Offshore Oilfield Development Planning under Uncertainty and Fiscal Considerations

Vijay Gupta*, Ignacio E. Grossmann†,

Department of Chemical Engineering, Carnegie Mellon University
Pittsburgh, PA 15213

Abstract

The development planning of an offshore oil and gas field infrastructure involves critical investment and operating decisions at an early stage of the project that impact the overall project profitability. These strategic/tactical decisions need to be made considering sufficient reservoir details, fiscal contracts with the government and uncertainty in the field parameters to be useful in practice. However, it makes the optimization problem difficult to model and solve. With this motivation, the objective of this paper is to present a comprehensive review of a unified modeling framework and solution strategies to address the issues of complex fiscal rules and endogenous uncertainties in the development planning of offshore oil and gas field infrastructure that relies on our recent work in this area. In particular, the paper emphasizes the need to have as a basis an efficient deterministic model that can account for various alternatives in the decision making process for a multi-field site incorporating sufficient level of details, while being computationally tractable for large instances. Consequently, such a model can be effectively extended to include other complexities, for instance a production sharing agreement and endogenous uncertainties. Computational results on the deterministic as well as multistage stochastic instances of the problem are discussed.

Keywords: oil & gas exploration; production sharing agreements; ringfencing; multistage stochastic programming; endogenous uncertainties; Lagrangean decomposition

1 Introduction

The development planning of offshore oil and gas fields has received significant attention in recent years given the new discoveries of large oil and gas reserves in the last decade around the world. These have been facilitated by the new technologies available for exploration and production of oilfields in remote locations that are often hundreds of miles offshore. Surprisingly, there has been a net increase in the total oil reserves in the last decade because of these discoveries despite increase in the total demand (BP, Statistical review Report 2011). Therefore, there is currently a strong focus on exploration and development activities for new oil fields all around the world, specifically at offshore locations. However, installation and operating decisions in these projects involve very large investments that potentially can lead to large profits, but also to losses if these decisions are not made carefully.

*E-mail: vijaygup@andrew.cmu.edu

†To whom all correspondence should be addressed. E-mail: grossmann@cmu.edu

In particular, the planning of offshore oil and gas field development represents a very complex problem and involves multi-billion dollar investments (Babusiaux et al., 2007). The major decisions involved in the oilfield development planning phase are the following:

(a) Selecting platforms to install and their sizes
(b) Deciding which fields to develop and what should be the order to develop them
(c) Deciding which wells and how many are to be drilled in the fields and in what sequence
(d) Deciding which fields are to be connected to which facility
(e) Determining how much oil and gas to produce from each field

Therefore, there are a very large number of alternatives that are available to develop a particular field or group of fields. However, these decisions should account for the physical and practical considerations, such as the following: a field can only be developed if a corresponding facility is present; nonlinear profiles of the reservoir to predict the actual flowrates of oil, water and gas from each field; limitation on the number of wells that can be drilled each year due to availability of the drilling rigs; and long-term planning horizon that is the characteristics of the these projects. Therefore, optimal investment and operating decisions are essential for this problem to ensure the highest return on the investments over the time horizon considered. By including all the considerations described n an optimization model, this leads to a large-scale multiperiod MINLP problem that is difficult to solve to global optimality. The extension of this model to the cases where we explicitly consider the fiscal rules and the uncertainties can further lead to a very complex problem to model and solve.

With the motivation described above, the paper presents a comprehensive review of the optimal development planning of offshore oil and gas fields based on our recent work in this area, highlighting the models and solution approaches that we proposed, and the key issues that may be still be present. In particular, a unified modeling framework is presented for this problem starting with a basic deterministic model that includes sufficient level of detail to be realistic as well as computationally efficient as proposed in Gupta and Grossman (2012a). Moreover, we discuss the extension of the model for incorporating fiscal rules defined by the terms of the contract between oil companies and governments (Gupta and Grossmann, 2012b), and uncertainty based on a multistage stochastic programming framework (Gupta and Grossmann, 2014a). A detailed literature review of the various models and solution approaches for oil/gas field development and operations can be found in Gupta (2013).

We first present a brief background on the oilfield development projects, the basic structure of an offshore oilfield site, fiscal contracts that are used in the industry and uncertain parameters are described. The specific problem under consideration involving nonlinear reservoir profiles, production sharing agreements and endogenous uncertainties, and the assumptions involved are presented. A basic MINLP model from Gupta and Grossman (2012a) relying on this description is reviewed and its extension to include fiscal rules (Gupta and Grossman, 2012b) and/or uncertain field parameters using multistage stochastic programming approach (Gupta and Grossman (2014a), and the corresponding solution strategies are highlighted next. Numerical results on several examples ranging from deterministic to stochastic cases for oilfield planning problem are considered to emphasize the efficiency of the models and the solution methods.

2 Background

In this section, we present a general background on the offshore oil/gas field infrastructure, fiscal rules that are usually associated, and uncertainties in the various parameters. These elements will be used as a basis to describe the problem under considerations in the next section.

The life cycle of a typical offshore oilfield project consists of following five steps:

1) *Exploration:* This activity involves geological and seismic surveys followed by exploration wells to determine the presence of oil or gas.
2) *Appraisal:* It involves drilling of delineation wells to establish the size and quality of the potential field. Preliminary development planning and feasibility studies are also performed.
3) *Development:* Following a positive appraisal phase, this phase aims at selecting the most appropriate development plan among many alternatives. This step involves capital-intensive investment and operating decisions that include facility installations, drilling, sub-sea structures, etc.
4) *Production:* After facilities are built and wells are drilled, production starts where gas or water is usually injected in the field at a later time to enhance productivity.
5) *Abandonment:* This is the last phase of an oilfield development project and involves the decommissioning of facility installations and subsea structures associated with the field.

Given that most of the critical investments are usually associated with the development planning phase of the project, this paper focuses on the key strategic/tactical decisions during this phase of the project.

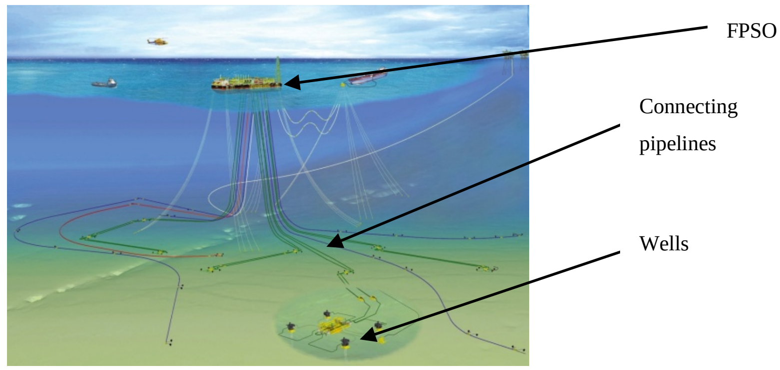


Figure 1: A Complex Offshore Oilfield Infrastructure

An offshore oilfield infrastructure (Figure 1) is usually very complex and involves various production facilities such as Floating Production, Storage and Offloading (FPSO), Tension Leg platform (TLP), fields, wells and connecting pipelines to produce oil and gas from the reserves. Each oilfield consists of a number of potential wells to be drilled using drilling rigs, which are then connected to the facilities through pipelines to produce oil. The field to facility connection involves trade-offs associated to the production rate of oil and gas, piping costs, and possibility of other fields to connect to that same facility. The number of wells that can be drilled in a field depends on the availability of the drilling rig that can drill a certain number of wells each year.

There is a multi-phase flow in the connecting pipelines due to the presence of oil, water and gas. Therefore, there may be several components present in the produced oil, and their relative amounts depend on certain parameters like cumulative oil produced. The facilities and piping connections in the offshore infrastructure are often in operation over many years. It is therefore important to anticipate future conditions when designing an initial infrastructure or any expansions. This can be accomplished by dividing the planning horizon, for example, 20 years, into a number of time periods with a length of 1 year, and allowing investment and operating decisions in each period, which leads to a multi-period planning problem.

There are a variety of contracts that are used in the offshore oil and gas industry (Babusiaux et al., 2007; Johnston, 1994; Sunley et al., 2002, and Tordo, 2007). Although the terms of a particular agreement are usually negotiated between both the entities in practice, these contracts can broadly be classified into two main categories:

1) *Concessionary System:* A concessionary (or tax and royalty) system usually involves royalty, cost deduction and tax. Royalty is paid to the government at a certain percentage of the gross revenues. The net revenue after deducting costs becomes taxable income on which a pre-defined percentage is paid as tax which may include both corporate income tax and a specific profit tax. The total contractor's share involves gross revenues minus royalty and taxes in each year. The basic difference as compared to the production sharing agreement is that the oil company keeps the right to all of the oil and gas produced at the wellhead and pays royalties, bonuses, and other taxes to the government. These contracts are used in countries such as Canada, USA and the UK.

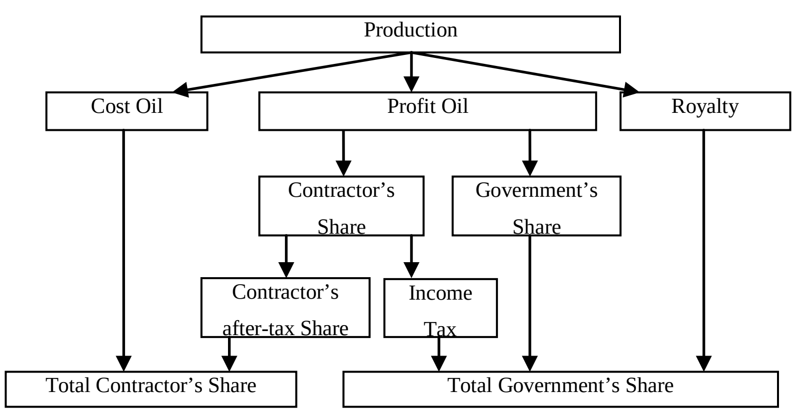


Figure 2: Revenue flow for a typical Production Sharing Agreement

2) *Production Sharing Agreements (PSAs):* The revenue flow in a typical Production Sharing Agreement can be seen as in Figure 2 (World Bank, 2007). First, in most cases, the company pays royalty to the government at a certain percentage of the total oil produced. After paying the royalties, some portion of the remaining oil is treated as cost oil by the oil company to recover its costs. There is a ceiling on the cost oil recovery to ensure revenues to the government as soon as production starts. The remaining part of the oil, called profit oil, is divided between oil company and the host government at a certain percentage. The oil company needs to further pay income tax on its share of profit oil. Hence, the total contractor's (oil company) share in the gross revenue comprises of cost oil and contractor's profit oil share after tax. The other important feature of a PSA is that the government keeps rights to the oil produced at wellhead, and transfers title to a portion of the extracted oil and gas to oil company that works as a contractor at an agreed delivery point. Notice that the cost oil limit is one of the key differences with a concessionary system (World Bank, 2007). These contracts are used in countries such as Cambodia, China, Egypt, India, Angola and Nigeria.

The specific rules defined in such a contract (either concessionary or PSA, hybrid) between oil company and host government determine the profit that the oil company can keep, as well as the royalties and profit oil share that are paid to the government. These profit oil fractions, royalty rates define the fiscal terms of a particular contract and can be either "regressive or progressive". Regressive fiscal terms are not directly linked to the profitability of the project, e.g. fixed percentage of royalty or profit oil share for the entire planning horizon. However, the progressive fiscal terms (e.g. profit oil shares, royalty rates) are based on the profitability of the project, i.e. these terms penalize higher production rates, where cumulative oil produced, daily production, rate of return, R-factor, are the typical profitability measures that determine the tier structure (levels) for these contract terms. For instance, if the cumulative production is in the range of the first tier, $0 \le xc_t \le 200$, the contractor receives 50% of the profit oil, while if the cumulative production reaches in tier 2, $200 \le xc_t \le 400$, the contractor receives 40% of the profit oil, and so on (see Figure 3). In practice, as we move to the higher tier, the percentage share of contractor decreases.

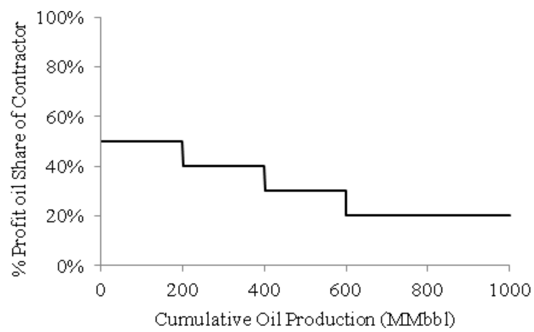


Figure 3: Progressive profit oil share of the contractor

Ringfencing is an important concept that is usually part of the fiscal contracts and imposed by the government, which affects the cash flows over the planning horizon. In a typical ringfencing provision, investment and operational costs for a specified group of fields or block can only be recovered from the revenue generated from those fields or block (see Figure 4). It means that the set of particular fields are "ring-fenced". Therefore, income derived from one contract area or project cannot be offset against losses from another contract area or project. In financial terms, a ringfencing provision basically defines the level at which all fiscal calculations need to be done, and restricts the oil companies to balance the costs and revenues across various projects/blocks for minimizing the tax burden. For example, fiscal calculations for Fields 1-3 (Ringfence 1) and Field 4-5 (Ringfence 2) in Figure 4 cannot be consolidated at one place. Ringfencing provisions are more popular in production sharing contracts.

These fiscal contracts, terms and ringfencing provisions are the backbone of most of the contracts that are currently used, and can have significant impact on the revenues. In addition, there can be some other fiscal considerations for a particular contract of interest, but for simplicity we only consider the important financial elements as described above.

In addition, there are multiple sources of uncertainty in these oil/gas field development projects. The market price of oil/gas, quantity and quality of reserves at a field are the most important sources of the uncertainty in this context. The uncertainty in oil prices is influenced by the political, economic or other market factors and it belongs to the exogenous uncertainty problems. The uncertainty in the reserves on the other hand, is linked to the accuracy of the reservoir data (technical uncertainty). While the existence of oil and gas at a field is indicated by seismic surveys and preliminary exploratory tests, the actual amount of oil in a field, and the efficacy of extracting the oil will only be known after capital investment have been made at the field (Goel and Grossmann, 2004), i.e. endogenous uncertainty. Both, the price of oil and the quality of reserves directly affect the overall profitability of a project, and hence it is important to consider the impact of these uncertainties when formulating the decision policy. However, in this paper we only address the uncertainty in the field parameters that is already a very challenging problem to model and solve. In the next section, we outline the specific problem under consideration that incorporates nonlinear reservoir behavior, fiscal rules and endogenous uncertainties.

3 Problem Description

In this paper, we summarize our recent work on the development planning of an offshore oil and gas field infrastructure under complex fiscal rules and endogenous uncertainties. In particular, a multi-field site, $F = \{1,2,\ldots f\}$, with potential investments in the floating production storage and offloading (FPSO) facilities, $FPSO = \{1,2,\ldots fpso\}$ with continuous capacities and ability to expand them in the future is considered (Figure 4). The extension for including Tension Leg Platform (TLP) is straightforward but for simplicity we only consider FPSOs (Figure 5). These FPSO facilities cost multi-billion dollars depending on their sizes, and have the capability of operating in remote locations for very deep offshore oilfields (200m-2000m) where seabed pipelines are not cost effective. FPSOs are large ships that can process the produced oil and store it until it is shipped to the onshore site or sales terminal. Processing includes the separation of oil, water and gas into individual streams using separators located at these facilities. Each FPSO facility has a lead time between the construction or expansion decision, and its actual availability. The wells are subsea wells in each field that are drilled using drilling ships. Therefore, there is no need to have a facility present to drill a subsea well. The only requirement to recover oil from it is that the well must be connected to a FPSO facility.

The connection of a field to an installed FPSO facility and a number of wells need to be drilled to produce oil from these fields for the given planning horizon. The planning horizon is discretized into a number of time periods t, each with 1 year of duration. The location of each FPSO facility and its possible connections to the given fields are assumed to be known. Notice that each FPSO facility can be connected to more than one field to produce oil, while a field can only be connected to a single FPSO facility due to engineering requirements and economic viability of the project. The water produced with the oil is usually re-injected after separation, while the gas can be sold in the market. In this case, we consider natural depletion of the reserves i.e. no water or gas re-injection. The location of production facilities and possible field and facility allocation itself is a very complex problem. In this work, we assume that the potential location of facilities and field-facility connections are given. In addition, the potential number of wells in each field is also given. There are three main complexities in the problem considered here:

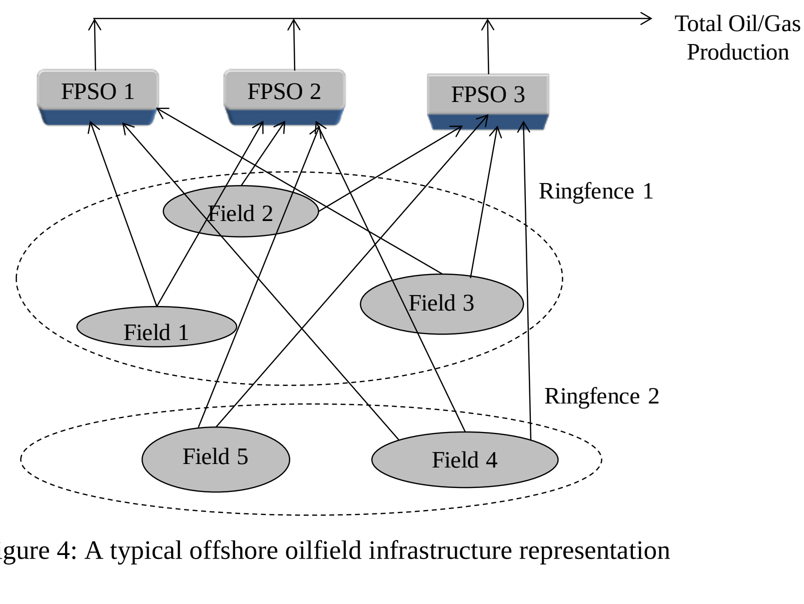
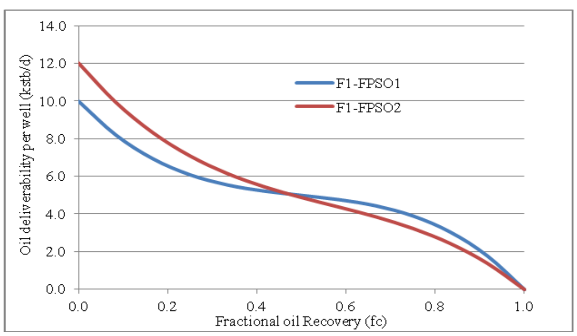


Figure 4: A typical offshore oilfield infrastructure representation

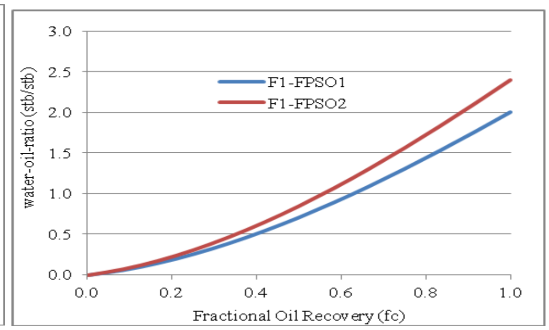
Figure 5: FPSO (Floating Production Storage and Offloading) facility

3.1 Nonlinear Reservoir Profiles

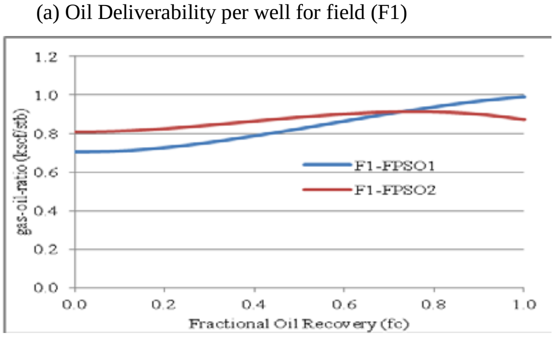
We consider three components (oil, water and gas) explicitly during production from a field. Field deliverability, i.e. maximum oil flowrate from a field, water-oil-ratio (WOR) and gas-oil-ratio (GOR) are approximated by a cubic equation (a)-(c) (see Figure 6), while cumulative water produced and cumulative gas produced from a field are represented by fourth order separable polynomials, eq. (d)-(e), that are derived in Gupta and Grossmann (2012a). The motivation for using polynomials for cumulative water produced and cumulative gas produced, eq. (d)-(e), as compared to WOR and GOR, eq. (b)-(c), is to avoid bilinear terms, eq. (f)-(g), in the formulation and allow converting the resulting model into an MILP formulation. All the wells in a particular field *f* are assumed to be identical for the sake of simplicity leading to the same reservoir profiles, eq. (a)-(g), for each of these wells.



(a) Oil Deliverability per well for field (F1)



(b) Water to oil ratio for field (F1)



(c) Gas to oil ratio for field (F1)

Figure 6: Nonlinear Reservoir profiles for field (F1) and 2 FPSOs (FPSO1- 2)

$$\hat{Q}_f^d = a_{1,f}(fc_{f,t})^3 + b_{1,f}(fc_f)^2 + c_{1,f}\, fc_f + d_1 \qquad \forall f \qquad \text{(a)}$$

$$w\hat{o}r_f = a_{2,f}(fc_f)^3 + b_{2,f}(fc_f)^2 + c_{2,f}\, fc_f + d_{2,f} \qquad \forall f \qquad \text{(b)}$$

$$g\hat{o}r_f = a_{3,f}(fc_f)^3 + b_{3,f}(fc_f)^2 + c_{3,f}\, fc_f + d_{3,f} \qquad \forall f \qquad \text{(c)}$$

$$\hat{w}c_f = a_{4,f}(fc_f)^4 + b_{4,f}(fc_f)^3 + c_{4,f}\, fc_f^2 + d_{4,f}\, fc_f \qquad \forall f \qquad \text{(d)}$$

$$\hat{g}c_f = a_{5,f}(fc_f)^4 + b_{5,f}(fc_f)^3 + c_{5,f}\, fc_f^2 + d_{5,f}\, fc_f \qquad \forall f \qquad \text{(e)}$$

$$w_f = w\hat{o}r_f . x_f \qquad \forall f \qquad \text{(f)}$$

$$g_f = g\hat{o}r_f . x_f \qquad \forall f \qquad \text{(g)}$$

3.2 Production Sharing Agreements

We consider progressive (sliding scale) production sharing agreements with ring-fencing provisions which are widely used in several countries (Figure 2). In particular, we assume a sliding scale profit oil share of the contractor linked to the cumulative oil produced (see Figure 3). Notice that this tier structure is a step function, which requires additional binary variables to model and makes the problem harder to solve. Moreover, the cost recovery ceiling is considered to be a fraction of the gross revenues in each time period t. For simplicity, the cost recovery ceiling fraction and income tax rates are assumed to be a fixed percentages (no sliding scale), and there are no explicit royalty provisions which is a straightforward extension. A set of ringfences *RF = {1,2,…}* among the given fields is specified (see Figure 4) to ensure that fiscal calculations are done for each ringfence separately (see Gupta and Grossmann, 2012b). Qualitatively, a typical ringfencing provision states that the investment and operational costs for a specified group of fields or block can only be recovered from the revenue generated from those fields or block. Notice that in general a field is associated to a single ringfence, while a ringfence can include more than one field. In contrast, a facility can be connected to multiple fields from different ringfences for producing oil and gas. These ringfences may or may not have the same fiscal rules.

3.3 Endogenous Uncertainties

We consider the uncertainty in the field parameters i.e. field size, oil deliverability per well, water-oil ratio and gas-oil ratio. These are the endogenous uncertain parameters since our investment and operating decisions affect the stochastic process (Jonsbraten et al., 1998; Goel and Grossmann, 2006; Tarhan et al., 2008; and Gupta and Grossmann, 2011). In particular, the uncertainty in the field parameters can only resolved when an investment is made in that field and we start producing from it. Therefore, our decisions determine the timing of the uncertainty realization i.e. decision-dependent uncertainty.

The average profile in Figure 7 represents the oil deliverability per well for a field as a nonlinear polynomial in terms of the fractional oil recovery (eq. (a)) under the perfect information. However, due to the uncertainty in the oil deliverability, the actual profile is assumed to be either lower or upper side of the average profile with a given probability. In particular, eq. (h) represents the oil deliverability per well for a field under uncertainty where parameter α_{oil} is used to characterize this uncertainty. For instance, if $\alpha_{oil} > 1$, then we have a higher oil deliverability than expected ($\alpha_{oil} = 1$), whereas for $\alpha_{oil} < 1$ a lower than expected oil deliverability is observed. Since, the uncertain field size is an inverse function of the fraction oil recovery, a higher field size will correspond to the low fractional oil recovery whereas a small field size will correspond to the higher factional oil recovery for a given amount of the cumulative oil production. Similarly, eqs. (i) and (j) correspond to the uncertain field profiles for water-oil-ratio and gas-oil-ratio that are characterized by the uncertain parameters α_{wor} and α_{gor}, respectively. Notice that since cumulative water produced (eq. (d)) and cumulative gas produced (eq. (e)) profiles are used in the model, instead of water-oil-ratio (eq. (b)) and gas-oil-ratio (eq. (c)), the uncertainty in the parameters α_{wor} and α_{gor} is transformed into the corresponding uncertainty in the parameters α_{wc} and α_{gc} as in eqs. (k) and (l), respectively (Gupta and Grossmann, 2014a).

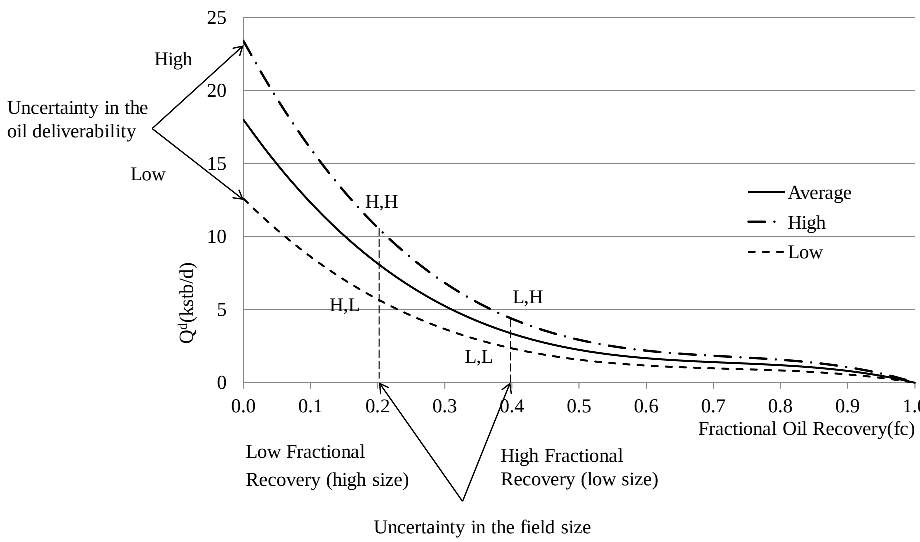
$$Q_f^d = \alpha_{oil} \cdot \hat{Q}_f^d \qquad \forall f \tag{h}$$

$$wor_f = \alpha_{wor} \cdot w\hat{o}r_f \qquad \forall f \tag{i}$$

$$gor_f = \alpha_{wor} \cdot g\hat{o}r_f \qquad \forall f \tag{j}$$

$$wc_f = \alpha_{wc} \cdot \hat{w}c_f \qquad \forall f \tag{k}$$

$$gc_f = \alpha_{wc} \cdot \hat{g}c_f \qquad \forall f \tag{l}$$



$$fc \propto \frac{1}{REC_f}$$

Figure 7: Oil deliverability per well for a field under uncertainty

Moreover, the uncertain parameters for every field i.e. $\theta_f = \{REC_f, \alpha_{oil}, \alpha_{wor}, \alpha_{gor}\}$ are considered to have a number of possible discrete realizations $\tilde{\theta}_f^k$ with a given probability. Therefore, all the possible combinations of these realizations yield a set of scenarios $s \in S^{\sup}$ where each scenario has the corresponding probability p^s. However, the total number of scenarios in set $S^{\sup}$ grows exponentially with the number of uncertain parameters and their possible realizations, which makes the problem intractable. To overcome this limitation, we assume that the uncertain parameters for a field $\theta_f = \{REC_f, \alpha_{oil}, \alpha_{wor}, \alpha_{gor}\}$ are correlated using practical considerations. Therefore, only a subset of the possible scenarios $S \subset S^{\sup}$ is sufficient to represent the uncertainty (Gupta and Grossmann, 2014a). We also assume that the uncertainty in the parameters of a field is resolved if we drill N_1

number of wells in that field and produce from it for a duration of N_2 years. These assumptions on uncertainty resolution rules are flexible and can be changed depending on the available field information.

The discrete planning horizon and a discrete set of the selected scenarios for each field with given probabilities can be represented by the scenario trees. However, since the timing of the uncertainty realization for a field (or its corresponding scenarios) depends on the drilling and operating decisions, the resulting scenario tree is decision-dependent. For instance, if we consider a set of two uncertain fields $F = \{1,2\}$ where each field has 4 uncertain parameters $\theta_f = \{REC_f, \alpha_{oil}, \alpha_{wor}, \alpha_{gor}\}$. The sampled scenario set based on the parameter correlations for each field has 2 elements i.e. $\{\tilde{\theta}_f^1, \tilde{\theta}_f^2\}$ with equal probability. Therefore, the problem involves only 4 scenarios $S = \{1:(\tilde{\theta}_1^1, \tilde{\theta}_2^1); 2:(\tilde{\theta}_1^1, \tilde{\theta}_2^2); 3:(\tilde{\theta}_1^2, \tilde{\theta}_2^1); 4:(\tilde{\theta}_1^2, \tilde{\theta}_2^2)\}$ each with a probability of 0.25. Figure 8 represents the scenario tree for this problem, where the uncertainty in the first field is resolved at the end of first year, since we drill N_1 wells in the field at the beginning of year 1 and produce from this field during that year (N_2 =1). The system can be in two states in year 2 depending on the value of the $\tilde{\theta}_1^k$. Similarly, uncertainty in the field 2 is resolved in year 4 under the scenarios 3 and 4 due to drilling and operating decisions, whereas it remains uncertain in the scenarios 1 and 2. Therefore, the resulting scenario tree depends on the optimization decisions, which are not known a priori, requiring to model the superstructure of the all possible scenario trees that can occur based on our decisions, and therefore, increase the complexity of the model.

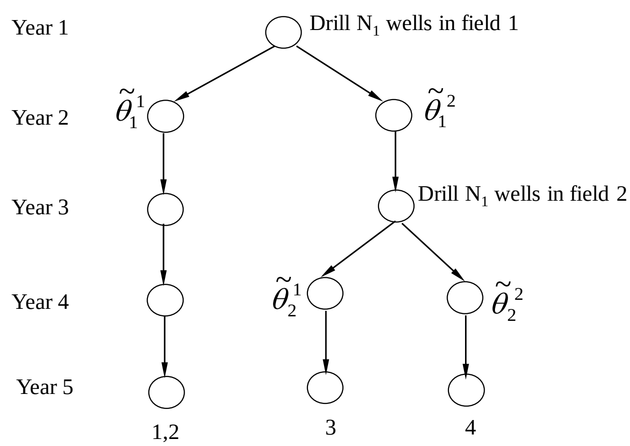


Figure 8: Decision-dependent scenario tree for two fields

The problem is to determine the optimal investment and operating decisions to maximize the contractor's expected NPV for a given planning horizon considering the above nonlinear profiles, production sharing agreements and endogenous uncertainties. In particular, investment decisions in each time period *t* and scenario s include FPSO facilities installation or expansion, and their respective installation or expansion capacities for oil, liquid and gas; fields-FPSO connections; and the number of wells that need to be drilled in each field *f* given the restrictions on the total number of wells that can be drilled in each time period *t* over all the given fields. Operating decisions include the oil/gas production rates from each field *f* in each time period *t* under every scenario s. It is assumed that the installation and expansion decisions occur at the beginning of each time period *t*, while operations take place throughout the time period. There is a lead time of l_1 years for each FPSO facility initial installation, and a lead time of l_2 years for the expansion of an earlier installed FPSO facility. Once installed, we assume that the oil, liquid (oil and water) and gas capacities of a FPSO facility can only be expanded once. These assumptions are made for the sake of simplicity, and both the model and the solution approaches are flexible enough to incorporate more complexities. In the Appendix A, we outline the basic oilfield development planning model proposed in Gupta and Grossmann (2012a) that relies on this description, which is further extended to include fiscal rules and/or uncertainties in the next sections.

4 Incorporating Fiscal Contracts in Oilfield Planning

In this section, we incorporate the complex fiscal rules in the MINLP/MILP models as described in Gupta and Grossmann (2012b) emphasizing the unified framework. Particularly, we consider the progressive production sharing agreements (PSAs) with ringfencing provisions (Figure 2) that is one of the most generic forms of fiscal rules. Notice that the motivation for including progressive PSAs with ringfencing provisions is to consider the key elements of the most of the available contracts. Particular fiscal rules of interest can be modeled as the specific case of this representation as derived in Gupta and Grossmann (2012b).

The objective function becomes to maximize the contractor's NPV which is the difference between total contractor's revenue share and total cost occurred over the planning horizon, taking discounting into consideration. The idea of cost recovery ceiling is included in terms of min function (m) to limit the amount of total oil produced each year that can be used to recover the capital and operational expenses. In particular, it represents the minimum of the cost oil recovery and a given fraction ($f_{rf,t}^{CR}$) of the revenues. Constraint (m) is further converted to mixed-integer linear constraint using big-M formulation (see Gupta and Grossmann, 2012b for details). This ceiling on the cost oil recovery is usually enforced to ensure early revenues to the government as soon as production starts.

$$CO_{rf,t} = \min(CR_{rf,t}, f_{rf,t}^{CR} \cdot REV_{rf,t}) \qquad \forall rf, t \qquad \text{(m)}$$

Moreover, a sliding scale based profit oil share of contractor linked to the cumulative oil production, is also included in the model (Figure 3). In particular, disjunction (n) is used to model this tier structure for profit oil split which states that the variable $Z_{rf,i,t}$ will be true if cumulative oil production by the end of time period t for ringfence rf lies between given tier thresholds $L_{rf,i}^{oil} \le xc_{rf,t} \le U_{rf,i}^{oil}$, i.e. tier i is active and split fraction $f_{rf,i}^{PO}$ is used to determine the contractor share in that time period for ringfence rf. The disjunction (n) in the model is further reformulated into linear and mixed-integer linear constraints using the convex-hull formulation. In addition, the total cost in each year need to be disaggregated for each ringfence separately for the fiscal calculations that further increase the complexity of the model (see Gupta and Grossmann, 2012b for details). The other constraints and features remain the same as the MINLP and MILP models described in the Appendix A.

$$\bigvee_i \begin{bmatrix} Z_{rf,i,t} \\ ConSh_{rf,t}^{beforetax} = f_{rf,i}^{PO} \cdot PO_{rf,t} \\ L_{rf,i}^{oil} \le xc_{rf,t} \le U_{rf,i}^{oil} \end{bmatrix} \qquad \forall rf, t \qquad \text{(n)}$$

The resulting MINLP/MILP models consider the trade-offs involved between investment and operating decisions and resulting royalties, profit shares that are paid to the government, and yields the maximum overall NPV for the contractor due to the improved decisions, as can be seen from the computational results in section 6.

To improve the computational efficiency of these models, the logic constraints (o) and (p) that defines the tier sequencing are included in the model to tighten its relaxation as proposed in Gupta and Grossmann (2012b). These constraints can be expressed as integer linear inequalities. In addition, the valid inequalities (q), are also included that bounds the cumulative contractor's share in the cumulative profit oil by the end of time period t based on the sliding scale profit oil share and cost oil that has been recovered (see Gupta and Grossmann, 2012 for derivation).

$$Z_{rf,i,t}^{s} \Rightarrow \bigwedge_{\tau=t}^{T} \neg Z_{rf,i',\tau}^{s} \qquad \forall rf, i, i' < i, t, s \qquad \text{(o)}$$

$$Z_{rf,i,t}^{s} \Rightarrow \bigwedge_{\tau=1}^{t} \neg Z_{rf,i',\tau}^{s} \qquad \forall rf, i, i' > i, t, s \qquad \text{(p)}$$

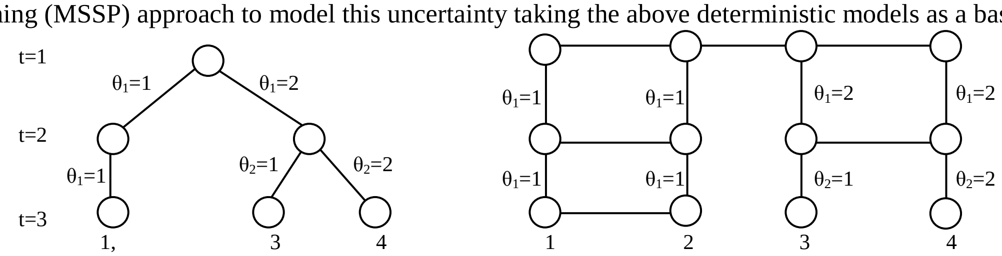
$$\sum_{\tau \le t} (Contsh_{rf,\tau}^{beforetax,s} / \alpha_\tau) \le \sum_{i'=1}^{i' \le i} (f_{rf,i'}^{PO} - f_{rf,i'-1}^{PO}) \cdot (xc_{rf,t}^{s} - L_{rf,i'}) - f_{rf,i^{end}}^{PO} \cdot \sum_{\tau \le t} (CO_{rf,\tau}^{s} / \alpha_\tau) \qquad \forall rf, i, t, s \qquad \text{(q)}$$

However, due to the presence of additional binary variables in the model for the disjunction (n) and a weak relaxation of the problem, the large-scale instances still become intractable. To solve these large instances involving fiscal contracts, Gupta and Grossmann (2012b) also proposed reformation technique for this fiscal model that does not rely on disjunction (n) and yields a valid upper bound and decisions to be fixed in the fullspace model to obtain a feasible solution. Moreover, a heuristic approach is also proposed that yields near optimal solution in reasonable time for the intractable instances of the problem.

Notice that the deterministic model with fiscal considerations presented here can also be used as the basis for the stochastic programming approaches explained in the next section to incorporate uncertainty in the model parameters under the unified framework. Optimal investment and operations decisions, the computational impact of adding a typical progressive production sharing agreement (PSA) terms and effectiveness of the proposed approaches is demonstrated in the results section with a small example.

5 Incorporating Endogenous Uncertainty in Oilfield Development Planning

In this section, we consider the oilfield development planning problem under endogenous uncertainties in the field parameters with/without fiscal terms as described in section 3. In particular, we use a multistage stochastic programming (MSSP) approach to model this uncertainty taking the above deterministic models as a basis.



(a) Standard Scenario Tree with uncertain parameters θ_1 and θ_2 (b) Alternative Scenario Tree

Figure 9: Tree representations for discrete uncertainties over 3 stages.

Given that the uncertainty can only be revealed once we invest in a field, the resulting scenario-tree is decision dependent (Figure 9-a). Therefore, an alternative representation of the scenario tree is used (Ruszczynski, 1997), where each scenario is represented by a set of unique nodes (Figure 9-b). The horizontal lines connecting nodes in time period *t*, mean that nodes are identical as they have the same information, and those scenarios are said to be indistinguishable in that time period. These horizontal lines correspond to the non-anticipativity (NA) constraints in the model that link different scenarios and prevent the problem from being decomposable. The alternative scenario tree representation allows to model the uncertainty in the problem more effectively (Goel and Grossmann, 2006; and Gupta and Grossmann, 2011).

5.1 Multistage Stochastic Formulation

The mixed-integer linear disjunctive multistage stochastic program with endogenous uncertainties for oilfield development problem using the alternative scenario-tree can be represented in the compact form **(MD)**. The detailed model and solution approaches are proposed in our recent paper, Gupta and Grossmann (2014a).

(MD)

$$\max \quad z = \sum_{s \in S} p^s \sum_{t \in T} c_t x_t^s \tag{1}$$

$$s.t. \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \qquad \forall t, s \tag{2}$$

$$x_t^s = x_t^{s'} \quad \forall t \in T_I, \forall s, s' \in S \tag{3}$$

$$Z_t^{s,s'} \Leftrightarrow F(x_1^s, x_2^s .... x_{t-1}^s) \quad \forall t \in T_C, \forall s, s' \in S \tag{4}$$

$$\begin{bmatrix} Z_t^{s,s'} \\ x_t^s = x_t^{s'} \end{bmatrix} \vee \begin{bmatrix} \neg Z_t^{s,s'} \end{bmatrix} \qquad \forall t \in T_C, \forall s, s' \in S \tag{5}$$

$$x_{jt}^s \in \{0,1\} \qquad \forall t, s, \forall j \in J' \tag{6}$$

$$x_{jt}^s \in R \qquad \forall t, s, \forall j \in J \setminus J' \tag{7}$$

The objective function (1) in the above model **(MD)** maximizes the expectation of an economic criterion such as NPV or total contractor share over the set of scenarios $s \in S$, and over a set of time periods $t \in T$. For a particular scenario *s*, inequality (2) represents constraints that govern decisions x_t^s in time period *t* and link decisions across time periods. Notice that these constraints correspond to the constraints (A2)-(A50) in the basic deterministic model we outlined in Appendix A. Moreover, in the case of the model with fiscal contracts, additional constraints as in section 4 need to be included (see Gupta and Grossmann, 2014a for details). Non-anticipativity (NA) constraints for initial time periods $T_I \subset T$ are given by equations (3) for each scenario pair *(s,s')* to ensure the same decisions in all the scenarios. The conditional NA constraints are written for the later time periods $T_C \subset T$ in terms of logic propositions (4) and disjunctions (5). Notice that the set of initial time periods T_I may include first few years of the planning horizon until uncertainty cannot be revealed, while T_C represents the rest of the time periods in the planning horizon. The function $F(x_1^s, x_2^s....x_{t-1}^s)$ in eq. (4) is an uncertainty resolution rule for a given pair of scenarios *s* and *s'* that determines the value of the corresponding boolean variable $Z_t^{s,s'}$ based on the decisions that have been implemented so far. The variable $Z_t^{s,s'}$ is further used in disjunction (5) to ensure the same decisions in scenarios *s* and *s'* if these are still indistinguishable in time period *t*. Eqs. (6)-(7) define the domain of the discrete and continuous variables in the model.

Notice that the model with reduced number of scenario pairs *(s,s')* that are sufficient to represent the non-anticipativity constraints can be obtained from model (MD) after applying the three properties presented in the paper by Gupta and Grossmann (2011). These properties are defined on the basis of symmetry, adjacency and transitivity relationship among the scenarios. The reduced model **(MDR)** can be formulated as follows, where P_3 is the set of minimum number of scenario pairs that are required to represent non-anticipativity in each time period *t*,

(MDR)

$$\min \quad z = \sum_{s \in S} p^s \sum_{t \in T} c_t x_t^s \tag{1}$$

$$s.t. \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \qquad \forall t, s \tag{2}$$

$$x_t^s = x_t^{s'} \qquad \forall t \in T_I, \forall (s, s') \in P_3 \tag{3a}$$

$$Z_t^{s,s'} \Leftrightarrow F(x_1^s, x_2^s....x_{t-1}^s) \qquad \forall t \in T_C, \forall (s, s') \in P_3 \tag{4a}$$

$$\begin{bmatrix} Z_t^{s,s'} \\ x_t^s = x_t^{s'} \end{bmatrix} \vee \begin{bmatrix} \neg Z_t^{s,s'} \end{bmatrix} \qquad \forall t \in T_C, \forall (s, s') \in P_3 \tag{5a}$$

$$x_{jt}^s \in \{0,1\} \qquad \forall t, s, \forall j \in J' \tag{6}$$

$$x_{jt}^s \in R \qquad \forall t, s, \forall j \in J \setminus J' \tag{7}$$

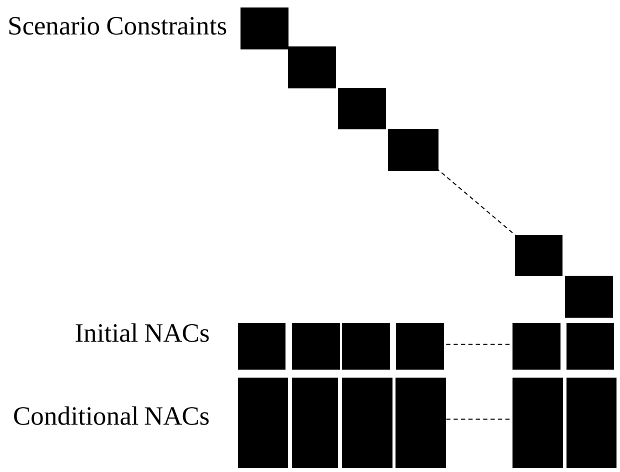


Figure 10: Structure of a typical Multistage Stochastic Program with Endogenous uncertainties

The mixed-integer linear disjunctive model (MDR) can further be converted to a mixed-integer linear programming model **(MLR)**. First, the logic constraints (4a) are re-written as the mixed-integer linear constraints eq. (4b) based on the uncertainty resolution rule, where $z_t^{s,s'}$ is a binary variable that takes a value of 1 if scenario pair *(s,s')* is indistinguishable in time period *t*, and zero otherwise. The disjunction (5a) can then be converted to mixed-integer linear constraints (5b) and (5c) using the big-M formulation. The resulting mixed-integer linear model (MLR) includes constraints (1), (2), (3a), (4b), (5b), (5c), (6) and (7).

$$B_t^s x_t^s + C_t^s z_t^{s,s'} \le d_t^s \qquad \forall t \in T_C, \forall (s,s') \in P_3 \tag{4b}$$

$$-M(1 - z_t^{s,s'}) \le x_t^s - x_t^{s'} \quad \forall t \in T_C, \forall (s,s') \in P_3 \tag{5b}$$

$$M(1 - z_t^{s,s'}) \ge x_t^s - x_t^{s'} \quad \forall t \in T_C, \forall (s,s') \in P_3 \tag{5c}$$

Figure 10 represents the block angular structure of model (MLR), where we can observe that the initial (eq. (3a)) and conditional (eqs. (4b), (5b) and (5c)) non-anticipativity constraints link the scenario subproblems (eq. (2)), i.e. these are the complicating constraints in the model. However, this structure allows decomposing the fullspace problem into smaller subproblems by relaxing the linking constraints. It should be noted that the NACs (especially conditional NACs) represent a large fraction of the total constraints in the model.

5.2 Solution Approach

The reduced model (MLR) is composed of scenario subproblems connected through initial (eq. (3a)) and conditional (eq. (4b), (5b) and (5c)) NA constraints. If these NA constraints are either relaxed or dualized using Lagrangean decomposition, then the problem decomposes into smaller subproblems that can be solved independently for each scenario within an iterative scheme for the multipliers as described in Caroe and Schultz (1999) and in Gupta and Grossmann (2011). In this way, we can effectively decompose the large scale problems in this class.

$$\textbf{(L1-MLR)} \qquad \max \quad \sum_{s \in S} p^s \sum_{t \in T} c_t x_t^s + \sum_{t \in T_1} \sum_{(s,s') \in P_3} \lambda_t^{s,s'} (x_t^s - x_t^{s'}) \tag{1a}$$

$$s.t. \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \qquad \forall t, s \tag{2}$$

$$x_{jt}^s \in \{0,1\} \qquad \forall t, s, \forall j \in J' \tag{6}$$

$$x_{jt}^{s} \in R \qquad \forall t, s, \forall j \in J \setminus J' \tag{7}$$

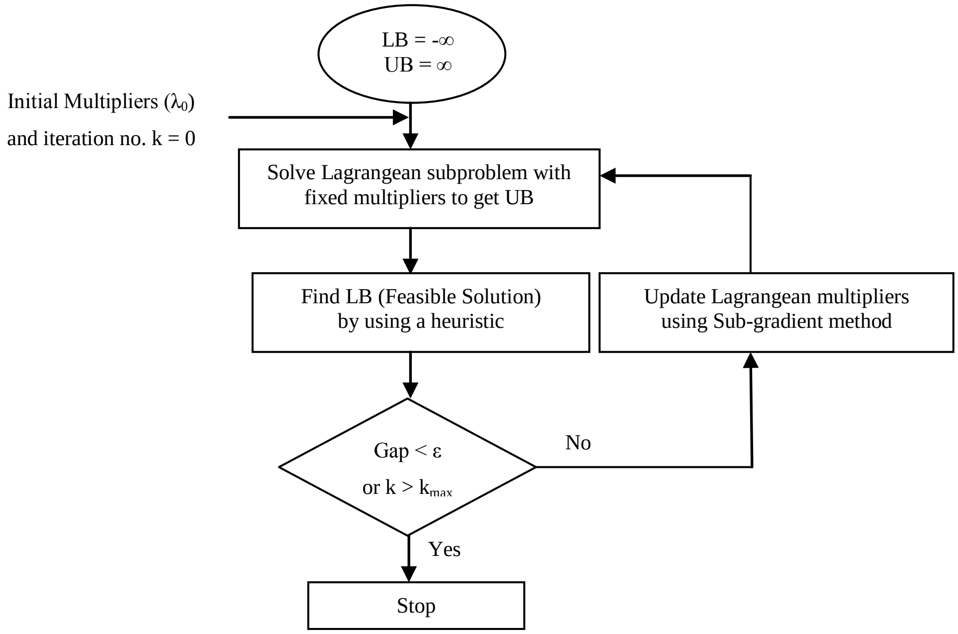


Figure 11: Lagrangean Decomposition algorithm

The Lagrangean decomposition algorithm of Figure 11 for MSSP with endogenous uncertainties as proposed in Gupta and Grossmann (2011) involves obtaining the upper bound (UB) by solving the Lagrangean problem **(L1-MLR)** with fixed multipliers $\lambda_t^{s,s'}$. Whereas, the Lagrangean problem (L1-MLR) is formulated from the mixed-integer linear reduced model (MLR) by relaxing all the conditional NA constraints (4b), (5b) and (5c) and dualizing all the initial NA constraints (3a) as penalty terms in the objective function. It gives rise to the subproblems for each scenario $s \in S$ **(L1-MLR^{s})** that can be solved in parallel.

(L1-MLR^{s})
$$\max \sum_{t \in T} p^s c_t x_t^s + \sum_{t \in T_1} x_t^s \Big(\sum_{\substack{(s,s') \in P_3 \\ s<s'}} \lambda_t^{s,s'} - \sum_{\substack{(s',s) \in P_3 \\ s>s'}} \lambda_t^{s',s} \Big) \tag{1b}$$

$$s.t. \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \qquad \forall t \tag{2a}$$

$$x_{jt}^{s} \in \{0,1\} \qquad \forall t, \forall j \in J' \tag{6a}$$

$$x_{jt}^{s} \in R \qquad \forall t, \forall j \in J \setminus J' \tag{7a}$$

The lower bound (LB) or feasible solution is generated by using a heuristic based on the solution of the Lagrangean problem (L1-MLR). In this heuristic, we fix the decisions obtained from the above problem (L1-MLR) in the reduced problem (MLR) such that there is no violation of NA constraints and solve it to obtain the lower bound. The sub-gradient method by Fisher (1985) or an alternative update scheme (see Mouret et al., 2011; Oliveira et al., 2013; and Tarhan et al. 2013) is used during each iteration to update the Lagrangean multipliers. The algorithm stops when either a maximum iteration/time limit is reached, or the difference between the lower and upper bounds, LB and UB, is less than a pre-specified tolerance. Notice that the extended form of this method relying on duality based branch and bound search has also been proposed in Goel and Grossmann (2006); Tarhan et al. (2009), and Tarhan et al. (2011) to close the gap between the upper and the lower bounds. Moreover, a special

Lagrangean decomposition algorithm that aggregates scenarios is proposed in Gupta and Grossmann (2014b) to further improve the quality of the dual bound at the root node.

6 Examples

In this section, we consider a variety of the examples for the oilfield development planning problem that covers deterministic, complex fiscal rules and stochastic features as discussed in the earlier sections.

6.1 Example 1: Deterministic Case

We first present an example of the oilfield planning problem assuming that there are no fiscal contracts and no uncertainty in the model parameters. In particular, we compare the computational results of the various deterministic MINLP and MILP models proposed in Gupta and Grossmann (2012a).

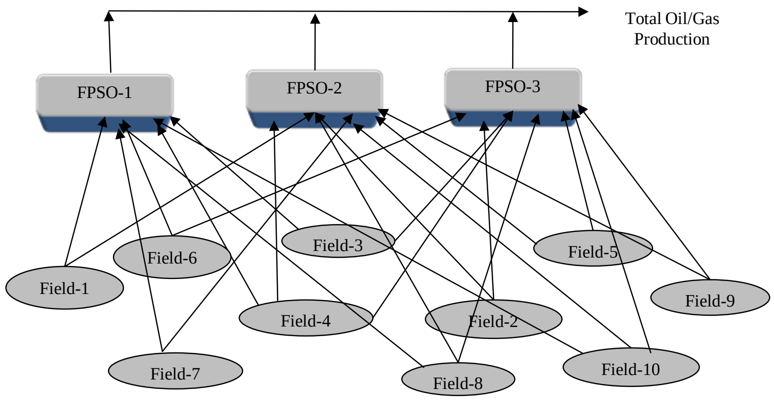
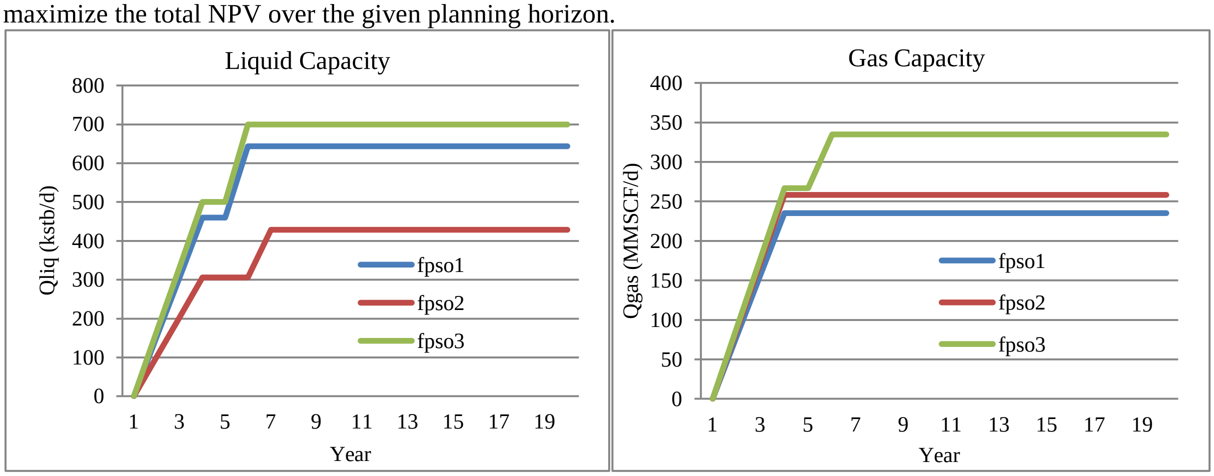


Figure 12: Deterministic Example 1 (10 Fields, 3 FPSOs, 20 years)

An oilfield infrastructure is considered with 10 oil fields (Figure 12) that can be connected to 3 FPSOs with 23 possible connections. There are a total of 84 wells that can be drilled in all of these 10 fields, and the planning horizon considered is 20 years, which is discretized into 20 periods of each 1 year of duration. The goal is to determine which of the FPSO facilities is to be installed or expanded, in what time period, and what should be its capacity of oil, liquid and gas, to which fields it should be connected and at what time, and the number of wells to be drilled in each field during each time period. Other than these installation decisions, there are operating decisions involving the flowrate of oil, water and gas from each field in each time period. The objective function is to maximize the total NPV over the given planning horizon.



(a) Liquid capacities of FPSO facilities *(b) Gas capacities of FPSO facilities*

Figure 13: FPSO installation and expansion schedule

The problem is solved using the DICOPT 2x-C solver for MINLP Models 1 and 2. These models were implemented in GAMS 23.6.3 and run on Intel Core i7 machine. The optimal solution of this problem that corresponds to the reduced MINLP Model 2-R solved with DICOPT 2x-C, suggests to install all the 3 FPSO facilities in the first time period with their respective liquid (Figure 13-a) and gas (Figure 13-b) capacities. These FPSO facilities are further expanded in the future when more fields come online or liquid/gas flow rates increases as can be seen from these figures. After initial installation of the FPSO facilities by the end of time period 3, these are connected to the various fields to produce oil in their respective time periods for coming online as indicated in Figure 14. The well drilling schedule for these fields in Figure 15 ensures that the maximum number of wells drilling limit and maximum potential wells in a field are not violated in each time period t. We can observe from these results that most of the installation and expansions are in the first few time periods of the planning horizon.

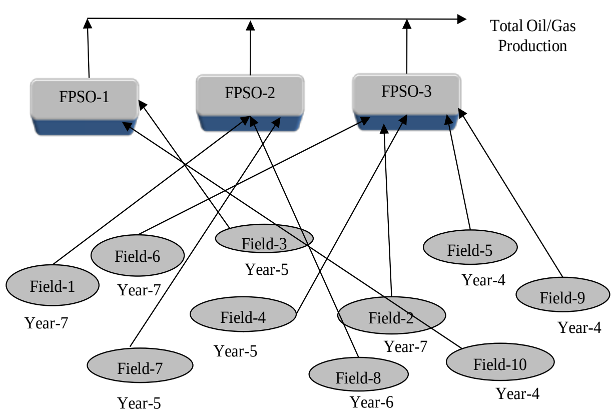


Figure 14: FPSO-field connection schedule

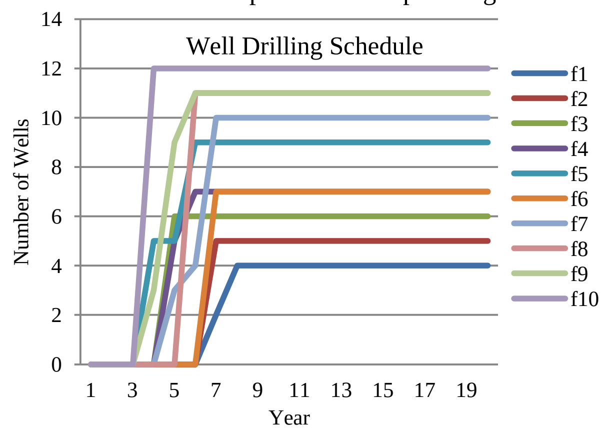
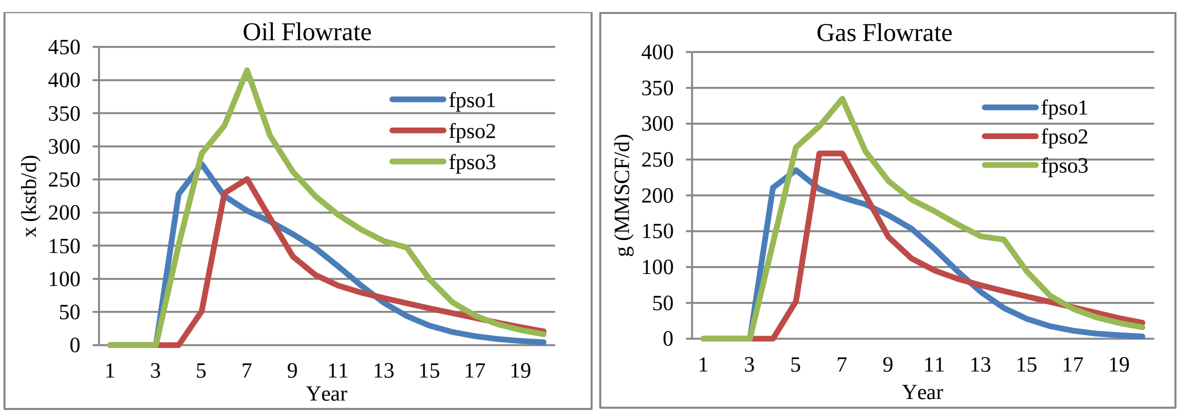


Figure 15: Well drilling schedule for fields



(a) *Total oil flowrates from FPSO's* (b) *Total gas flowrates from FPSO's*

Figure 16: Total flowrates from each FPSO facility

Table 1: Comparison of various models and solvers for Example 1

	Model 1		**Model 2**	
Constraints	5,900		10,100	
Continuous Var.	4,681		6,121	
Discrete Var.	851		851	
Solver	Optimal NPV (million$)	Time (s)	Optimal NPV (million$)	Time (s)
DICOPT	31297.94	132.34	30562.95	114.51
SBB	30466.36	4973.94	30005.33	18152.03
BARON	31297.94	>72,000	30562.95	>72,000

Other than these investment decisions, the operating decisions are the production rates of oil and gas from each of the fields, and hence, the total flow rates for the installed FPSO facilities that are connected to these fields as can be seen from Figures 16 (a)-(b). Notice that the oil flow rates increases initially until all the fields come online and then they start to decrease as the oil deliverability decreases when time progresses. Gas flow rate, which depends on the amount of oil produced, also follows a similar trend. The total NPV of the project is $30946.39M.

Tables 1-2 represent the results for the various model types (see Appendix A) considered for this example. DICOPT performs best in terms of solution time and quality compared to other MINLP solvers as can be seen from Table 1. There are significant computational savings with the reduced models as compared to the original ones for all the model types in Table 2. Even after binary reduction of the reformulated MILP, Model 3-R becomes expensive to solve, but yields global solutions, and provides a good discrete solution to be fixed/initialized in the MINLPs for finding better solutions. The optimal NPV that come from the Models 1 and 2 after fixing discrete variables based on MILP solution (even though it was solved within 10% of optimality tolerance) are $31329.81 M and $31022.48M, respectively. These are the best solutions among all other solutions obtained in Table 1 for the respective MINLPs. Therefore, while the MINLP model may sometimes lead to near optimal solutions, the MILP approximation is an effective way to consistently obtain these solutions. Notice also that the optimal discrete decisions for Models 1 and 2 are very similar even though they are formulated in a different way. However, only Model 2 can be reformulated into an MILP problem that gives a good estimate of the near optimal decisions to be used for these MINLPs.

Table 2: Comparison of models 1, 2 and 3 with and without binary reduction

	Model 1	**Model 1-R**	**Model 2**	**Model 2-R**	**Model 3-R**
Constraints	5,900	5,677	10,100	9,877	17,140
Continuous Var.	4,681	4,244	6,121	5,684	12,007
Discrete Var.	851	483	851	483	863
SOS1 Var.	0	0	0	0	800
NPV(million$)	31297.94	30982.42	30562.95	30946.39	30986.22
Time(s)	132.34	53.08	114.51	67.66	16295.26

*Model 1 and 2 solved with DICOPT 2x-C, Model 3 with CPLEX 12.2

Notice that although the advantage from using the MILP formulation in terms of the NPV value is not very apparent for this example since the required solution time is large, but it does yield a global solution that is difficult to rigorously obtain for the MINLPs. In addition, when we increase the complexity of the basic deterministic model to fiscal contracts and/or stochastic model, the advantage of MILP formulation becomes more apparent due to the availability of the robust MILP solvers compared to MINLP. In that case, the solutions from the local MINLP solvers are significantly sub-optimal and they cannot provide any guarantees of the valid upper bound in the Lagrangean decomposition unless each subproblem is solved to global optimality. Therefore, we use the MILP model for examples 2 and 3 that includes complex fiscal rules and/or uncertainties.

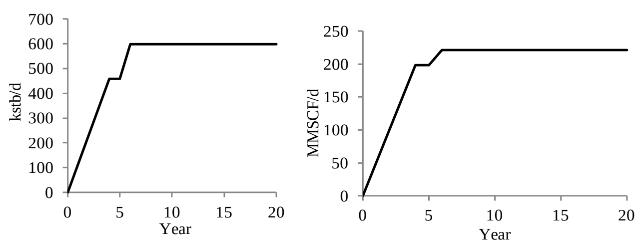
6.2 Example 2: Development planning with Complex Fiscal Rules

We demonstrate here the impact of adding a typical progressive production sharing agreement (PSA) terms for a deterministic case of the oilfield planning problem in terms of the investment and operating decisions, and the required computational effort. In this example, we consider 5 oilfields (Figure 4) that can be connected to 3 FPSOs with 11 possible connections, see Gupta and Grossmann (2012b) for details. There are a total of 31 wells that can be drilled in these 5 fields, and the planning horizon considered is 20 years. There is a cost recovery ceiling of 50% and 4 tiers (see Figure 3) that are defined for profit oil fraction between the contractor and host government based on the cumulative oil production but there are no ringfences present. The problem is solved to maximize the NPV of the contractor’s share after paying taxes and the corresponding optimal investment/operating decisions. Table 3 compares the performance of the MILP (Model 3F) involving detailed connections and reduced MILP model (Model 3RF) that are the extension of the deterministic Models 3 and 3R, respectively, with progressive PSAs. The models are implemented in GAMS 23.6.3 and run on Intel Core i7, 4GB RAM machine using CPLEX 12.2. We can observe that there is significant increase in the computational time with fiscal consideration for the MILP Model 3F, which takes more than 10 hours with a 14% of optimality gap as compared to the reduced MILP model (Model 3RF), which terminates the search with a 2% gap in reasonable time.

The optimal solution from Model 3RF suggests installing 1 FPSO facility (FPSO3) with expansions in the future (see Figure 17), while Figure 18 represents the well drilling schedule for this example. The tiers 2, 3 and 4 for profit oil split become active in years 6, 8 and 12, respectively, based on the cumulative oil production profile during the given planning horizon. Notice that the optimal solution of this problem does not develop field 1, which is not intuitive. The reason for not developing field 1 is that the size of the field 1 is quite small as compared to the other fields and the superstructure we consider does not allow connecting field 1 to FPSO 3, which is the only FPSO that is installed. Therefore, based on the superstructure and field size, it is not worth to install an additional FPSO to produce from this field after paying government share. In contrast, the solution from the simple NPV based optimization suggests exploring field 1 as well since it is worth in that case to install 2 small FPSO facilities and also produce from field 1 given that the trade-offs due to fiscal rules are neglected. Whereas, the total NPV of the contractor's share in this case is lower than the optimal solution of Model 3RF ($1,914.71M vs. $2,228.94M). Therefore, we can observe that incorporating fiscal terms within development planning can yield significantly different investment and operations decisions, and higher profit compared to a simple NPV based optimization.

Table 3. Computational Results for Example 2 (Model 3F vs. Model 3RF)

Model	**# of constraints**	**# of continuous variables**	**# of discrete variables**	**NPV ($Million)**	**Time (s)**	**Optimality Gap**
Model 3F	9,474	6,432	727	2,183.63	>36,000	<14%
Model 3RF	9,363	6,223	551	2,228.94	1,164	<2%



(a) Liquid capacity (b) Gas capacity

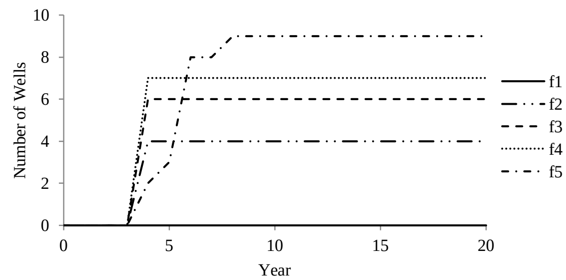


Figure 17. Optimal liquid and gas capacities of FPSO 3 (Example 2) Figure 18. Well drilling schedule (Example 2)

Note that fiscal terms without tier structure, for instance fixed percentage of profit share, royalty rates, often reduces the computational expense of solving the deterministic model directly without any fiscal terms instead. Surprisingly, the problem with flat 35% of the profit share of contractor is solved in 73s which is even smaller than the solution time for deterministic case without any fiscal terms (190s). On the other hand, the problem with 2 tiers instead of 4 as considered above is solved in 694s which is more than the model without fiscal terms and less than the model with 4 tiers as can be seen in Table 4. Therefore, the increase in computational time while including fiscal rules within development planning, is directly related to the number of tiers (levels) that are present in the model to determine the profit oil shares or royalties.

Table 4. Comparison of number of tiers vs. solution time for Model 3RF

# of tiers	**Time (s)**
4	1,164
2	694
1	73
No fiscal rules	190

Table 5 compares the further improvements in the solution time for Model 3RF (1,164s) after using the reformulation/approximation techniques and strategies that are proposed in Gupta and Grossmann (2012b), and reviewed in section 4. In particular, the tighter formulation Model 3RF-L that is obtained after including logic constraint (o-p) and valid inequalities (q) is solved in one fourth of the time than Model 3RF. Notice that these MILP models are solved with a 2% optimality tolerance yielding a slightly different objective values for Model 3RF and Model 3RF-L. Model 3RI, which relaxes the disjunction (n), can be solved more than 20 times faster than the original Model 3RF. Although the solution obtained is a relaxed one (upper bound of 2,591.10), it gives the optimal investment decisions that result in the same solution as we obtained from solving Model 3RF directly. The approximate version of this Model 3RI-A, takes only 82s as compared to Model 3RF (1164s) and yields the optimal solution after we fix the decisions from this model in the original one. Notice that the quality of the approximate solution itself is very good (~1.5% accurate) and both relaxed/approximate models are even ~3 times faster than the model without any fiscal terms (Model 3R) that takes 190s. Therefore, the approximate model can be use as a good heuristic to solve the large instances of the problem.

Table 5. Results for Example 2 after using various solution strategies

Model	# of constraints	# of continuous variables	# of discrete variables	NPV ($Million)	NPV after fixing decisions in Model 3RF ($Million)	Time (s)
Model 3RF	9,363	6,223	551	2,228.94	-	1,164
Model 3RF-L	11,963	6,223	551	2,222.40	-	275
Model 3RI-A	8,803	5,903	471	2,197.63	2,228.94	82
Model 3RI	8,803	5,903	471	2,591.10	2,228.94	48

In addition, when we consider two ringfences for this example 2 (see Figure 4) where progressive PSA terms are defined for each of these ringfences separately. Based on the computational performance of the Model 3RF as compared to Model 3F, we only show the results for Model 3RF, which is more efficient. Table 6 compares the results for various models for this oilfield development example involving ringfencing provisions. We can observe that including ringfencing provisions makes Model 3RF expensive to solve (>10 hrs), compared to the previous instance without any ringfences that required only 1,164s. This is due to the additional binary variables that are required in the model for each of the two ringfences, their trade-offs and FPSO cost disaggregation. In contrast, since Models 3RI and 3RI-A do not need binary variable for the sliding scale in disjunction, they solve much faster than Model 3RF (>300 times faster) and Model 3RF-L (~30 times faster). Notice that even after including ringfencing provisions, these two models are faster than the simple NPV based Model 3R. This is due to the trade-off from the fiscal part in the simple NPV based model without binary variables for the sliding scale. Notice that Model 3RI and 3RI-A are solved here in one of the most generic forms of the fiscal terms where the solutions may not be the global optimal, but the relaxed Model 3RI, which provides a valid upper bound, also allows to compare the solution quality. The optimal NPV after ringfencing provisions is lower as compared to the one without ringfencing provisions due to the additional restrictions it imposes on the revenue and cash flows.

Table 6. Results for Example 2 with ringfencing provisions

Model	# of constraints	# of continuous variables	# of discrete variables	NPV ($Million)	NPV after fixing decisions in Model 3RF ($Million)	Time (s)	Optimality gap (%)
Model 3RF	14,634	9,674	651	2,149.39	-	>36,000	<15.4%
Model 3RF-L	19,834	9,674	651	2,161.27	-	3,334	<2%
Model 3RI-A	13,514	9,034	491	2,148.90	2,142.75	134	<2%
Model 3RI	13,514	9,034	491	2,533.06	2,151.75	112	<2%

6.3 Example 3: Stochastic Case

In this example, we consider the planning of offshore oilfield under decision-dependent uncertainty in the field parameters, which resolves as a function of investment and operating decisions as described in section 5. Moreover, we also extend this MSSP example to include the complex fiscal rules.

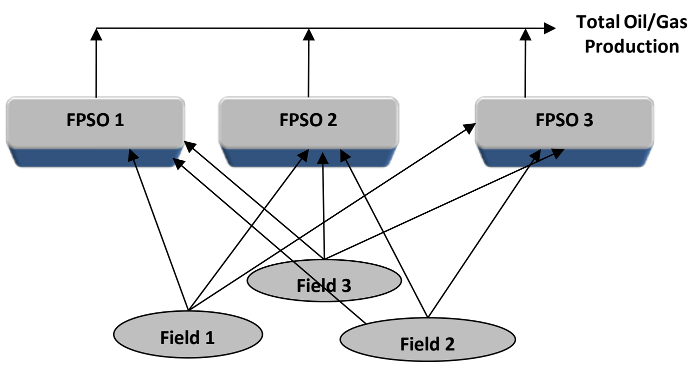


Figure 19: Stochastic Example 3 (3 oilfields, 4 Scenarios)

We consider the multistage stochastic MILP model by Gupta and Grossmann (2014a) as outlined in section 5 for maximizing the expected NPV in the development planning of an offshore oilfield, which is an extension of the deterministic MILP model (Model 3) presented in Gupta and Grossmann (2012a). The model for this oilfield planning example are implemented in GAMS 23.6.3 and run on Intel Core i7, 4GB RAM machine using CPLEX 12.2 solver. In particular, we consider 3 oilfields, 3 potential FPSO's and 9 possible connections among field-FPSO (Figure 19). A total of 30 wells can be drilled in the fields and the planning horizon is 10 years. Field 3 has a recoverable oil volume (field size) of 500 MMbbls. However, there is uncertainty in the size of fields 1 and 2 where each one has two possible realizations (low, high) with equal probability. Therefore, there are a total of 4 scenarios each with a probability of 0.25 (see Table 7). The problem is to determine the investment (FPSO installations and expansions, field-FPSO connections and well drilling) and operating decisions (oil production rate) for this infrastructure with the objective to maximize the total expected NPV (ENPV) over the planning horizon.

Table 7: Stochastic Example 3 (3 oilfields, 4 Scenarios)

Scenarios	s1	s2	s3	s4
Field 1 Size (MMbbls)	57	403	57	403
Field 2 Size (MMbbls)	80	80	560	560
Scenario Probability	0.25	0.25	0.25	0.25

The optimal ENPV for this problem is \$11.50 x10^9 when the reduced model (MLR) is solved in fullspace, and requires 1184s. Table 8 represents the model statistics for this example. The solution suggests installing only FPSO 3 in the first year (see Figure 20) and when the facility is available to produce at the beginning of year 4 due to lead time, 3, 5 and 12 wells are drilled in field 1,2 and 3, respectively. Since, field 1 and 2 are uncertain, therefore based on the realization of the uncertainty in their field sizes, more wells are drilled in these fields in the future for the favorable scenarios compared to the unfavorable outcomes, whereas no more wells are drilled in field 3. We can observe that the optimal scenario-tree is decision-dependent which is not known a-priori (Figure 20).

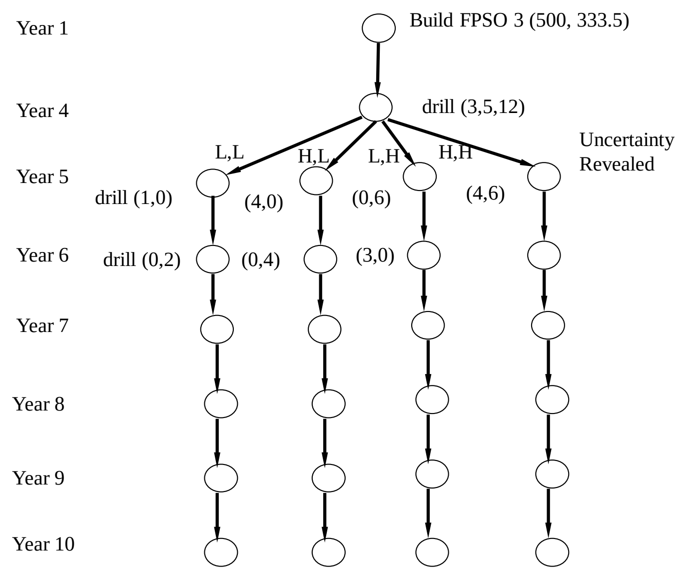


Figure 20: Optimal solution for 3 oilfield example (Example 3)

Table 8: Model statistics for the 3 oilfield example (Example 3)

Problem Type	Number of Constraints	Continuous Variables	Discrete Variables	SOS1 Variables
Reduced Model (MLR)	16,473	9,717	876	240
Individual Scenario	3,580	2,390	179	60

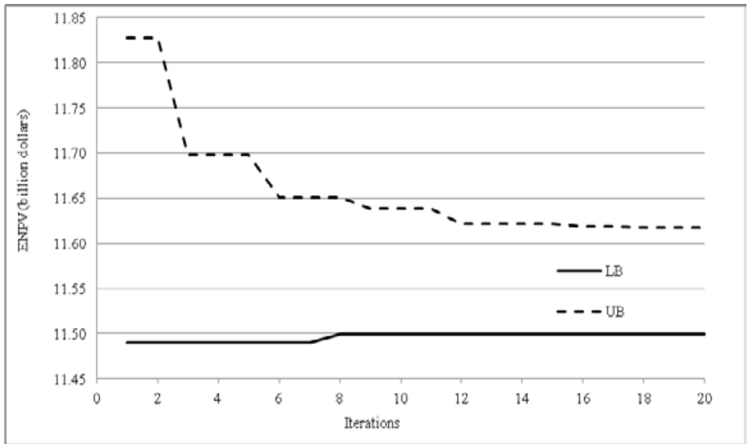


Figure 21: Lagrangean decomposition results for 3 oilfield example (Example 3)

Figure 21 demonstrates the progress of the bounds obtained at the root node using Lagrangean decomposition described in section 5 that relies on dualizing the initial NACs and removing the conditional NACs. A termination criteria of either 1% gap or 20 iterations is used. We can observe that the problem can be solved to ~1% optimality tolerance in only 466s compared to the fullspace model that takes 1184s. However, the solution of the expected value problem considering the mean value of the field sizes is \$11.28 x10^9. Therefore, the value of stochastic solution for this case is ~2% and the solution is more robust as compared to the deterministic approach (Gupta and Grossmann, 2014a).

In addition, we also consider the extension of this 3 oilfield example to the case where we include the progressive production sharing agreements with 15 years of planning horizon, and uncertain field sizes (field 1 and 2) having a total of 4 scenarios as described above. The resulting fullspace model becomes very difficult to solve (see Table 9). In particular, the best solution obtained after 10 hours is \$2.97x10^9 with more than 21% of optimality gap. On the other hand, Lagrangean decomposition can solve this problem in approximately 2 hrs for sequential implementation of the scenario subproblem solutions and in about 1 hr for parallel implementation, and yields a higher ENPV \$3.04x10^9 within 0.7% optimality tolerance.

Table 9: Multistage stochastic instance with fiscal rules for 3 oilfield example (Example 3)

Fullspace Model					Lagrangean Decomposition		
# Constraints	# Dis. Var.	# Cont. Var.	ENPV (\$$10^9$)	Time (s)	ENPV (\$$10^9$)	Sequential Time (s)	Parallel Time (s)
27,113	1,536	15,857	\$2.97 (>21%)	>36,000	\$3.04 (0.7%)	8,990	4,002

Therefore, the importance of the decomposition algorithm, especially parallel solution of the scenario subproblems increases as more complexities are added to the deterministic problem such as fiscal contracts. Several multistage stochastic instances of the problems in this class with/without fiscal contracts involving many scenarios are presented in Gupta and Grossmann (2014a). In addition, an improved Lagrangean decomposition approach that yields a tighter dual bound at the root node has also been proposed in our work (Gupta and Grossmann, 2014b).

7 Conclusions

In this paper, we have addressed oil and gas field development problems with a particular focus on the type of infrastructure, fiscal rules and uncertainties. We first outlined the recently proposed deterministic multi-period MINLP model by Gupta and Grossmann (2012a) for offshore oil and gas field infrastructure investment and operational planning. The model considers realistic details to be useful in practice such as multiple fields, three components (oil, water and gas), facility expansions decisions, well drilling schedules and nonlinear reservoir profiles. The reformulation of this model without bilinear terms and its further conversion into an MILP formulation that allow solving the problem to global optimality has also been summarized. Furthermore, a unified modeling framework has been presented on the extensions of this basic deterministic model to incorporate fiscal rules (Gupta and Grossmann, 2012b) and/or endogenous uncertainties (Gupta and Grossmann, 2014a). Numerical results on development planning problems with and without fiscal contracts and uncertainties are included to illustrate the efficiency of the models and solution approaches that are proposed in afore-mentioned papers. It is hoped that this paper has shown that there has been very significant progress in the mathematical programming models and solution algorithms for offshore oilfield development planning. In the future, these approaches can be used as a basis to incorporate additional complexities such as uncertain oil/gas prices.

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