unattractive, but can have very favorable fiscal terms resulting in very large profits in the long-term.

With this motivation for optimal investment and operations decisions in a realistic situation for offshore oil and gas field planning project, we incorporate the generic fiscal terms within development planning models (MINLP and MILP) presented in the previous sections with objective of maximizing the total contractor's (oil company) share. In particular, we include the cost recovery ceiling in terms of min function (30) to limit the amount of total oil produced each year that can be used to recover the capital and operational expenses. This ceiling on the cost oil recovery is usually enforced to ensure early revenues to the Govt. as soon as production starts.

$$CO_t = \min(CR_t, f_t^{CR} \cdot REV_t) \quad (30)$$

Moreover, sliding scale based profit oil share of contractor that is linked to the cumulative oil production is also included in the model. In particular, disjunction (31) is used to model this tier structure for profit oil split which says that variable $Z_{i,t}$ will be true if cumulative oil production by the end of time period t lies between given tier thresholds $L_i \leq xc_t \leq U_i$ i.e. tier i is active and split fraction f_i^{PO} is used to determine the contractor share in that time period. The disjunction (31) in the model is further reformulated into linear and mixed-integer linear constraints using convex-hull formulation.

$$\vee_i \left[\begin{array}{l} Z_{i,t} \\ ConSh_t^{beforetax} = f_i^{PO} \cdot PO_t \\ L_i^{oil} \leq xc_t \leq U_i^{oil} \end{array} \right] \quad (31)$$

Furthermore, the proposed model is also extended to include the ring-fencing which is the provisions that are usually part of fiscal terms and have significant impact on the NPV calculations. These provisions determine that all the costs associated with a given block (which may be a single field or a group of fields) or license must be recovered from revenues generated within that block, i.e. the block is "ring-fenced". Optimal investment and operations decisions and computational impact of adding a typical progressive Production Sharing Agreement (PSA) terms are demonstrated in the next section with a small example.

Example 2

In this instance of oilfield planning problem, we consider 5 oilfields that can be connected to 3 FPSO's with 11 possible connections. There are a total of 31 wells that can be drilled in all of these 5 fields and the planning horizon considered is 20 years. There is a cost recovery ceiling and 4 tiers (see. Fig. 3) for profit oil split between the contractor and host Government that are linked to cumulative oil production which defines the fiscal terms of a typical progressive Production Sharing Agreement.

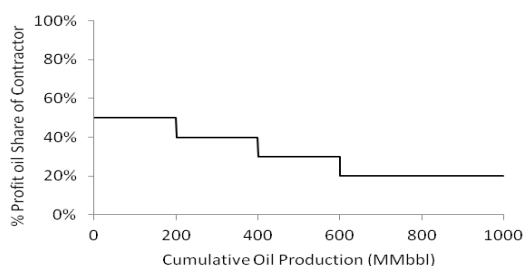


Figure 3. Contractor's Profit oil share for Example 2

Table 3 compares the results of the MILP (P2) and reduced MILP (P2-R) with progressive PSAs for this example. We can observe that there is significant increase in the computational time with fiscal consideration for the original MILP formulation (P2) which takes more than 10 hours to be within 14% of optimality gap as compared to the reduced MILP model (P2-R) that reaches within 2% of gap in reasonable computational time. Therefore, including fiscal rules within development planning can make the problem much harder to solve due to the additional binary variables that are required.

Table 3. Computational Results for Example 2

Model	Const.	Cont. var.	Dis. var.	NPV Millions	Time (s)
P2 with PSA	9,474	6432	727	2,183.63	>36,000
P2-R with PSA	9,363	6,223	551	2,228.94	1,163.7

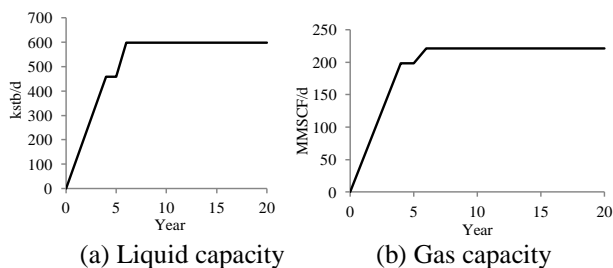


Figure 4. Liquid and Gas capacities of FPSO 3 facility

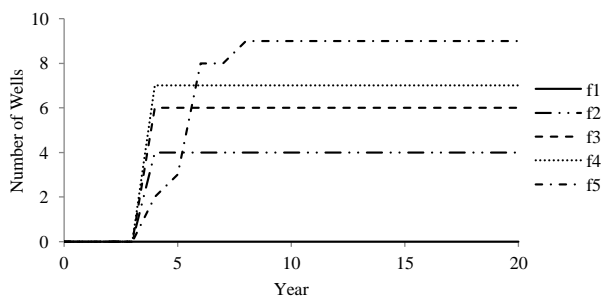


Figure 5. Well drilling schedule for Example 2

The optimal solution from model (P2-R) with fiscal considerations suggests installing 1 FPSO facility with expansions in the future (see Fig. 4) while Fig. 5 represents the well drilling schedule for this example. The tiers 2, 3 and 4 for profit oil split gets active in years 6, 8 and 12, respectively, based on the cumulative oil production profile during the given planning horizon.

Conclusions

In this paper, we have proposed a new generic MINLP model for offshore oil and gas field infrastructure investment and operational planning considering multiple fields, three components (oil, water and gas) explicitly in the formulation, facility expansions decisions, well drilling schedules and nonlinear reservoir profiles. The MINLP model yields good solutions to realistic instances when solving with DICOPT directly. Furthermore, the model can be reformulated into an MILP with which the problem can be solved to global optimality. The proposed MINLP and MILP formulations are further improved by using a binary reduction scheme resulting in the significant computational savings. Complex fiscal terms are included in the proposed models and the results on an example show significant increase in the computational time for the original MILP formulation as compared to the reduced one. The models will further be extended to include more complex fiscal rules in an efficient way.

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