Decision Automation for Oil and Gas Well Startup Scheduling Using MILP

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Abstract

A novel approach to scheduling the startup of oil and gas wells in multiple fields over a decade-plus discrete-time horizon is presented. The major innovation of our formulation is to treat each well or well type as a batch-process with time-varying yields or production rates that follow the declining, decaying or diminishing curve profile. Side or resource constraints such as process plant capacities, utilities and rigs to place the wells are included in the model. Current approaches to this long-term planning problem in a monthly time-step use manual decision-making with simulators where many scenarios, samples or cases are required to facilitate the development of possible feasible solutions. Our solution to this problem uses mixed-integer linear programming (MILP) which automates the decision-making of deciding on which well to startup next to find optimized solutions. Plots of an illustrative example highlight the operation of the well startup system and the decaying production of wells.

Keywords: Well startup optimization, oil and gas production, long-term planning.

1. Introduction

Long-term production planning of raw materials from oil and gas reserves determines the scheduling of well startups considering a set of resources to be shared among multiple fields. To meet raw material production profile from these wells, efficient modelling and solution capabilities can properly represent the problem and automate the search of a well exploration chart, typically for a decade-plus time horizon considering a monthly or quarterly time-step. However, current approaches to solve such problems count on simulation of scenarios instead of optimization due to the combinatorics of the long-term planning horizon plays with many different well production that must be started-up, sequenced and shutdown subject to considering constraint limiting equipment, workforce and other resources.

We cover in this paper order, placement, timing, capacities and allocations of new wells and well types, along with well production profiles, although the literature in the optimization of oil and gas production also includes surface facilities details. These are manifolds, surface centres, and their interconnections, plus injection profiles of drillings considering pressure, porosity, among other properties and conditions (Flores-Salazar et al., 2011; Gupta and Grossmann, 2012; Tavallali and Karimi, 2016). Although their integrated approach, investigation of medium- to short-term planning or with a year as time-step cannot support strategic decisions to be defined for a decade time-horizon as the proposition of this paper. This aims to guide both the supply of processing plants (in symbiosis with the well production fields) as well as long-term selling contracts of hydrocarbon raw materials such as natural gas, condensates and crude-oils. The optimization considers key resource bottlenecks as equipment, utilities, skilled and unskilled manpower, shared over multiple types of wells in various locations.

2. Problem statement

The problem consists of determining the well startup schedule to maintain production of raw feed materials to the finite-capacity processing plant *P*. The well production system involves a sequencer unit *S* and a rig capacity tank *R* connected to a well *W* in order to model the fact that the rig equipment is shared across multiple well startups. The sequencer *S* is a hypothetical unit to order the drilling of wells with regards to manpower resources. Considering a well unit as a batch-process without replenishment of new materials, is our novel approach to represent the oil and gas well reservoir, where the network in Figure 1 is constructed using the unit-operation-port-state superstructure (UOPSS) formulation (Kelly, 2005; Zyngier and Kelly, 2012). The system is composed of the following objects: a) unit-operations *m* for continuous-processes (⊠), batch-processes (□) and tanks (Δ), and b) their connectivity involving arrows (→), in-ports *i* (○) and out-ports *j* (⊗). Unit-operations and arrows have binary and continuous variables (*y* and *x*, respectively) and their ports hold the states for the relationships among the objects.

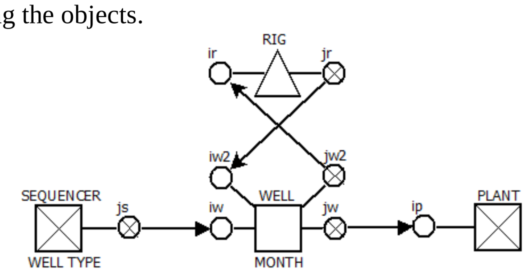

Figure 1. Sequencer, well, rig and plant system.

In the proposed model, the objective function (1) maximizes the production or the flow $x_{j,i,t}$ of raw material from well out-port to the plant's in-port considering the *j*,*i* pairs ∈ JI_{WP}. The semi-continuous constraints of the unit-operations *m* for the sequencer and plant continuous-processes are with respect to $LB_{m,t}\, y_{m,t} \leq x_{m,t} \leq UB_{m,t}\, y_{m,t}$, e.g., if the binary variable $y_{m,t}$ is true, the throughput flow $x_{m,t}$ is between the unit-operation lower and upper bounds; otherwise it is zero. For tank object rig, the flow $x_{m,t}$ is replaced by the holdup variable and bounds $xh_{m,t}$ in the previous semi-continuous constraint. For the well batch-process *W*, its holdup value $xh_{m,t}$ is taken when the unit-operation *m* starts up ($zsu_{m,t}$ = 1) considering the respective bounds ($LB_{m,t}\, zsu_{m,t} \leq xh_{m,t} \leq UB_{m,t}\, zsu_{m,t}$). In the case of well production, both bounds are the same as the well holdup since it is a reservoir. The arrows, representing the connection-flows from out-ports to in-ports, are bounded as $LB_{j,i,t} y_{j,i,t} \leq x_{j,i,t} \leq UB_{j,i,t} y_{j,i,t}$. The UOPSS formulation, given by the objects and their connectivity as in Figure 1, is specified in Eqs. (2) to (13).

Equations (2) and (3) represent, respectively, the sum of the arrows leaving from the out-ports *j* (or splitters) and arriving in the in-ports *i* (or mixers) and their summation must be between the bounds of the unit-operation *m* connected to them. In the case of the sequencer *S* operation in Eq. (2), its out-port to in-port pairs *j*,*i* are not flow of material as in Eq. (3) (for the ports of the well *W* to the plant *P*), instead they are considered a resource, tool or what we refer to as a utensil to be assigned in a startup of a well production. Equations (2) and (3) can be considered semi-continuous constraints for the summation of the flows in- and out- of a port connected to a unit-operation or more specifically to the setup or binary variable of the unit-operation. The arriving or departing flows of a unit occur if it is operating ($y_{m,t}$ = 1). The utensil operation of equipment in Eq. (4) controls the use of the sequencer *S* among all wells to be explored. When the sequencer is operating in a well type, the summation of the arrows leaving its out-port of a unit-operating mode *m* is equal to the sequencer flow $x_{m,t}$ for $m \in M_S$.

Equations (5) to (9) consider bounds on yields, both inverse ($LB_{i,t}$ and $UB_{i,t}$) in the in-ports *i* and direct ($LB_{j,t}$ and $UB_{j,t}$) in the out-ports *j*, since the unit-operations *m* ($m \notin M_R$) can have more than one stream arriving in or leaving from their connected ports. Equations (5) and (6) are related to inverse yields of the utensil in-ports of the well *W* connecting the sequencer *S* and rig *R* through their out-ports. When the well starts up ($zsu_{m,t}$ = 1), its holdup $xh_{m,t}$ is taken as a whole at this moment, then the sequencer and the rig operate. The production from the well startup is governed by Eq. (9) with the new proposition of considering a well as a batch-process with a decaying plot $r_{j,tt}$ defined as a piecewise linear interpolation of the reservoir time-life with different ratios at each pre-defined intervals of the full production time horizon.

The direct yield of the utensil out-port *js* connecting the sequencer *S* to the well *W* is defined in Eq. (8). In Eq. (9), the out-port *jw2* connects the wells to the rig to control its availability. The quantity balance of the inventory or holdup for unit-operations of tanks ($m \in M_R$) is defined in Eq. (10) and manages the availability of the rig *R*. Equations (11) and (12) are the structural transition constraints to facilitate the setup $y_{m,t}$ or startup $zsu_{m,t}$ of different unit-operations interconnected by out-ports *j* and in-ports *i*. If the setup of unit-operations *m* and *m'* are true in Eq. (11), then the setup variable $y_{j,i,t}$ of the arrow stream between them are implicitly turned-on. In Eq. (12), the setup variable of *m'* is replaced by the summation of the startups as wells are treated as batch-processes. These logic valid cuts reduce the tree search in branch-and-bound methods. In Eq. (13), for all physical well units with more than one procedure, mode or task, at most one unit-operation *m* is allowed to startup at a time.

$$Max\ Z = \sum_{t} \sum_{(j,i) \in JI_{WP}} x_{j,i,t} \tag{1}$$

$$\sum_{i \in I_W} x_{j,i,t} \geq LB_{m,t}\, y_{m,t} \quad , \quad \sum_{i \in I_W} x_{j,i,t} \leq UB_{m,t}\, y_{m,t} \quad \forall \quad (m,j) \in M_S, t \tag{2}$$

$$\sum_{j \in J_W} x_{j,i,t} \geq LB_{m,t}\, y_{m,t} \quad , \quad \sum_{j \in J_W} x_{j,i,t} \leq UB_{m,t}\, y_{m,t} \quad \forall \quad (i,m) \in M_P, t \tag{3}$$

$$\sum_{i \in I_W} x_{j,i,t} = x_{m,t} \quad \forall \quad (j,m) \in M_S, t \tag{4}$$

$$\sum_{j \in J_S} x_{j,i,t} \geq LB_{i,t}\, xh_{m,t} \quad , \quad \sum_{j \in J_S} x_{j,i,t} \leq UB_{i,t}\, xh_{m,t} \quad \forall \quad (i,m) \in M_W, t \tag{5}$$

$$\sum_{j \in J_R} x_{j,i,t} \geq LB_{i,t}\, xh_{m,t} \quad , \quad \sum_{j \in J_R} x_{j,i,t} \leq UB_{i,t}\, xh_{m,t} \quad \forall \quad (i,m) \in M_W, t \tag{6}$$

$$\sum_{tt<t} r_{j,tt}\, xh_{m,tt} = \sum_{i \in I_P} x_{j,i,t} \quad \forall \quad (m,j) \in M_W, t \tag{7}$$

$$\sum_{i \in I_W} x_{j,i,t} \geq LB_{j,t}\, x_{m,t} \quad , \quad \sum_{i \in I_W} x_{j,i,t} \leq UB_{j,t}\, x_{m,t} \quad \forall \quad (m,j) \in M_S, t \tag{8}$$

$$\sum_{i \in I_R} x_{j,i,t} \geq LB_{j,t}\, xh_{m,t} \quad , \quad \sum_{i \in I_R} x_{j,i,t} \leq UB_{j,t}\, xh_{m,t} \quad \forall \quad (m,j) \in M_W, t \tag{9}$$

$$xh_{m,t} = xh_{m,t-1} + \sum_{j \in J_W} x_{j,i,t} - \sum_{i \in I_W} x_{j,i,t} \quad \forall \quad (i,m,j) \in M_R, t \tag{10}$$

$$y_{m',t} + y_{m,t} \geq 2y_{j,i,t} \quad \forall \quad (m',j,i,m), t \tag{11}$$

$$\sum_{tt<t} zsu_{m',tt} + y_{m,t} \geq 2y_{j,i,t} \quad \forall \quad (m',j,i,m), t \tag{12}$$

$$\sum_{m \in W} zsu_{m,t} \leq 1 \quad \forall \quad t \tag{13}$$

$$x_{m,t}, x_{j,i,t}, xh_{m,t} \geq 0;\ y_{j,i,t}, y_{m,t} = \{0,1\};\ zsu_{m,t} = [0,1] \tag{14}$$

3. Sequence-dependent, run-length and number of startups

Equations (15) to (17) are the temporal transition constraints from Kelly and Zyngier (2007). They are applied to the sequencer *S* and well *W*, since their operation (turn-on or turn-off) is part of the optimization problem. For the plant *P* and rig *R*, their binary variables are fixed as they can always operate. In these sequence-dependent relationships, setup or binary variables $y_{m,t}$ manage the dependent startup, switch-over-to-itself and shutdown variables ($zsu_{m,t}$, $zsw_{m,t}$ and $zsd_{m,t}$, respectively) that are relaxed in the interval [0,1] instead of considering them as logic variables. Equation (17) is necessary to guarantee the integrality of the relaxed variables.

$$y_{m,t} - y_{m,t-1} - zsu_{m,t} + zsd_{m,t} = 0 \quad \forall \quad m \in M_S \ \vee \ M_W, t \tag{15}$$

$$y_{m,t} + y_{m,t-1} - zsu_{m,t} - zsd_{m,t} - 2zsw_{m,t} = 0 \quad \forall \quad m \in M_S \ \vee \ M_W, t \tag{16}$$

$$zsu_{m,t} + zsd_{m,t} + zsw_{m,t} \leq 1 \quad \forall \quad m \in M_S \ \vee \ M_W, t \tag{17}$$

Equations (18) and (19) model the run-length or uptime considering *UT* as the lower bound of using the sequencer (the workforce), meaning that when the exploration starts in a well the length of the operation cannot exceed the total number of time-periods (time horizon divided by Δt as time-step); more details on these constraints can be found in Kelly and Zyngier (2007) and Zyngier and Kelly (2009). Equation (20) controls the total number of startups allowed per well *W*.

$$\frac{UT}{\Delta t}\sum_{t} zsu_{m,t} \le n_{periods} \quad \forall \quad m \in M_S \tag{18}$$

$$\sum_{tt=t-UT}^{t-1} z_{m,tt} \le y_{m,t} \quad \forall \quad m \in M_S, t > 1 \tag{19}$$

$$\sum_{t} zsu_{m,t} \le n_{startups}^{W} \quad \forall \quad m \in M_W \tag{20}$$

4. Illustrative example

The illustrative example in Figure 2 defines the well exploration and production of 60 wells (10 Type A, 20 Type B and 30 Type C) with a time-horizon duration of 14-years and 1-month time-period durations (totally 168 time-periods). The well production flowrate profile uses a declining or decaying curve equation to profile its relative-time production variation after its start-up using standard barrel equivalents (SBE) for its oil, gas and condensate. There is a production plant that has time-varying upper capacity from 0 to 10 kSBE/day and a rig that allows one well startup per month. The well type A has different rate of production by varying the drilling inclination, size, pressure, etc.

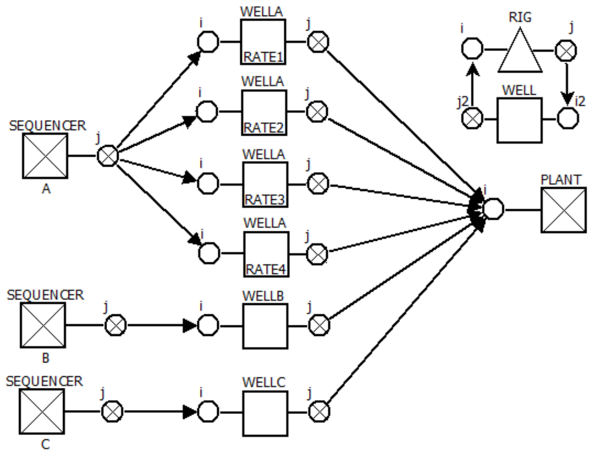

Figure 2. Illustrative example for 60 wells in 14 years with 1-month as time-step.

This example uses the structural-based unit-operation-port-state superstructure (UOPSS) found in the semantic-oriented platform IMPL (Industrial Modeling and Programming Language) using Intel Core i7 machine at 2.7 Hz with 16GB of RAM. Figure 3 shows the unit-operation Gantt chart for the entire problem found in Figure 2. The future time-horizon is 168-months discretized into 1-month time-period. The MILP objective yields 987.5 kSBE of raw material processed by the plant *P*. The problem has 9,566 continuous and 5,670 binary variables and 8,054 equality and 16,077 inequality constraints (degrees-of-freedom = 7,182) and is solved in 146.8 seconds using 8 threads in CPLEX 12.6. The results of the exploration of the wells starts in B, then C and finally A. The overall processing feed during the total horizon of 168 months is maximum if the well production starts after 20 months. Problems with minimal plant feed from the initial time had lower objective functions. The in situ small plots highlight the decaying curves of well B and well A (rate3) with respect to the startups.

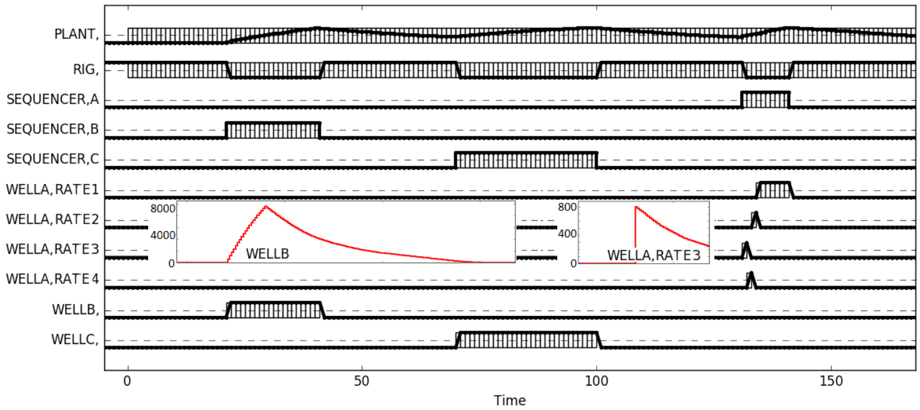

Figure 3. Gantt chart for the illustrative example.

5. Conclusions

The oil and gas well startup scheduling presented here is an important guidance for strategic decisions into long-term selling contracts and related investment strategies balancing processing plant capacities and associated logistics for raw materials distribution in pipelines. The modelling of each well or well type as a batch-process is the novel feature of this paper.

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