Article

Synthesis of heat-integrated water networks using a modified heat exchanger network

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Abstract: This work presents the synthesis of heat-integrated water networks (HIWNs) by using mathematical programming. A new superstructure is synthesised combining a water network and a modified heat exchanger network. Based on the proposed superstructure, a mixed-integer nonlinear programming (MINLP) model is developed. The model is solved by using a one-step solution strategy enabling different initialisations and generation of multiple solutions, from which the best one is chosen. The results show that the proposed model can be effectively used for solving HIWN problems of different complexity, including large-scale problems.

Keywords: water network; heat integration; heat-integrated water network.

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1. Introduction

Sustainable utilisation of natural resources, including water and energy, is very important in our daily life as well as within the industrial sector. This sector uses large amounts of these resources and generates waste streams and emissions discharged into the environment. The increased consumption of natural resources, their future scarcity, greenhouse gas emissions and environmental pollution will influence the environment and climate change. It is necessary to be in line with the adopted targets and sustainable development goals to contribute to the climate and energy framework to achieve economic efficiency and environmental sustainability.

Water and energy are used for various purposes within industrial processes. The minimisation of consumption of these resources can be achieved by systematically exploring their interconnections in combined water and energy networks or heat-integrated water networks (HIWNs). Various systematic methods (conceptual, mathematical programming and their combinations) have been used to achieve this goal. The conceptual methods are based on pinch analysis (PA), and mathematical programming (MP) methods are based on superstructure optimization. These methods have been applied to minimise water and energy consumption in various processes. The MP methods compared to PA methods can better address HIWNs, including large-scale problems, multiple contaminants, multiple freshwater sources, and water and heat losses and gains. MP methods have been successfully applied to find the best trade-offs between investment and operating costs in HIWNs. A work by Budak Duhbacı *et al.* [1] presented a review of papers considering water and energy minimisation and the improvements achieved in different industrial processes by applications of MP methods. Ahmetović *et al.* [2] presented a review of systematic methods (PA and MP) along with the results of case studies in Kraft pulp mills and reported typical savings in freshwater and energy consumption. The reader is also referred to review papers by Zhang *et al.* [3], Kermani *et al.* [4], and Ahmetović *et al.* [5] for more information about various methods used for solving HIWN problems.

The focuses of several papers have been related to proposing superstructures, models and efficient solution strategies that can be used for solving various HIWN problems including large-scale problems. Leewongtanawit and Kim [6] proposed the simultaneous design methodology using an MINLP model and solved a large-scale HIWN problem with multiple contaminants and multiple freshwater sources of different temperatures and quality. Yang and Grossmann [7] proposed the simultaneous water and heat targeting model combining water network with heat targeting model [8] for the flowsheet optimisation. This targeting model can be applied to HIWN problems including large-scale problems. Liu *et al.* [9] proposed a generalised disjunctive programming model (GDP) for the simultaneous water and energy integration in HIWNs. This model enables the trade-offs between freshwater and utility consumption, and non-isothermal mixing reducing the complexity of the subsequent heat exchanger network (HEN) design. Ibrić *et al.* [10] proposed a compact superstructure and an MINLP model for the simultaneous synthesis of HIWNs. The proposed heuristic rules are used for superstructure simplification. The proposed model is solved using a two-step solution strategy including targeting and design steps. Hong *et al.* [11] proposed a systematic simultaneous optimisation approach including a three-step solution strategy that can be used for solving industrial large-scale HIWN problems. In this strategy, a combination of NLP and MINLP models is used to design the HIWN. Ibrić *et al.* [12] proposed a modified compact superstructure of their previous work [10], an MINLP model, and a three-step solution strategy for solving large-scale problems of HIWNs. The modified superstructure includes a simplified heat integration block enabling non-isothermal mixing and indirect heat exchange within an overall water network with reduced complexity. Recently, Dong *et al.* [13] proposed a superstructure considering splitters and mixers for every water-using operation and heat exchange unit to simultaneously design HIWN. The results are presented for a different number of heat exchange units starting from the minimum number of heat exchange units and increasing this number in each iteration.

Based on the published papers, it can be concluded that the vast majority of them have considered small and medium scale HIWN problems, and only several works addressed large-scale HIWN problems. Also, based on the results reported in the literature for HIWN problems (small, medium, and large-scale), it can be concluded that the proposed optimal designs have a relatively small number of heat exchangers compared to traditional HEN designs in which hot and cold process water streams are involved in heat integration. In most cases, it is less than ten heat exchangers in optimal HIWN designs reported in the literature. One of the reasons is that hot and cold water streams in HIWNs can be involved in indirect heat transfers through heat exchangers as well as direct heat transfers by mixing various water streams. Accordingly, due to the mixing of various hot and cold water streams (direct heat transfer), the number of heat exchangers in a final HIWN design can be significantly smaller compared to a classical HEN design in which hot and cold process streams are mainly involved in an indirect heat integration.

This work presents a new superstructure and a simultaneous optimisation MINLP model of HIWN, which can be solved for a given limiting number of heat exchangers. In this model, a water network is combined with a modified HEN model instead considering a stage-wise HEN superstructure model [14,15] to reduce the overall network complexity regarding the number of heat exchangers and consider additional opportunities for splitting and mixing of hot and cold water streams and various patterns of heat exchangers (serial, parallel and their combination) and bypasses. The MINLP model is solved by a one-step solution strategy to minimise the total annualised cost (TAC) of the overall HIWN. This strategy also enables a random generation of initial points, an iterative solving of the MINLP model, and the generation of multiple solutions from which the best one can be chosen. The results obtained for various HIWN problems including large-scale problems are in good agreement with the reported literature results.

2. Materials and Methods

2.1. Problem formulation

Process industries usually require process water that is used in different process operations (process water-using units). Process operations require water of different temperatures and quality and water within process operations gets contaminated in contact with process streams. The wastewater leaving process is discharged into the environment concerning specified temperature constraints. Figure 1 shows a generic representation of the HIWN problem. The synthesis problem of HIWN to be solved is stated as follows. The generic process consists of process water-using units ($p \in P$) requiring water of different quality and temperature. Within unit p water gets contaminated in direct contact with a process stream and thus the contaminant ($c \in C$) is transferred to the water stream. The water is supplied from external water sources as freshwater ($s \in S$). Freshwater sources can have different temperatures and contaminants concentrations. Also, water can be reused from the internal water sources (from process unit P_1 to process unit P_p and otherwise) within the water network (WN). To supply water at required temperatures of process units p, water heating and cooling is required within the HEN. The HEN consists of heat exchangers ($e \in E$) enabling the maximum heat recovery between hot and cold water streams and thus the minimum utility consumption at the specified exchanger minimum approach temperature (EMAT). The hot and cold utility is available for additional heating and cooling. The wastewater is discharged into the environment at the specified temperature. It is required to determine the optimum design of the HIWN to minimise the TAC. The network design should include the optimal connections between freshwater sources, process water-using units and heat exchangers.

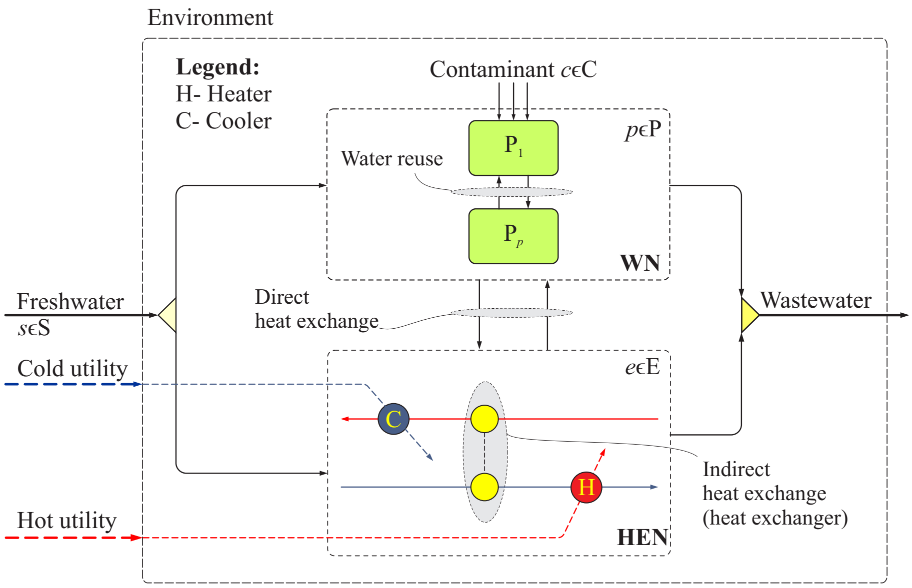

Figure 1. The schematic representation of the HIWN problem.

2.2. Motivation Example

As a motivation example, a simple problem is demonstrated consisting of two process water-using units and a single contaminant. Freshwater is at temperature 20 °C and wastewater is discharged into the environment at 30 °C. Data for this example are presented in Table 1. A mass load of contaminant c in the process unit p ($L_{p,c}$), the maximum inlet $x_{p,c}^{(in,max)}$ and outlet $x_{p,c}^{(out,max)}$ concentration of contaminant c and operating temperature t_p of the process water-using unit p are given. Cost data required for this problem and all other problems solved in this paper are the same. If water integration is not considered, freshwater is used in P_1 and P_2 in total amount of 87.5 kg/s. Water required by water-using units is at 100 °C and 75 °C and heating of freshwater is necessary if no heat integration is considered. Also, to satisfy the temperature constraints of the wastewater, cooling of streams leaving process water-using units is required. Thus, the total amount of hot and cold utility is 25,462.5 kW and 21,787.5 kW. The basic design without water and heat integration is given in Figure 2.

Table 1. Process water-using units data for motivation example.

Unit	$L_{p,c}$ (g/s)	$x_{p,c}^{(in,max)}$ (ppm)	$x_{p,c}^{(out,max)}$ (ppm)	t_p (°C)
P_1	5	50	100	100
P_2	30	50	800	75

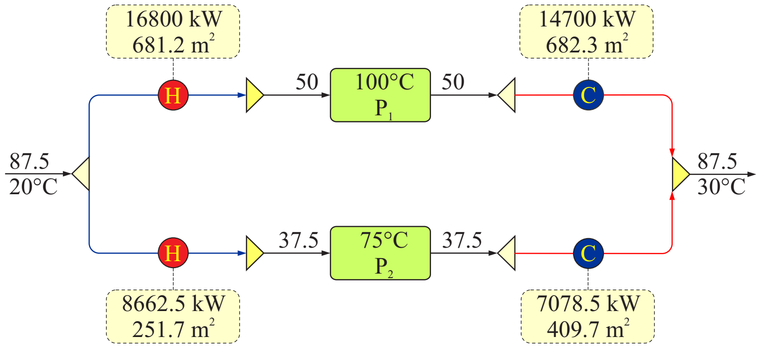

Figure 2. Base case for the motivation example without water and heat integration.

To reduce freshwater use, water reuse opportunities between process water-using units need to be considered. The water leaving P_1 with contaminant concentration 100 ppm can be partially reused in P_2 when mixed with freshwater to reduce contaminant concentration to 50 ppm required at the P_2 inlet. It is evident from Figure 2 that the energy of hot streams leaving process units can be used in a certain amount, depending on the temperature approach, to preheat cold freshwater streams. Thus, the heat integration opportunities can also be considered to maximise heat recovery. Performing optimisation of HIWN using mathematical programming by minimising the TAC, an optimal design shown in Figure 3 can be obtained with reduced freshwater consumption (70 kg/s vs. 87.5 kg/s). The Freshwater consumption is reduced due to water reuse $P_1 \rightarrow P_2$ in the amount of 20 kg/s. The utilities consumption is significantly lower for hot (2940 kW vs. 25,462.5 kW) and cold utility (0 kW vs. 21,787.5 kW) due to heat integration. The energy of the effluent stream (14,112 kW) is utilised to preheat freshwater from 20 °C to 68 °C. Also, the energy of stream leaving P_1 (3780 kW) is used to additionally preheat freshwater from 68 °C to 86 °C. Hot utility (2940 kW) is required to achieve the operating temperature of P_1 (100 °C). An investment cost of heat exchangers for the integrated design is somewhat higher (248,188 $/y vs. 229,751 $/y) compared to the base case due to higher heat exchange areas as a result of lower temperature difference at the hot and cold side of heat exchangers. The TAC of the integrated design is 2,112,569 $/y obtained by the later proposed model and solution approach. The solution obtained in the literature [16] by using the MINLP model (538 equations, 427 continuous variables, 92 discrete variables) proposed by Ahmetović and Kravanja [17] and global optimisation solver BARON exhibits the same network design. The proposed model for this example consist of 118 single equations, 228 continuous variables and 9 discrete variables.

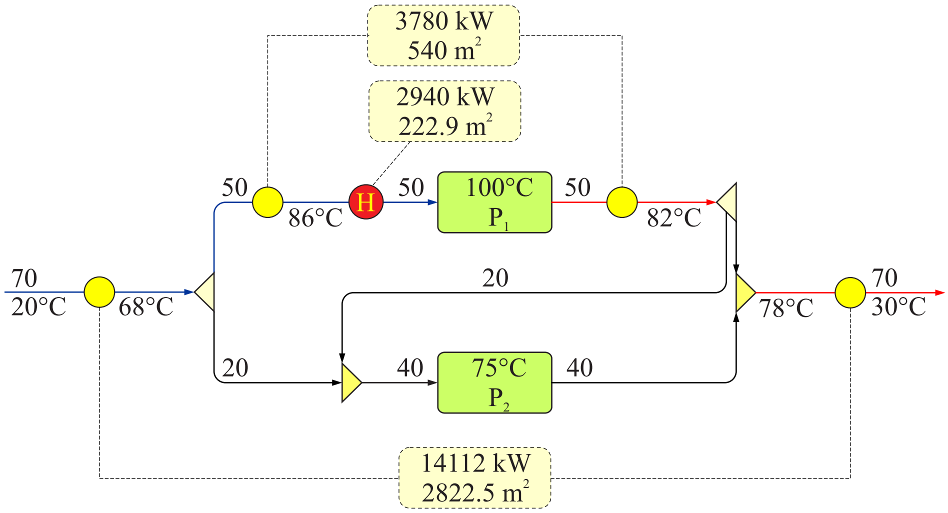

Figure 3. Optimal design of HIWN.

2.3. *Superstructure*

Figure 4 shows a generic design of the proposed HIWN superstructure. Superstructure consist of:

- freshwater sources $s \in S = \{S_1, S_2, \ldots S_s\}$;
- proces water-using units $p \in P = \{P_1, P_2, \ldots P_p\}$;
- heat exchanger units $e \in E = \{E_1, E_2, \ldots E_e\}$ with the corresponding heaters and coolers;
- wastewater discharge into the environment.

Elements of the superstructure within each of the blocks consist of mixers and/or splitters enabling their connections within and between the blocks. These connections enable multiple options for water reuse. Options for heat exchange include direct heat exchange through the non-isothermal mixing as well as indirect heat exchange within heat exchangers, heaters and coolers. To each heat exchanger $e \in E$, one heater and one cooler are assigned for additional heating and cooling of water streams.

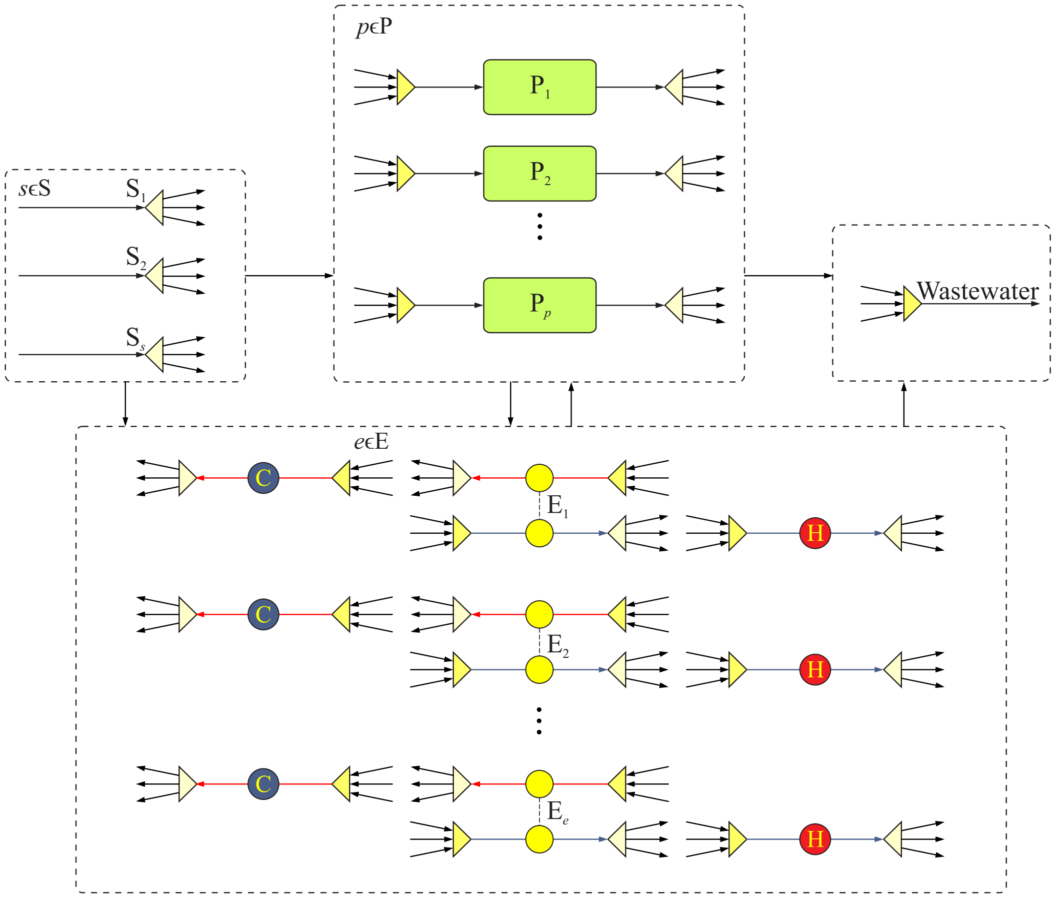

Figure 4. The proposed HIWN superstructure.

2.4. Model

This section describes the proposed model for the HIWN superstructure, shown in Figure 4. The model consists of the mass and heat balance of the WN and HEN superstructure elements as well as additional heat transfer constraints within the HEN. The model is developed with the following assumptions:

- Temperature and contaminant concentrations of the freshwater source s are constant;
- The temperature at the inlet/outlet of process unit p is constant;
- The specific heat capacity of water streams is constant;
- The flowrate of water through process unit p is constant under the assumption that the mass of contaminants in the water streams is much lower that the overall mass of the water stream;
- Process unit p operates at constant mass loads of contaminants transferred to the water streams;
- Constant individual heat transfer coefficients of hot and cold water streams and utilities;
- Heat losses to the environment are neglected;
- Single hot and cold utility is available at constant inlet/outlet temperatures;
- Counter-current heat exchange is assumed.

Freshwater splitter

The freshwater splitter with the corresponding streams is shown in Figure 5. The mass balance of the freshwater splitter $s \in S$ is given by Equation (1).

$$m_s = \sum_p m_{s,p} + \sum_e m_{s,e}^{(hs)} + \sum_e m_{s,e}^{(cs)} + \sum_e m_{s,e}^{(H)} + \sum_e m_{s,e}^{(C)} \quad , \forall s \in S \tag{1}$$

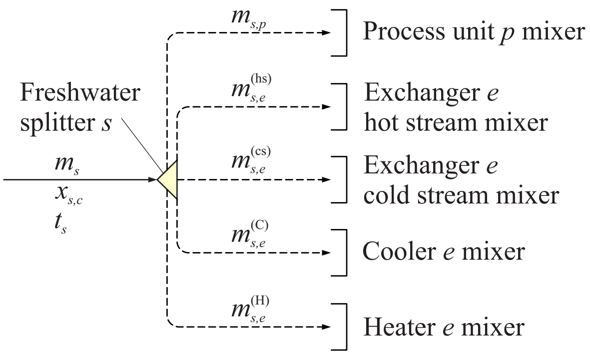

Figure 5. Freshwater splitter with corresponding streams.

Process water-using unit

The process water-using unit $p \in P$ with the corresponding water streams is shown in Figure 6. The overall mass balance and the mass balance of contaminant $c \in C$ for the process unit $p \in P$ mixer is given by Equations (2) and (3).

$$m_p^{(in)} = \sum_s m_{s,p} + \sum_e m_{e,p}^{(hs)} + \sum_e m_{e,p}^{(cs)} + \sum_{\substack{p' \\ p \neq p'}} m_{p',p} + \sum_e m_{e,p}^{(C)} + \sum_e m_{e,p}^{(H)} \quad ,\forall p \in P \tag{2}$$

$$m_p^{(in)} x_{p,c}^{(in)} = \sum_s m_{s,p} x_{s,c} + \sum_e m_{e,p}^{(hs)} x_{e,c}^{(hs)} + \sum_e m_{e,p}^{(cs)} x_{e,c}^{(cs)} + \sum_{\substack{p' \\ p \neq p'}} m_{p',p} x_{p',c}^{(out)} + \sum_e m_{e,p}^{(C)} x_{e,c}^{(C)} + \sum_e m_{e,p}^{(H)} x_{e,c}^{(H)} \quad ,\forall p \in P, \forall c \in C \tag{3}$$

The heat balance of the process unit p mixer is given by Equation (4).

$$m_p^{(in)} t_p^{(in)} = \sum_s m_{s,p} t_s + \sum_e m_{e,p}^{(hs)} t_e^{(hs,out)} + \sum_e m_{e,p}^{(cs)} t_e^{(cs,out)} + \sum_{\substack{p' \\ p \neq p'}} m_{p',p} t_{p'}^{(out)} + \sum_e m_{e,p}^{(C)} t_e^{(C,out)} + \sum_e m_{e,p}^{(H)} t_e^{(H,out)} \quad ,\forall p \in P \tag{4}$$

Under the assumption that the mass of contaminants in the water streams is much lower than the overall mass, the flow-rate of water at the inlet and outlet from the process water-using unit p is equal, as given by Equation (5). However, the mass of contaminant c is increased as the mass load of contaminant $L_{p,c}$ is transferred to the water stream within the process unit p, as given by Equation (6).

$$m_p^{(in)} = m_p^{(out)} \quad ,\forall p \in P \tag{5}$$

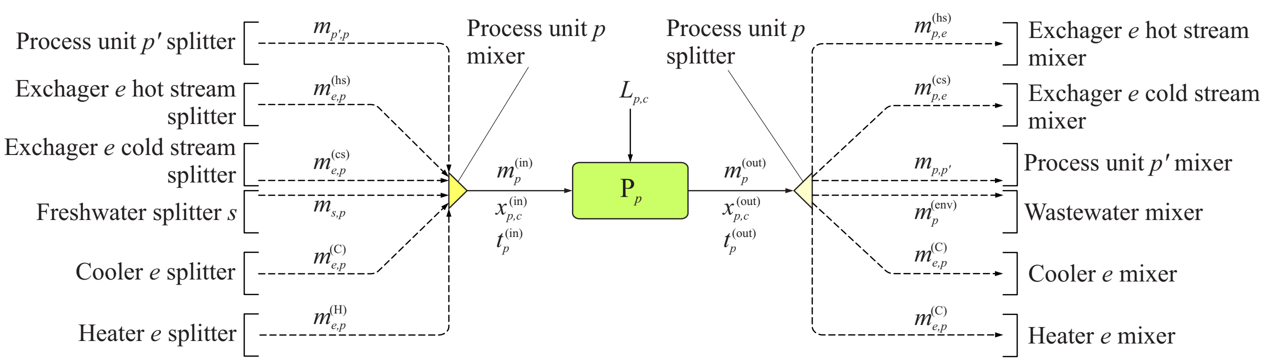

Figure 6. Process water-using unit with corresponding streams.

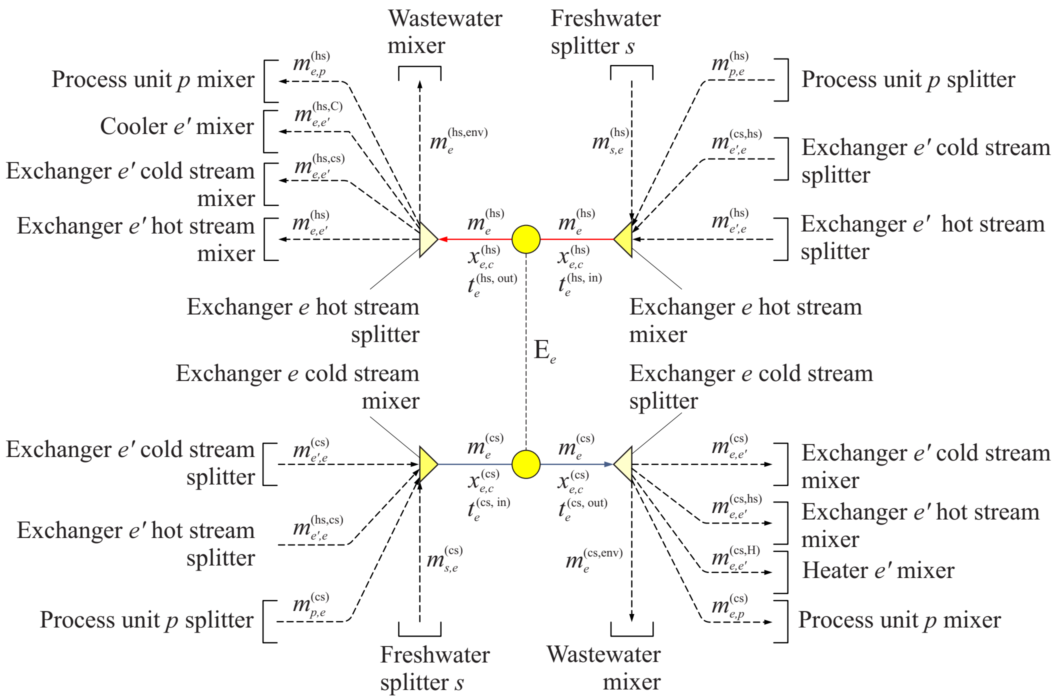

Figure 7. Heat exchanger unit with corresponding streams.

$$m_p^{(in)} x_{p,c}^{(in)} + L_{p,c} = m_p^{(out)} x_{p,c}^{(out)} \quad , \forall p \in P, \forall c \in C \tag{6}$$

The mass balance of the process unit p splitter is given by Equation (7).

$$m_p^{(out)} = \sum_{\substack{p' \\ p \neq p'}} m_{p,p'} + \sum_e m_{p,e}^{(hs)} + \sum_e m_{p,e}^{(cs)} + \sum_e m_{p,e}^{(C)} + \sum_e m_{p,e}^{(H)} \quad + m_p^{(env)}, \forall p \in P \tag{7}$$

Heat exchangers

Figure 7 shows a counter-current heat exchanger with the corresponding hot and cold streams. The exchanger $e \in E$ consist of an exchanger hot stream mixer and splitter as well as a cold stream mixer and splitter. The existence of mixers in front of heat exchangers enables the possibility that any water stream within the network can be directed towards any heat exchanger as a hot and/or cold stream. In addition, a number of heat exchangers are set arbitrarily and the existence of multiple heat exchangers enables many options for heat exchange (serial, parallel, bypass and their combinations) as there are interconnections between individual heat exchangers. The overall mass balance and the mass balance for each contaminant $c \in C$ for the exchanger $e \in E$ hot stream mixer is given by Equations (8) and (9).

$$m_e^{(hs)} = \sum_p m_{p,e}^{(hs)} + \sum_s m_{s,e}^{(hs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs,hs)} \quad , \forall e \in E \tag{8}$$

$$m_e^{(hs)} x_{e,c}^{(hs)} = \sum_p m_{p,e}^{(hs)} x_{p,c}^{(out)} + \sum_s m_{s,e}^{(hs)} x_{s,c} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs)} x_{e',c}^{(hs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs,hs)} x_{e',c}^{(cs)} \quad , \forall e \in E, \forall c \in C \tag{9}$$

The heat balance of exchanger $e \in E$ hot stream mixer is given by Equation (10).

$$m_e^{(hs)} t_e^{(hs,in)} = \sum_p m_{p,e}^{(hs)} t_p^{(out)} + \sum_s m_{s,e}^{(hs)} t_s + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs)} t_{e'}^{(hs,out)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs,hs)} t_{e'}^{(cs,out)} \quad , \forall e \in E \tag{10}$$

The exchanger $e \in E$ hot stream splitter mass balance is given by Equation (11).

$$m_e^{(hs)} = m_e^{(hs,env)} + \sum_p m_{e,p}^{(hs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(hs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(hs,cs)} + \sum_{e'} m_{e,e'}^{(hs,C)} \quad ,\forall e \in E \tag{11}$$

199 The overall mass balance and the mass balance for each contaminant $c \in C$ for the exchanger
200 $e \in E$ cold stream mixer is given by Equations (12) and (13). The heat balance of the
201 exchanger $e \in E$ cold stream mixer is given by the Equation (14).

$$m_e^{(cs)} = \sum_p m_{p,e}^{(cs)} + \sum_s m_{s,e}^{(cs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs,cs)} \quad ,\forall e \in E \tag{12}$$

$$m_e^{(cs)} x_{e,c}^{(cs)} = \sum_p m_{p,e}^{(cs)} x_{p,c}^{(out)} + \sum_s m_{s,e}^{(cs)} x_{s,c} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs)} x_{e',c}^{(cs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs,cs)} x_{e',c}^{(hs)} \quad ,\forall e \in E, \forall c \in C \tag{13}$$

$$m_e^{(cs)} t_e^{(cs,in)} = \sum_p m_{p,e}^{(cs)} t_p^{(out)} + \sum_s m_{s,e}^{(cs)} t_s + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(cs)} t_{e'}^{(cs,out)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(hs,cs)} t_{e'}^{(hs,out)} \quad ,\forall e \in E \tag{14}$$

The exchanger $e \in E$ cold stream splitter mass balance is given by Equation (15).

$$m_e^{(cs)} = m_e^{(cs,env)} + \sum_p m_{e,p}^{(cs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(cs)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(cs,hs)} + \sum_{e'} m_{e,e'}^{(cs,H)} \quad ,\forall e \in E \tag{15}$$

202 Coolers and heaters

203 According to the proposed superstructure shown in Figure 4 to each heat exchanger
204 $e \in E$ one heater (H) is assigned for additional heating of the cold streams and one
205 cooler (C) for additional cooling of the hot streams. Figure 8 shows cooler and heater
206 with the corresponding streams. The overall mass balance and the mass balance for each
207 contaminant $c \in C$ for cooler $e \in E$ mixer is given by Equations (16) and (17). The heat
208 balance of the cooler $e \in E$ mixer is given by Equation (18).

$$m_e^{(C)} = \sum_p m_{p,e}^{(C)} + \sum_s m_{s,e}^{(C)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(C)} + \sum_{e'} m_{e',e}^{(hs,C)} \quad ,\forall e \in E \tag{16}$$

$$m_e^{(C)} x_e^{(C)} = \sum_p m_{p,e}^{(C)} x_{p,c}^{(out)} + \sum_s m_{s,e}^{(C)} x_{s,c} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(C)} x_{e',c}^{(C)} + \sum_{e'} m_{e',e}^{(hs,C)} x_{e,c}^{(hs)} \quad ,\forall e \in E, \forall c \in C \tag{17}$$

$$m_e^{(C)} t_e^{(C,in)} = \sum_p m_{p,e}^{(C)} t_p^{(out)} + \sum_s m_{s,e}^{(C)} t_s + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(C)} t_{e'}^{(C,out)} + \sum_{e'} m_{e',e}^{(hs,C)} t_{e'}^{(hs,out)} \quad ,\forall e \in E \tag{18}$$

The mass balance of the cooler $e \in E$ splitter is given by the Equation (19).

$$m_e^{(C)} = \sum_p m_{e,p}^{(C)} + m_e^{(C,env)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(C)} \quad ,\forall e \in E \tag{19}$$

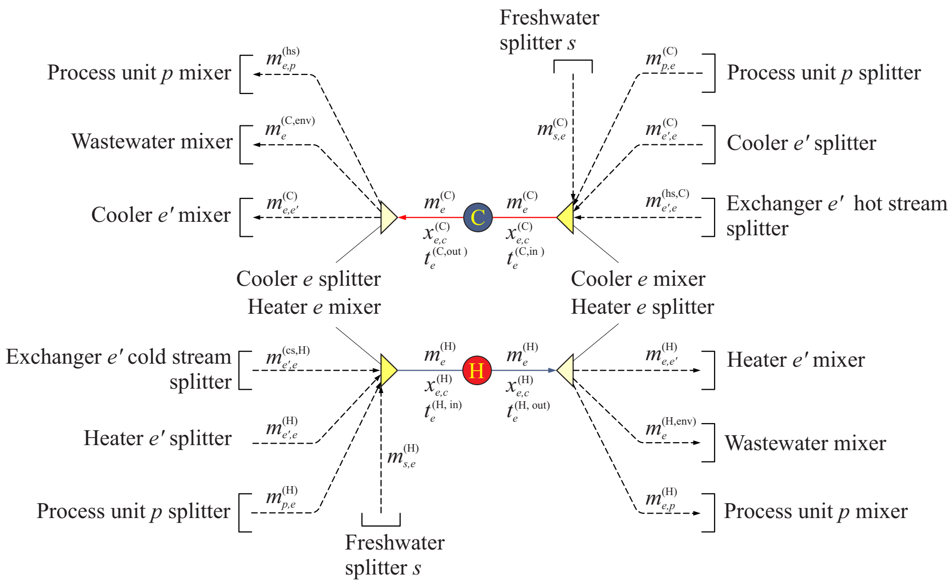

Figure 8. Cooler and heater with corresponding streams.

The overall mass balance and the mass balance for each $c \in C$ for the heater $e \in E$ mixer is given by Equations (20) and (21). The heat balance of the heater $e \in E$ mixer is given by Equation (22).

$$m_e^{(H)} = \sum_p m_{p,e}^{(H)} + \sum_s m_{s,e}^{(H)} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(H)} + \sum_{e'} m_{e',e}^{(cs,H)} \quad ,\forall e \in E \tag{20}$$

$$m_e^{(H)} x_e^{(H)} = \sum_p m_{p,e}^{(H)} x_{p,c}^{(out)} + \sum_s m_{s,e}^{(H)} x_{s,c} + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(H)} x_{e',c}^{(H)} + \sum_{e'} m_{e',e}^{(cs,H)} x_{e,c}^{(cs)} \quad ,\forall e \in E, \forall c \in C \tag{21}$$

$$m_e^{(H)} t_e^{(H,in)} = \sum_p m_{p,e}^{(H)} t_p^{(out)} + \sum_s m_{s,e}^{(H)} t_s + \sum_{\substack{e' \\ e \neq e'}} m_{e',e}^{(H)} t_{e'}^{(H,out)} + \sum_{e'} m_{e',e}^{(cs,H)} t_{e'}^{(cs,out)} \quad ,\forall e \in E \tag{22}$$

The mass balance of the heater $e \in E$ splitter is given by Equation (23).

$$m_e^{(H)} = \sum_p m_{e,p}^{(H)} + m_e^{(H,env)} + \sum_{\substack{e' \\ e \neq e'}} m_{e,e'}^{(H)} \quad ,\forall e \in E \tag{23}$$

Wastewater mixer

The streams entering the wastewater mixer are those leaving all the splitters in the water network except the freshwater splitter as shown in Figure 9. The overall mass balance of the wastewater mixer and the mass balance for the contaminant $c \in C$ are given by Equations (24) and (25). The heat balance of the wastewater mixer is given by the Equation (26).

$$m^{(env)} = \sum_p m_p^{(env)} + \sum_e m_e^{(cs,env)} + \sum_e m_e^{(hs,env)} + \sum_e m_e^{(C,env)} + \sum_e m_e^{(H,env)} \quad ,\forall e \in E \tag{24}$$

$$m^{(env)} x_c^{(env)} = \sum_p m_p^{(env)} x_{p,c}^{(out)} + \sum_e m_e^{(cs,env)} x_{e,c}^{(cs)} + \sum_e m_e^{(hs,env)} x_{e,c}^{(hs)} + \sum_e m_e^{(C,env)} x_{e,c}^{(C)} + \sum_e m_e^{(H,env)} x_{e,c}^{(H)} \quad ,\forall e \in E, \forall c \in C \tag{25}$$

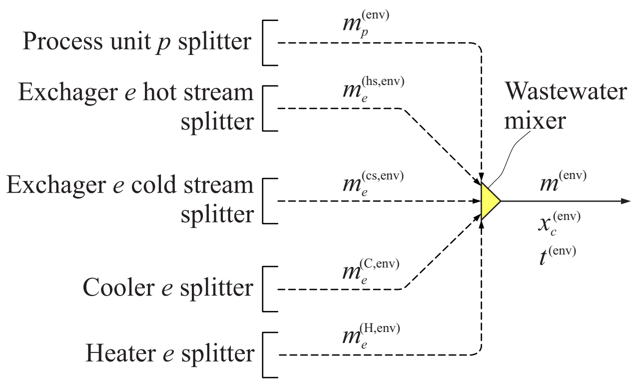

Figure 9. Wastewater mixer with corresponding streams.

$$m^{(env)}t^{(env)} = \sum_p m_p^{(env)} t_p^{(out)} + \sum_e m_e^{(cs,env)} t_e^{(cs,out)} + \sum_e m_e^{(hs,env)} t_e^{(hs,out)} + \sum_e m_e^{(C,env)} t_e^{(C,out)} + \sum_e m_e^{(H,env)} t_e^{(H,out)} \quad ,\forall e \in E \tag{26}$$

Global mass balance 218

According to Karuppiah and Grossmann [18], the global mass balance equations can improve the lower bounds of the relaxed model. The overall mass balance and the contaminant mass balance are given by Equations (27) and (28).

$$\sum_s m_s = m^{(env)} \tag{27}$$

$$\sum_s m_s x_{s,c} + \sum_p L_{p,c} = m^{(env)} x_c^{(env)} \quad ,\forall c \in C \tag{28}$$

Heat transfer constraints 219

Within the heat exchanger $e \in E$, heat transfer occurs between the hot and cold water streams. The heat allocated from the hot water stream (q_e) is given by Equation (29). The same amount of heat (q_e) is allocated to the cold water stream as the heat losses are neglected as given by Equation (30).

$$q_e = m_e^{(hs)} c_p \left(t_e^{(hs,in)} - t_e^{(hs,out)} \right) \tag{29}$$

$$q_e = m_e^{(cs)} c_p \left(t_e^{(cs,out)} - t_e^{(cs,in)} \right) \tag{30}$$

The heat balance of cooler and heater is given by Equations (31) and (32). 220

$$q_e^{(C)} = m_e^{(C)} c_p \left(t_e^{(C,in)} - t_e^{(C,out)} \right) \tag{31}$$

$$q_e^{(H)} = m_e^{(H)} c_p \left(t_e^{(H,out)} - t_e^{(H,in)} \right) \tag{32}$$

By assuming counter-current heat exchangers with the upper bound on temperature differences Γ, the temperature differences at the hot and cold sides of heat exchangers, coolers and heaters are given by Equations (33)-(38).

$$\Delta th_e \leq t_e^{(hs,in)} - t_e^{(cs,out)} + \Gamma(1 - z_e) \tag{33}$$

$$\Delta tc_e \leq t_e^{(hs,out)} - t_e^{(cs,in)} + \Gamma(1 - z_e) \tag{34}$$

$$\Delta th_e^{(C)} \leq t_e^{(C,in)} - t^{(CU,out)} + \Gamma\left(1 - z_e^{(C)}\right) \tag{35}$$

$$\Delta tc_e^{(C)} \leq t_e^{(C,out)} - t^{(CU,in)} + \Gamma\left(1 - z_e^{(C)}\right) \tag{36}$$

$$\Delta th_e^{(H)} \leq t^{(HU,in)} - t_e^{(H,out)} + \Gamma\left(1 - z_e^{(H)}\right) \tag{37}$$

$$\Delta tc_e^{(H)} \leq t^{(HU,out)} - t_e^{(H,in)} + \Gamma\left(1 - z_e^{(H)}\right) \tag{38}$$

Equations (39)-(42) describe the feasibility of temperatures entering and leaving heat exchangers, heaters and coolers.

$$t_e^{(hs,in)} \geq t_e^{(hs,out)} \tag{39}$$

$$t_e^{(cs,in)} \leq t_e^{(cs,out)} \tag{40}$$

$$t_e^{(H,in)} \leq t_e^{(H,out)} \tag{41}$$

$$t_e^{(C,in)} \geq t_e^{(C,out)} \tag{42}$$

With the binary variables $z_e, z_e^{(C)}, z_e^{(H)}$, denoting the existence of heat exchangers, coolers and heaters, the transfer is constrained by the upper bounds by the following Equations (43)-(45).

$$q_e - q_e^{(up)} z_e \leq 0 \tag{43}$$

$$q_e^{(C)} - q_e^{(C,up)} z_e^{(C)} \leq 0 \tag{44}$$

$$q_e^{(H)} - q_e^{(H,up)} z_e^{(H)} \leq 0 \tag{45}$$

Objective function of the model

The objective function of the model minimises the TAC of the network consisting of freshwater cost, hot and cold utility cost and the investment cost of HEN as given by Equation (46)

$$\begin{aligned} \min TAC = &\sum_s m_s CFW_s H + \sum_e q_e^{(H)} C_{HU} + \sum_e q_e^{(C)} C_{CU} + \sum_e (a z_e + b A_e^n) \\ &+ \sum_e \left(a z_e^{(C)} + b\left(A_e^{(C)}\right)^n \right) + \sum_e \left(a z_e^{(H)} + b\left(A_e^{(H)}\right)^n \right) \end{aligned} \tag{46}$$

where heat exchanger, cooler and heater heat exchange area parameters are given as follows:

$$A_e = \frac{q_e\left(\dfrac{1}{h_e^{(hs)}} + \dfrac{1}{h_e^{(cs)}}\right)}{\left(\Delta th_e \Delta tc_e \dfrac{\Delta th_e + \Delta tc_e}{2}\right)^{1/3}}$$

$$A_e^{(C)} = \frac{q_e^{(C)}\left(\dfrac{1}{h_e^{(hs)}} + \dfrac{1}{h_{CU}}\right)}{\left(\Delta th_e^{(C)} \Delta tc_e^{(C)} \dfrac{\Delta th_e^{(C)} + \Delta tc_e^{(C)}}{2}\right)^{1/3}}$$

$$A_e^{(H)} = \frac{q_e^{(H)}\left(\dfrac{1}{h_e^{(cs)}} + \dfrac{1}{h_{HU}}\right)}{\left(\Delta th_e^{(H)} \Delta tc_e^{(H)} \dfrac{\Delta th_e^{(H)} + \Delta tc_e^{(H)}}{2}\right)^{1/3}}$$

2.5. Solution strategy

The model is implemented in General Algebraic Modeling System (GAMS) v. 24.6.1 [19] and solved on a laptop computer with a 2.80 GHz processor (4 cores) and 16 GB RAM. The proposed model is solved by using a one-step iterative strategy. For the solution of the MINLP model, SBB solver is used with CONOPT as NLP solvers for the root node and sub-nodes. Multiple solutions are generated by solving the same model in iterations by generating different initialization points for all variables on each solve run. Initialization points are randomly generated for variables between zero and upper bound for each variable using GAMS function `uniform` and generating different initial values in each solve run (`execseed = gmillisec(jnow)`). Generic upper bounds are obtained based on the initial data from the problem formulation, as proposed by Ibrić *et al.* [10].

3. Results and Discussion

To demonstrate the capabilities of the proposed model, nine examples are solved in increasing order of complexity concerning a number of process water-using units, contaminants and freshwater sources. The following data are used for the solution of the problems.

Freshwater: Freshwater is at a temperature of either 20 °C or 30 °C with a unit cost of 0.375 \$/t. Secondary water is at temperature 80 °C with a unit cost of 0.45 \$/t and 100 °C with a unit cost 0.5 \$/t. The specific heat capacity of all water streams is 4.2 kJ/(kgK). The freshwater is free of contaminants except in Example 9 where two freshwater sources of different qualities are assumed.

Utility: Hot utility is fresh either steam at 120 °C or 150 °C with unit costs 377 \$/(kWy) and 388 \$/(kWy), respectively. The inlet and outlet temperatures of the hot utility is the same. The cold utility is cooling water at inlet temperature 10 °C and outlet temperature 20 °C. The unit cost of cooling water is 189 \$/(kWy).

Heat exchangers data: Individual heat transfer coefficients for hot and cold water streams, hot and cold utilities are 1 kW/(m^2K). The heat exchangers are counter-current with annualised investment cost (\$/y) given as a function of heat exchange area (m^2) as follows: $8000 + 1200(A)^{0.6}$.

The plant operates continuously at 8000 h/y.

3.1. Example 1

This example demonstrates a solution of a small-scale multi-contaminant problem consisting of two process water-using units. In addition, heat losses/gains are considered within the process units (different temperatures at the inlet and outlet of the process units). The data for the process water-using units are taken from the literature [20] and given in Table 2. The EMAT is set to 1 °C.

Table 2. Process water-using units data for example 1.

Unit	$L_{p,c}$ (kg/h)			$x_{p,c}^{(in,max)}$ (ppm)			$x_{p,c}^{(out,max)}$ (ppm)			t_p (°C)	
	A	B	C	A	B	C	A	B	C	In	Out
P_1	6	3	4	5	150	100	50	200	200	25	35
P_2	5	8	1	150	120	60	300	150	300	100	85

For the solution of the problem, a set of heat exchangers is arbitrary for all examples. Note, that as the number of heat exchangers increases, the complexity of the proposed superstructure also increases. However, solutions obtained in the literature for examples of different complexities suggest that only few heat exchangers are selected in the final design. For this example, the maximum number of heat exchangers is three with a corresponding number of heaters and coolers. Bogataj and Bagajewicz [20] solved this example considering wastewater treatment and without constraints on the concentration of the contaminants in the effluent stream. However, an optimal solution obtained later in the literature [21] exhibited network design without wastewater treatment and a freshwater consumption of 33.333 kg/s. The hot and cold utility consumption was 1485.45 kW and 369.18 kW, respectively.

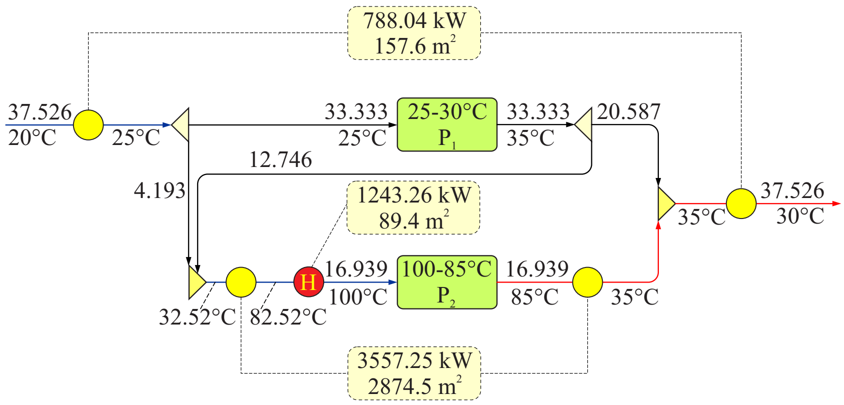

Figure 10. Optimal network design for example 1.

The optimal network design shown in Figure 10 exhibits a freshwater consumption of 37.526 kg/s and is increased compared to the solution in the literature [21] due to the lower freshwater cost (0.375 vs. 2.5 $/t). The optimal design requires only hot utility (1243.26 kW). Two heat exchangers and one heater are included in the final network design with the HEN investment cost of 209,437 $/y. The TAC of the network is 1,083,423 $/y.

3.2. Example 2

This example considers a single contaminant problem with three process water-using units studied in the literature [10,22–25]. Process water-using units data for example 2 are given in Table 3. The EMAT is set to 1 °C.

Table 3. Process water-using units data for example 2.

Unit	$L_{p,c}$ (g/s)	$x_{p,c}^{(in,max)}$ (ppm)	$x_{p,c}^{(out,max)}$ (ppm)	t_p (°C)
P_1	5	50	100	100
P_2	30	50	800	75
P_2	50	800	1100	100

The optimal network design obtained by using the proposed model in this paper exhibited freshwater consumption of 77.273 kg/s and a total consumption of hot utility of 3245.2 kW distributed within two heaters. Heat integration occurs between the wastewater stream at maximum temperature 100 °C and the freshwater at 20 °C. In addition, the part of the stream leaving P_1 (11.668 kg/s) is conditioned with preheated freshwater and used to preheat an internal water stream with a minimum temperature of 75 °C within a heat exchanger with a heat load of 2023.9 kW. The optimal network design is the same as that reported by Ibrić *et al.* [10]. Table 4 shows a comparison of the results with those reported in the literature for example 2 comprising freshwater and utility consumption, the number of heat exchangers (HE), the HEN investment and the TAC of the network.

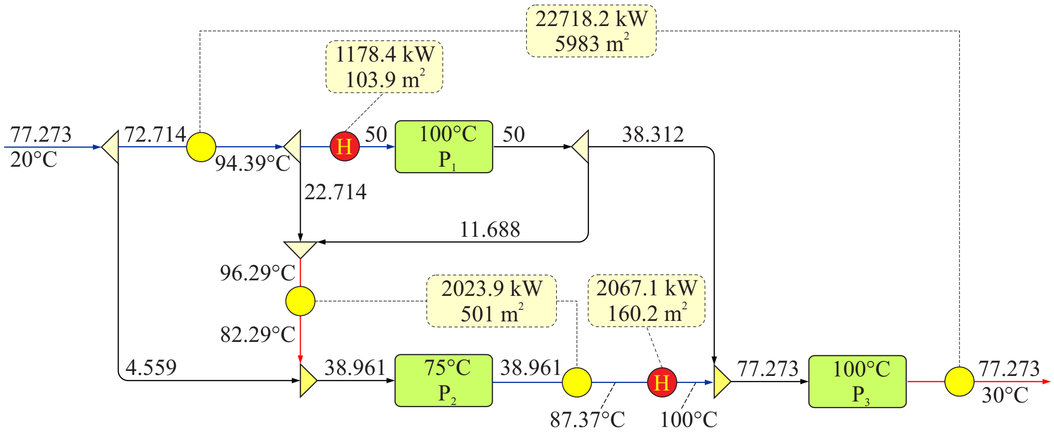

Figure 11. Optimal network design for example 2.

Table 4. Comparison of the results with those from the literature for example 2.

Reference	Freshwater (kg/s)	Hot utility (kW)	No of HEs	HEN investment ($/y)	TAC ($/y)
Dong *et al.* [22]	87.2	3671.4	4	305,913	2,631,805
Ahmetović and Kravanja [26]	77.273	3245.5	5	364,450	2,422,531
Yan *et al.* [25]	77.273	3245.5	5	398,349	2,456,431
Ibrić *et al.* [10]	77.273	3245.5	4	348,184	2,406,266
Jagannath and Almansoori [24]	77.273	3245.2	5	395,694	2,453,697
This paper	77.273	3245.2	5	348,184	2,406,266

3.3. Example 3

This example demonstrates a solution of the HIWN problem considering multiple freshwater sources. A multi-contaminant problem (three contaminants) with three process water-using units was modified from the literature [22]. Freshwater source is available at 20 °C and two secondary water sources at 80 and 100 °C all without contaminants are considered. Data for the process water-using units are given in Table 5. The temperature of wastewater discharged into the environment is 30 °C without limitation on the contaminants concentrations in the effluent stream. The EMAT is 1 °C.

Table 5. Process water-using units data for example 3.

Unit	$L_{p,c}$ (g/s)			$x_{p,c}^{(in,max)}$ (ppm)			$x_{p,c}^{(out,max)}$ (ppm)			t_p (°C)
	A	B	C	A	B	C	A	B	C	
P_1	3.0	2.4	1.8	0	0	0	100	80	60	100
P_2	4.0	3.0	3.6	50	40	15	150	115	105	75
P_3	1.5	0.6	2.0	50	50	30	125	80	130	35

An optimal network design for example 3 is shown in Figure 12. Two freshwater sources are selected in the final design with a total freshwater consumption of 71.961 kg/s compared to the minimum of 70 kg/s. Most of the freshwater used is at 20 °C (63.232 kg/s) with an additional amount of freshwater available at 100 °C (8.729 kg/s). Only hot utility is required for freshwater preheating (89.34 kW). A HEN consists of two heat exchangers and a heater with the HEN investment cost of 291,489 $/y. The TAC of the network is 1,133,777 $/y.

3.4. Example 4

This example studies a single-contaminant problem with four process water-using units previously studied in the literature [17,20,25,27]. The freshwater is at 20 °C without

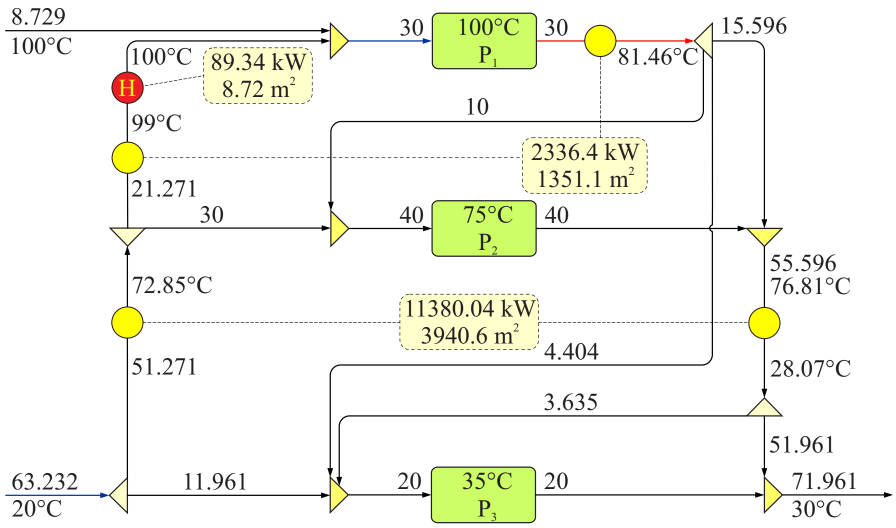

Figure 12. Optimal network design for example 3.

contaminants and the wastewater is discharged into the environment at 30 °C. The data for the process water-using units are given in Table 6.

Table 6. Process water-using units data for example 4.

Unit	$L_{p,c}$ (g/h)	$x_{p,c}^{(in,max)}$ (ppm)	$x_{p,c}^{(out,max)}$ (ppm)	t_p (°C)
P_1	2000	0	100	40
P_2	5000	50	100	100
P_3	30000	50	800	75
P_4	4000	400	800	50

The optimal network design, shown in Figure 13, exhibited the minimum freshwater consumption (25 kg/s) and consumption of only hot utility (1050 kW). The optimal HEN design consists of two heat exchangers and one heater with the HEN investment cost of 131,525 \$/y. This is a somewhat improved solution compared to the solution obtained by using a more complex stage-wise HEN superstructure combined with the water network superstructure [27]. The same network design and TAC were obtained by Yan *et al.* [25] by using the NLP model and the global optimisation solver BARON. A comparison of the results with those from the literature for example 4 is given in Table 7.

Table 7. Comparison of the results with those from the literature for example 4.

Reference	Freshwater (kg/s)	Hot utility (kW)	No of HEs	HEN investment (\$/y)	TAC (\$/y)
Bogataj and Bagajewicz [20] *	25	1050	3	146,748	812,598
Ahmetović and Kravanja [17]	25	1050	2	134,227	800,077
Yan *et al.* [25]	25	1050	3	131,525	797,375
Ibrić *et al.* [27]	25	1050	2	134,227	800,077
This paper	25	1050	3	131,527	797,377

* Recalculated from Ahmetović and Kravanja [17].

3.5. Example 5

In this example, a problem originally solved as an isothermal water network problem [18] is modified as a non-isothermal water network problem, as reported in the literature [21]. The problem includes four process-water using units with two contaminants and a single freshwater source at 20 °C without contaminants. The wastewater is discharged into the environment at 30 °C with no restrictions on contaminants concentrations in the

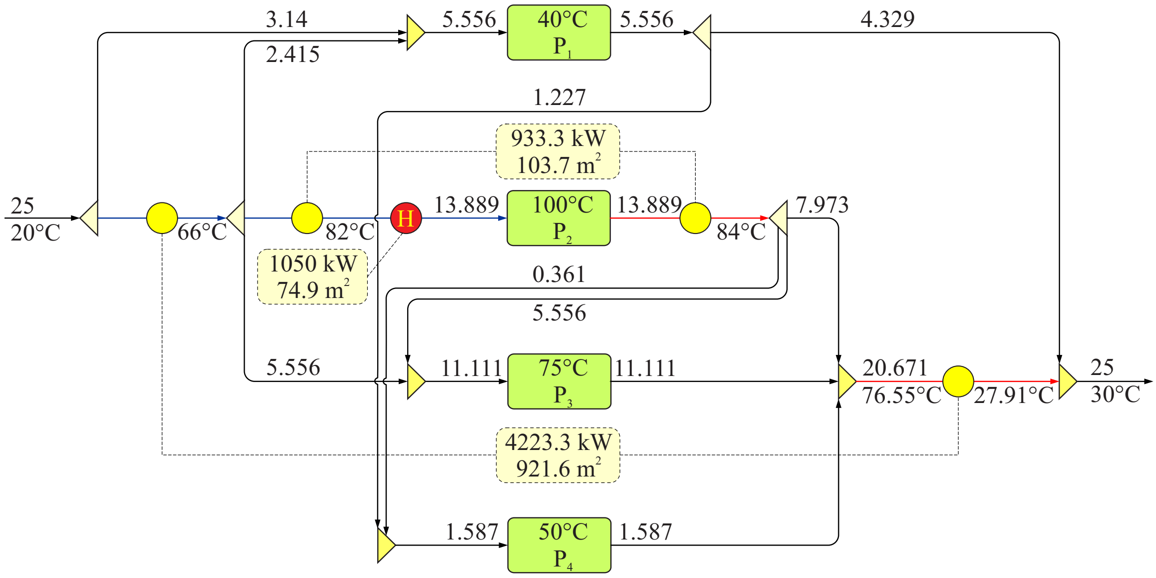

Figure 13. Optimal network design for example 4.

effluent stream. Data for the process water-using units are given in Table 8. The EMAT is 1 °C.

Table 8. Process water-using units data for example 5.

Unit	$L_{p,c}$ (kg/h)		$x_{p,c}^{(in,max)}$ (ppm)		$x_{p,c}^{(out,max)}$ (ppm)		t_p (°C)
	A	B	A	B	A	B	
P_1	1	1.5	0	0	25	37.5	40
P_2	1	1	50	50	70	70	50
P_3	1	1	50	50	66.667	66.667	75
P_4	2	1	50	50	78.571	78.571	100

Ibrić *et al.* [21] reported the solution with a freshwater consumption of 19.444 kg/s and consumption of hot utility of 816.67 kW. The network exhibited a design comprising of two heat exchangers and one heater with the HEN investment cost of 188,801 \$/y. The TAC of the network was 706,687.7 \$/y. An optimal design shown in Figure 14 exhibited slightly increased consumption of freshwater compared to the literature [21] (20.698 vs. 19.444 kg/s). Consequently, the consumption of hot utility is also increased (869.33 vs. 816.67 kW). However, as a trade-off exist, the HEN investment cost is reduced by ≈ 41 % (111,207 vs. 188,801 \$/y). The TAC of the network is 662,489 \$/y and is reduced by 6.25 % compared to the literature [21].

3.6. Example 6

This example is slightly more complex than the previous one considering a number of contaminants. Problem consists of four process-water using units and three contaminants originally solved in the literature [20] and later studied by other authors [10,25,26]. The data for the process water-using units are given in Table 9. Freshwater is available at 20 °C without contaminants and wastewater is discharged into the environment at 30 °C. The EMAT is 1 °C.

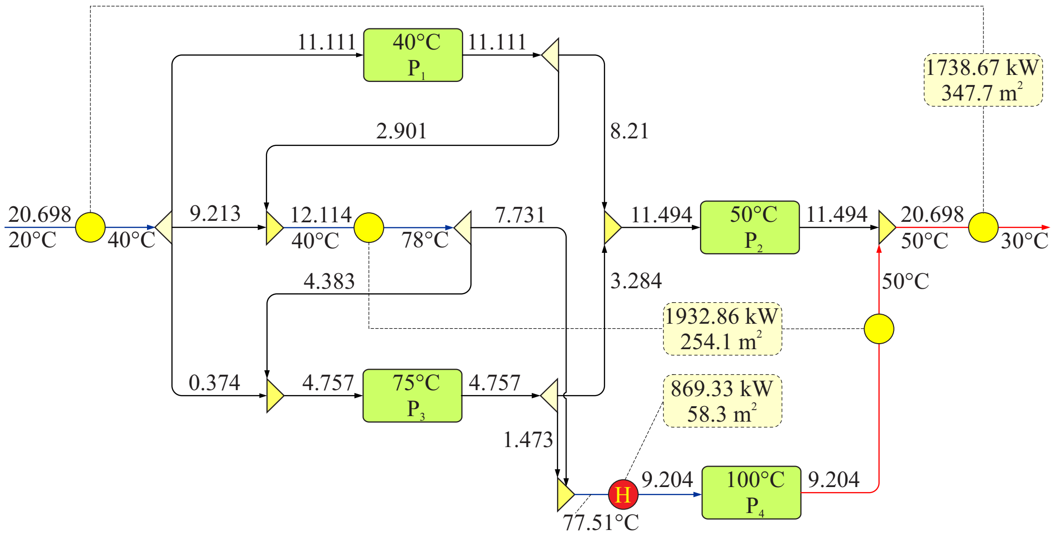

Figure 14. Optimal network design for example 5.

Table 9. Process water-using units data for example 6.

Unit	$L_{p,c}$ (kg/h)			$x_{p,c}^{(in,max)}$ (ppm)			$x_{p,c}^{(out,max)}$ (ppm)			t_p (°C)
	A	B	C	A	B	C	A	B	C	
P_1	2	1	3	0	15	0	100	100	100	40
P_2	5	0	15	50	100	30	100	200	250	100
P_3	30	4	0	100	100	100	800	750	600	75
P_4	4	22	17	400	380	250	800	800	800	50

The optimal network design obtained by using the model proposed in this paper is shown in Figure 15. Compared to the network design in the literature [10], the basic design, freshwater and hot utility consumption and heat integration opportunities are the same. The only difference is related to the water reuse opportunities related to P_4 due to the increased flowrate in P_4. Table 10 shows a comparison of the results with those from the literature for example 6.

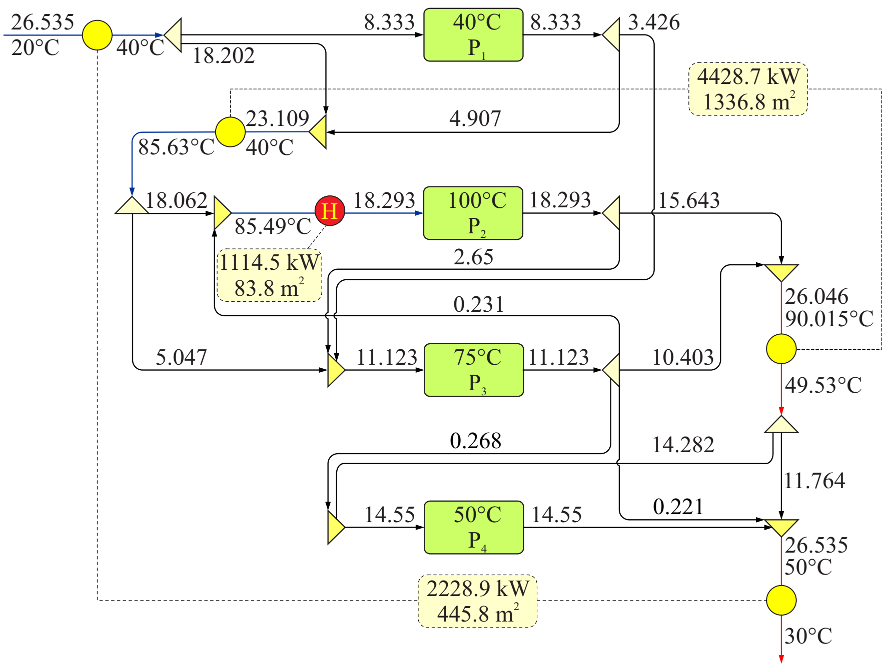

Figure 15. Optimal network design for example 6.

Table 10. Comparison of the results with those from the literature for example 6.

Reference	Freshwater (kg/s)	Hot utility (kW)	No of HEs	HEN investment ($/y)	TAC ($/y)
Bogataj and Bagajewicz [20]*	26.535	1115.7	4	199,861	907,066
Ahmetović and Kravanja [26]	26.535	1114.5	4	183,063	889.772
Yan *et al.* [25]	26.716	1122.1	4	183,107	894,856
Ibrić *et al.* [10]	26.535	1114.5	3	177,859	884,595
This paper	26.535	1114.5	3	177,859	884,595

* Recalculated data taken from Ahmetović and Kravanja [26].

3.7. Example 7

The complexity of the studied problems of HIWNs significantly increases with the increasing number of process water-using units. This example considers a single contaminant HIWN problem with eight process units. Data for the example are taken from the literature [28] and presented in Table 11. Problem is solved for EMAT of 1 °C.

Table 11. Process water-using units data for example 7.

Unit	$L_{p,c}$ (g/s)	$x_{p,c}^{(in,max)}$ (ppm)	$x_{p,c}^{(out,max)}$ (ppm)	t_p (°C)
P_1	2	25	80	40
P_2	2.88	25	90	100
P_3	4	25	200	80
P_4	3	50	100	60
P_5	30	50	800	50
P_6	5	400	800	90
P_7	2	400	600	70
P_8	1	0	100	50

In recent publication Ibrić *et al.* [12] solved this example by using a three-step solution strategy. However, the HEN superstructure still exhibits a large number of possible heat-integration opportunities even with a limited number of hot and cold streams. With the new modified HEN superstructure proposed in this paper, the HEN combinatorial problem is reduced and a good solution is obtained. Dong *et al.* [13] proposed a solution to this problem by fixing a number of heat exchangers, heaters and coolers and solving problems for their different numbers. In this way, a set of solutions is obtained with different numbers of exchangers. An optimal network design obtained in this work is shown in Figure 16. The network design exhibited the same heat integration opportunities as in the literature [13] and also the same freshwater and utilities consumption. A difference is in the internal water distribution of water streams with a very similar network design. A comparison of the results with those from the literature for example 7 is presented in Table 12.

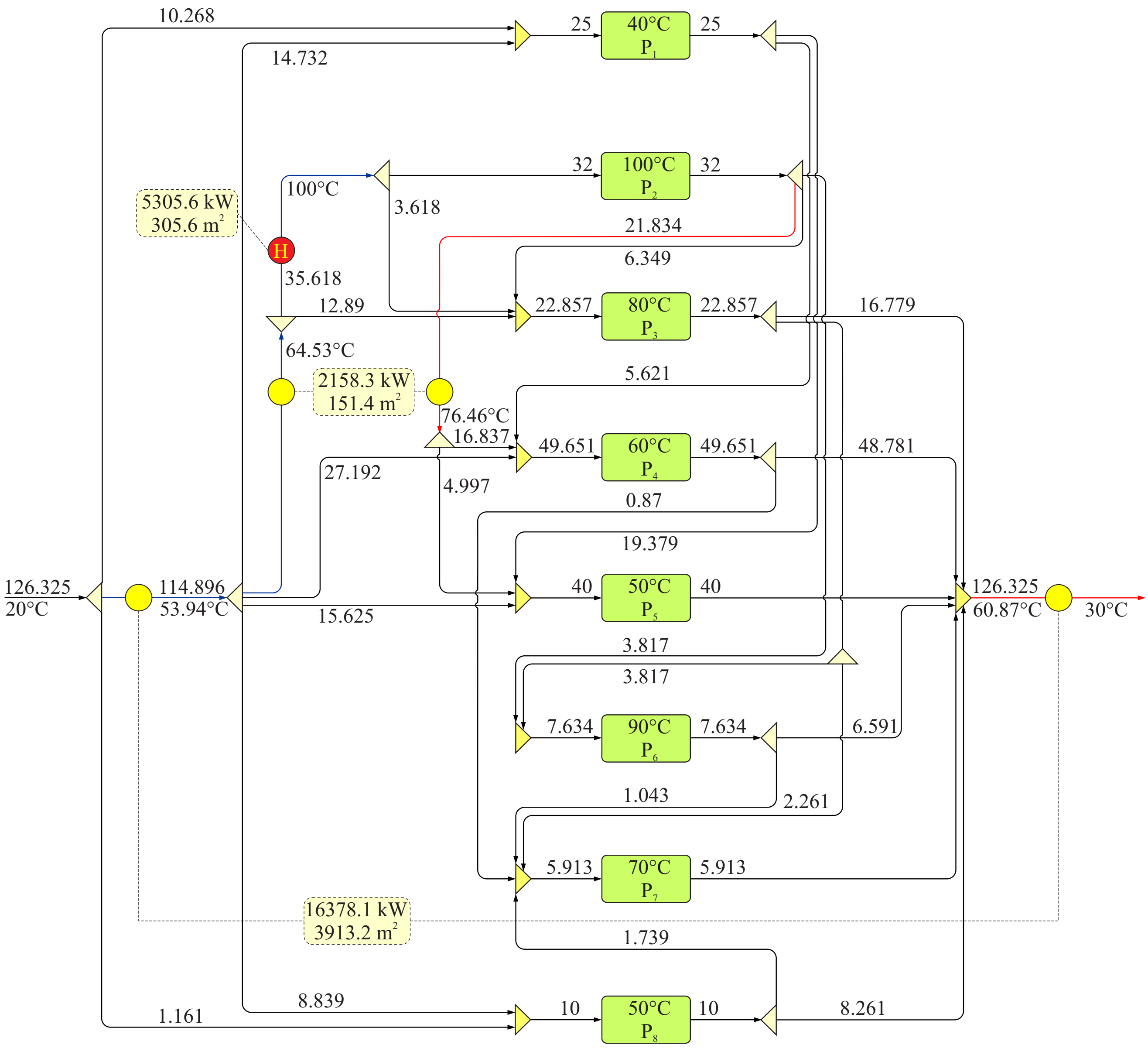

Figure 16. Optimal network design for example 7.

Table 12. Comparison of the results with those from the literature for example 7.

Reference	Freshwater (kg/s)	Hot utility (kW)	No of HEs	HEN investment ($/y)	TAC ($/y)
Bagajewicz *et al.* [28]	125.94	5289.6	12	NA	NA
Hong *et al.* [29]	127.713	5363.94	4	274,471	3,651,247
Hong *et al.* [30]	126.426	5325.12	3	274,471	3,641,653
Ibrić *et al.* [27]	125.943	5289.6	4	273,660	3,628,022
Ibrić *et al.* [12]	125.943	5289.6	4	271,907	3,626,269
Dong *et al.* [13]	126.33	5305.6	3	257,245	3,621,773
This paper	126.325	5305.6	3	257,247	3,621,775

3.8. Example 8

In the previous example, only hot utility was required and thus a problem is considered as a threshold problem concerning utility consumption. This problem occurs when the temperature of the freshwater source (t_s) is lower than the temperature of wastewater

($t^{(env)}$) and the maximum temperature of process water-using units ($t_p^{(in,max)}$). If $t_s \geq t^{(env)}$ a cooling utility is also required and thus a problem is considered to be a pinched HIWN problem. In this example, a single contaminant pinched problem is considered with fifteen process water-using units. The temperature of the freshwater source is 30 °C and is the same as the temperature of wastewater discharged into the environment. The data for the process water-using units are taken from the literature [9] and given in Table 13. Hot utility temperature is 150 °C. The EMAT is 10 °C.

Table 13. Process water-using units data for example 8.

Unit	$L_{p,c}$ (g/s)	$x_{p,c}^{(in,max)}$ (ppm)	$x_{p,c}^{(out,max)}$ (ppm)	t_p (°C)
P_1	1	0	100	50
P_2	2	0	100	80
P_3	4	0	200	110
P_4	4.5	0	300	90
P_5	0.5	0	100	100
P_6	2	250	400	70
P_7	2.25	150	300	100
P_8	1.5	200	250	120
P_9	1	300	350	80
P_{10}	1.8	350	380	95
P_{11}	1	300	350	110
P_{12}	1.2	350	400	95
P_{13}	6	0	200	100
P_{14}	0.3	380	400	100
P_{15}	0.8	350	800	60

Figure 17 shows an optimal network design for example 8. The optimal design exhibited a minimum freshwater consumption of 100 kg/s as well and a minimum consumption of hot (4200 kW) and cold (4200 kW) utility. When compared to a recent work [12] hot and cold utility consumption is reduced (4200 kW vs. 4246.2 kW) at the expense of increasing heat exchanger investment cost (352,273 \$/y vs. 339,573 \$/y). Compared to the solution presented by Dong *et al.* [13], the freshwater and utilities consumption are the same. Also, heat exchangers' heat load and heat exchange areas are identical as well as for an additional cooler. However, the distribution of heat loads and heat exchange areas for the heaters are different causing a decrease in HEN investment (352,273 \$/y vs. 360,448 \$/y). A comparison of the results with those from the literature for example 8 is presented in Table 14.

Table 14. Comparison of the results with those from the literature for example 8.

Reference	Freshwater (kg/s)	Hot utility (kW)	Cold utility (kW)	No of HE	HEN investment (\$/y)	TAC (\$/y)
Liu *et al.* [9]	100	4200	4200	9	492,900	3,918,600
Hong *et al.* [29]	100	4200	4200	5	369,344	3,872,744
Hong *et al.* [30]	100	4200	4200	7	447,976	3,951,376
Ibrić *et al.* [12]	100	4246.2	4246.2	5	339,573	3,869,655
Dong *et al.* [13]	100	4200	4200	6	360,448	3,863,848
This paper	100	4200	4200	6	352,273	3,855,672

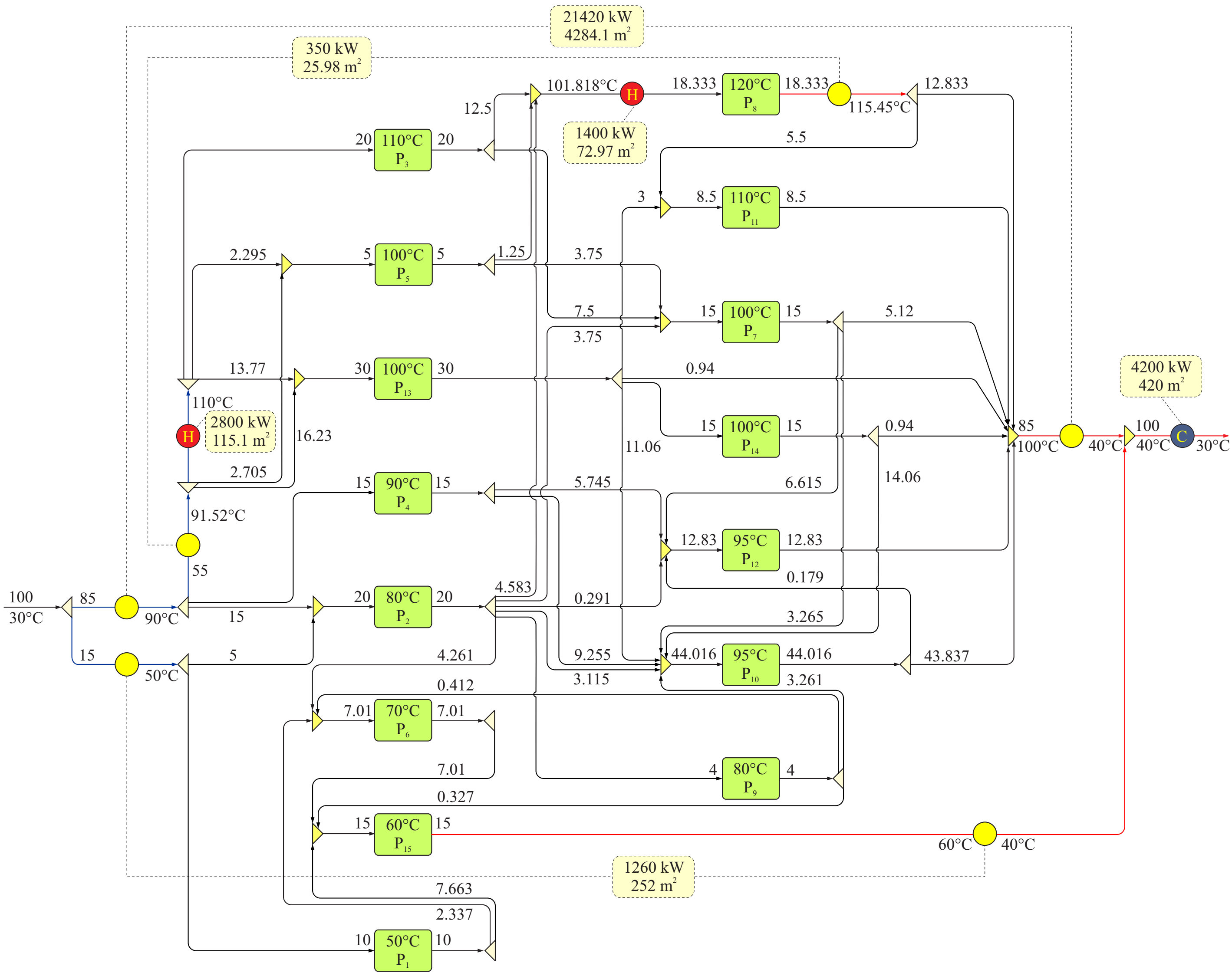

Figure 17. Optimal network design for example 8.

3.9. Example 9

In this example, a large scale multi-contaminant problem was solved including ten process water-using units and four contaminants. Data for the process water-using unit are taken from the literature [6] and given in Table 15. Two freshwater sources are available. The first freshwater source is at 20 °C and 0 ppm contaminants concentration and the second freshwater source at 30 °C with 10 ppm contaminants concentration with a unit cost of 0.1 $/t.

An optimal network design obtained by solving the proposed model is shown in Figure 18. Only one freshwater source is selected at 20 °C in the amount of 160.682 kg/s. The results obtained in the literature [12] exhibited consumption of freshwater from both sources (120.128 kg/s and 44.447 kg/s, respectively) and the total amount slightly increased compared to the solution in this paper. The explanation is the different prices of freshwater and utilities. As the freshwater cost in this paper (0.387 $/t) is lower than the freshwater cost in the literature (0.5 $/t), freshwater at 20 °C is forced. Due to the increased cost of hot (377 $/(kWy) vs. 120 $/(kWy)) and cold (189 $/(kWy) vs. 30 $(kWy)) utility it's consumption is significantly reduced compared to the solutions in the literature [7,12]. Also, a lower number of heat exchangers exists in the optimal design presented in this paper with the HEN investment cost of 268,801 $/y. The TAC of the optimal network design is 4,548,400 $/y. The results show that a significant change in the water network design can be expected with the change in cost for freshwater and utilities, as shown in Example 1 and in the recent research [31]. Detailed sensitivity analyses are required for a different type of HIWN problems to analyse the impact of freshwater and utility cost as well as investment cost on the HIWN design. This can be interesting to explore in the future research.

Table 15. Process water-using units data for example 9.

Unit	$L_{p,c}$ (g/h)				$x_{p,c}^{(in,max)}$ (ppm)				$x_{p,c}^{(out,max)}$ (ppm)				t_p (°C)
	A	B	C	D	A	B	C	D	A	B	C	D	
P_1	616,776	484,965	706,308	5,682,795	200	500	100	1500	25,000	20,000	28,500	230,000	70
P_2	313,497	245,880	966,308.4	106,548	350	3000	500	400	8000	9000	24,080	3000	60
P_3	123,480	80,360	52,920	39,200	350	450	150	500	3500	2500	1500	1500	90
P_4	56,800	17,400	1000	4800	800	650	450	300	15,000	5000	700	1500	80
P_5	2744	19,600	27,440	23,520	1300	2000	2000	4000	2000	7000	9000	10,000	70
P_6	1,237,500	1,100,000	1,086,250	27,500	3000	2000	100	0	12,000	10,000	8000	200	100
P_7	450,988	872,880	218,220	3,302,396	450	0	250	650	2000	3000	1000	12,000	40
P_8	797,63.5	89,287.5	11,905	153,574.5	100	250	200	550	3450	4000	700	7000	80
P_9	55,624	35, 992	65,440	0	150	450	3000	100	1000	1000	4000	100	50
P_{10}	400	400	400	400	0	0	0	0	100	100	100	100	60

3.10. Model statistics

Table 16 shows basic model statistics including model sizes and computational times for the given number of solve iterations. A number of iterations can be increased in search of a better solution if the best one can not be found in a given smaller number of iterations. In a recent publication by Ibrić *et al.* [12], a large scale example is solved, including fifteen process water-using units (Example 8). Their proposed model for this example consisted of 645 equations, 1028 continuous variables and 96 discrete variables with a total computational time of 3924 s in only 3 iterations. The proposed model in this paper for the same example exhibits a reduced number of equations with a slightly increased number of continuous variables (1254 vs. 1028). However, the number of discrete variables is reduced (15 vs. 96). Total computational time is 1596 s for a total of 1000 solve iterations which is on average less than 2 s per iteration .

Table 16. Model statistics for the studied examples.

Example	No. of equations	No. of continuous variables	No. of discrete variables	CPUs	No. of iterations
1	154	262	9	26	100
2	124	262	9	38	100
3	207	458	12	80	100
4	165	458	12	492	1000
5	189	441	12	59	100
6	215	466	12	103	200
7	220	778	15	218	100
8	262	1254	15	922	500
9	359	1058	15	1596	1000

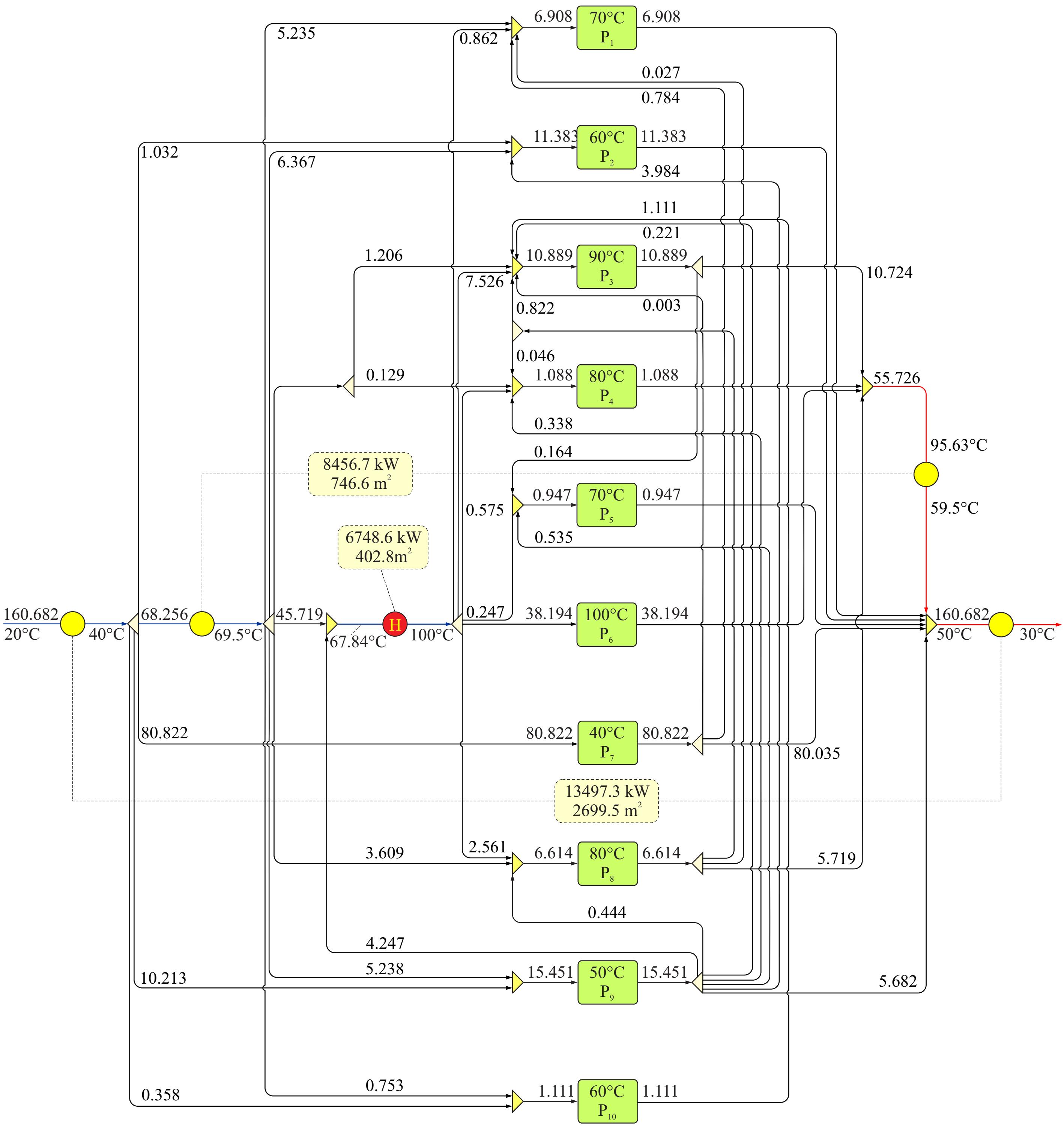

Figure 18. Optimal network design for example 9.

4. Conclusions 423

This paper presents a mathematical programming model for the synthesis of heat-integrated water networks with a modified heat exchanger network. The heat exchanger network consists of a given maximum number of heat exchangers with corresponding heaters and coolers with different heat exchange opportunities. A mixed-integer nonlinear programming model is solved by using a one-step solution strategy and a local solver SBB within GAMS software. Examples of different complexities are solved to demonstrate the capabilities of the model and the results obtained are in good agreement with those in the literature. The model can be extended considering practical constraints and different types of heat exchangers to consider maximum and minimum heat exchange areas. Also, additional model extension is necessary to comprise minimum practical water flowrates and to manage and control the network complexity. The model can be extended to include wastewater treatment units and enable options for water regeneration reuse and recycling.

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Abbreviations

The following abbreviations are used in this manuscript:

EMAT	Exchanger Minimum Approach Temperature
GAMS	General Algebraic Modeling System
GDP	Generalised Disjunctive Programming
HEN	Heat Exchanger Network
HIWN	Heat Integrated Water Network
MINLP	Mixed Integer Nonlinear Programming
MP	Mathematical Programming
NLP	Nonlinear Programming
PA	Pinch Analysis
TAC	Total Annualised Cost
WN	Water Network

Indices

c	Contaminant
e	Heat exchanger
p	Proces water-using unit
s	Freshwater source

Sets

C	Contaminants
E	Heat exchangers
P	Proces water-using units
S	Freshwater sources

Superscripts

C	Cooler
CU	Cold utility
cs	Cold stream
env	Environment
H	Heater
hs	Hot stream
HU	Hot utility
in	Inlet
max	Maximum
out	Outlet
up	Upper bound

Parameters

a	Fixed cost for heat exchanger, \$/y
b	Area cost coefficient for heat exchanger, \$/m^2
CFW_s	Freshwater cost, \$/kg
C_{CU}	Cold utility cost, \$/(kWy)
C_{HU}	Hot utility cost, \$/(kWy)
c_p	Specific heat capacity of water, kJ/(kgK)
h	Individual heat transfer coefficient , kW/(m^{2}K)
H	Plant annual operating hours , h/y
$L_{p,c}$	Mass load of contaminant c in process unit p , kg/s
n	Cost exponent for heat exchanger
$t_p^{(in)}$	Inlet temperature of of the process unit p, °C
$t_p^{(in)}$	Outlet temperature of of the process unit p, °C
t_s	Freshwater temperature of the source s, °C
$t^{(env)}$	Temperature of wastewater discarged into the environment, °C
$x_{s,c}$	Concentration of contaminant c in freshwater source s, ppm
$x_{p,c}^{(in,max)}$	Maximum concentration of contaminant c at the inlet to process unit p, ppm
$x_{p,c}^{(out,max)}$	Maximum concentration of contaminant c at the outlet of process unit p, ppm
Γ	Upper bound for temperature driving force, °C

Continuous variables

A	Heat exchange area, m^2
m	Water mass flowrate, kg/s
x	Contaminant concentration, ppm
t	Stream temperature, °C
q	Heat flowrate (load), W
Δth	Temperature difference at the hot side of heat exchanger , °C
Δtc	Temperature difference at the cold side of heat exchanger , °C

Binary variables

z Existence of heat exhanger, -

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