ORIGINAL ARTICLE

Journal Section

Shale Gas Pad Development Planning under Price Uncertainty

Can Li[1] | John P. Eason[2] | Markus G. Drouven[2] | Ignacio E. Grossmann[1]

[1]Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

[2]Optimization Engineering, EQT Corporation, Pittsburgh, PA 15222, USA

Correspondence
Ignacio E. Grossmann, Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA
Email: grossmann@cmu.edu

Funding information
EQT Corporation; Scott Energy Institute, Carnegie Mellon University

In this paper, we study shale gas pad development under natural gas price uncertainty. We optimize the sequence of operations, gas curtailment and storage on a single pad to maximize the net present value (NPV). The optimization problem is formulated as an mixed-integer linear programming (MILP) model, which is similar to the one proposed by Ondeck et al. [1]. We investigate how natural gas price uncertainty affects the operation strategy in the pad development. Both two-stage and multi-stage stochastic programming are used as the mathematical framework to hedge against uncertainty. Our case study shows that there is value of using stochastic programming when the price variance is high. However, when the variance of the price is low, solving the stochastic programming problems does not create additional value compared with solving the deterministic problem.

KEYWORDS
Shale gas, Pad development, Price uncertainty, Stochastic programming

1 | INTRODUCTION

According to the Annual Energy Outlook 2019 published by the EIA [2], the production of natural gas is expected to grow by 50% in the next 30 years. The growth in natural gas production supports increasing domestic consumption, particularly in the industrial and electric power sectors, and higher levels of natural gas exports. In Figure 1, the prediction of the consumption and the production of dry natural gas in the U.S. is shown under different sensitivity cases. We can observe that there will likely be an increase in both the production and the consumption of dry natural gas even in the case of low economic growth and low oil price.

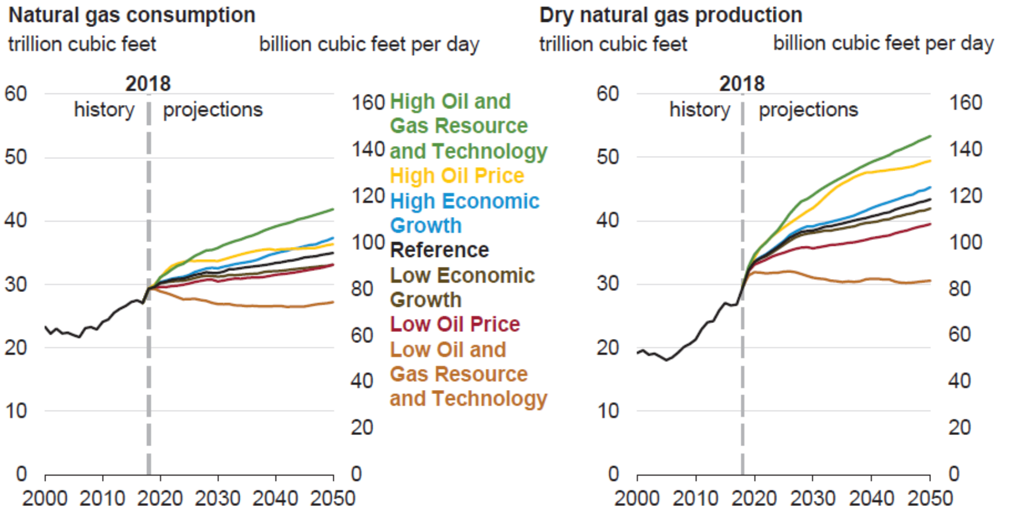

FIGURE 1. The prediction of natural gas production and consumption in different sensitivity cases

The increase in natural gas production is driven by continued development of lower-cost shale gas resources. The Annual Energy Outlook 2019 [2] shows that dry natural gas production from shale gas and tight oil continues to grow in both share and absolute volume because of the sheer size of the associated resources, which extend over nearly 500,000 square miles, and because of improvements in technology that allow for the development of these resources at lower costs. Shale gas is expected to account for 90% of U.S. dry natural gas production in 2050.

Despite the significant role that shale gas plays in the energy industry, the shale gas industry is still young [3]. Recently, the optimization of shale gas systems has drawn increasing attention in academia. Many shale gas companies [4] also start to apply optimization models to make better use of their resources. A common problem that arises in the shale gas industry is the operation of a single shale gas wellpad as it affects the whole natural gas supply chain. Recently, Ondeck et al. [1] proposed an MILP (mixed-integer linear programming) model that considers the economical development of a shale gas pad and the optimal production of shale gas.

To develop a well to production (Figure 2), four operations must take place in the following order: 1) top setting (TS), 2) horizontal drilling (HZ), 3) hydraulic fracturing (FRAC), and 4) turning in line (TIL). The first operation, top setting, is the process of drilling a well down to the selected shale gas formation and properly encasing the well to prevent the release of gas and other chemicals into the ground surrounding the well bore. Once the vertical part of the well has been developed, the next step is horizontal drilling (HZ), which is followed by fracturing (FRAC). Fracturing refers to the injection of a fracturing fluid into a geologically tight formation under high pressure of up to 70 MPa. Once the fracturing is complete, the well can be turned in line to release the gas. Based on the desired production, the entire well or sections of the well can be turned in line. From start to finish, the process of completing a well can take anywhere from a few weeks to two months, based on the geology of the subsurface, the length of the well, and the availability of resources.

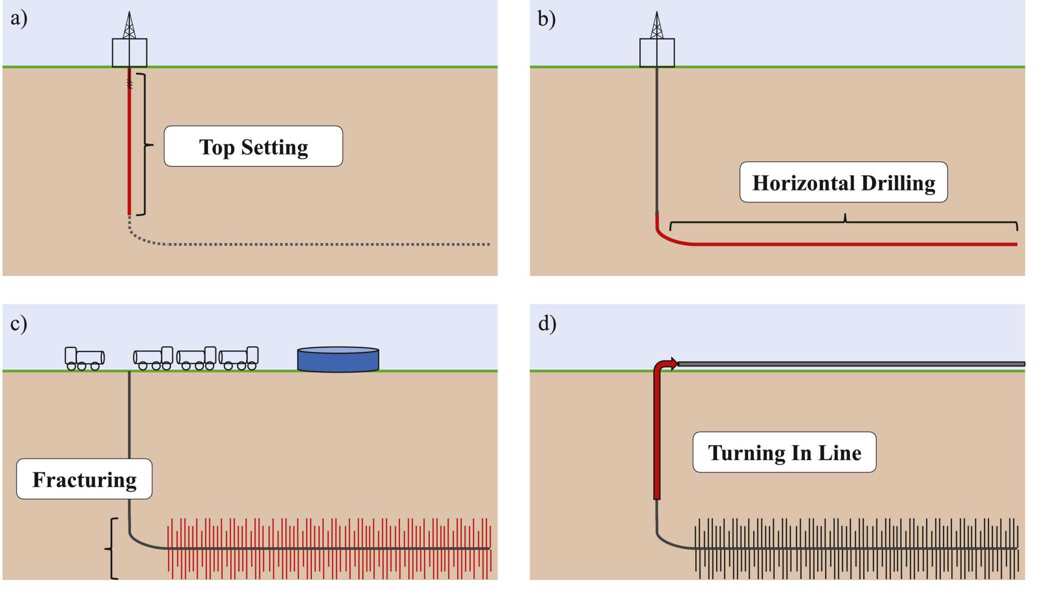

FIGURE 2. The four operations necessary to develop a well to completion: (a) top setting, (b) horizontal drilling, (c) hydraulic fracturing, and (d) turning in line.

Ondeck et al. [1] point out that traditionally, upstream operators complete one operation for all the wells in a single wellpad before moving to the next operation to reduce the mobilization of their crew and equipment. However, there are several drawbacks for this type of operating strategy. First, since all the wells are turned in line around the same time, there is a dramatic increase of natural gas production in this single pad. Therefore, pipelines with large capacities are needed in order to deliver the natural gas to the customers. Second, since the wells cannot be turned in line until all the first three operations are completed for all the wells in the pad, the lateness of turning in line incurs a loss in the net present value (NPV) compared with the strategy in which some of the wells are turned in line before all the first three operations are completed for all the three wells. Third, the production of a typical shale gas well decreases sharply after the first year of turning in line. Therefore, if all the wells are turned in line around the same time, there will be a period with little gas production.

From our analysis on the conventional strategy of single pad operations, there is a tradeoff between avoiding the mobilization costs, and turning the well in line earlier to maximize the production NPV. In the paper by Ondeck et al. [1], the authors perform a sensitivity analysis by varying the mobilization costs. Considering mobilization costs tends to affect the schedule of the single pad development.

This paper follows the work of Ondeck et al. [1] and investigates whether price uncertainty affects the optimal strategy of single pad development. Stochastic programming [5] is used as the mathematical framework to model decision-making under uncertainty. This paper is organized as follows. In section 2, we review some related work on the optimization models in the shale gas industry. The deterministic model for single pad planning is described in section 3. We provide the motivation for using stochastic programming to model the single pad development problem in section 4. A brief introduction to stochastic programming is given in section 5. Several case studies are given in section 6 to demonstrate whether using stochastic programming can create additional value compared with the deterministic model. We draw the conclusions in section 7.

2 | RELATED WORK

In this section, we review the literature related to the optimization of shale gas systems. We first review the deterministic models. The models that consider uncertainty are relatively few and are reviewed at the end of this section. The deterministic optimization models for shale gas development can be grouped into three levels based on the time scale: design/planning, scheduling, and operating.

The design/planning models correspond to long term decisions, usually for more than a decade. Cafaro and Grossmann [6] present a mixed-integer nonlinear programming (MINLP) model for the strategic planning, design, and development of the shale gas supply chain network where they determine planning decisions such as the number of wells to drill at every location, and design decisions, such as the size of gas processing plants, the length and diameter of the pipelines so as to maximize the net present value of the project. Following the work of Cafaro and Grossmann [6], Drouven and Grossmann [7] propose an MINLP model that involves planning, design, and strategic decisions such as where, when, and how many shale gas wells to drill, where to lay out gathering pipelines, as well as which delivery agreements to arrange. Guerra et al. [8] propose an optimization framework for the integration of water management and shale gas supply chain design. Gao and You [9] propose an MINLP model that addresses the life cycle economic and environmental optimization of shale gas supply chain network design and operations. Arredondo-Ramírez et al.[10] use a disjunctive programming-based approach to account for complex logical relationships in the optimal planning of shale gas exploitation and infrastructure development.

The optimal scheduling of operations for shale gas development considers shorter time period than in planning/design. Knudsen et al. [11] formulate an MILP model, which is solved using Lagrangean decomposition, for shut-in scheduling in large, multi-well shale-gas systems. Cafaro et al. [12] propose a continuous time MINLP model and a discrete time MILP model for planning shale gas well refracture treatments. Cafaro et al. [13] propose a continuous time optimization model for planning multiple refracture treatments over the lifespan of a shale gas well. Yang et al. [14] propose an MILP model that addresses source water acquisition, waste water production, reuse and recycle, and subsequent transportation, storage, and disposal in shale gas production. Ondeck et al. [1] consider the scheduling of the four operations in a single wellpad.

At the operational level, decisions are made on a daily to weekly time scale. Forouzanfar and Reynolds [15] formulate a continuous optimization model to simultaneously optimize the number of wells, their locations and controls. Wilson and Durlofsky [16] develop a surrogate model to accurately model complex shale gas reservoirs and further use the model for shale gas field development optimization. Others [17, 18] have studied the placement of hydraulic fracture stages.

The models for shale gas development that consider uncertainty are relatively few. Drouven et al. [19] apply two-stage stochastic programming in a moving horizon approach for optimal shale well development and refracturing planning under exogenous gas price uncertainty and endogenous well performance uncertainty. Gao and You [20] develop a two-stage stochastic mixed integer linear fractional programming (SMILFP) model to optimize the levelized cost of energy generated from shale gas under uncertainty of estimated ultimate recovery (EUR). Guerra et al. [21] develop a two-stage stochastic model embedded in a moving horizon strategy to dynamically solve the planning of shale well development and refracturing. Zeng and Cremaschi [22] propose stochastic programming models to the artificial lift infrastructure planning for shale gas producing wells.

3 | PROBLEM STATEMENT FOR THE DETERMINISTIC MODEL

The deterministic problem addressed in this article is similar to the one proposed by Ondeck et al. [1]. We consider the shale gas development problem for a single wellpad represented in Figure 3. The potential wells are identified *a priori*. The productivity profile for each well is also given. In order to complete a shale gas well, four operations, i.e., top setting, horizontal drilling, fracturing, turning in line, need to be done sequentially. The corresponding equipment needs to be disassembled once the operation on the wellpad is changed and there is a mobilization cost for the change of operation.

We consider gas curtailment in the model, i.e., the the amount of gas produced can be less than the amount determined by the production curve. The curtailed gas is stored in the well and can be released in future time periods. Instead of modeling the storage of curtailed gas in each well, we simplify the problem by considering virtual storage of the whole wellpad.

The decisions in the model are: 1) when and which wells should be developed to completion, 2) the timing of the operations performed on the well, 3) when and how much gas should be released from the completed wells. 4) the amount of gas coming in and out of storage at each time.

The major assumptions in this work are:

1. The locations and the information related to the development and the production of the prospective wells are known *a priori*. This includes the lengths of the wells as well as the production curves, which are functions of the wells' horizontal drilling lengths. The wells are optimally placed laterally such that well interference does not occur.
2. Well shut-in is not allowed.
3. The development cost for all four operations and the time to complete each operation are known for each well.
4. Every operation is performed for the entire well. A well cannot be "partially" completed. Note that this requirement is removed in the stochastic programming models.
5. Every operation can be performed at most once at a well. There are no refracturing of wells at a later time.
6. The development resource mobilization cost is a one time fee that includes transportation, assembly, and disassembly. The initial value is an assumed estimate, calculated based on past experience.
7. The optimization is solved using a discrete time model with time intervals of one week.

Since the mathematical formulation of the deterministic model is similar to [1], we include the description of the MILP model in Appendix A. The focus of this paper is on the extensions of the deterministic MILP model, which consider uncertainty in natural gas price.

4 | MOTIVATION FOR A STOCHASTIC PROGRAMMING MODEL

Although the deterministic model for the single pad shale gas development problem has been studied by [1], there is no model for single pad planning that considers uncertainty in the parameters to the planning model.

Natural gas price is a major source of uncertainty in the planning problem. Henry Hub natural gas spot prices from 1998-2018 [23] are shown in Figure 4. It is easy to observe that the natural gas price has fluctuated significantly in the past 20 years. On the one hand, upstream operators would like to develop more wells when the price is high. On the other hand, there are also scenarios where natural gas is no longer a profitable business.

Therefore, in this paper, we investigate the effect of the uncertainties from natural gas price on the optimal decision-making in the pad development problem. Since the decisions involved in the pad development are long-term decisions,

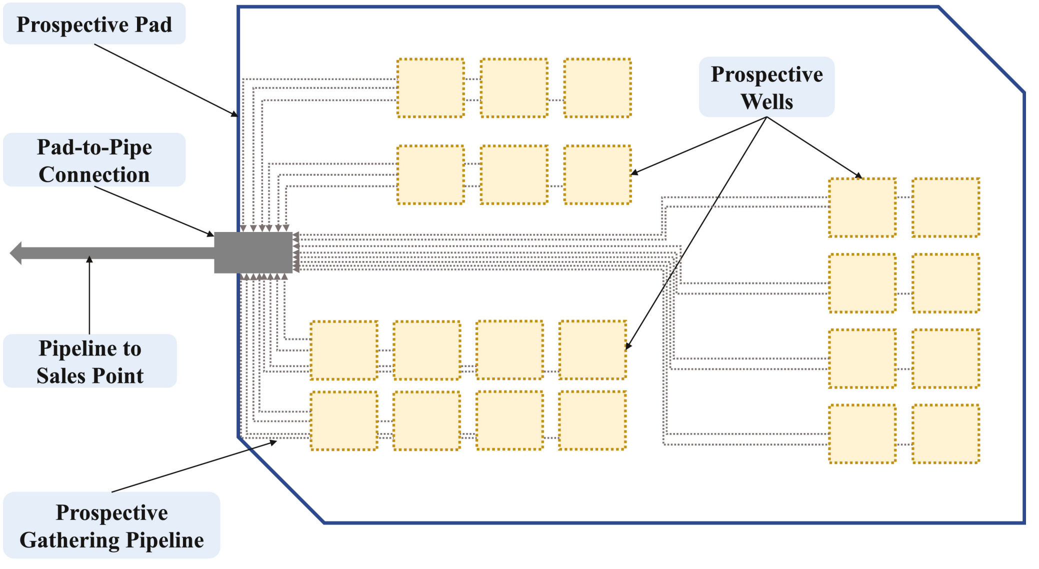

FIGURE 3. Prospective pad

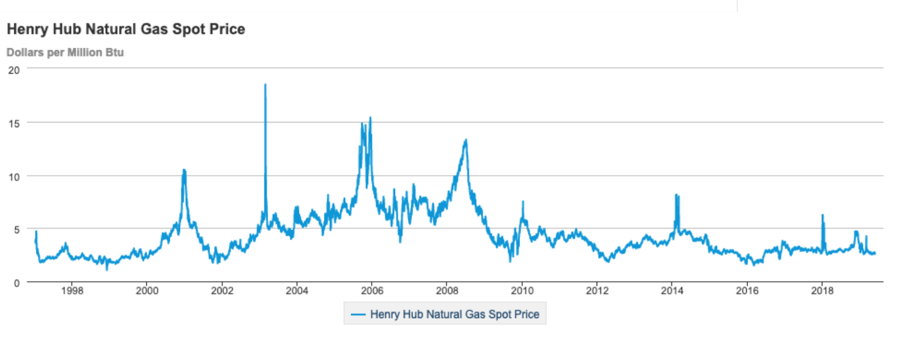

FIGURE 4. Historical natural gas price

usually ranging from a few months to years, we use stochastic programming to maximize the expected net present value. A brief introduction to stochastic programming is given in the next section.

5 | INTRODUCTION TO STOCHASTIC PROGRAMMING

Stochastic programming is an optimization framework that deals with decision making under uncertainty [5]. In stochastic programming, it is assumed that the probability distributions of the uncertain parameters are known *a priori*. The uncertainties are usually characterized by some discrete realizations of the uncertain parameters as an approximation to the continuous probability distribution. For example, the realizations of the demand of a product can have three different values which represent high, medium, and low demand, respectively. Each realization is defined as a *scenario*. The objective of stochastic programming is to optimize the expected value of an objective function (e.g., the

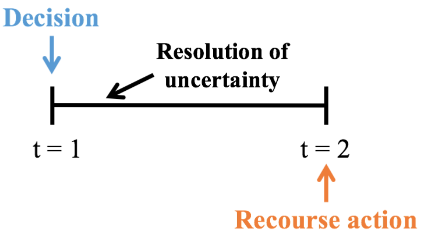

FIGURE 5. Two-stage problem

expected cost) over all the scenarios.

5.1 | Two-stage stochastic programming

A special case of stochastic programming is two-stage stochastic programming (Figure 5). Specifically, stage 1 decisions are made 'here and now' at the beginning of the period, and are then followed by the resolution of uncertainty. Stage 2 'wait and see' decisions, or recourse decisions, are taken as corrective actions at the end of the period. One common type of a two-stage stochastic program is the mixed-integer linear program presented in equation (1). Set Ω is the set of scenarios. Parameter $\tau(\omega)$ is the probability of scenario ω. The n-dimensional vector x represents the first stage decisions, while the m-dimensional vector $y(\omega)$ represents the second stage decisions in scenario ω. Both x and $y(\omega)$ variables can be mixed-integer. Without loss of generality, we assume that the first n_1 ($n_1 \leq n$) variables of the first stage decisions and the first m_1 ($m_1 \leq m$) variables of the second stage decisions are binary. The uncertainties are reflected in the matrices (vectors), $W(\omega)$, $T(\omega)$, $h(\omega)$. Equation (1) is often referred to as the *deterministic equivalent* of the two-stage stochastic program. This problem can be solved directly if the number of scenarios is modest; if the number of scenarios is large, special decomposition algorithms, such as Benders decomposition [24], can be used. However, if we have (mixed) integer stage two variables, Benders decomposition is not applicable.

$$\min \quad c^T x + \sum_{\omega \in \Omega} \tau(\omega) d^T(\omega) y(\omega) \tag{1a}$$

$$\text{s.t.} \quad Ax \leq b \tag{1b}$$

$$W(\omega) y(\omega) \leq h(\omega) - T(\omega) x \quad \forall \omega \in \Omega \tag{1c}$$

$$x \in \left\{ x = (x_1, x_2) : x_1 \in \{0, 1\}^{n_1}, x_2 \geq 0 \right\} \tag{1d}$$

$$y(\omega) \in \left\{ y = (y_1, y_2) : y_1 \in \{0, 1\}^{m_1}, y_2 \geq 0 \right\} \tag{1e}$$

5.2 | The value of stochastic solution

The value of stochastic solution (VSS) [5] is used to quantify the value that stochastic programming yields compared with the deterministic model. We need to define some notation before we present the mathematical expression for VSS. Let $\xi(\omega)$ be the vector that represents the random parameters involved in the two-stage stochastic programming problem for scenario ω. Define $Q(x, \xi(\omega))$

$$Q(x, \xi(\omega)) = c^T x + \min_y d^T(\omega) y \tag{2a}$$

$$\text{s.t.} \quad W(\omega) y \leq h(\omega) - T(\omega) x, \quad y \in \left\{ y = (y_1, y_2) : y_1 \in \{0, 1\}^{m_1} \quad y_2 \geq 0 \right\} \tag{2b}$$

as the optimization problem associated with one particular realization of random parameter ξ. The expected value of ξ is defined as $\bar{\xi}$. The *expected value solution* is defined as

$$\bar{x}(\bar{\xi}) = \arg\min_x Q\left(x, \bar{\xi}\right) \tag{3}$$

where the parameter ξ is fixed at its expected value $\bar{\xi}$. In order to quantify how the expected value solution performs in different scenarios, we define *the expected results of using the expected value solution* (EEV) as

$$EEV = \mathbb{E}_\xi\left[Q\left(\bar{x}(\bar{\xi}), \xi\right) \right] \tag{4}$$

The *recourse problem* (RP), i.e., the two-stage stochastic program, is defined as

$$RP = \min_x \mathbb{E}_\xi Q(x, \xi) \tag{5}$$

which chooses the first stage decisions x that minimizes the expected value of $Q(x, \xi)$. The difference of EEV and RP can quantify the difference of the expected cost between the expected value solution and the stochastic solution. Therefore, it is reasonable to define $EEV - RP$ as the *value of stochastic solution* (VSS),

$$VSS = EEV - RP \tag{6}$$

5.3 | Multistage stochastic programming

The two-stage decision-making process can be generalized to account for multiple stages. Multistage stochastic programming models allow recourse decisions in each stage coming after stage one. Hence, they are also fully adaptive to the uncertainty realization. An example of a multistage scenario tree is shown in Figure 6, where we have 3 different realizations of the uncertainty parameters for stage two and three, and end up with $3^2 = 9$ scenarios in total. In stage two, the decision-maker is only aware of the uncertainties realized at stage two; the parameters in stage three cannot be realized until the time goes to stage three.

The general multistage stochastic programming formulation is given as follows [5]:

$$\min c_1 x_1 + \mathbb{E}_{\xi_2}[\min c_2(\omega_2) x_2(\omega_2) + \ldots + \mathbb{E}_{\xi_H}[\min c_H(\omega_H) x_H(\omega_H)] \ldots] \tag{7a}$$

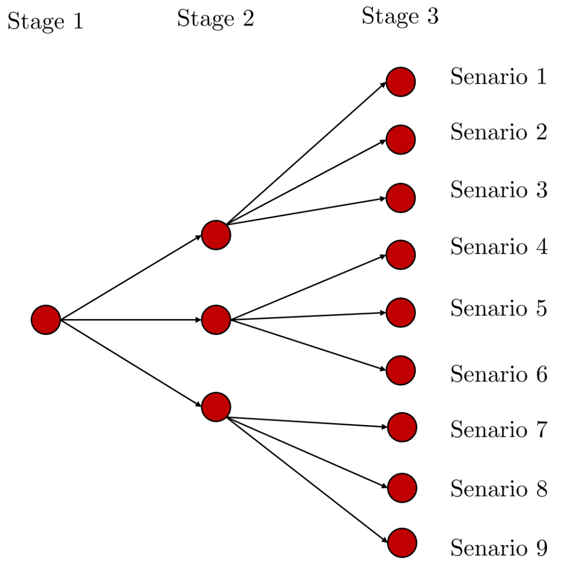

FIGURE 6. Illustration of a scenario tree with 3 stages and 3 realizations per stage

$$\text{s.t.} \quad W_1 x_1 = h_1 \tag{7b}$$

$$T_1(\omega_2)x_1 + W_2 x_2(\omega_2) = h_2(\omega) \tag{7c}$$

$$\dots \tag{7d}$$

$$T_{H-1}(\omega_H)x_{H-1}(\omega_{H-1}) + W_H x_H(\omega_H) = h_H(\omega) \tag{7e}$$

$$x_1 \in X_1; x_t(\omega_t) \in X_t, t = 2, ..., H; \tag{7f}$$

where we have H stages. The set of scenarios in stage t is represented as ω_t. The *deterministic equivalent* of the general multistage stochastic programming is then defined as follows [5]:

$$\min_{x \in X_1} \{c_1 x_1 + Q_2(x_1) : W_1 x_1 = h_1\} \tag{8}$$

where the expected value function for stage $t+1$ is given by:

$$Q_{t+1}(x_t) = \mathbb{E}_{\xi_{t+1}}[Q_{t+1}(x_t, \xi_{t+1}(\omega))] \tag{9}$$

for all t to obtain the recursion for $t = 2, ..., H-1$,

$$Q_t(x_{t-1}, \xi_t(\omega)) = \left\{ \min_{x(\omega) \in X_t} c_t(\omega)x_t(\omega) + Q_{t+1}(x_t) : W_t x_t(\omega) = h_t(\omega) - T_{t-1}(\omega)x_{t-1} \right\} \tag{10}$$

For the terminal condition $t = H$, we have:

$$Q_H(x_{H-1}, \xi_H(\omega)) = \left\{ \min_{x(\omega) \in X_H} c_H(\omega)x_H(\omega) : W_H x_H(\omega) = h_H(\omega) - T_{H-1}(\omega)x_{H-1} \right\} \tag{11}$$

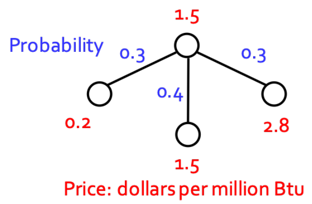

FIGURE 7. The scenario tree for two-stage stochastic programming with high price variance

6 | CASE STUDY

In the case study, we use a data set that has 9 wells in a single wellpad. We assume that none of the operations have been performed on the wells. All the wells are allowed to be developed at any time in the planning horizon. Each of the four operations takes one to two weeks to finish. If all the operations for the 9 wells are performed, 41 weeks are needed. Therefore, we assume that the whole planning horizon is slightly over 41 weeks in the case studies. The details of the length of the planning horizon can be found in the subsections below. We assume that a well can be “partially" completed, which is different from the assumption in the problem statement for the deterministic model in section 3 because when the price becomes low, it can be unprofitable to complete the wells.

To showcase whether using stochastic programming can create value, we apply both two-stage and multistage stochastic programming to the single pad development problem. We also evaluate the impact of the variance of the prices in the scenarios on the optimal solution of the two-stage stochastic programs. The details can be found in the following subsections.

6.1 | Two-stage stochastic programming with high price variance

We investigate price uncertainty by formulating a two-stage stochastic programming problem. The whole planning horizon is 45 weeks with the decisions of the first 20 weeks corresponding to the first stage decisions and the decisions of week 21 - week 45 corresponding to the second stage decisions. The scenario tree for this problem is shown in Figure 7. We assume that the prices are 0.2, 1.5, 2.8 dollars per million Btu for the three scenarios, which are shown in red in Figure 7. The probabilities for each scenario are shown in blue in Figure 7, being 0.3, 0.4, 0.3, respectively. It should be noted that the prices here are lower than the natural gas spot price shown in Figure 4. This is because in the proposed MILP model we do not consider all the costs that are needed to deliver natural gas to the customers, for example, transportation costs are not considered in the model. Therefore, the natural gas prices should be discounted in order to correctly characterize whether developing the wells is profitable or not. We also assume that the prices are constant after the first stage, i.e., the price outside the planning horizon is the same as the price in the second stage of the stochastic programming problem.

All the problems are solved using CPLEX 12.7 [25] on the 12 processors of an Intel Xeon (2.67GHz) machine with 64 GB RAM. The walltime limit is set to 12 hours. The computational statistics of the deterministic problem and the stochastic programming problem including the number of binary and continuous variables, the number of constraints, walltime, optimality gap, are shown in Table 1. The stochastic model cannot be solved to optimality within our time limit.

To compare the difference between the deterministic model and the stochastic programming model, we first solve the deterministic model with the price of natural gas fixed at 1.5 dollars per Btu. The Gantt chart for the deterministic problem is shown in Figure 8. All the 9 wells are completed in the deterministic solution. Five wells are turned in line

TABLE 1. Computational statistics of the deterministic and the stochastic model with high price variance

	Binary Var	Continuous Var	Constraints	Walltime	gap
Deterministic	3655	867	4524	1,112 secs	0.01%
Stochastic	9,963	1,859	12,828	12 hrs	2.41%

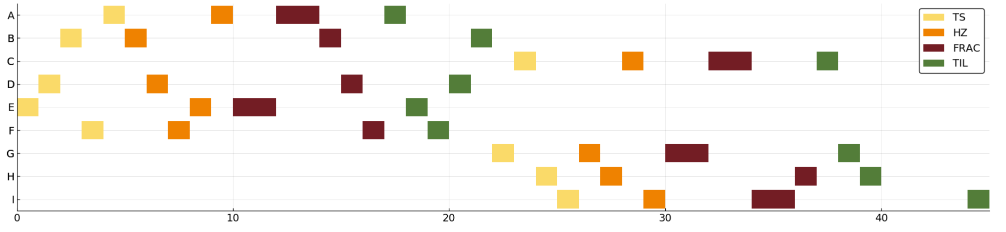

FIGURE 8. Gantt Chart of the deterministic problem

around week 20. The other four wells are turned in line at the end of the planning horizon.

Then we fix the first stage decisions and check how it performs in different scenarios, i.e., the decisions from week 1 to week 20 are fixed at the corresponding optimal solution of the deterministic model and we only optimize the decisions from week 21 to week 45 for the three scenarios, respectively. The Gantt charts for prices equal to 0.2, 1.5, 2.8 dollars per Btu, are shown in Figures 9, 10, and 11, where the first stage decisions are obscured to denote that they are fixed based on the deterministic model.

In the scenario when the price equals to 0.2 dollar per Btu, the optimal solution only turns in line the two wells that have not been completed in the first stage. No new wells are developed in stage two when the price is low. However, the scenarios when the price is 1.5 or 2.8 dollars per Btu, all the 9 wells are completed within the planning horizon.

The stochastic solutions for the three scenarios are shown in Figures 12, 13, and 14. Compared with the deterministic solution that fractures five wells in the first stage, the stochastic solution only fractures three wells in the first stage. The stochastic solution tries to wait until the second stage to decide if more wells should be fractured. In the scenario when the price is 0.2 dollars per Btu, no more wells are fractured in the second stage. When the prices are greater than or equal to 1.5 dollars per Btu, all the wells are completed in stage 2. Note that two wells are top set at the end of stage 1 so that the wells can be turned in line faster once the operators realize that the price is going up. Moreover, although the two top set wells are turned in line in the low price scenario, this does not sacrifice the overall expected NPV significantly since top setting is the cheapest operation among the four operations.

In order to demonstrate the value that stochastic programming can create, we show the NPVs of the expected

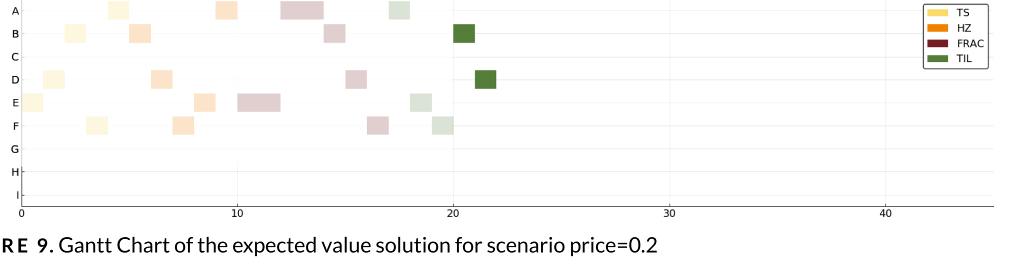

FIGURE 9. Gantt Chart of the expected value solution for scenario price=0.2

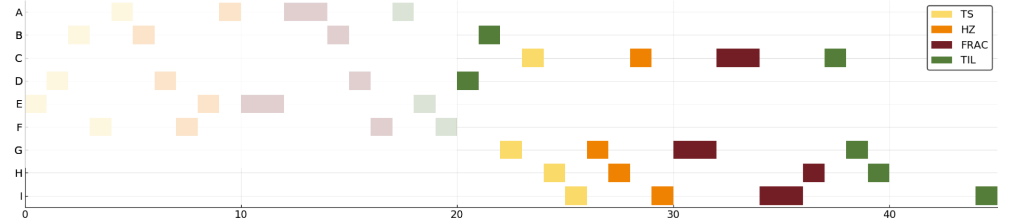

FIGURE 10. Gantt Chart of the expected value solution for scenario price=1.5

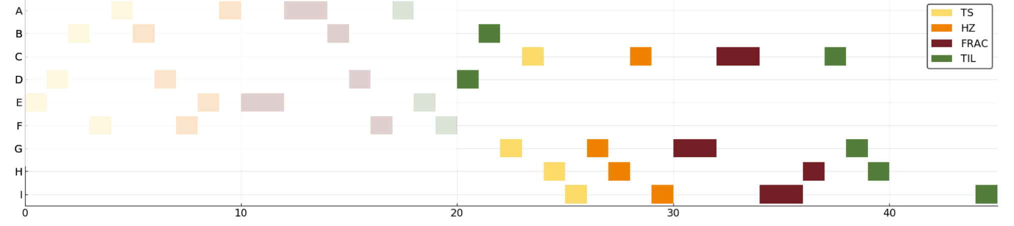

FIGURE 11. Gantt Chart of the expected value solution for scenario price=2.8

solution and the stochastic solution in all the three scenarios in Table 2. While the expected value solution has slightly larger NPVs when the prices are medium or high, it has a highly negative NPV when the price is 0.2 dollars per Btu. On the other hand, the stochastic solution tries to delay development decisions to stage two as much as possible and has higher NPV than the expected value solution in the low price scenario. By taking the expected NPV, we can calculate that the value of stochastic solution is 3.50 million dollars.

6.2 | Two-stage stochastic programming with low price variance

In this subsection, we change the assumption of the scenario tree slightly compared with subsection 6.1. The scenario tree is shown in Figure 10 when the price has smaller variance compared with the scenario tree in Figure 9. The prices in the three scenarios are 0.75, 1.5, and 2.25 dollars per Btu.

We solve the same two-stage stochastic programming problem with week 1 to week 20 as the first stage, and week 21 to week 45 as the second stage. It turns out that the optimal solutions of the three scenarios are all similar to the expected value solution shown in Figure 8. Hence, there is no value in solving the stochastic programming model. The main reason is that in all the three scenarios natural gas is profitable. Therefore, all the wells are completed as in the deterministic model for all the three scenarios.

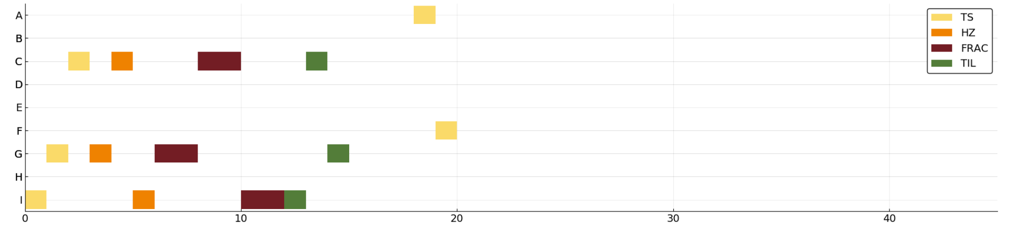

FIGURE 12. Gantt Chart of the stochastic solution for scenario price=0.2

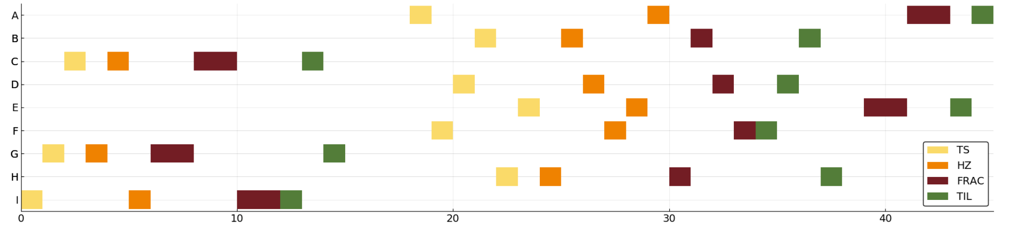

FIGURE 13. Gantt Chart of the stochastic solution for scenario price=1.5

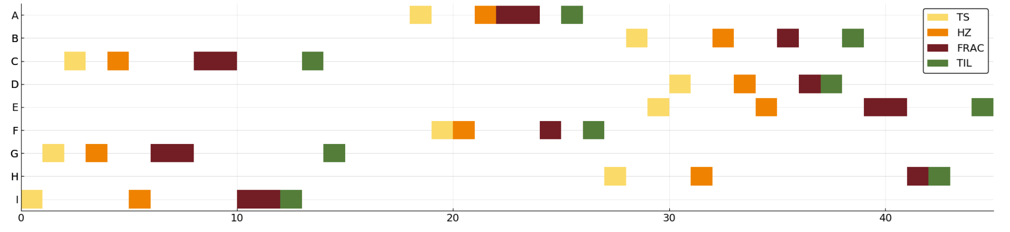

FIGURE 14. Gantt Chart of the stochastic solution for scenario price=2.8

6.3 | Multistage stochastic programming with high price variance

Multistage stochastic programming is a more accurate way to model the single pad development problem since it considers a sequence of decisions that react to outcomes that evolve over time. We apply multistage stochastic programming to our single pad development problem with high price variance. We assume that the whole planning horizon is 48 weeks. The planning horizon is equally divided into three stages. In each stage, the realizations of price can be 0.2, 1.5, and 2.8 dollars per Btu with probabilities 0.3, 0.4, and 0.3, respectively. The scenario tree for the multistage problem is shown in Figure 16. We also assume that the prices outside the planning horizon remain equal to the prices in the third stage.

The stochastic programming model has 31,213 binary variables, 4,153 continuous variables, and 49,801 constraints. It is solved using CPLEX 12.7 [25] on the 12 processors of an Intel Xeon (2.67GHz) machine with 64 GB RAM for 12 hours. The CPLEX solver can obtain an optimality gap of 3.76% within the time limit. The best expected NPV from CPLEX is 79.94 million dollars. To quantify the value of using stochastic programming, we use the concept of the value of stochastic solution for multistage stochastic programming from Escudero et al. [26]. In their definition, the expected results of using the expected value solution at stage t (EEV_t) is obtained by fixing the first $t-1$ stage decisions to the optimal solution of the expected value problem and solve the rest of the stochastic programming problem. The value of stochastic solution at stage t (VSS_t) is defined as $VSS_t = EEV_t - RP$. For the detailed definition, we refer the readers to

TABLE 2. NPVs of the expected value and the stochastic solution (million dollars)

Scenario	Price	NPV(Expected value)	NPV(Stochastic Solution)
1	0.2	-24.28	-7.74
2	1.5	71.45	70.86
3	2.8	182.58	178.48

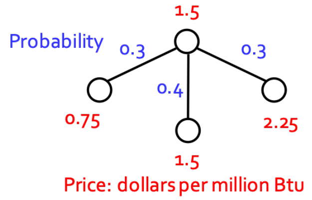

FIGURE 15. The scenario tree for two-stage stochastic programming with low price variance

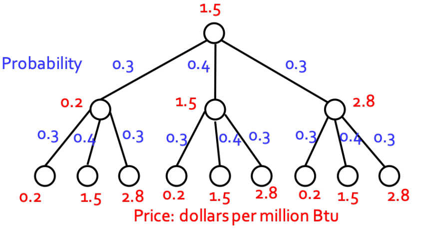

FIGURE 16. The scenario tree for multistage stochastic programming with high price variance

[26]. In this case, $EEV_2 = 74.62$ million dollars, $EEV_3 = 73.40$ million dollars, $VSS_2 = 5.33$ million dollars, $VSS_3 = 6.55$ million dollars.

In Figures 17 to 20, we show the Gantt charts of some scenarios. In both Figure 17 and Figure 19 when the price in the third stage and outside the planning horizon is only 0.2 dollar per Btu, not all the wells are completed in stage three because the natural gas price is not high enough to make the business profitable. On the other hand, when the price is greater than or equal to 1.5 dollars, it is preferable to complete all the 9 wells, which corresponds to the scenarios shown in Figures 18 and 20. Since the price in the third stage determines whether natural gas is profitable outside the planning horizon, all the optimal solutions try to delay the decisions to the third stage, i.e., no operation is performed in the last few days of stage two.

We can also observe that the price in stage two affects the number of wells that are turned in line. When the price is 0.2 dollar per Btu in stage two, only one well is turned in line (see Figures 17 and 18). When the price is greater than or equal to 1.5 dollars per Btu in stage two (see Figures 19 and 20), two wells are turned in line in stage two. Because of the higher price in stage two in Figures 19 and 20, one more well is turned in line to obtain the revenue of natural gas early in stage two.

The similarity of the multistage and the two-stage stochastic programming solution is that both of them try to delay

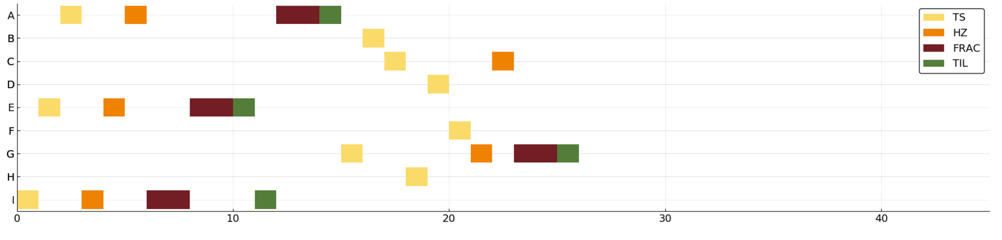

FIGURE 17. Gantt Chart of the stochastic solution for scenario price=0.2, 0.2, at stage two and three, respectively

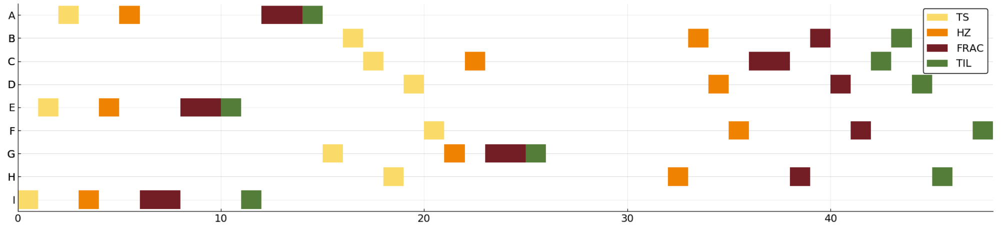

FIGURE 18. Gantt Chart of the stochastic solution for scenario price=0.2, 1.5, at stage two and three, respectively

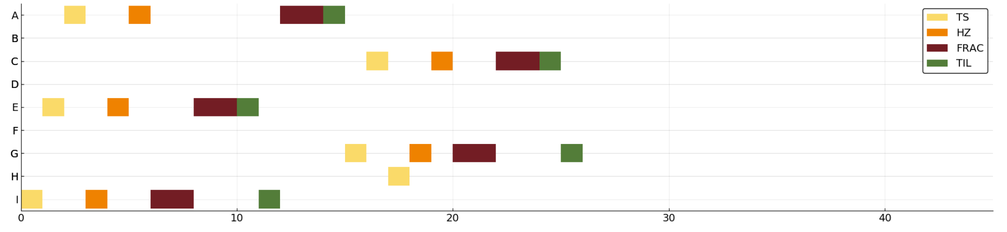

FIGURE 19. Gantt Chart of the stochastic solution for scenario price=1.5, 0.2, at stage two and three, respectively

the decisions to the last stage so that the uncertainty of the prices are fully realized. The major difference is that the multistage solution allows the uncertainties to evolve every quarter, which is a better approximation of what happens in practice.

7 | CONCLUSIONS

In this paper, we study the shale gas pad development problem under price uncertainty. The deterministic model is similar to the MILP model proposed by Ondeck et al. [1] where a wellpad is given with several prospective wells and the upstream operator needs to determine the sequence of operations on the wellpad. To extend the work of Ondeck et al.[1], we investigate how price uncertainties can affect the decision-making in this context. The mathematical framework that we use in this paper is stochastic programming, which is regarded as a risk-neutral approach to hedge against uncertainty. We introduce some concepts to quantify the value that stochastic programming can create, such as the value of stochastic solution (VSS). We apply both two-stage and multistage stochastic programming to the development of a wellpad with 9 wells under price uncertainty. When the variance of price is high, we can obtain values using stochastic programming on the order of million dollars. The stochastic solutions in both two-stage and multistage

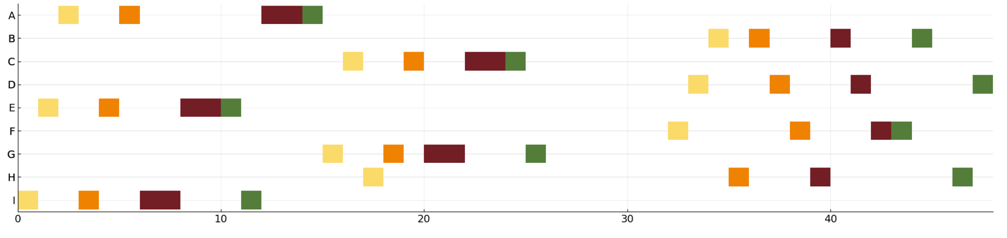

FIGURE 20. Gantt Chart of the stochastic solution for scenario price=1.5, 1.5, at stage two and three, respectively

try to delay the development decisions to the last stage when the uncertainty of the prices are fully realized. We also demonstrate that multistage stochastic programming is a better approximation of what happens in the real world than two-stage stochastic programming since it allows the uncertainties to evolve every quarterly.

However, when the price variance is low, there is no value of using stochastic programming since all the scenarios have the same recourse decisions. This gives us some insights on when to use stochastic programming. There may not be any value of using stochastic programming even if there are uncertain parameters in the model. Stochastic programming can only add value if there is something that we can do differently than the deterministic solution.

Future work will concentrate on extending the model to multiple pads development problem, and considering the mobilization of crew and equipment between the wellpads. We expect the optimization problem to grow even larger once more wellpads are taken into account. Therefore, better solution techniques need to be developed to solve the large-scale stochastic programming problem with higher efficiency.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge financial support from EQT Corporation, the Scott Energy Institute, Carnegie Mellon University, Center for Advanced Process Decision-making, Carnegie Mellon University. The authors would like to thank Prof. Diego Cafaro for discussions about this problem.

APPENDIX A: MATHEMATICAL FORMULATION FOR THE DETERMINISTIC MODEL

Nomenclature

Sets

$t \in T$ = Time points in the scheduling horizon

$w \in W$ =Wells

$u \in U$ =well operation periods

$o \in O$ =operations $\{TS, HZ, FRAC, TIL\}$

$a \in A$ =well age

Parameters

NRI_w= NRI of well w

ϕ_t= Discounted rate time period for time period t

ADR =Annual discount rate

l_w=Lateral length of well w

$P^{NPV}_{t,w}$ =NPV of a well w where t is the starting week of TIL

π_t= Natural gas price forecast for period t

$\hat{T}$= Total number of time periods used to generate revenue

$\gamma_{a,w}$=Gas production forecast per foot for well w of age a

$t^{dev}_{w,o}$ =Development time for operation o at well w

$Store^{max}$ = Maximum capacity of storage

P^{max}_t = Maximum capacity of gas that can be produced for time period t

Mob_o=Mobilization cost for operation o

$WOC_{w,o}$=Operation cost for operation o at well w

$T_{planning}$=Total number of time periods used for planning horizon

$pDiscProRev^{out}_{w,t}$=Discounted revenue per ft for well outside planning horizon if well is turned in line at time t

P_t^{max}=Maximum capacity of gas that can be produced for time period t

Binary Variables

s_t^{in} = If gas is being put into storage at time period t

$y_{t,w,o}^{start}$ = If operation o starts in time period t at well w

$z_{t,w,o}^{active}$=If operation o is active during time period t at well w

Continuous Variables

NPV=Net present value

$Disc.Rev$=Discounted revenue

$Disc.MC$=Discounted mobilization cost

$Disc.OC$=Discounted operation cost

$Disc.Rev_t^{in}$=Discounted revenue inside the scheduling horizon for time period t

$Disc.Rev_w^{out}$=Discounted revenue outside the scheduling horizon for well w

$MCO_{t,p,o}$=Mobilization cost for time period t at pad p for operation o

$P_{t,w}$=Amount of gas produced at well w during time period t

$Store_t^{out}$ = Amount of gas out of storage during time period t

$Store_t^{in}$ = Amount of gas put into storage during time period t

$StoreLevel_t$ = Storage level during time period t

<u>**Parameter Calculations**</u> The parameter $pDiscProdRevOutside_{w,t}$ is calculated prior to optimizing. The parameter represents the revenue per foot outside the planning horizon if a well w is turned in line at time period t. To calculate the parameter, we need to sum over the time outside the planning horizon ($a+t>T$) while the summation is within the time horizon that revenues are considered ($a+t<\hat{T}$). Each term in the summation represents the discount rate ϕ_{a+t} times the gas price π_{a+t} times the production $\gamma_{a,w}$ when this well is at age a.

$$pDiscProdRevOutside_{w,t} = \sum_{a:\{T<a+t<\hat{T}\}} (\phi_{a+t} \cdot \pi_{a+t} \cdot \gamma_{a,w}) \quad \forall t \in T, w \in W \tag{12}$$

7.1 | Constraints

We outline the constraints used in the model. We show different types of reformations for some of the constraints.

<u>**General Constraints**</u>

Single Operation Per Well: At most one operation can start at each time period t for each well w,

$$\sum_{o \in O} y_{t,w,o}^{start} \leq 1 \quad \forall w \in W, t \in T \tag{13}$$

Operation Done at Most Once: Each operation can only be performed at most once for each well throughout the planning horizon.

$$\sum_{t \in T} y_{t,w,o}^{start} \leq 1 \quad \forall w \in W, o \in O \tag{14}$$

Sequencing Operations (ordering): Operation o cannot start until operation *o-1* is complete. Since each operation can be performed at most once in the entire planning horizon, the time that operation o starts, represented by the left hand side of (15), must be greater than or equal to the time that operation $o-1$ starts plus the development time of operation $o-1$, which is represented by the right hand side of (15).

$$\sum_{t \in T} \left(t \cdot y^{start}_{t,w,o} \right) \geq \sum_{t \in T} \left(\left[t + t^{dev}_{w,o-1} \right] \cdot y^{start}_{t,w,o-1} \right) \quad \forall w \in W, o \in O, o > 1 \tag{15}$$

Sequencing Operations (ordering) r1:An alternative way to model this constraint could be using equation (16). Here variable $y^{start}_{t,w,o}$ is forced to zero if operation $o-1$ has not been completed at time t.

$$y^{start}_{t,w,o} \leq \sum_{t' \leq t - t^{dev}_{w,o-1}} y^{start}_{t',w,o-1} \quad \forall t \in T, w \in W, o \in O, o > 1 \tag{16}$$

Sequencing Operations (tightening constraint): Operation o cannot happen before operation $o-1$, nor can operation $o-1$ happen after operation o,

$$\sum_{t' \leq t} y^{start}_{t',w,o} + \sum_{t' \geq t} y^{start}_{t',w,o-1} \leq 1 \quad \forall w \in W, t \in T, o \in O, o > 1 \tag{17}$$

Operations Completion: Equation (18) forces that the operations on well w must be completed in the planning horizon if the first operation starts in the planning horizon, i.e, no well can be left unfinished within the planning horizon.

$$\sum_{t \in T} y^{start}_{t,w,o} = \sum_{t \in T} y^{start}_{t,w,o-1} \quad \forall w \in W, o \in O, o > 1 \tag{18}$$

Single Operation Per Time Period Per Pad: At most one operation can be active at each time period t in the wellpad,

$$\sum_{w \in W} \sum_{o \in O} z^{active}_{t,w,o} \leq 1 \quad \forall t \in T \tag{19}$$

where $z^{active}_{t,w,o}$ is a binary variable that decides whether operation o is performed on well w at time t,

$$z^{active}_{t,w,o} = \sum_{t' : \{t' \leq t,\ t - t^{dev}_{w,o} < t'\}} y^{start}_{t',w,o} \quad \forall t \in T, w \in W, o \in O \tag{20}$$

Base Production

The following equation gives production for well w at time t. If well w is turned in line at time $t-a$, the age of the well at time t is a. The production can be calculated based on the production curve.

$$P_{t,w} = \sum_{a:\{a<t\}} y^{start}_{t-a,w,TIL} \cdot \gamma_{a,w} \cdot I_w \quad \forall t \in T, w \in W \tag{21}$$

Mobilization Constraints

Cost of Mobilization: If operation o is not active at time $t-1$ but is active at time o, we need to move the corresponding equipment and crew for operation o to the wellpad at time t and a mobilization cost is incurred.

$$MCO_{t,p,o} \geq Mob_o \cdot \left(\sum_{w \in W} \left(z^{active}_{t,w,o} \right) - \sum_{w \in W} \left(z^{active}_{t-1,w,o} \right) \right) \quad \forall t \in T, p \in P, o \in O \tag{22}$$

Since the mobilization constraints make the problem hard to solve, we propose several reformulations to account for the mobilization costs.

Cost of Mobilization r1: We define new binary variables $z^{change}_{t,o}$ which decide whether the wellpad change to perform operation o from a different operation $o' \neq o$ at time t or not. Similar to (22), variable $z^{change}_{t,o}$ can be calculated by,

$$z^{change}_{t,o} \geq \sum_{w \in W} \left(z^{active}_{t,w,o} \right) - \sum_{w \in W} \left(z^{active}_{t-1,w,o} \right) \tag{23}$$

The mobilization cost at time t can be calculated by summing over the potential operations change at time t.

$$MCO_{t,p,o} = \sum_{o \in O} Mob_o z^{change}_{t,o} \tag{24}$$

Since the optimization model always try to minimize the costs, variables $z^{change}_{t,o}$ always equal to zero if the corresponding operation is not changed at time t.

Cost of Mobilization r2: Mobilization cost occurs if we start operation o at time t at any well w and there is no operation o ends at time t.

$$MCO_{t,p,o} \geq Mob_o \cdot \left(\sum_{w \in W} \left(y^{start}_{t,w,o} \right) - \sum_{w \in W} \left(y^{start}_{t-t^{dev}_{w,o},w,o} \right) \right) \quad \forall t \in T, p \in P, o \in O \tag{25}$$

Capacity Constraints

Capacity Constraint: For every time period t, the production plus the gas released out of storage minus the gas curtailed in the storage must be less than or equal to the maximum gas that can be produced at time t. Note that the virtual storage is considered for the whole wellpad. We do not distinguish the storage for each well.

$$\left(\sum_{w \in W} P_{t,w} \right) + Store^{out}_t - Store^{in}_t \leq P^{max}_t \quad \forall t \in T \tag{26}$$

No Storage In/Out: If storage is being loaded, no gas can be released from storage. If the storage is not being loaded, we constrain the amount of gas released to be less than the maximum (based on number of wells *W*). The number 91,000 is what works in practice to provide a valid upper bound.

$$\left(1 - s_t^{in}\right) \cdot W \cdot 91000 \geq Store_t^{out} \quad \forall t \in T \tag{27}$$

Storage In Used: Maximum amount of gas put into storage, based on number of wells and if storage is allowed

$$s_t^{in} \cdot W \cdot 91000 \geq Store_t^{in} \quad \forall t \in T \tag{28}$$

Storage Level: For all periods greater than 1, the storage level at time *t* is equal to the storage level at time *t-1* plus any gas added to storage at time *t*, minus any gas removed from storage at time *t*

$$StoreLevel_t = StoreLevel_{t-1} + Store_t^{in} - Store_t^{out} \quad \forall t \in T, t > 1 \tag{29}$$

Initial Storage Level: At initial time period (*t = 1*), the storage level is equal to any gas added to storage minus any gas removed from storage

$$StoreLevel_1 = Store_1^{in} - Store_1^{out} \tag{30}$$

<u>**Revenues**</u>

The **discounted revenue inside the scheduling horizon at time** t **(**$Disc.Rev_t^{in}$ **)** with capacity constraints is,

$$Disc.Rev_t^{in} = \phi_t \cdot \pi_t \cdot \left(\sum_w \left(P_{t,w} \cdot NRI_w\right) + NRI\left(Store_t^{out} - Store_t^{in}\right)\right) \quad \forall t \in T \tag{31}$$

where we sum over the sales of gas production at each well w plus the revenue from the gas storage.

The **discounted revenue outside the scheduling horizon (** $Disc.Rev_w^{out}$ **)** is:

$$Disc.Rev_w^{out} = NRI_w \cdot l_w \sum_t pDiscProdRevOutside_{w,t} y_{t,w,TIL}^{start} \quad \forall w \in W \tag{32}$$

where parameter $pDiscProdRevOutside_{w,t}$ has been precalculated in equation (12). It calculates the revenue well w can generate outside the planning horizon if it is turned in line at time t.

The **discounted revenue for gas left in fictional storage (*Disc. Rev*^{left})** is calculated with equation (33). There can be some gas left in the storage at the end of the planning horizon. We assume that the sales of the left gas is evenly distributed outside the planning horizon, i.e., from time T to $\hat{T}$.

$$Disc.Rev^{left} = \sum_{u:\{T<u<\hat{T}\}} \left(\phi_u \cdot \pi_u \cdot N\bar{R}I \cdot \frac{StoreLevel_T}{(\hat{T} - T)} \right) \tag{33}$$

The **discounted revenue (*Disc. Rev*)** is equal to the summation of the three types of revenues described above,

$$Disc.Rev = \sum_t Disc.Rev_t^{in} + \sum_w Disc.Rev_w^{out} + Disc.Rev^{left} \tag{34}$$

Costs

The **discounted operating cost (*Disc. OC*)** is calculated by the following:

$$Disc.OC = \sum_t \phi_t \cdot \sum_o \sum_w \left(y_{t,w,o}^{start} \cdot WOC_{w,o} \right) \tag{35}$$

The **discounted mobilization cost (*Disc. MC*)**, calculated based on the **mobilization costs (*$MCO_{t,p,o}$*)**

$$Disc.MC = \sum_t \phi_t \cdot \sum_p \sum_o MCO_{t,p,o} \tag{36}$$

Objective

Maximizing NPV

$$NPV = Disc.Rev - Disc.OC - Disc.MC \tag{37}$$

REFERENCES

[1] Ondeck A, Drouven M, Blandino N, Grossmann IE. Multi-Operational Planning of Shale Gas Pad Development. Computers & Chemical Engineering 2019;http://www.sciencedirect.com/science/article/pii/S0098135418307920.

[2] U S Energy Information Administration, Annual Energy Outlook 2019; 2019. https://www.eia.gov/outlooks/aeo/pdf/aeo2019.pdf.

[3] Morris A, Optimizing Shale Gas Production from Well to Wire; 2015. http://www.mccormick.northwestern.edu/news/articles/2015/06/optimizing-shale-gas-production-from-well-to-wire.html.

[4] Markus G Drouven, Optimization Engineering at EQT; 2018. https://pubsonline.informs.org/do/10.1287/orms.2018.06.07/full/.

[5] Birge JR, Louveaux F. Introduction to stochastic programming. Springer Science & Business Media; 2011.

[6] Cafaro DC, Grossmann IE. Strategic planning, design, and development of the shale gas supply chain network. AIChE Journal 2014;60(6):2122–2142.

[7] Drouven MG, Grossmann IE. Multi-period planning, design, and strategic models for long-term, quality-sensitive shale gas development. AIChE Journal 2016;62(7):2296–2323.

[8] Guerra OJ, Calderón AJ, Papageorgiou LG, Siirola JJ, Reklaitis GV. An optimization framework for the integration of water management and shale gas supply chain design. Computers & Chemical Engineering 2016;92:230–255.

[9] Gao J, You F. Shale gas supply chain design and operations toward better economic and life cycle environmental performance: MINLP model and global optimization algorithm. ACS Sustainable Chemistry & Engineering 2015;3(7):1282–1291.

[10] Arredondo-Ramírez K, Ponce-Ortega JM, El-Halwagi MM. Optimal planning and infrastructure development for shale gas production. Energy Conversion and Management 2016;119:91–100.

[11] Knudsen BR, Grossmann IE, Foss B, Conn AR. Lagrangian relaxation based decomposition for well scheduling in shale-gas systems. Computers & Chemical Engineering 2014;63:234–249.

[12] Cafaro DC, Drouven MG, Grossmann IE. Optimization models for planning shale gas well refracture treatments. AIChE Journal 2016;62(12):4297–4307.

[13] Cafaro DC, Drouven MG, Grossmann IE. Continuous-time formulations for the optimal planning of multiple refracture treatments in a shale gas well. AIChE Journal 2018;64(5):1511–1516.

[14] Yang L, Grossmann IE, Manno J. Optimization models for shale gas water management. AIChE Journal 2014;60(10):3490–3501.

[15] Forouzanfar F, Reynolds A. Joint optimization of number of wells, well locations and controls using a gradient-based algorithm. Chemical Engineering Research and Design 2014;92(7):1315–1328.

[16] Wilson KC, Durlofsky LJ. Optimization of shale gas field development using direct search techniques and reduced-physics models. Journal of Petroleum Science and Engineering 2013;108:304–315.

[17] Ma X, Plaksina T, Gildin E. Optimization of placement of hydraulic fracture stages in horizontal wells drilled in shale gas reservoirs. In: Unconventional Resources Technology Conference Society of Exploration Geophysicists, American Association of Petroleum; 2013. p. 1479–1489.

[18] Yu W, Sepehrnoori K. Optimization of multiple hydraulically fractured horizontal wells in unconventional gas reservoirs. Journal of Petroleum Engineering 2013;2013.

[19] Drouven MG, Grossmann IE, Cafaro DC. Stochastic programming models for optimal shale well development and refracturing planning under uncertainty. AIChE Journal 2017;63(11):4799–4813.

[20] Gao J, You F. Deciphering and handling uncertainty in shale gas supply chain design and optimization: Novel modeling framework and computationally efficient solution algorithm. AIChE Journal 2015;61(11):3739–3755.

[21] Guerra OJ, Calderón AJ, Papageorgiou LG, Reklaitis GV. Integrated shale gas supply chain design and water management under uncertainty. AIChE Journal 2019;65(3):924–936.

[22] Zeng Z, Cremaschi S. Artificial lift infrastructure planning for shale gas producing horizontal wells. Proceedings of the FOCAPO/CPC, Tuscan, AZ, USA 2017;p. 8–12.

[23] U S Energy Information Administration, Henry Hub Natural Gas Spot Price; 2019. `https://www.eia.gov/dnav/ng/hist/rngwhhdD.htm`.

[24] Benders JF. Partitioning procedures for solving mixed-variables programming problems. Numerische mathematik 1962;4(1):238–252.

[25] CPLEX I, 12.7. 0 User's Manual. IBM; 2016.

[26] Escudero LF, Garín A, Merino M, Pérez G. The value of the stochastic solution in multistage problems. Top 2007;15(1):48–64.