

# Long-term scheduling of a single-stage multi-product continuous process to manufacture high performance glass

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## **Abstract**

In this paper the long-term scheduling of a real world multi-product single stage continuous process for manufacturing glass is studied. This process features long minimum run lengths, and sequence dependent changeovers of the order of days, with high transition costs. The long-term scheduling involves extended time horizons that lead to large scale mixed integer linear programming (MILP) scheduling models. In order to address the difficulties posed by the size of the models, three different rolling horizon algorithms based on different models and time aggregation techniques are studied. The models used are based on the continuous time slot MILP model, and on the traveling salesman model proposed by Erdirik-Dogan and Grossmann (2008). Due to the particular characteristics of the process under study, several new features are proposed, which include: a) carry-over changeovers across the due dates; b) minimum run lengths across the due dates; c) a rigorous aggregation of the products based on the type of changeovers; d) definition of minimum inventory levels at the end of the time horizon. Several case studies are formulated in order to compare different scenarios, and assess the proposed rolling horizon algorithms.

*Keywords:* Planning; Scheduling; Multi-product continuous plants; MILP; Glass production

## **1 Introduction**

This paper addresses the long-term scheduling of a real world multi-product continuous process for manufacturing value added glass products. The long-term scheduling is embedded in a simul-

taneous planning and scheduling framework motivated by strategic business goals, and by specific features of the process, such as long sequence-dependent changeovers, and relatively large minimum run lengths. The simultaneous planning and scheduling is approached through the formulation of a mixed-integer linear programming (MILP) model, using exact methods based on linear programming (LP) based Branch & Bound (B&B) to solve the problem. In addition, time decomposition strategies and rolling horizon algorithms are applied to solve the scheduling over a long time horizon.

The simultaneous planning and scheduling aims to integrate the planning decisions with the scheduling details into one model, where the planning economic objectives are merged with the scheduling of the production over the time horizon of the planning. This approach has the advantage of generating feasible planning decisions for the scheduling in one step. However, this integration may lead to large scale MILP models that require efficient models and solution approaches as discussed by several authors in the literature (Maravelias and Sung, 2009).

In this work two formulations are used and integrated into a solution approach to be described later: a) a slot-based continuous time formulation; and b) a traveling salesman problem (TSP) sequence based model. These two models are based on the models proposed by Erdirik-Dogan and Grossmann (2006) and Erdirik-Dogan and Grossmann (2008). These authors proposed a bi-level decomposition algorithm where in the upper level a relaxation of the scheduling model is used to define the products to be produced in the lower level. At the lower level a slot-based continuous time formulation model is used to determine the detailed timing operations. The relaxation model proposed in the first work is based on the underestimation of the sequence dependent-changeovers, while in the latter work the relaxed model is based on using TSP sequence based constraints. Liu et al. (2008) have also proposed a TSP-based model for multi-product continuous process, with different sequence constraints and sub-tour elimination constraints. In their studies, their model compared favorable with a modified model of Erdirik-Dogan and Grossmann (2008). In other related work, Sung and Maravelias (2007) have proposed a novel approach, whereby feasible regions for the production and cost are derived in off-line from the scheduling model, and then integrated within the planning model resulting in only one planning model. Jia and Ierapetritou (2003) developed a continuous time MILP model for the simultaneous planning for gasoline blending and short-term scheduling of the distribution, where small and efficient models are used for real case problems.

Optimal short term scheduling of continuous processes has been motivated by the economic pressures to increase the efficiency of continuous processes that produce multiple products. Four types of optimization models have been used for the short term scheduling of continuous processes: a) Resource-Task Network (RTN) models (Castro et al., 2004, 2009; Schilling and Pantelides, 1996; Ierapetritou and Floudas, 1998; Zhang and Sargent, 1998); b) State-Task Network with unit-

specific event-based continuous time representation (Ierapetritou and Floudas, 1998); c) slot-based continuous time models (Karimi and McDonald, 1997; Lamba and Karimi, 2002; Lim and Karimi, 2003; Erdirik-Dogan and Grossmann, 2006; Lee et al., 2002); and d) TSP-based models (Alle and Pinto, 2002); e) proportional lot-sizing and scheduling problem (PLSP) (Suerie, 2005).

Recently, medium term scheduling models have been proposed in the literature (Shaik et al., 2009; Erdirik-Dogan and Grossmann, 2008; Liu et al., 2008, 2009; Chen et al., 2008), using additional solution strategies to cope with the increased size of the MILP models to solve.

The concept of long-term scheduling is not common in the literature because of two main reasons: 1) scheduling is usually associated with batch operations over a one or two weeks horizon; and 2) there is no clear distinction between long-term scheduling and planning. However, the process under consideration in this work has specific features that lead us to use the idea of long-term scheduling.

In order to address the computational burden that may prevent exact methods to be used in real world applications, some of the approaches that have been devised include: a) heuristics to reduce the size of the models; b) decomposition and aggregation techniques; and c) improvement optimization-based strategies. A review of these approaches can be found elsewhere Mendez et al. (2006).

Regarding decomposition approaches, Bassett et al. (1996) proposed time-based decompositions concepts for scheduling of batch processes that are also valid for continuous processes. Their decompositions rely on the aggregation of time at the planning level with time periods of one month, and disaggregation of the time at the scheduling level with time periods of one week. Dimitriadis et al. (1997) presented two rolling horizon algorithms that are characterized by using at each level a detailed and an aggregate model in order to reduce the dimensionality of the problem by solving smaller subproblems. Their algorithms will be further analyzed later in this paper. Recent works have used rolling horizon algorithms to reduce the dimensionality of the problems to solve. Liu et al. (2009) extended their previous model to multiple continuous production lines, and applied a rolling horizon algorithm where in each subproblem the time horizon is extended and some binary variables are fixed. Shaik et al. (2009) proposed a bi-level decomposition scheme where in each subproblem a different model is solved. This decomposition is integrated into a rolling horizon algorithm that they have applied to a real case study.

In addition, algorithmic advances involving the integration of heuristics methods within B&B solvers, while maintaining the logic inherent to the upper and lower bounds, may contribute to tackle larger MILP problems. Examples of these heuristics are Local Branching (LB) (Fischetti and Lodi, 2003), Relaxation Induced Neighborhood Search (RINS) (Danna et al., 2005), and evolutionary algorithms for polishing MILP solutions (Rothberg, 2007).

In this work we address the long-term scheduling of continuous manufacturing of high perfor-

mance glass using extensions of the models proposed by Erdirik-Dogan and Grossmann (2008) in order to cope with the specific features of the glass production line, and to improve their applicability. The main extensions are the following: a) implementation of minimum run lengths across due dates; b) changeovers across due dates; c) an aggregation strategy for the products; and d) terminal constraints for inventory levels. Three rolling horizons algorithms are proposed based on time decomposition strategies, and based on different models. These strategies are featured with terminal constraints for inventory levels at the end of the time horizon to provide feedback to earlier time periods of the demand after the time horizon.

## **2 Process description**

The industrial process under consideration is a multi-product continuous process used to manufacture high performance glass products, with high levels of transmitted daylight, and reduced heat losses. From the point of view of production scheduling, the furnace and an online coater are the only relevant units.

In the furnace raw materials are melted with specific compositions that define the color of the substrate of the product, which defines the product type. The changeover from one product with a given substrate to another product with a different substrate is made through a continuous operation, where the new raw materials are added to the furnace in order to dilute or remove some of the previous raw materials from the furnace. This is a complex operation that may take several days in order to reach the desired composition. After the changeover from one substrate to other substrate, the process must produce the same substrate for a minimum run length.

The online coater is used to apply a coating to the surface of the glass that changes its color, defining a new product. This task involves a changeover without a significant transition time.

The complex features of this process in terms of long changeovers between some products with high transition costs, and no changeovers between some of them, associated with minimum run lengths of several days in order to ensure process stability, motivated the development of a mathematical programming approach to optimize the scheduling of the production.

## **3 Problem statement**

Given is a time horizon of 18 months over which the following items are specified: deterministic product demands; initial, minimum, and maximum inventory levels of products; production rates for each product; sequence dependent transitions, operating, inventory, and transition costs, and selling prices. The due dates are set at the end of each month. Products with a coating have different processing rates and costs, and selling prices. The scheduling problem consists in determining the

production time for each product, sequence of production, and inventory levels that maximize the profit given by the difference between revenues and inventory holding costs, operating costs, and transition costs.

The sales are assumed to be equal to the forecasted demand. This means that if the capacity of the process is greater than the demand for a given time horizon, the extra production is added to the inventory. However, if the capacity of the process is not enough to satisfy the forecasted demand, inventories are used for meeting demands. The model includes slack variables and penalties for the violation of safety stocks, and the violation of the maximum storage capacity.

## 4 Solution approaches

The dynamics involved in this process in terms of the length of the transitions and minimum run lengths require that horizons of the order of 18 to 24 months be considered. This has led us to adopt the definition of long-term scheduling for this problem. The time horizon is divided into two parts: a time horizon of 18 months which is of interest for determining the production, and an additional time horizon of six months to provide feedback on the future demand to avoid a "myopic" inventory policy at the end of the scheduling period of 18 months.

Time decompositions integrated into a rolling horizon framework are used in order to address two issues. The first one is the size of the MILP model that is generated when long time horizons are specified, and the associated difficulties posed to commercial MILP solvers. Using a rolling horizon strategy smaller subproblems are solved in sequence in order to cover the specified time horizon. The second issue is related to the "myopic" solution from a model that does not take into consideration the demand pattern after the time horizon of interest, and consequently drives the inventory levels of each product to the safety stock level at the end of the time horizon. This aspect is particularly important in this process due to the combination of minimum run lengths, long transition times and number of products that may result in one product not being produced for several months. As a first approach the rolling horizon algorithm was applied over a time horizon longer than the time horizon of interest with the objective of providing feedback information about the demand after the time horizon of interest. However, our results showed that this is not enough for the model to avoid dropping significantly the inventory levels during the time horizon of interest. In order to overcome this issue, terminal constraints for the inventory levels at the end of an extended time horizon are proposed in this work to avoid the depletion of inventory. This situation will be clear with one of the case studies presented later. Figure 1 shows the extended time horizon that includes the time horizon of interest,  $1.5T$  (i.e. 18 months), and the additional time of  $0.5T$  (i.e. 6 months) at the end to provide feedback to the initial horizon. In addition, imposing the constraints beyond the time  $1.5T$  smooths the response over the time horizon of interest.

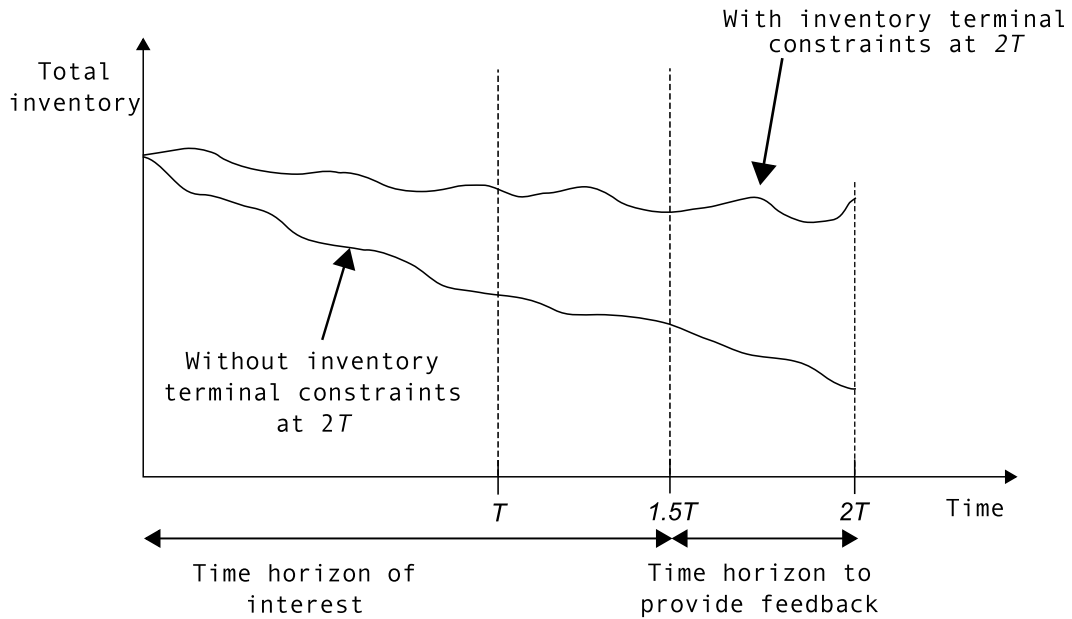


Figure 1: Total inventory profile for two different models, regarding inventory terminal constraints.

Figure 1 illustrates the trend of total inventory profiles that may result from the optimization of two models, one with terminal constraints for the inventory, and the other without terminal constraints at the end of the time horizon. The difference between the two profile trends in Figure 1 is explained by the trade-offs of the transition costs, inventory holding costs, and production cost in each case. The solution of the bottom curve leads to a larger number of transitions, reducing the production cost, and with the demand satisfied by depleting inventory, decreasing it to significantly low values.

In this work three rolling horizon algorithms are implemented: 1) a hybrid rolling horizon involving a planning model and a scheduling model; 2) an asynchronous rolling horizon, where the time periods in the planning level are aggregated into 28 days, and the scheduling models are disaggregated into time periods of one week; and 3) a detailed rolling horizon strategy based only on the scheduling model.

#### 4.1 Hybrid rolling horizon

The hybrid rolling horizon strategy integrates a planning and a scheduling model with expansions and shrinking of the time intervals where the models are applied, see Figure 2. The planning model is a simplified representation of the scheduling model, which does not consider the detailed timing of the operations and uses the sequencing constraints proposed by Erdirik-Dogan and Grossmann (2007). The scheduling model is a slot based continuous time representation with a detailed timing for the operations. The advantage of the planning model is that it does not require specifying the

number of slots per time period which is required by the scheduling model.

This rolling horizon strategy involves three steps, where two subproblems are solved in each step (see Figure 2). The subproblems are solved for the same time horizon, and between the steps the time horizon is expanded. In step number one, in the first subproblem the planning model is applied over the time horizon of one year, while in the second subproblem the scheduling model is applied over the first half of the year and the planning model shrinks to half of the time horizon. The solution of the planning model from the first subproblem is used to restrict the set of products that can be assigned in each time period of the scheduling model, and consequently the number of slots defined for each time period. From the solution of the scheduling model in the second subproblem the binary variables associated with the assignments of products to the time periods and changeovers are fixed, while the continuous variables remain free variables.

In the second step, the time horizon is expanded to 18 months, and in the first subproblem the planning model is solved again over the time period of one year. In the second subproblem, the scheduling model is applied over the time period of one year, and the planning model over six months. In the scheduling model, in the second half of the time horizon all variables are free and the products and slots are restricted by the solution of the planning model from the previous subproblem.

In the last step, the time horizon is expanded to 24 months, where the scheduling model with the binary variables fixed is applied over the first year and the planning model is applied over the second year. In the second subproblem, the scheduling model is expanded to more six months with the products and slots restricted from the planning model from the previous subproblem.

## **4.2 Asynchronous rolling horizon**

The asynchronous rolling horizon strategy uses the same structure of the previous strategy, see Figure 3, but with time periods aggregated in the planning model that are then disaggregated in the scheduling model. The objective of this approach is to improve the accuracy of the balances to the inventory levels by increasing the time resolution in the scheduling models down to weeks, while keeping the lower time resolution in the planning models. Like in the previous approach, the solution of the planning model is used to restrict the products and slots that are assigned to each time period in the scheduling model. Here, the products that are not assigned in a given time period of time in the planning model, are not considered in the scheduling model in the time periods that cover the same time of the period of the planning model. In the planning model the length of the time periods is 28 days, while in the scheduling model the length is 7 days. These lengths allow a simple definition of the products that are not assigned in the time periods of the scheduling model, which would not be possible with time periods of 31 days in the planning model and 7 days in the scheduling model. It should be noted that due dates are at the end of every 7 days in the scheduling

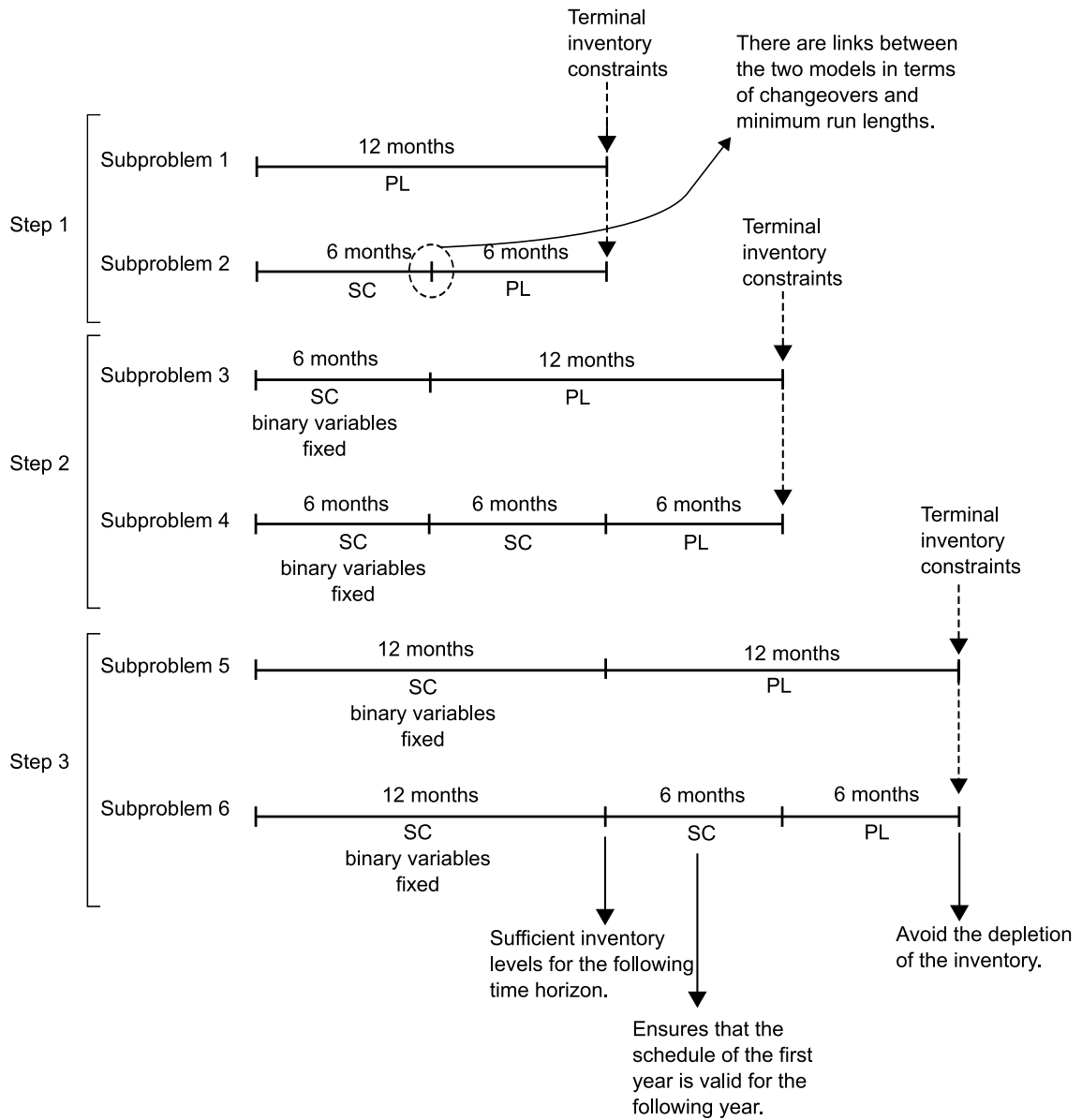


Figure 2: Hybrid rolling horizon strategy, steps, sub-problems and models used. SC - scheduling model with time periods of one month, PL - planning model with time periods of one month



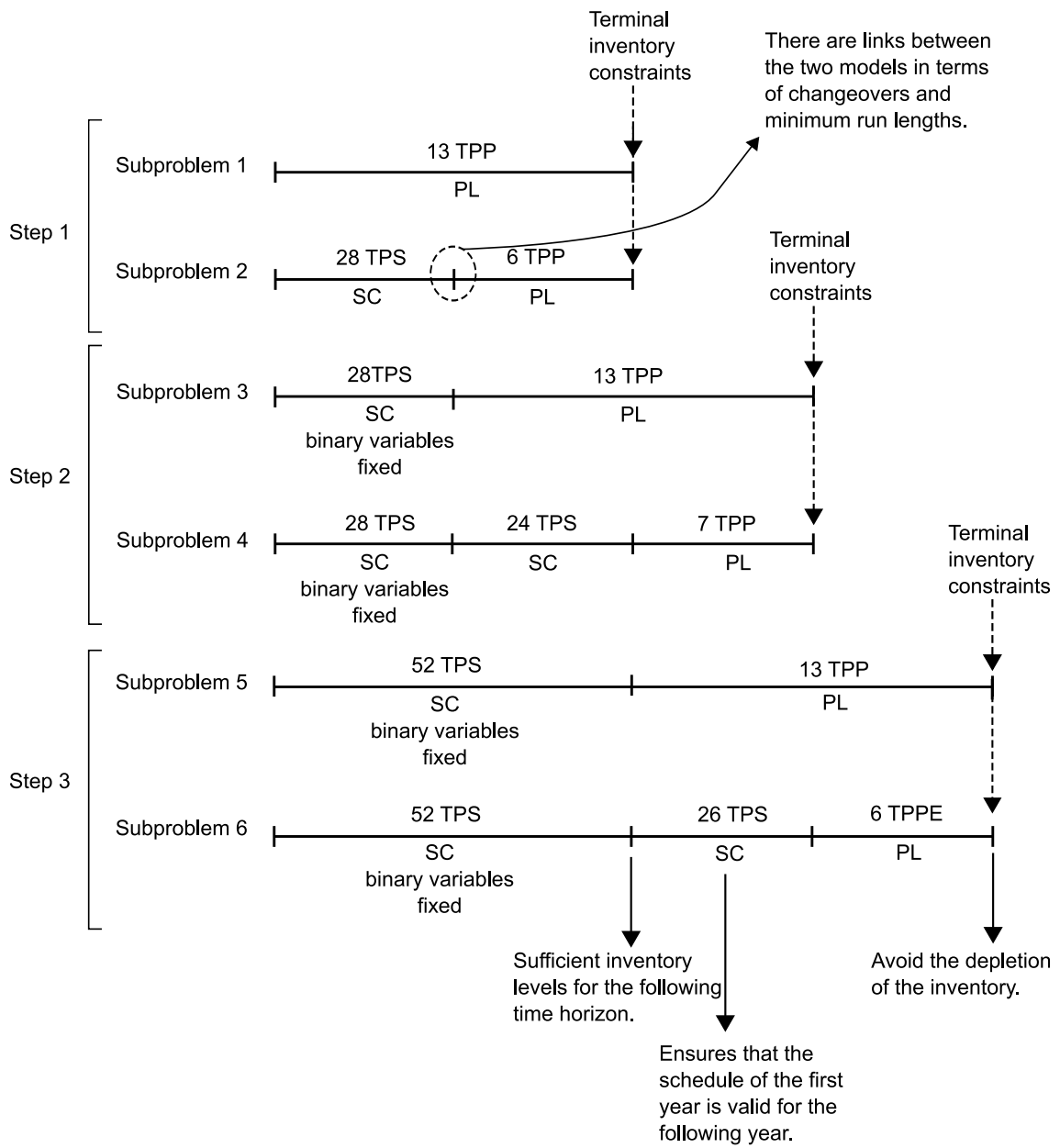


Figure 3: Asynchronous rolling horizon, steps, sub-problems and models used. SC - scheduling model with time periods TPS equal to 7 days, PL - planning model with time periods TPP equal to 28 days, and TPPE equal to one month.

model.

### **4.3 Detailed rolling horizon**

The objective of the detailed rolling horizon is to evaluate the performance of the scheduling model when applied to large time horizons, and compare it with the planning model. The detailed rolling horizon uses the scheduling model with time periods of one month for each subproblem as illustrated in Figure 4. This strategy involves the solution of three subproblems. In the first subproblem, the scheduling model is applied over the time horizon of one year, and the solution for the first six months is used to restrict the products and number of slots assigned to the first six months of the next step. In the second step the time horizon is expanded to one year and half with a reduced model in the first half year. In the third step the time horizon is expanded to two years, where in the first six months the binary variables are fixed, and in the second semester the products and number of slots are restricted according to the solution of the previous subproblem. Terminal constraints for the inventory levels are used at the end of the time horizon on each subproblem.

The rolling horizon algorithms described have some conceptual similarities with the forward rolling horizon proposed by Dimitriadis et al. (1997): a) aggregated and detailed models are used; b) the detailed model is applied over a period of time in the beginning of the time horizon, and the aggregated model over the remaining time; c) the period of time over which the detailed model is applied increases and the period of time over which the aggregated model is applied shrinks; d) after solving the detailed model the binary variables are fixed. However, in this work the following additional features are considered: a) the time horizon moves forward between each step in order to cover an extended time horizon; b) in the asynchronous rolling horizon algorithm, the planning model represents a simplification of the scheduling model, and also uses aggregated time periods in order to decrease the dimensionality of the problem; c) terminal constraints are added at the end of the time horizon, in order to prevent the depletion of the inventory. In the proposed rolling horizon algorithms the binary variables of the detailed models are also fixed, but the continuous variables are free. This allows the algorithms not only to correct part of the decisions made by the planning models (Dimitriadis et al., 1997), but work also to revise the continuous variables when moving forward the terminal constraints for the inventory levels at the end of the time horizon.

## **5 Mathematical formulation**

### **5.1 Detailed MILP model**

The detailed MILP model used in this work is an extension of the continuous time slot-based scheduling model proposed by Erdirik-Dogan and Grossmann (2006). In this model the represen-

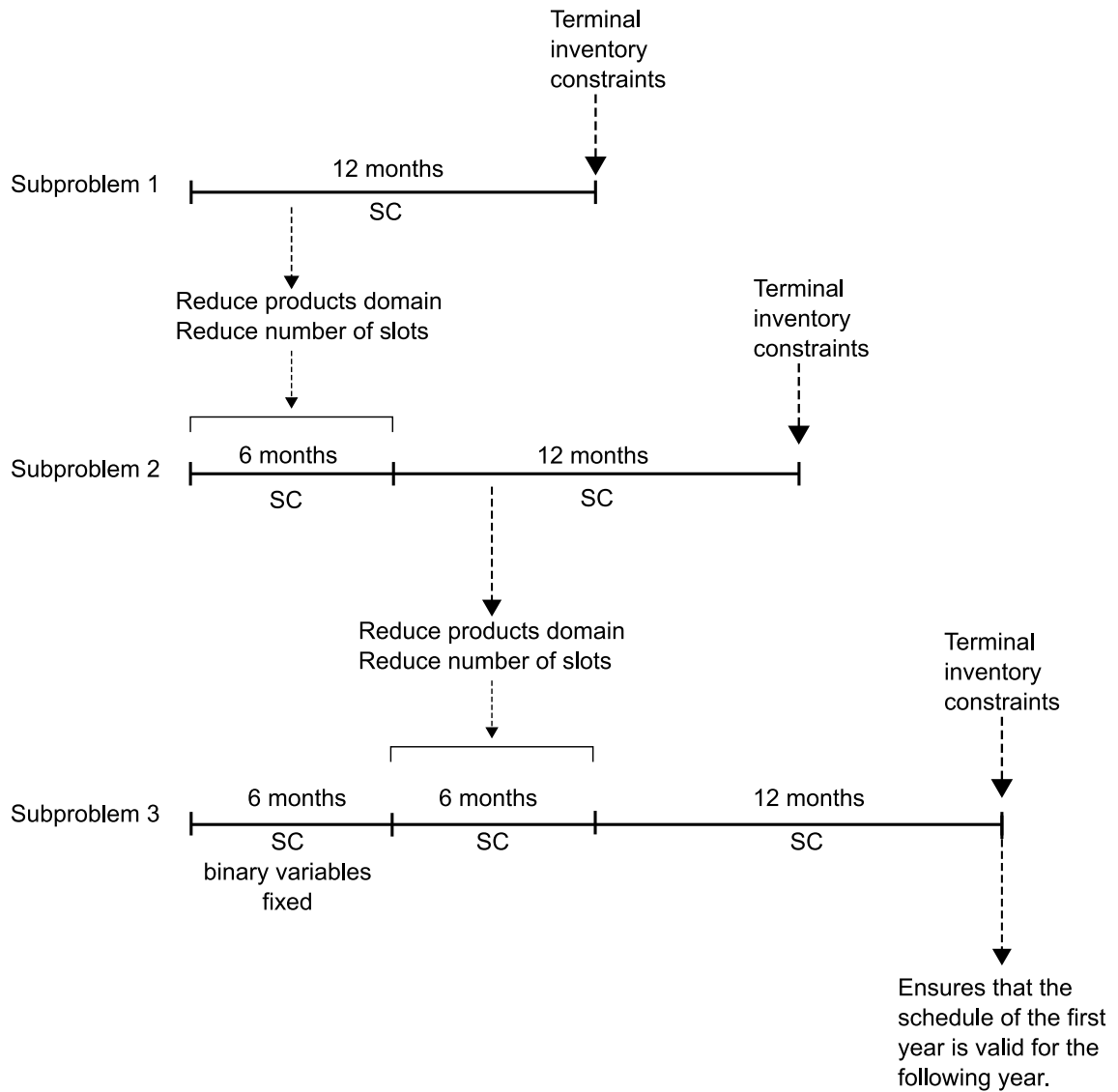


Figure 4: Detailed rolling horizon, sub-problems and models used. SC - scheduling model with time periods of 1 month.

tation of the time is divided into time periods, with the demand due at the end of the time period, i.e. one month. The time periods have fixed length, each one is divided in slots with the following features (see Figure 5):

1. all slots except the last slot of the time period involve both a production time and a transition time;
2. the last slot of each time period is only composed by the processing time;
3. the slots have variable length;
4. only one product is assigned per slot;
5. products may be assigned to more than one slot per time period;
6. there is a fixed number of slots per time period.

The problem under study in this work has some significant differences when compared with the case studies solved by Erdirik-Dogan and Grossmann (2006). First, in this process there are changeovers with transition times that can have the same duration of a time period, or take a significant part of the time period. Second, there is a minimum run length for a subset of products that can be similar to the length of the time periods. In this case the model of Erdirik-Dogan and Grossmann (2006) may lead to infeasible solutions. Another characteristic of our problem is the presence of changeovers between some products with transition time equal to zero, which can introduce degeneracy in the solutions. Erdirik-Dogan and Grossmann (2006) have assumed in their case studies that the sales could be greater or equal than the forecasted demand, considering that the production surplus after the demand is fulfilled can be sold. However, if the sales are assumed to be equal to the forecasted demand, the surplus capacity of the process is stored as inventory, since the process does not stop. Nevertheless, this is only relevant whenever the capacity of the process exceeds the total forecasted demand. When this is not the case, the trade-offs between the inventory holding cost, production cost, and transition cost dictate the amount of inventory that is depleted and the capacity of the process, which depends on the products to manufacture, number of transitions, and production runs duration. In addition, because of the minimum run lengths and long transition times, the model cannot drive all the inventory levels to the safety stock values at the end of the time horizon. Otherwise, at the beginning of the next time horizon the inventory levels of some of the products would go below the safety stock levels.

In order to address these issues the following extensions to the model of Erdirik-Dogan and Grossmann (2006) are proposed: a) aggregate the products to eliminate the products without changeovers from the scheduling part of the model; b) allow minimum run lengths across due dates; c) allow changeovers across due dates; d) terminal inventory constraints at the end of the

time horizon. As will be shown these extensions are non-trivial. The MILP model is described in detail in the next subsections.

### 5.1.1 Aggregation of the products

In the process under study the products are distinguished mainly by color. Their color depends on the substrate, which depends on the composition of the raw materials fed to the furnace, and depends on the online application of a coating. Therefore, one product with a given substrate has a different color from another product with the same substrate plus a coating. In this process there are two types of changeovers: 1) sequence-dependent changeovers with long transition times; and 2) changeovers with no transition time. The first type of changeovers correspond to change in the composition of the melt in the furnace in order to change from one substrate to other substrate. A

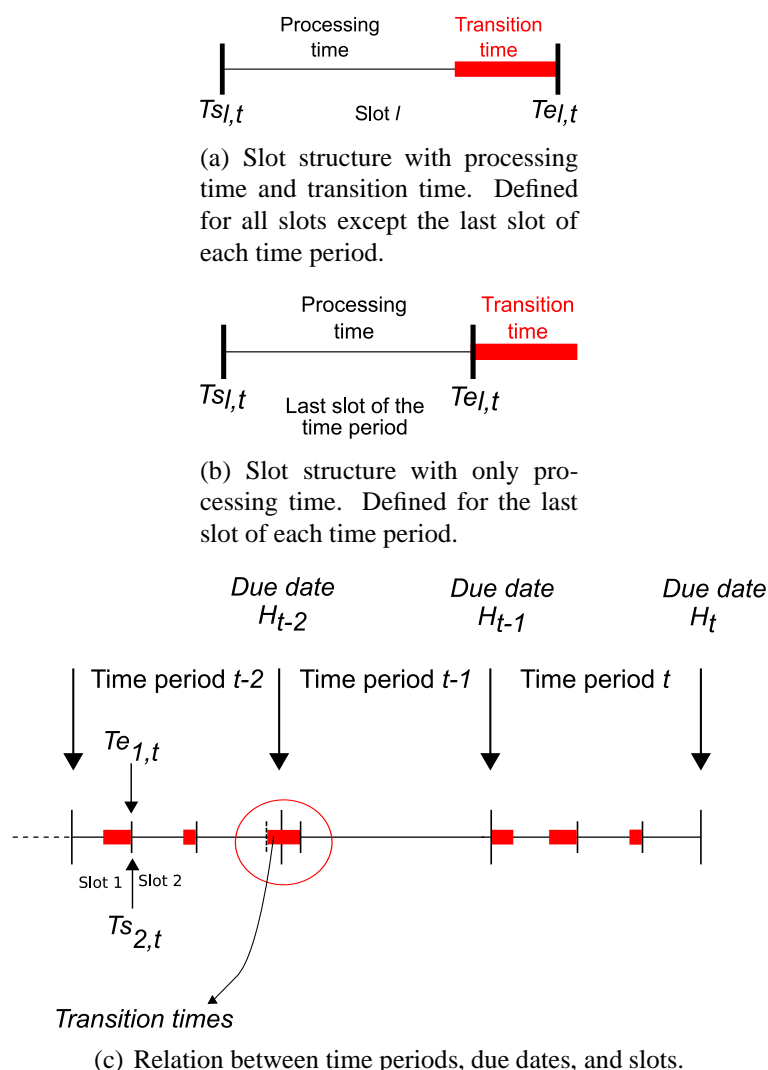


Figure 5: Slot based continuous time representation.

changeover without transition time occurs when the thickness is changed or a coating is applied to a given substrate. Figure 6 illustrates the relation between the different products in terms of changeovers. In each row, the products with the same substrate are presented, and in each column the products that derive from the product in the same column by application of a coating. The last column corresponds to products with the same substrate but with a different thickness. In order to produce the product C1S1 (substrate 1 plus coating 1), due to process constraints the process must produce first the product S1 (only with substrate 1) and then make a changeover with zero length to the product C1S1. Based on the fact that in the production sequence the products with no changeovers always appear together, we aggregate them into one pseudo-product, and the production sequence is only determined using the pseudo-products. Therefore, the processing time assigned to one pseudo-product in the schedule must be disaggregated into the processing times of the products aggregated into it. The processing time for each pseudo-product,  $\tilde{\theta}_{i,t}$ ,  $\forall i \in IM$ , is equal to the production time for the product defined by only the substrate,  $\tilde{\theta}_{2_{i',t}}$ ,  $\forall i' \in IS$ , plus the processing time of the product  $k$  that has coating 1,  $\tilde{\theta}_{2_{k,t}}$ , plus processing time of the product  $j$  that has coating 2,  $\tilde{\theta}_{2_{j,t}}$ , plus the processing time of the product  $n$  with a different thickness,  $\tilde{\theta}_{2_{n,t}}$ ,

$$\tilde{\theta}_{i,t} = \tilde{\theta}_{2_{i',t}} + \tilde{\theta}_{2_{k,t}} + \tilde{\theta}_{2_{j,t}} + \tilde{\theta}_{2_{n,t}} \quad \forall i \in IM, i' \in IS_{p,i'}, k \in CT1_{i,k}, j \in CT2_{i,j}, n \in THK_{i,n}, t \in TS \quad (1)$$

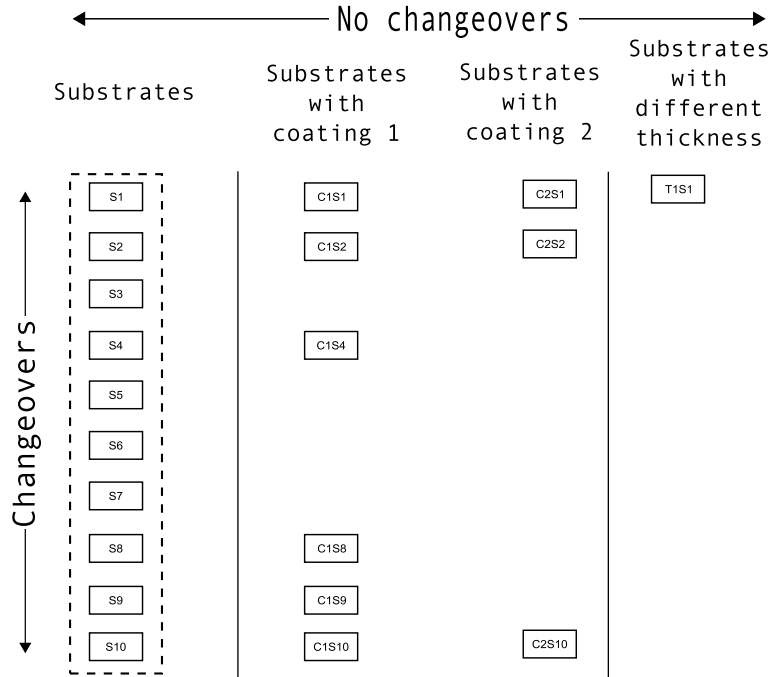


Figure 6: Changeover matrix between products. Each box represents a product, and products in the same row have the same substrate.

The set of pseudo-products is defined as  $IM$ ,  $IS$  is the set of products with only the substrate, and the sets of products with coatings 1, coatings 2, and a different thickness are represented by  $CT1_{i,k}$ ,  $CT1_{i,j}$ ,  $THK_{i,n}$ , respectively. The set of all products is  $P := \{IS \cup CT1 \cup CT2 \cup THK\}$ . Note that the products aggregated into one pseudo-product have different production rates, production costs, demand and selling prices. Therefore, the processing times denoted by  $\tilde{\theta}_{2_{i',t}}$ ,  $\tilde{\theta}_{2_{k,t}}$ ,  $\tilde{\theta}_{2_{j,t}}$ ,  $\tilde{\theta}_{2_{n,t}}$  are then used in the inventory balances, and objective function to account for the inventory levels, and processing costs, respectively, for each product.

The binary variables associated with the assignments are  $YOP_{i,t}$ ,  $W_{i,l,t}$ ,  $Z_{i,k,l,t}$  and  $TRR_{i,k,t}$  are defined below, where the index  $i$  is used for pseudo-products in the assignment equations and the index  $p$  for the disaggregated products in the inventory balances.

$$YOP_{i,t} = \begin{cases} 1 & \text{if product } i \text{ is assigned to the time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$W_{i,l,t} = \begin{cases} 1 & \text{if product } i \text{ is assigned to slot } l \text{ during time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{i,k,l,t} = \begin{cases} 1 & \text{if product } i \text{ assigned to slot } l \text{ is followed by product } k \text{ assigned to slot } l + 1 \\ & \text{during time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$TRR_{i,k,t} = \begin{cases} 1 & \text{if product } i \text{ assigned to the last slot of } l \text{ during time period } t \text{ is followed} \\ & \text{by product } k \text{ assigned to slot the first slot of time period } t + 1 \\ 0 & \text{otherwise} \end{cases}$$

The sets of products, slots, and time periods used in each scheduling model are defined as:

$$LM_t := \text{set of slots active for the time period } t$$

$$TS := \text{set of time periods active for the scheduling model}$$

These are subsets of the sets  $P$ ,  $L$ , and  $T$  that are defined over all products, slots, and time periods.

### 5.1.2 Assignment and processing times

Equation 2 enforces that only one product be assigned to each slot, and Equation 3 sets the processing time,  $\theta_{i,l,t}$ , to zero if product  $i$  is not assigned to slot  $l$  during the time period  $t$ .

$$\sum_{i \in IM} W_{i,l,t} = 1 \quad \forall l \in LM_t, t \in TS \quad (2)$$

$$\theta_{i,l,t} \leq H_t W_{i,l,t} \quad \forall i \in IM, l \in LM_t, t \in TS \quad (3)$$

The processing time of each product during the time period  $t$  is given by the sum of the processing times of product  $i$  in all slots during the time period,

$$\tilde{\theta}_{i,t} = \sum_{l \in LM_t} \theta_{i,l,t} \quad \forall i \in IM, t \in TS \quad (4)$$

Note that  $\tilde{\theta}_{i,t}$  is not equal to the length of the total production run of product  $i$ , because the production of a given product  $i$  can take place across several time periods.

### 5.1.3 Transitions within time periods

The changeover from product  $i$  to product  $k$  within a time period  $t$  is formulated through the following equations (Erdirik-Dogan and Grossmann, 2008):

$$\sum_{k \in IM} Z_{i,k,l,t} = W_{i,l,t} \quad \forall i \in IM, l \in LM_t \setminus LML, t \in T \quad (5)$$

$$\sum_{i \in IM} Z_{i,k,l,t} = W_{k,l+1,t} \quad \forall i \in IM, l \in LM_t \setminus LML, t \in T \quad (6)$$

Equations 5 and 6 state that if product  $i$  is assigned to slot  $l$  during time period  $t$  then there is one changeover from product  $i$  to product  $k$ , and if product  $k$  is assigned to slot  $l+1$  during time period  $t$  then there is a changeover from product  $i$  in slot  $l$  to product  $k$  in slot  $t+1$ . Notice that if only one product is assigned to period  $t$  then  $i = k$ .

### 5.1.4 Transitions between adjacent periods

The changeovers across the due dates are modeled using Equations 7 and 8.

$$\sum_{k \in IM} TRR_{i,k,t} = W_{i,l,t} \quad \forall i \in IM, l \in LML, t \in TS \setminus TL \quad (7)$$

$$\sum_{i \in IM} TRR_{i,k,t} = W_{k,l,t+1} \quad \forall k \in IM, l \in LMF, t \in TS \setminus TL \quad (8)$$

As an extension of the model proposed by Erdirik-Dogan and Grossmann (2006), the changeovers in this model can occur before, across, and after the due date, see Figure 7. In order to model the changeover across the due date, the transition time,  $\tau_{i,k}$ , is disaggregated into two variables,  $\tau e_{i,k,t}$  and  $\tau s_{i,k,t+1}$ ,

$$\tau_{i,k} TRR_{i,k,t} = \tau e_{i,k,t} + \tau s_{i,k,t+1} \quad \forall i, k \in IM, t \in TS \setminus TL \quad (9)$$

where  $TRR_{i,k,t}$  is a 0-1 variable to denote if product  $i$  is followed by product  $k$  at the end of period  $t$ .  $\tau e_{i,k,t}$  is assigned at the end of the time period, and  $\tau s_{i,k,t+1}$  is the part of the transition time that is assigned at the beginning of the next time period. See the time balances in Equations 12 and 13.



### 5.1.5 Timing balances

Within a time period  $t$ , each slot except the last slot, includes a processing time and a transition time. Thus, the end time of a slot,  $T_{e_{l,t}}$  is equal to the start time of the slot,  $T_{s_{l,t}}$ , plus the processing time plus the transition time:

$$T_{e_{l,t}} = T_{s_{l,t}} + \sum_i \theta_{i,l,t} + \sum_{i \in IM} \sum_{k \neq i \in IM} \tau_{i,k} Z_{i,k,l,t} \quad \forall l \in LM_t \setminus LML, t \in TS \quad (10)$$

In the last slot of each time period the transition time is not included within the time period. Therefore, the end time of these slots is given by,

$$T_{e_{l,t}} = T_{s_{l,t}} + \sum_{i \in IM} \theta_{i,l,t} \quad \forall l \in LML, t \in TS \quad (11)$$

The end time of the last slot of each time period plus the potential transition time across time periods is equal to the time at the end of time period,  $HT_t$ ,

$$T_{e_{l,t}} + \sum_{i \in IM} \sum_{k \neq i \in IM} \tau_{e_{i,k,t}} = HT_t \quad \forall l \in LML, t \in TS \quad (12)$$

The starting time of the first slot of each time period is equal to the potential transition time from the product of the last slot of the previous time period plus the time of the end of the previous time period.

$$T_{s_{1,t}} = \sum_{i \in IM} \sum_{k \in IM} \tau_{s_{i,k,t}} + HT_{t-1} \quad \forall l \in LMF, t \in TS \setminus TF \quad (13)$$

The end time of each slot, except the last slot of each time period, is equal to the starting time of the next slot:

$$T_{e_{l,t}} = T_{s_{l+1,t}} \quad \forall l \in LM_t \setminus LML, t \in TS \quad (14)$$

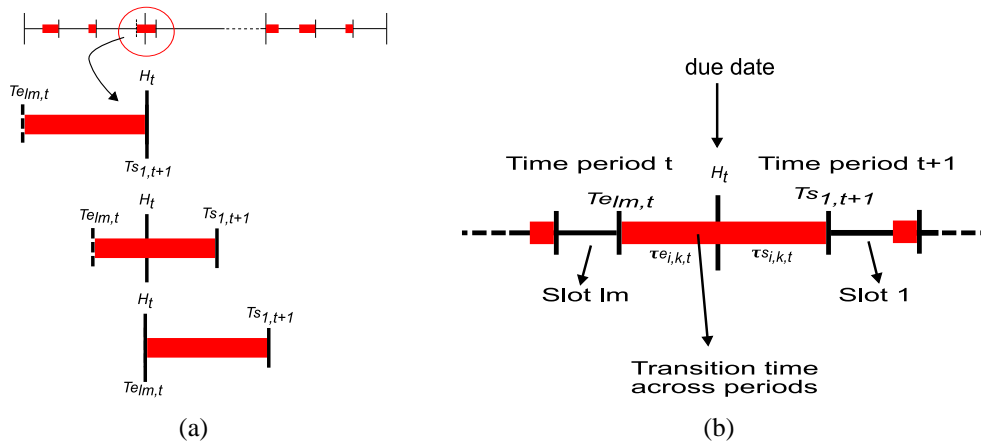


Figure 7: Transition times across time periods. In the figure  $lm$  denotes the last slot of the time period.

Equations 15, 16, and 17 ensure that the length of each time period is equal to the sum of the transition times plus the sum of the processing times. In each equation the equality ensures that there are no idle times.

$$\sum_{i \in IM} \sum_{k \neq i} \tau e_{i,k,t} + \sum_{i \in IM} \sum_{l \in LM} \theta_{i,l,t} + \sum_{i \in IM} \sum_{k \neq i} \sum_{l \in LM_t \setminus LML} \tau_{i,k} Z_{i,k,l,t} = H_t \quad \forall t \in TF \quad (15)$$

$$\begin{aligned} \sum_{i \in IM} \sum_{k \in IM} \tau s_{i,k,t} + \sum_{i \in IM} \sum_{l \in LM} \theta_{i,l,t} + \sum_{i \in IM} \sum_{k \in IM} \sum_{l \in LM_t \setminus LML} \tau_{i,k} Z_{i,k,l,t} + \\ + \sum_{i \in IM} \sum_{k \in IM} \tau e_{i,k,t} = H_t \quad \forall t \in TS \setminus (TF \cup TL) \quad (16) \end{aligned}$$

$$\sum_{i \in IM} \sum_{k \neq i \in IM} \tau s_{i,k,t} + \sum_{i \in IM} \sum_{l \in LM} \theta_{i,l,t} + \sum_{i \in IM} \sum_{k \neq i \in IM} \sum_{l \in LM \setminus LML} \tau_{i,k} Z_{i,k,l,t} = H_t \quad \forall t \in TSL \quad (17)$$

### 5.1.6 Minimum run length

A minimum run length is enforced in the model in order to keep the process stable. Several authors (Karimi and McDonald, 1997; Suerie, 2005; Lee et al., 2002; Lim and Karimi, 2003; Lamba and Karimi, 2002; Ierapetritou et al., 1999) have developed equations to model this concept. The minimum run length can be defined in a simplistic form by the following equation:

$$\theta_{i,l,t} \geq MRL_i W_{i,l,t} \quad \forall i \in IM, l \in LM_t, t \in TS \quad (18)$$

where  $\theta_{i,l,t}$  is the processing time of product  $i$  in slot  $l$  during the time period  $t$  and  $MRL_i$  denotes the minimum run length of product  $i$ . However, Equation 18 does not consider minimum run lengths across due dates, which removes flexibility to the model.

In this section a general model to enforce minimum run lengths across multiple time periods is presented. The main idea is that the production time of product  $i$ ,  $\tilde{\theta}_{i,t}$ , plus the production times of the same product in the next time periods that belong to the production run in the time period  $t$ , plus the production times of the same product in the previous time periods that belong to the same production run, must be greater or equal than the minimum run length if  $YOP_{i,t} \geq 1$ . This is represented by the disjunction in Equation 19.

$$\left[ \tilde{\theta}_{i,t} + \sum_{t' \in B_{i,t}} TPB_{i,t'} + \sum_{t' \in A_{i,t}} TPA_{i,t'} \geq MRL_i \right] \vee \left[ \tilde{\theta}_{i,t} = 0 \right] \quad \forall i \in IM, t \in TS \quad (19)$$

The above disjunction can be modeled as:

$$\tilde{\theta}_{i,t} + \sum_{t' \in B_{i,t}} TPB_{i,t'} + \sum_{t' \in A_{i,t}} TPA_{i,t'} \geq MRL_i YOP_{i,t} \quad \forall i \in IM, t \in TS \quad (20)$$

$$\tilde{\theta}_{i,t} \leq H_t YOP_{i,t} \quad (21)$$

where  $A_{i,t} \subseteq T$ , and is formally defined as,

$$A_{i,t} := \{t' : t' \geq t + 1, t' \leq t^* : HT_t + MRL_i \leq HT_{t^*}, HT_t + MRL_i > HT_{t^*-1}\} \quad (22)$$

and where  $HT_t$  denotes the time at the end of the time period  $t$ .  $B_{i,t} \subseteq T$ , and is defined as

$$B_{i,t} := \{t' : t' \leq t - 1, t' \geq t^* : HT_{t-1} - MRL_i < HT_{t^*}, HT_{t-1} - MRL_i \leq HT_{t^*-1}\} \quad (23)$$

The members of  $A_{i,t}$  are the time periods after  $t$ , such that the sum of the length of the time periods in  $A_{i,t}$  plus the production run in the time period  $t$  is greater or equal than the minimum run length. The members of  $B_{i,t}$  are the time periods before  $t$ , such that the sum of the length of the time periods in  $B_{i,t}$  plus the production run in the time period  $t$  is greater or equal than the minimum run length. As an example, consider the sequence of production in Figure 8, with

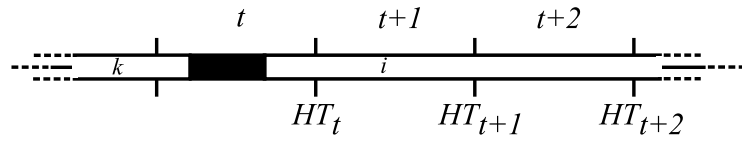


Figure 8: Sequence of production from the product  $k$  to product  $i$ . The black box denotes a changeover.

$MRL_i = 20$  days, and the duration of each interval is 10 days. The elements of  $A_{i,t}$  are given by  $t' \geq t + 1$  and  $t' \leq t^* : HT_t + MRL_i \leq HT_{t^*}, HT_{t^*-1} < HT_t + MRL_i$ , thus  $t^* = t + 2$ , and  $A_{i,t} := \{t + 1, t + 2\}$ . Considering now as reference the time period  $t + 2$ , the elements of  $B_{i,t+2}$  are given by  $t' \leq (t + 2) - 1$ , and  $t' \geq t^* : HT_{t+1} - MRL_i \geq HT_{t^*-1}, HT_{t^*} > HT_{t+1} - MRL_i$ , thus  $t^* = t$ , and  $B_{i,t+2} := \{t, t + 1\}$ .

$TPA_{i,t'}$  is equal to the production time of the product  $i$  in the time period  $t'$  if there is a transition from product  $i$  in time period  $t' - 1$  to product  $i$  in the time period  $t'$ ,  $TRR_{i,i,t'-1}$ , and  $TPA_{i,t'}$  is equal to zero otherwise.  $TPB_{i,t'}$  is equal to the production time of the product  $i$  in the time period  $t'$  if there is a transition from product  $i$  in time period  $t'$  to product  $i$  in the time period  $t' + 1$ ,  $TRR_{i,i,t'}$ , and  $TPB_{i,t'}$  is equal to zero otherwise. This is represented by the disjunctions in the Equations 24 and 25. Therefore in Equation 20, the term corresponding to the sum of  $TPA_{i,t'}$  is the sum of the production times of product  $i$  for  $t' \geq t + 1$  that belong to the same production run of the product  $i$  in the time period  $t$ . In the same equation the sum of  $TPB_{i,t'}$  is the sum of the production times of product  $i$  for  $t' \leq t - 1$  that belong to the same production run of the product  $i$  in the time period  $t$ .

$$\left[ \begin{array}{c} TRR_{i,i,t} \\ TPB_{i,t} = \tilde{\theta}_{i,t} \end{array} \right] \vee \left[ \begin{array}{c} -TRR_{i,i,t} \\ TPB_{i,t} = 0 \end{array} \right] \quad \forall i \in IM, t \in TS \setminus TL \quad (24)$$

$$\left[ \begin{array}{c} TRR_{i,i,t-1} \\ TPA_{i,t} = \tilde{\theta}_{i,t} \end{array} \right] \vee \left[ \begin{array}{c} -TRR_{i,i,t-1} \\ TPA_{i,t} = 0 \end{array} \right] \quad \forall i \in IM, t \in TS \setminus TF \quad (25)$$

These two disjunctions can be modeled using a convex-hull formulation (Balas, 1985), and after

some algebraic manipulations the minimum run length can be formulated as:

$$\tilde{\theta}_{i,t} + \sum_{tt \in B_{i,t}} \tilde{\theta}b1_{i,tt} + \sum_{tt \in A_{i,t}} \tilde{\theta}a1_{i,tt} \geq MRL_i YOP_{i,t} \quad \forall i \in IM, t \in TS \quad (26)$$

$$\tilde{\theta}_{i,t} = \tilde{\theta}b1_{i,t} + \tilde{\theta}b2_{i,t} \quad \forall i \in IM, t \in TS \quad (27)$$

$$\tilde{\theta}b1_{i,t} \leq H_t TRF_{i,i,t} \quad \forall i \in IM, t \in TS \quad (28)$$

$$\tilde{\theta}b2_{i,t} \leq H_t (1 - TRF_{i,i,t}) \quad \forall i \in IM, t \in TS \quad (29)$$

$$\tilde{\theta}_{i,t} = \tilde{\theta}a1_{i,t} + \tilde{\theta}a2_{i,t} \quad \forall i \in IM, t \in TS \quad (30)$$

$$\tilde{\theta}a1_{i,t} \leq H_t TRF_{i,i,t-1} \quad \forall i \in IM, t \in TS \quad (31)$$

$$\tilde{\theta}a2_{i,t} \leq H_t (1 - TRF_{i,i,t-1}) \quad \forall i \in IM, t \in T \quad (32)$$

where the variables  $TPB_{i,tt}$  and  $TPA_{i,tt}$  were eliminated. This formulation is tighter than a big-M formulation, and uses the disaggregated variables  $\tilde{\theta}a1_{i,t}$ ,  $\tilde{\theta}a2_{i,t}$ ,  $\tilde{\theta}b1_{i,t}$ , and  $\tilde{\theta}b2_{i,t}$ .

Equation 26 is active when  $YOP_{i,t} \geq 1$ , and redundant otherwise. This means that the equation is active for all the time periods that belong to a production run. For a production run with the minimum run length that uses the time periods  $t$ ,  $t + 1$ , and  $t + 2$ , the application of Equation 26 to the time period  $t + 2$  might consider the processing time of a production run that starts in  $t + 3$ . However, for this production run the minimum run length is enforced by the application of Equation 26 to the first time period of this production run. An alternative formulation could be defined replacing the right hand side (RHS) of Equation 26, leading to

$$\tilde{\theta}_{i,t} + \sum_{tt \in B_{i,t}} \tilde{\theta}b1_{i,tt} + \sum_{tt \in A_{i,t}} \tilde{\theta}a1_{i,tt} \geq MRL_i \left[ YOP_{i,t} - \left( TRF_{i,i,t} + \sum_{l \in LM} Z_{i,i,l,t} \right) \right] \quad (33)$$

with the above RHS the constraint would be only active for the first time period of the production run. However, from computational experience this formulation did not provide tighter linear relaxations.

### 5.1.7 Inventory balances

Equations 34 and 35 define the inventory balances at the end of the time periods for each product,

$$INVO_{p,t} - BCKL_{p,t} = INVI_p - BCKLI_p + \eta_p r_p \tilde{\theta}2_{p,t} - S_{p,t} \quad \forall p \in P, t \in TF \quad (34)$$

$$INVO_{p,t} - BCKL_{p,t} = INVO_{p,t-1} - BCKL_{p,t-1} + \eta_p r_p \tilde{\theta}2_{p,t} - S_{p,t} \quad \forall p \in P, t \in TS \setminus TF \quad (35)$$

where  $INVO_{p,t}$  and  $BCKL_{p,t}$  denote the inventory and backlog of product  $p$  at the end of the time period after the sales,  $INVI_p$ , and  $BCKLI_p$  are the initial inventory and backlog at the beginning of the time horizon,  $\eta_p$ , and  $r_p$  are the process yield and gross production rates of product  $p$ , and

$S_{p,t}$  denotes the sales of product  $p$  during time period  $t$ . Note that the processing times used in the inventory balances are given by  $\tilde{\theta}_{2,p,t}$  defined over all the products, while  $\tilde{\theta}_{i,t}$  are defined over the pseudo-products used in the assignment equations to defined the scheduling. The inventory holding costs given by the area under the curve of the inventory vs time, represented by  $Area_{p,t}$ , is overestimated as discussed in Erdirik-Dogan and Grossmann (2006).

$$Area_{p,t} \geq INVI_p H_t + \eta_p r_p \tilde{\theta}_{2,p,t} H_t \quad \forall p \in P, t \in TF \quad (36)$$

$$Area_{p,t} \geq INVO_{p,t-1} H_t + \eta_p r_p \tilde{\theta}_{2,p,t} H_t \quad \forall p \in P, t \in TS \setminus TF \quad (37)$$

The safety stock bounds, and an upper bound for the maximum storage capacity are enforced by Equations 38 and 39,

$$INVO_{p,t} - BCKL_{p,t} + s_{p,t} \geq INVMIN_p \quad \forall p \in P, t \in TS \quad (38)$$

$$\sum_{p \in P} INVO_{p,t} + S_{p,t} \leq INVMAX + q_t \quad \forall t \in TS \quad (39)$$

where  $s_{p,t}$  and  $q_t$  are slack variables,  $INVMIN_p$  is a constant representing the safety stock,  $INVMAX$  is the maximum storage capacity. The sales are set equal to the deterministic forecast demand by the equation:

$$S_{p,t} = d_{p,t} \quad \forall p \in P, t \in TS \quad (40)$$

Equation 41 represents the terminal inventory constraints that are used to avoid the model driving the inventory levels to the safety stock levels at the end of the time horizon.

$$INVO_{p,t} - BCKL_{p,t} + slth_p \geq INVMIN_p + \sum_{tt \in TT} d_{p,tt} \quad \forall p \in P, t \in TL \quad (41)$$

$slth_p$  is a slack variable, and  $TT$  is the set of time periods used to estimate the demand after the end of the time horizon.  $TT$  is a parameter that must be defined (e.g. 4 months).

### 5.1.8 Symmetry breaking constraints

Erdirik-Dogan and Grossmann (2006) have proposed symmetry breaking constraints to prevent degenerate solutions when one product can be assigned to more than one slot in the same time period. Their constraints, Equations 42 to 47 impose that only one product can be assigned to more than one slot, and this product must be assigned to the first slot. These constraints are represented by the following equations:

$$YOP_{i,t} \geq W_{i,l,t} \quad \forall i \in IM, l \in LM, t \in TS, \quad (42)$$

$$YOP_{i,t} \leq NY_{i,t} \quad \forall i \in IM, t \in TS \quad (43)$$

$$NY_{i,t} = \sum_{l \in LM} W_{i,l,t} \quad \forall i \in IM, t \in TS \quad (44)$$

$$NY_{i,t} \leq NSLOTS_t YOP_{i,t} \quad \forall i \in IM, t \in TS \quad (45)$$

$$NY_{i,t} \geq NSLOTS_t - \left( \sum_{k \in IM} YOP_{k,t} - 1 \right) - Ms(1 - W_{i,l,t}) \quad \forall i \in IM, l \in LMF, t \in TS \quad (46)$$

$$NY_{i,t} \leq NSLOTS_t - \left( \sum_{k \in IM} YOP_{k,t} - 1 \right) + Ms(1 - W_{i,l,t}) \quad \forall i \in IM, l \in LMF, t \in TS \quad (47)$$

where  $NSLOTS_t$  denotes the number of slots specified for the time period  $t$ , and  $NY_{i,t}$  the number of slots assigned to product  $i$ ,  $Ms$  represents big-M constants defined as a function of the number of slots. Note that Equations 46 and 47 are only valid for the first slot of each time period.

In the derivation of Erdirik-Dogan and Grossmann (2006) it is assumed that by optimality the product that is assigned to more than one slot uses consecutive slots. However, in the problem of this paper this may not be true because the model may be driven to introduce transitions to reduce processing costs and increase transition cost if this improves the objective function. In order to force the slots to be consecutive the following symmetry breaking constraints are proposed:

$$\left[ \begin{array}{c} YF_{i,t} \\ NY_{i,t} \geq 2 \\ \sum_i \sum_l \sum_t Z_{i,i,l,t} \geq NY_{i,t} - 1 \end{array} \right] \vee \left[ \begin{array}{c} \neg YF_{i,t} \\ NY_{i,t} \leq 1 \end{array} \right] \quad \forall i \in IM, t \in TS \quad (48)$$

The above disjunction states that if one product is assigned to more than one slot then the number of changeovers must be greater or equal than the number of slots minus one. Through a big-M reformulation, Equation 48 yields:

$$NY_{i,t} \geq 2 - M_2(1 - YF_{i,t}) \quad \forall i \in IM, t \in TS \quad (49)$$

$$NY_{i,t} \leq 1 + M_f YF_{i,t} \quad \forall i \in IM, t \in TS \quad (50)$$

$$\sum_{l \in LM_t \setminus LML} Z_{i,i,l,t} \geq NY_{i,t} - 1 - M_f(1 - YF_{i,t}) \quad \forall i \in IM, t \in TS \quad (51)$$

$$Z_{i,i,l,t} \leq YF_{i,t} \quad \forall i \in IM, l \in LM_t \setminus LML, t \in TS \quad (52)$$

where  $M_2$  and  $M_f$  are big-M parameters with tight values.

### 5.1.9 Coatings

The following constraint is imposed by the operation of the online coater, which states that the processing time of a product without coating must be greater or equal than the processing time for

the products with the same substrate but with a coating applied.

$$\tilde{\theta}_{2_{i',t}} + \tilde{\theta}_{2_{n,t}} \geq \tilde{\theta}_{2_{k,t}} + \tilde{\theta}_{2_{j,t}} \quad \forall i' \in IS, k \in CT1, j \in CT2, n \in THK, t \in TS \quad (53)$$

### 5.1.10 Objective function

The objective of the MILP model is to maximize the profit given by the sum of the sales revenues minus the inventory holding costs, operating costs, and transition costs. In addition, in the objective function penalties are considered for the violation of safety stocks, maximum storage capacity, and target inventory levels at the end of the time horizon.

$$\begin{aligned} profit = & \sum_{p \in P} \sum_{t \in TS} p_{p,t} S_{p,t} - \sum_{p \in P} \sum_{t \in TS} cinv_{p,t} Area_{p,t} - \sum_{t \in TS} \sum_{p \in P} coper_{p,t} \eta_p r_p \tilde{\theta}_{2_{p,t}} \\ & - \sum_{i \in IM} \sum_{k \in IM} \sum_{l \in LM \setminus LML} \sum_{t \in TS} ctran_{i,k} Z_{i,k,l,t} - \sum_{i \in IM} \sum_{k \in IM} \sum_{t \in TS \setminus TL} ctran_{i,k} TRT_{i,k,t} \quad (54) \\ & - \sum_{t \in TS} PEN1_t q_t - \sum_{p \in P} \sum_{t \in TS} PEN2_{p,t} s_{p,t} - \sum_{p \in P} PEN3 slth_p \end{aligned}$$

In the above objective function the transition costs are calculated over the changeovers within the time periods and the changeovers across the due dates.

## 5.2 Planning model

The planning model used in this work is based on the model proposed by Erdirik-Dogan and Grossmann (2008) with extensions similar to the ones of the scheduling model. It is a TSP-based model, where the binary variables associated with the assignments are  $YP_{i,t}$ ,  $ZP_{i,k,t}$ ,  $ZP_{i,k,t}$ , and  $ZZZ_{i,k,t}$ , defined as:

$$\begin{aligned} YP_{i,t} &= \begin{cases} 1 & \text{if product } i \text{ is assigned to the time period } t \\ 0 & \text{otherwise} \end{cases} \\ ZP_{i,k,t} &= \begin{cases} 1 & \text{if product } i \text{ is followed by product } k \text{ during time period } t \\ 0 & \text{otherwise} \end{cases} \\ ZZP_{i,k,t} &= \begin{cases} 1 & \text{if the link between product } i \text{ and product } k \text{ is broken during time period } t \\ 0 & \text{otherwise} \end{cases} \\ ZZZ_{i,k,t} &= \begin{cases} 1 & \text{if product } i \text{ assigned to the last slot of } l \text{ during time period } t \text{ is followed} \\ & \text{by product } k \text{ assigned to slot the first slot of time period } t + 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$XF_{i,t} = \begin{cases} 1 & \text{if product } i \text{ is assigned to the first position of the sequence in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XL_{i,t} = \begin{cases} 1 & \text{if product } i \text{ is assigned to the last position of the sequence in time period } t \\ 0 & \text{otherwise} \end{cases}$$

The sets of products, and time periods used in each planning model are defined as:

$IM$  := set of pseudo-products  $i$

$TP$  := set of time periods active for the planning model

These are subsets of the sets  $P$ , and  $T$  that are defined over all products, and time periods. The MILP model is described in detail in the next subsections.

### 5.2.1 Production and sequencing constraints

The processing time of the pseudo-products,  $\tilde{\theta}_{i,t}$  is bounded by the length of the time period, if the product is assigned to the respective time period.

$$\tilde{\theta}_{i,t} \leq H_t YP_{i,t} \quad \forall i \in IM, t \in TP \quad (55)$$

Equations 56 to 62 represent the sequence constraints proposed by Erdirik-Dogan and Grossmann (2008). The two first equations state that if product  $i$  is assigned to the time period  $t$  then there is a changeover from product  $i$  to product  $k$  during time period  $t$ , and if product  $k$  is assigned to time period  $t$  then there is a changeover from product  $k$  to product  $i$  during time period  $t$ .

$$YP_{i,t} = \sum_{k \in IM} ZP_{i,k,t} \quad \forall i \in IM, t \in TP \quad (56)$$

$$YP_{k,t} = \sum_{i \in IM} ZP_{i,k,t} \quad \forall k \in IM, t \in TP \quad (57)$$

These equations enforce a cycle between the products that is broken by using the following constraint:

$$\sum_{i \in IM} \sum_{k \in IM} ZP_{i,k,t} = 1 \quad t \in TP \quad (58)$$

Equations 59 and 60 indicate that only one product can be assigned to the first and last position, respectively.

$$\sum_{i \in IM} XF_{i,t} = 1 \quad \forall t \in TP \quad (59)$$

$$\sum_{i \in IM} XL_{i,t} = 1 \quad \forall t \in TP \quad (60)$$



Equations 61 and 62 state that if product  $i$  is the last product of the time period  $t$  then there is a transition from product  $i$  from time period  $t$  to product  $k$  in time period  $t + 1$ , and if product  $k$  is the first product of the time period  $t + 1$  then there is a transition from product  $i$  from time period  $t$  to product  $k$  in time period  $t + 1$ .

$$\sum_{i \in IM} ZZZ_{i,k,t} = XF_{k,t+1} \quad \forall k \in IM, t \in TP \setminus TL \quad (61)$$

$$\sum_{k \in IM} ZZZ_{i,k,t} = XL_{i,t} \quad \forall i \in IM, t \in TP \setminus TL \quad (62)$$

The following equations introduce relations between the binary variables associated with the assignments and changeovers, including the case when only one product  $i$  is assigned to a time period  $t$ .

$$ZZP_{i,k,t} \leq ZP_{i,k,t} \quad \forall i, k \in IM, t \in TP \quad (63)$$

$$YP_{i,t} \geq ZP_{i,i,t} \quad \forall i \in IM, t \in TP \quad (64)$$

$$ZP_{i,i,t} + YP_{k,t} \leq 1 \quad \forall i \neq k \in IM, t \in TP \quad (65)$$

$$ZP_{i,i,t} \geq YP_{i,t} - \sum_{k \neq i \in IM} YP_{k,t} \quad \forall i \in IM, t \in TP \quad (66)$$

$$XF_{k,t} \geq \sum_{i \in IM} ZZP_{i,k,t} \quad \forall k \in IM, t \in TP \quad (67)$$

$$XL_{i,t} \geq \sum_{k \in IM} ZZP_{i,k,t} \quad \forall i \in IM, t \in TP \setminus TL \quad (68)$$

$$YP_{i,t} \geq ZZP_{i,k,t} \quad \forall i, k \in IM, t \in TP \quad (69)$$

$$YP_{k,t} \geq ZZP_{i,k,t} \quad \forall i, k \in IM, t \in TP \quad (70)$$

$$YP_{i,t} \geq XF_{i,t} \quad \forall i \in IM, t \in TP \quad (71)$$

$$YP_{i,t} \geq XL_{i,t} \quad \forall i \in IM, t \in TP \quad (72)$$

### 5.2.2 Minimum run length

The equations to enforce a minimum run length follow the same approach used in the scheduling model. Their derivation starts by postulating the following disjunction:

$$\left[ \tilde{\theta}_{i,t} + \sum_{tt \in B_{i,t}} TPB_{i,tt} + \sum_{tt \in A} TPA_{i,tt} \geq MRL_i \right] \vee \left[ \neg YP_{i,t} \right] \quad \forall i \in IM, t \in TP \quad (73)$$

Following a similar reasoning as in the derivation of Equations 26 to 32, the minimum run length for the planning model can be formulated as:

$$\tilde{\theta}_{i,t} + \sum_{tt \in B_{i,t}} \tilde{\theta}b1_{i,tt} + \sum_{tt \in A} \tilde{\theta}a1_{i,tt} \geq MRL_i YP_{i,t} \quad \forall i \in IM, t \in TP \quad (74)$$

$$\tilde{\theta}_{i,t} = \tilde{\theta}b1_{i,t} + \tilde{\theta}b2_{i,t} \quad \forall i \in IM, t \in TP \quad (75)$$

$$\tilde{\theta}b1_{i,t} \leq H_t ZZZ_{i,i,t} \quad \forall i \in IM, t \in TP \quad (76)$$

$$\tilde{\theta}b2_{i,t} \leq H_t (1 - ZZZ_{i,i,t}) \quad \forall i \in IM, t \in TP \quad (77)$$

$$\tilde{\theta}_{i,t} = \tilde{\theta}a1_{i,t} + \tilde{\theta}a2_{i,t} \quad \forall i \in IM, t \in TP \quad (78)$$

$$\tilde{\theta}a1_{i,t} \leq H_t ZZZ_{i,i,t-1} \quad \forall i \in IM, t \in TP \quad (79)$$

$$\tilde{\theta}a2_{i,t} \leq H_t (1 - ZZZ_{i,i,t-1}) \quad \forall i \in IM, t \in T \quad (80)$$

where  $\tilde{\theta}a1_{i,t}$ ,  $\tilde{\theta}a2_{i,t}$ ,  $\tilde{\theta}b1_{i,t}$ , and  $\tilde{\theta}b2_{i,t}$  are additional auxiliary variables, and the  $TPB_{i,tt}$  and  $TPA_{i,tt}$  were eliminated.

### 5.2.3 Inventory balances

The equations applied for the inventory balances are similar to the equations used in the scheduling model.

$$\tilde{\theta}_{i,t} = \tilde{\theta}2_{i',t} + \tilde{\theta}2_{k,t} + \tilde{\theta}2_{j,t} + \tilde{\theta}2_{n,t} \quad \forall i \in IM, i' \in IS_{i,i'}, k \in CT1_{i,k}, j \in CT2_{i,j}, n \in THK_{i,n}, t \in TP \quad (81)$$

Note that in the following equations the set  $P$  is defined as  $P := \{IS \cup CT1 \cup CT2 \cup THK\}$ , which means all products.

$$INVO_{p,t} - BCKL_{p,t} = INVI_p - BCKL_p + \eta_p r_p \tilde{\theta}2_{p,t} - S_{p,t} \quad \forall p \in P, t \in TF \quad (82)$$

$$INVO_{p,t} - BCKL_{p,t} = INVO_{p,t-1} - BCKL_{p,t-1} + \eta_p r_p \tilde{\theta}2_{p,t} - S_{p,t} \quad \forall p \in P, t \in TP \setminus TF \quad (83)$$

$$Area_{p,t} \geq INVI_p H_t + \eta_p r_p \tilde{\theta}2_{p,t} H_t \quad \forall p \in P, t \in TF \quad (84)$$

$$Area_{p,t} \geq INVO_{p,t-1} H_t + \eta_p r_p \tilde{\theta}2_{p,t} H_t \quad \forall p \in P, t \in TP \setminus TF \quad (85)$$

$$\sum_{p \in P} INVO_{p,t} + S_{p,t} \leq INVMAX + q_t \quad \forall t \in TP \quad (86)$$

$$INVO_{p,t} - BCKL_{p,t} + s_{p,t} \geq INVMIN_p \quad \forall p \in P, t \in TP \quad (87)$$

$$S_{p,t} = d_{p,t} \quad \forall p \in P, t \in TP \quad (88)$$

$$INVO_{p,t} - BCKL_{p,t} + slth_p \geq INVMIN_p + \sum_{tt \in TT} d_{p,tt} \quad \forall p \in P, t \in TL \quad (89)$$

#### 5.2.4 Time balances

The planning model uses a simplified representation of the timing constraints that does not take into consideration possible changeovers between sub-cycles  $i$  within a time period. The total transition time within a time period,  $TRNP_t$ , is equal to the sum of all transition times minus the transition time associated with the link that is broken,

$$TRNP_t = \sum_{i \in IM} \sum_{k \neq i \in IM} \tau_{i,k} ZP_{i,k,t} - \sum_i \sum_{k \neq i \in IM} \tau_{i,k} ZZP_{i,k,t} \quad \forall t \in TP \quad (90)$$

$$\sum_{i \in IM} \tilde{\theta}_{i,t} + TRNP_t + \sum_{i \in IM} \sum_{k \neq i \in IM} \tau e_{i,k,t} = H_t \quad \forall t \in TF \quad (91)$$

$$\sum_{i \in IM} \sum_{k \neq i \in IM} \tau s_{i,k,t} + \sum_{i \in IM} \tilde{\theta}_{i,t} + TRNP_t + \sum_{i \in IM} \sum_{k \neq i \in IM} \tau e_{i,k,t} = H_t \quad \forall t \in TP \setminus (TF \cup TL) \quad (92)$$

$$\sum_{i \in IM} \sum_{k \neq i \in IM} \tau s_{i,k,t} + \sum_{i \in IM} \tilde{\theta}_{i,t} + TRNP_t = H_t \quad \forall t \in TL \quad (93)$$

The changeover between time periods is also modeled across the due dates by,

$$\tau_{i,k} ZZZ_{i,k,t} = \tau e_{i,k,t} + \tau s_{i,k,t+1} \quad \forall i, k \neq i \in IM, t \in TP \setminus TL \quad (94)$$

#### 5.2.5 Coatings

The following constraint states that the processing time of the product without coating must be greater or equal than the processing time for the products with the same substrate but with a coating applied.

$$\tilde{\theta}_{i',t} + \tilde{\theta}_{n,t} \geq \tilde{\theta}_{k,t} + \tilde{\theta}_{j,t} \quad \forall i' \in IS, k \in CT1, j \in CT2, n \in THK, t \in TP \quad (95)$$

### 5.2.6 Objective function

The objective function is given by,

$$\begin{aligned}
profit = & \sum_{p \in P} \sum_{t \in TP} p_{p,t} S_{p,t} - \sum_{p \in P} \sum_{t \in TP} cinv_{p,t} Area_{p,t} - \sum_{t \in TP} \sum_{p \in P} coper_{p,t} \eta_p r_p \tilde{\theta}_{p,t} \\
& - \sum_{i \in IM} \sum_{k \neq i \in IM} \sum_{t \in TP} ctran_{i,k} (ZP_{i,k,t} - ZZP_{i,k,t}) \\
& - \sum_{i \in IM} \sum_{k \neq i \in IM} \sum_{t \in TP} ctran_{i,k} ZZZ_{i,k,t} \\
& - \sum_{t \in TP} PEN1_t q_t - \sum_{p \in P} \sum_{t \in TP} PEN2_{p,t} s_{p,t} - \sum_{p \in P} PEN3 slth_p
\end{aligned} \tag{96}$$

Note that within the rolling horizon algorithms where the scheduling and planning models are used, the objective function is defined over the time periods of both model models.

### 5.2.7 Interface between scheduling and planning

The hybrid and the asynchronous rolling horizon algorithms integrate the scheduling and planning models in the same subproblem. Thus, at the interface of these models some variable must be linked in order to ensure that the minimum run lengths and changeovers across due dates are valid over the interface of the two models. Equations 97 and 98 ensure the link between the binary variables associated with the changeovers.

$$\sum_{i \in IM} TRF_{i,k,t} = XF_{k,t+1} \quad \forall i \in IM, l \in LML, t \in TSL, t \notin TL \tag{97}$$

$$TRF_{i,k,t} = ZZZ_{i,k,t} \quad \forall i, k \in IM, t \in TSL \setminus TL \tag{98}$$

In the asynchronous strategy the set of time periods contains first the time periods for the scheduling model and then the time periods for the planning model. This is illustrated with the following example. Consider 8 time periods of 7 days in the scheduling model and two time periods of 28 days in the planning model. Therefore, the set of time periods is defined as:

$$T := \{ w1, \dots, w8, t1, t2 \}$$

where  $w1 : w8$  are the time periods for the scheduling model and  $t1, t2$  denote the time periods for the planning model. For a subproblem with the scheduling model defined over  $w1 : w4$ , and the planning model over  $t2$ , the link between the scheduling and the planning model must be done between the time periods  $w4$  and  $t2$  (see Figure 9). Therefore, some of the equations presented have to be slightly modified in order to incorporate the link between the last time period of the scheduling model and the first period of the planning model for a given subproblem. In addition,

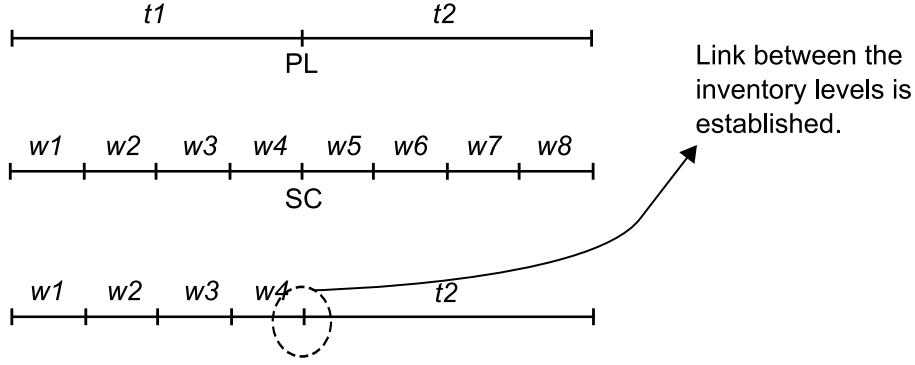


Figure 9: Time representation for the asynchronous rolling horizon strategy.

the link between the inventory levels is enforced with:

$$INVO_{p,t} = INVO_{p,tt-1} \quad \forall p \in P, t \in TSL, tt \in TPF \quad (99)$$

$$BCKL_{p,t} = BCKL_{p,tt-1} \quad \forall p \in P, t \in TSL, tt \in TPF \quad (100)$$

where  $TSL$  and  $TPF$  are singletons that represent the last time period of the scheduling and the first time period for the planning for a given subproblem. For the given example, we have  $TSL := w4$  and  $TPF := t2$ .

### 5.3 Remarks

1. At the interface of the scheduling and planning models the variables  $\tau e_{i,k,t}$ ,  $\tau e_{i,k,t}$ ,  $\theta b1_{i,t}$ ,  $\theta a1_{i,t}$ , are the same for both models, which together with Equations 97 and 98 ensure that minimum run lengths and transitions across due dates are valid at the interface.
2. The constraints proposed for modeling the changeovers across the due dates assume that the changeovers fit in the length of two time periods. Otherwise, these constraints must be reformulated.
3. The symmetry breaking constraints given by Equations 49 to 52 are only required if a minimum run length and two more products can fit within a time period.
4. For the rolling horizon algorithms where the planning model is applied, optimality of the solutions is not guaranteed because even if the planning model is solved to optimality, the solution is an overestimation of the profit.
5. In the planning model, sub-tour elimination constraints are not included, which may lead to an overestimation of the processing time at this level. However, the products assigned can then be re-adjusted at the scheduling level.

## 5.4 Alternative models

In order to compare the performance of the proposed models, three additional models are studied. The first and second correspond to the scheduling and planning models described before, but without considering the changeovers and minimum run lengths across the due dates. The aim is to assess the computational cost of introducing these constraints, and the production flexibility provided by these constraints. The third model relies on a variation of the proposed planning model with the sequence constraints, Equations 56 to 68, replaced by the sequence constraints used by Liu et al. (2008):

$$\sum_{i \in IM} XF_{i,t} = 1 \quad \forall t \in TP \quad (101)$$

$$\sum_{i \in IM} XL_{i,t} = 1 \quad \forall t \in TP \quad (102)$$

$$XF_{i,t} \leq YP_{i,t} \quad \forall i \in IM, t \in TP \quad (103)$$

$$XL_{i,t} \leq YP_{i,t} \quad \forall i \in IM, t \in TP \quad (104)$$

$$\sum_{i \neq k \in IM} ZP_{i,k,t} = YP_{k,t} - XF_{k,t} \quad \forall k \in IM, t \in TP \quad (105)$$

$$\sum_{k \neq i \in IM} ZP_{i,k,t} = YP_{i,t} - XL_{i,t} \quad \forall i \in IM, t \in TP \quad (106)$$

$$\sum_{i \in IM} ZZZ_{i,k,t} = XF_{k,t+1} \quad \forall k \in IM, t \in TP \setminus TPL \quad (107)$$

$$\sum_{k \in IM} ZZZ_{i,k,t} = XL_{i,t} \quad \forall i \in IM, t \in TP \setminus TPL \quad (108)$$

$$O_{k,t} - (O_{i,t} + 1) \geq -BIG_{MP} (1 - ZP_{i,k,t}) \quad \forall i \neq k \in IM, t \in TP \quad (109)$$

$$O_{i,t} \leq BIG_{MP} YP_{i,t} \quad \forall i \in IM, t \in TP \quad (110)$$

$$O_{i,t} \leq \sum_{k \in IM} YP_{k,t} \quad \forall i \in IM, t \in TP \quad (111)$$

$$O_{i,t} \geq XF_{i,t} \quad \forall i \in IM, t \in TP \quad (112)$$

where  $O_{i,t}$  denotes the order for the assignment of product  $i$  in the time period  $t$ . The remaining variables follow the nomenclature used in this work. Note that the last four equations represent sub-tour elimination constraints. The third model is used to evaluate the performance of the sequence constraints implemented in the planning model. The goal is not to perform a comparison with the model proposed by Liu et al. (2008), since their model does not include some of the features of the models presented in this work.

## 6 Case studies

In order to assess the performance of the proposed rolling horizon algorithms, and their applicability to support real world decisions, six case studies are presented. The case studies are as follows:

- **Case 1** - *hybrid rolling horizon algorithm with terminal constraints* for the inventory at the end of the time horizon of each subproblem. The time resolution is one month in the planning and scheduling models.
- **Case 2**- *asynchronous rolling horizon algorithm with terminal constraints* for the inventory at the end of the time horizon of each subproblem. The time resolution is 28 days in the planning model, and 7 days in the scheduling model until the 18th month. For the remaining time periods in the planning model the length is one month.
- **Case 3**- *detailed rolling horizon algorithm with terminal constraints* for the inventory at the end of the time horizon of each subproblem. The time resolution is one month using only the scheduling model with ten slots per time period.
- **Case 4** - *hybrid rolling horizon algorithm without terminal constraints* for the inventory at the end of the time horizon of each subproblem. The time resolution is one month in the planning and scheduling models.
- **Case 5**- *hybrid rolling horizon algorithm with terminal constraints* for the inventory at the end of the time horizon of each subproblem, *but without the changeovers and minimum run lengths across the time periods*. The time resolution is one month in the planning and scheduling models.
- **Case 6** - *hybrid rolling horizon algorithm with terminal constraints* for the inventory at the end of the time horizon of each subproblem, but a *modified planning model* using the sequence constraints from the work of Liu et al. (2008). The planning and scheduling models use time periods of one month.

The time horizon for each case is set to two years, which involves 18 months of detailed scheduling plus six additional months with the planning model to provide feedback on the final inventory level. The set of products to manufacture involves 10 different substrates, 9 coated products, and 1 substrate with a different thickness, resulting in 20 different products. The penalty weights used in the objective functions are set to large values in order to avoid the violation of the safety stocks, maximum storage capacity, and terminal constraints.

For each case study, three instances with different stopping criteria are considered: 1) minimum optimality gap of 5% and maximum time set to 3,600s; 2) minimum optimality gap of 1% and maximum time set to 3,600s; and 3) minimum optimality gap of 1% and maximum time set to 10,800s for all subproblems of Case 4, and 3,600s in the first three subproblems and 10,800s in the last three subproblems for the rolling horizon algorithms of Cases 1, 2, 3, 4 and 5. The goal of the first instance is to try to find a solution with a reasonable optimality gap within one hour. The second instance has a smaller minimum optimality gap without increasing the maximum time allowed. The last instance provides an idea of how much the solution can be improved by increasing the maximum time to 10,800s.

## 6.1 Overview of the results

For each case, the solution obtained from the rolling horizon algorithm corresponds to a period of two years. In terms of objective functions, they are a combination of the profit and the penalty terms for violation of the safety stocks, maximum storage capacity, and inventory terminal constraints. In order to provide a quantification of the terms involved, Table 1 shows the detailed economic and inventory results for the first year of the two years horizon. Table 1 shows that the values of the profit are different from case to case, and in addition do not present a consistent trend with the instances used. This is explained by the following reasons: 1) different constraints are used, for example Case 4 does not consider inventory terminal constraints, while the other cases consider; 2) models with different levels of accuracy, in Case 3 the rolling horizon algorithm uses only the scheduling, while the other cases use also the planning model; Case 2 also uses a different time discretization; 3) the solutions should be compared considering the profit and the violations of inventory constraints; and 4) for each subproblem in the rolling horizon algorithm, CPLEX with the heuristics and parallel options is used, which may lead to different solutions from run to run. In this section the results obtained in terms of scheduling and total inventory levels are discussed. The results for each case correspond to the second instance (maximum time of 3,600s and minimum optimality gap of 1%).

### 6.1.1 Impact of the inventory terminal constraints

The influence of the inventory terminal constraints on the results is studied in this section. The inventory terminal constraints set a minimum inventory level for each product at the end of the time horizon, to the safety stock,  $INVMIN_p$ , plus four months of demand,  $\sum_{tt \in TT} d_{p,tt}$ . We denote the sum over all the products of the minimum inventories at the end of the time horizon by  $INV_{730}$ ,

$$INV_{730} = \sum_{p \in P} \left( INVMIN_p + \sum_{tt \in TT} d_{p,tt} \right) \quad (113)$$



Table 1: Economic and production results for each case for the first year of the time horizon.

	Case 1 <sup>†</sup>			Case 2 <sup>‡</sup>		
	1	2	3	1	2	3
Instances*						
Profit (\$)	4.063	4.131	4.08	4.393	4.228	4.235
Sales (\$)	11.182	11.182	11.18	11.155	11.155	11.16
Inventory (\$)	1.189	1.171	1.18	1.007	1.047	1.04
Operating (\$)	5.324	5.242	5.24	4.924	5.195	5.17
Transitions (\$)	0.606	0.639	0.68	0.831	0.685	0.72
# Transitions:	17.0	19.0	20	26.0	21.0	22
# Transition days:	62.3	65.7	69.8	85.5	70.5	73.8
Tons produced in the first year	43,251	42,371	43421	40,588	42,959	42900
Amount below the safety stock (ton):	0	0	0	304	268	268
Total backlog (ton):	0	0	0	12	12	12
Amount above max capacity (ton):	0	0	0	0	0	0
Total violation of inventory terminal constraints (ton)	0	163	0	4732	720	2328
	Case 3 <sup>†</sup>			Case 4 <sup>†</sup>		
Instances*	1	2	3	1	2	3
Profit (\$)	4.044	4.168	4.112	4.706	4.711	4.778
Sales (\$)	11.182	11.182	11.182	11.182	11.182	11.182
Inventory (\$)	1.175	1.171	1.196	1.021	1.002	0.985
Operating (\$)	5.297	5.174	5.210	4.220	4.248	4.152
Transitions (\$)	0.666	0.669	0.664	1.236	1.220	1.267
# Transitions:	20.0	20.0	20.0	31.0	29.0	31.0
# Transition days:	68.5	68.9	68.3	127.1	125.5	130.3
Tons produced in the first year	43,892	41,910	43,028	34,810	35,214	34,421
Amount below the safety stock (ton):	0	0	0	78	0	0
Total backlog (ton):	0	0	0	0	0	0
Amount above max capacity (ton):	0	0	0	0	0	0
Total violation of inventory terminal constraints (ton)	745	12	142	-	-	-
	Case 5 <sup>†</sup>			Case 6 <sup>†</sup>		
Instances*	1	2	3	1	2	3
Profit (\$)	4.070	4.147	4.127	4.088	4.102	4.111
Sales (\$)	11.182	11.182	11.182	11.182	11.182	11.18
Inventory (\$)	1.185	1.164	1.174	1.192	1.160	1.17
Operating (\$)	5.333	5.186	5.221	5.279	5.279	5.27
Transitions (\$)	0.593	0.685	0.660	0.623	0.641	0.63
# Transitions:	17.0	21.0	20.0	18.0	19.0	20
# Transition days:	61.0	70.5	67.9	64.1	66.0	65
Tons produced in the first year	43,536	43,038	43,501	43,145	43,236	43029
Amount below the safety stock (ton):	55	55	55	0	0	0
Total backlog (ton):	5	5	5	0	0	0
Amount above max capacity (ton):	0	0	0	0	0	0
Total violation of inventory terminal constraints (ton)	132	398	17	404	108	46

These results are masked and do not represent real values. \* - Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP ={1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. <sup>†</sup> - Scenario with time periods of one month. <sup>‡</sup> - Scenario with time periods of one week.

Figure 10 presents the total inventory profile obtained with Cases 1, 2, 3, and 4 for  $INV_{730} = 22,645$  ton. In Cases 1, 2, and 3 the total inventory level decreases at the beginning of the time horizon, but afterwards there is not a clear trend to deplete the inventory. This is a result of the terminal constraints for the minimum inventory levels enforced at the end of the time horizon of each subproblem of the rolling horizon algorithms used. The reduction in the inventory level at the beginning of the time horizon is supported by the number of transitions and short production runs in that period of time.

The bottom curve represents the total inventory profile for Case 4 over the period of two years without the terminal constraints for the minimum inventory levels at the end of the time horizon of each subproblem of the hybrid rolling horizon algorithm. In Case 4, there is a clear trend to deplete the total inventory level during the first year of the time horizon. In the first year the safety stocks are not violated, while in the second year, because the total inventory level continues to decrease, it leads to violations of the safety stocks. The depletion of the inventory is explained by the large number of transition days in the first year determined by the model, 125.5 days, against the 65.7 days obtained for Case 1, (see Table 1).

The sensitivity of the total inventory profile to different values of  $INV_{730}$  is presented in Figure 11, using Case 1 with the first instance. This figure shows that increasing the value of  $INV_{730}$  the total inventory increases over the full time horizon, while the profit for the first year and objective function over the two years decrease (see Table 2). Analyzing the results from Table 2, the relation between  $INV_{730}$  and the objective function value suggests that an optimum value of  $INV_{730}$  may exist between  $INV_{730} = 0$  and  $INV_{730} = 15,907$  ton. As explained before, the relation between the value of  $INV_{730}$  and the profit is explained by the trade-off between the cost of the changeovers and the operating cost of the products. This shows that the results in terms of scheduling, inventory,

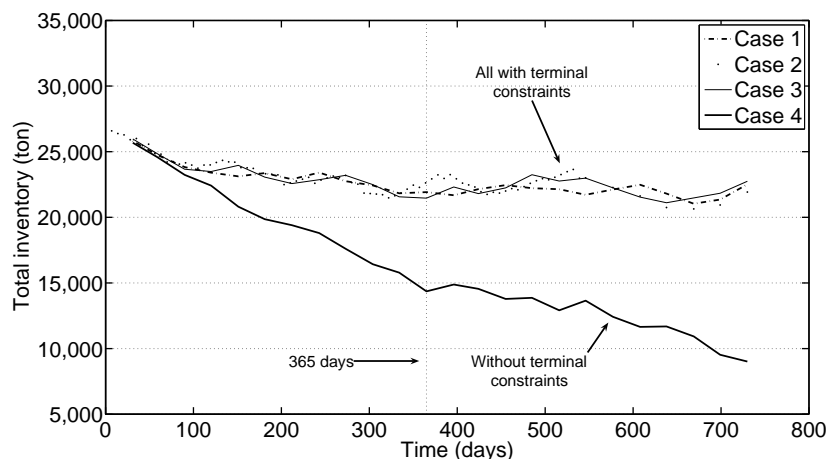


Figure 10: Total inventory profile for Case 1, 2, 3, and 4 over two years.

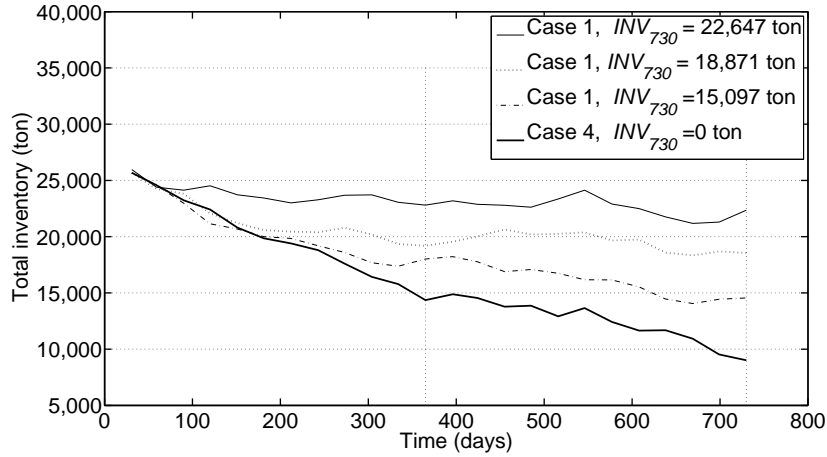


Figure 11: Impact of the inventory terminal constraints on the total inventory profile over two years.  $INV_{730}$  denotes the total inventory set by the terminal constraints on the day 730th.

and profit are sensitive to the value of  $INV_{730}$ , and therefore its definition depends on the decision maker. In this study a conservative approach is followed, using  $INV_{730} = 22,647$  ton.

Table 2: Results for Case 1 and 4 obtained with the first instance (minimum optimality gap = 5% and maximum CPU time = 3,600s).

	Case 4	Case 1		
$INV_{730}$ (ton)	0	15,097	18,871	22,645
<b>Results for day 365</b>				
Inventory (ton)	14,360	18,013	19,190	22,801
Profit (\$)	4.706	4.485	4.240	4.063
<b>Results for day 730</b>				
Inventory (ton)	9,011	14,554	18,553	22,356
Objective function (\$)	-83.539	8.505	8.278	7.870
Violations of safety stocks (ton)	739	0	0	0
Violations of terminal constraints (ton)	-	543	43	289

### 6.1.2 Gantt charts

Figures 12 and 13 show the Gantt charts obtained in Case 1 for the same schedule, but with a different level of detail in terms of products. In the first Gantt chart only the pseudo-products are shown, while in the second all products are represented. These results illustrate that the processing time of each run of the pseudo-products is equal to the sum of the processing times of the products aggregated into them. Figures 12 and 15 show the Gantt charts for Cases 1 and 4, respectively,

where the difference can be seen in terms of the number of transitions and length of the production runs between both cases. The large number of transitions in Case 4 is explained by the trade-off between the transition cost and the production cost.

In Figure 14 the Gantt chart obtained with Case 2 with the demand due at the end of every seven days is presented. Comparing the sequence of production with the sequence of Case 1 it is similar in the beginning of the time horizon, but after some time differences start to arise resulting in two more transitions in this case than in Case 1.

The Gantt chart from Case 5, without considering minimum run lengths and changeovers across due dates, is illustrated in Figure 16. Comparing this Gantt chart with the one from Case 1, is clear that the model from Case 5 does not have as much flexibility as the models with minimum run lengths and changeovers across due dates to fit changeovers and production runs within the time periods, which leads to the violation of the safety stocks in the first year (see Table 1).

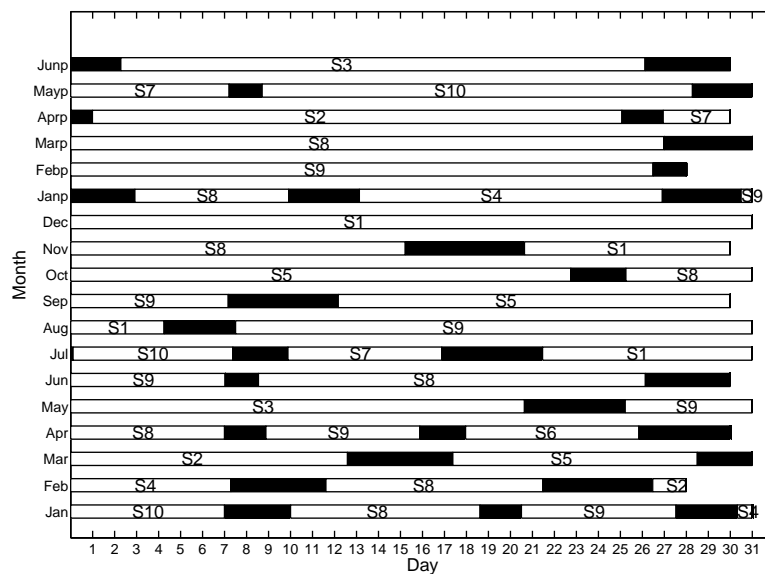


Figure 12: Schedule for Case 1, for the instance with minimum gap of 1% and maximum time of 3,600s. The black boxes represent changeovers. Only the pseudo-products are represented.

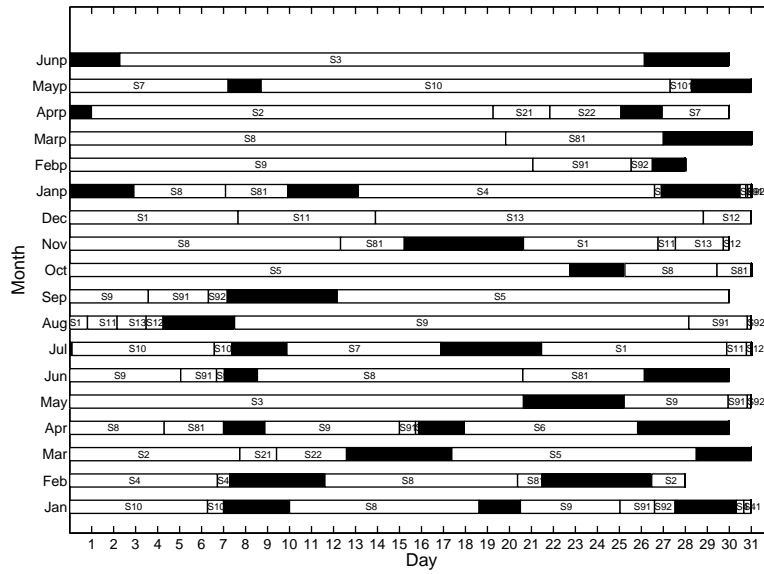


Figure 13: Schedule for Case 1, for the instance with minimum gap of 1% and maximum time of 3,600s. The black boxes represent changeovers. All products are represented.

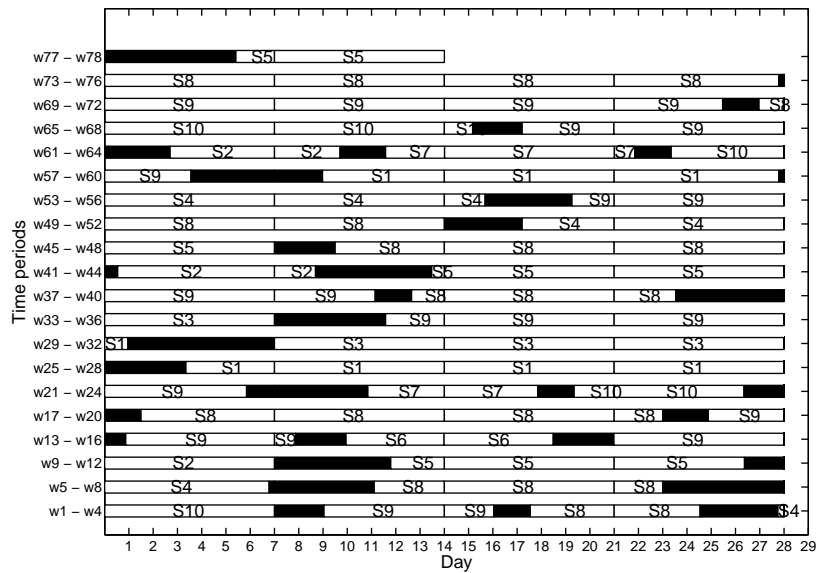


Figure 14: Schedule for Case 2, instance with minimum gap of 1% and maximum time of 3,600s. The black boxes represent changeovers. Only the pseudo-products are represented. Time periods of seven days are used.

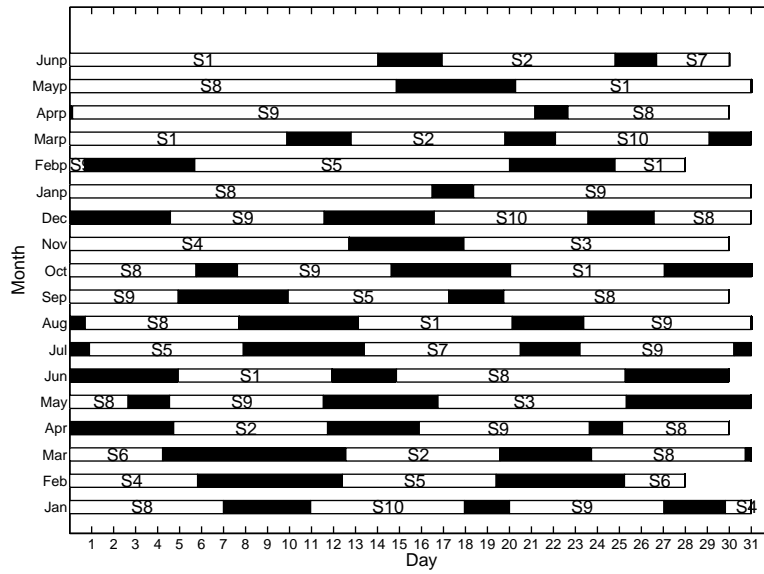


Figure 15: Schedule for Case 4, instance with minimum gap of 1% and maximum time of 3,600s. The black boxes represent changeovers. Only the pseudo-products are represented.

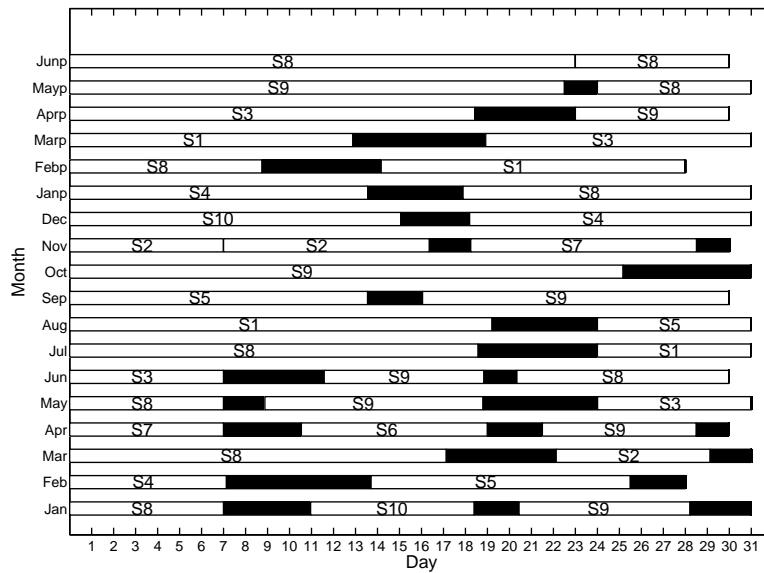
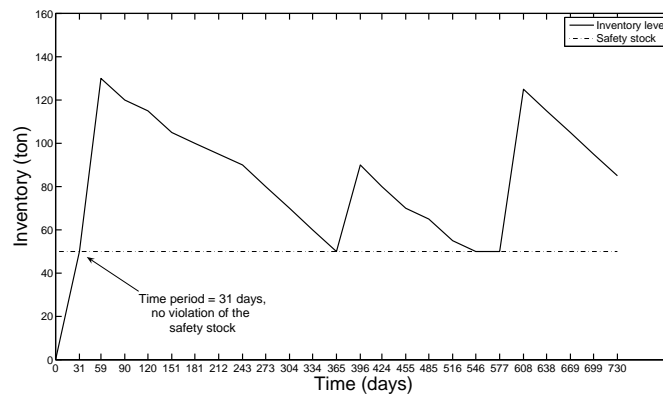


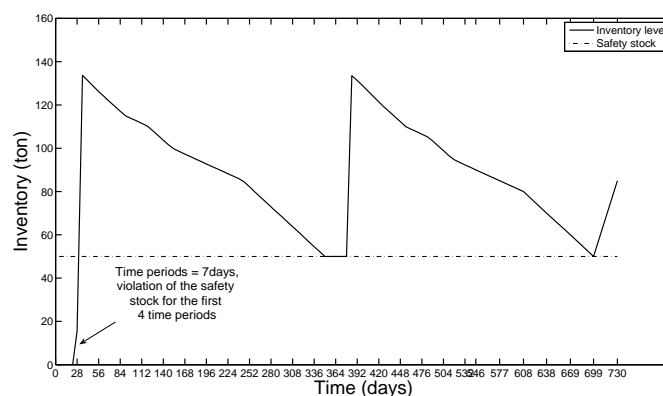
Figure 16: Schedule for Case 5, instance with minimum gap of 1% and maximum time of 3,600s. The black boxes represent changeovers. Only the pseudo-products are represented.

### 6.1.3 Analysis of the time resolution

The case studies with time periods of one month, and minimum run lengths and changeovers across the due dates are able to achieve results without violation of the safety stocks. However, when a finer time resolution is used, as in Case 2 with time periods of one week, violations of safety stocks and backlog occur, because the sample time is smaller. To illustrate this, Figures 6.1.3 and 6.1.3 show the inventory level of one product, designated by S41, obtained with Case 1 and Case 2, respectively. Case 1 has time periods of one month, and Case 2 has time periods of one week in the scheduling model. The due dates are specified at the end of the month in Case 1, and in Case 2 they are specified at the end of each week. Considering the first 31 days of the time horizon, the model in Case 1 does not consider a violation of the safety stock, because at the end of the first time period, the inventory is at the level of the safety stock. However, in Case 2 with inventory balances at the end of each week, the model determines the violation of the safety stocks for the first four time periods.



(a)



(b)

Figure 17: a) Inventory level of product S41, obtained with Case 1 with time periods of one month; b) Inventory level of product S41 obtained with Case 2 with time periods of one week.

## 6.2 Computational results

The MILP models and the rolling horizon algorithms are implemented in GAMS (Brooke et al., 1998) and solved on a machine running Linux with 8 processors Intel Xeon, 1.86GHz and 8GB of RAM. CPLEX 11.2.0 is used, with the polishing (Rothberg, 2007) option activated after 10 minutes of elapsed time, and using the opportunistic parallel option with 4 threads. The algorithm behind the polishing option is based on genetic operators that generate a vector of integer variables with a sub-set of variables fixed for any given integer vector. For each sub-set of variables fixed, a smaller sub-MILP is solved and a new incumbent may be found. This heuristic does not violate the logic of upper and lower bound in the B&B algorithm. Note that both options, opportunistic parallel and polishing, introduce random behavior in CPLEX that may lead to nonreproducible results. Nevertheless, the random search in the B&B tree may improve the incumbent, improving implicitly and explicitly the performance of the search.

### 6.2.1 Case 1

The best profit over two years is 8.075m.u., which is obtained with the third instance. The maximum optimality gaps obtained are 5.0%, 6.6%, and 1.7% for the three instances studied, respectively (see Table 3). A finer time resolution with time periods of one week was also studied. However the size of the model increases approximately four times posing a challenge to the solver to close the optimality gap. The first instance (minimum optimality gap of 5% and maximum time of 3,600s) is able to achieve the minimum gap set, with CPU times below 674s. While the second and third instances presented in Table 3 show that the gaps can be reduced by increasing the maximum time set. However, the relative difference of the objective function between the first and the third instance is only 0.47%, but requiring an additional 9,278s, (see Table 3). The number of slots considered for each scheduling model is determined by the number of products that are assigned by the planning model in the previous subproblem, for example the number of slots of the subproblem 2 in the first instance is 17, which is determined by the total number of products determined by the planning model for the first six months. Otherwise, if the scheduling model is used as a first approach the number of slots would be 120.

### 6.2.2 Case 2

For this case, the best profit over two years is -13.919m.u., which is obtained in the second instance. The maximum optimality gaps for each instance are 14.5%, 4.2%, and 11.7%, respectively, (see Table 4). The negative value of the objective function means that there are nonzero slack variables associated with the violation of the safety stocks. As mentioned before, this is due to the length of the time periods.



Table 3: Size of the models, and results for each subproblem and instances for Case 1 with time periods of one month.

Instances	SP	Equations	Total	Variables		Gap (%)	RMIP <sup>†</sup>	Profit (m.u.)	CPU (s)
				Binary	Slots				
1	Gap* = 5%								
	1	9,095	8,115	3,860	-	3.7	4.233	4.027	526
	2	5,961	5,300	2,267	17	5.0	4.212	3.923	165
	3	10,216	9,129	3,960	17	4.8	6.071	5.752	674
	4	6,697	5,868	2,081	28	4.8	6.031	5.699	540
	5	11,089	9,921	3,960	28	4.9	8.211	7.786	612
2	Gap* = 1%								
	1	9,095	8,115	3,860	-	2.3	4.233	4.085	3,600
	2	6,123	5,540	2,408	20	0.9	4.219	4.076	573
	3	10,278	9,171	3,960	20	6.6	6.070	5.649	3,600
	4	6,802	5,962	2,108	32	2.6	6.036	5.815	3,600
	5	11,220	10,009	3,960	32	2.1	8.234	8.002	3,600
3	Gap* = 1%								
	1	9,095	8,115	3,860	-	1.7	4.233	4.078	3,600
	2	5,973	5,329	2,289	17	0.8	4.219	4.071	554
	3	10,252	9,152	3,960	17	1.4	6.071	5.945	3,600
	4	6,827	6,016	2,149	30	1.0	6.025	5.919	96
	5	11,185	9,989	3,960	30	1.0	8.214	8.095	3,605
	6	7,755	6,837	2,140	43	0.8	8.177	8.075	384

Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP = {1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. † - Linear relaxation in the root node.

### 6.2.3 Case 3

For this case the best profit over two years is 8.093m.u., which is obtained in the second instance. The maximum optimality gaps obtained are 13.2%, 4.2%, and 4.5% for the three instances studied, respectively. In this case a finer time resolution was also investigated, but increasing the resolution of the time grid leads to large problems that cannot be solved by CPLEX.

In this case, the size of the models is considerably larger than the size of the models in Case 1. However, the detailed model is able to obtain solutions with optimality gaps below 5% within three hours, which is comparable with the performance obtained with the hybrid rolling horizon algorithm used in Case 1. Comparing the first subproblem of the second instance of Cases 1 and 3, the first has an objective function equal to 4.085m.u. with 2.3% gap obtained in 3,600s, and Case 3 has an objective function equal to 4.085m.u. with 2.5% gap obtained in 3,600s, (see Tables 3 and 5). This suggests that the planning model has a similar performance to the detailed scheduling model for problems of this dimension.

Table 4: Size of the models, and results for each subproblem and instances for Case 2 for three different instances with different stopping criteria.

Instances	SP	Equations	Total	Variables		Gap (%)	RMIP <sup>†</sup>	Profit (m.u.)	CPU (s)
				Binary	Slots				
1	Gap* = 5%								
	1	9,900	8,809	4,190	-	4.7	4.236	2.861	709
	2	11,280	10,144	3,306	80	4.5	4.263	-18.352	557
	3	14,339	12,887	4,290	80	5.3	-15.553	-16.459	3,600
	4	14,902	13,480	3,288	136	8.9	-15.528	-17.597	3,600
	5	17,839	16,148	4,290	136	14.5	-13.295	-15.589	3,600
2	Gap* = 1%								
	1	9,900	8,809	4,190	-	3.0	4.236	2.909	3,600
	2	11,477	10,435	3,506	84	0.3	4.266	-17.576	1,470
	3	14,329	12,888	4,290	84	2.9	-15.563	-16.063	3,600
	4	14,559	12,913	2,877	136	2.9	-15.529	-16.049	3,600
	5	17,892	16,178	4,290	136	4.2	-13.346	-13.970	3,600
3	Gap* = 1%								
	1	9,900	8,809	4,190	-	3.1	4.236	2.906	3,600
	2	11,684	10,684	3,634	88	1.0	4.268	-17.698	638
	3	14,389	12,923	4,290	88	4.8	-15.557	-16.365	3,600
	4	14,472	12,818	2,819	136	6.3	-15.538	-16.651	10,800
	5	17,863	16,166	4,290	136	7.2	-13.339	-14.400	10,800
	6	17,669	15,824	2,607	190	11.7	-13.309	-15.129	10,800

Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP ={1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. <sup>†</sup> - Linear relaxation in the root node.

Table 5: Size of the models, and results for each subproblem and instances for Case 3 with time periods of one month.

Instances	SP	Equations	Total	Variables		Gap (%)	RMIP <sup>†</sup>	Profit (m.u.)	CPU (s)
				Binary	Slots				
1	Gap* = 5%								
	1	10,421	19,134	13,340	120	2.8	4.232	4.076	2,599
	2	11,876	20,527	13,684	138	13.2	6.174	5.401	3,600
	3	12,939	21,381	13,718	148	9.9	8.385	7.582	3,600
2	Gap* = 1%								
	1	10,421	19,134	13,340	120	2.5	4.232	4.085	3,600
	2	11,867	20,498	13,659	138	4.2	6.179	5.868	3,600
	3	12,933	21,371	13,705	148	3.3	8.416	8.093	3,600
3	Gap* = 1%								
	1	10,421	19,134	13,340	120	2.5	4.232	4.082	10,800
	2	11,874	20,518	13,680	138	2.6	6.177	5.937	10,800
	3	12,912	21,361	13,707	147	4.5	8.389	7.959	10,800

Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP ={1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. <sup>†</sup> - Linear relaxation in the root node.

## 6.2.4 Case 4

For this case, the best objective function obtained is -83.539m.u., which corresponds to a penalized objective function due to violations of the safety stocks during the second year. For the same instance, for the first year the profit obtained is 4.706m.u with 127.1 days of transitions (see Table 1). Table 6 shows information about the size of the models and the performance of this approach. Results for the three instances with different stopping criteria are shown. In the three instances

Table 6: Size of the models, and results for each subproblem and instances for Case 4.

Instances	SP	Equations	Total	Variables		Gap (%)	RMIP <sup>†</sup>	Profit (m.u.)	CPU (s)
				Binary	Slots				
1									
	1	9,095	8,116	3,860	-	4.8	5.352	4.939	461
	2	6,171	5,612	2,446	21	4.5	5.325	4.851	14
	3	10,302	9,199	3,960	21	2.8	7.404	7.125	49
	4	7,141	6,400	2,340	40	0.7	7.361	7.221	33
	5	11,339	10,094	3,960	40	109.3	9.677	-102.764	3,600
	6	7,987	7,045	2,193	55	45.7	-45.286	-83.539	3,600
2									
	1	9,095	8,116	3,860	-	3.8	5.352	4.963	3,600
	2	6,059	5,439	2,337	19	1.0	5.322	4.952	353
	3	10,292	9,180	3,960	19	1.1	7.410	7.264	3,600
	4	7,134	6,392	2,340	38	1.0	7.381	7.233	115
	5	11,347	10,113	3,960	38	109.7	9.694	-98.829	3,600
	6	7,995	7,075	2,200	53	29.9	-66.082	-94.546	3,600
3									
	1	9,095	8,116	3,860	-	3.6	5.352	4.968	3,600
	2	6,059	5,439	2,337	19	0.9	5.324	4.966	254
	3	10,317	9,208	3,960	19	1.0	7.409	7.273	1,700
	4	7,104	6,341	2,306	37	0.9	7.381	7.251	29
	5	11,352	10,100	3,960	37	105.4	9.710	-176.759	10,800
	6	8,008	7,071	2,213	52	47.8	-81.580	-182.759	10,800

Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP = {1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. <sup>†</sup> - Linear relaxation in the root node.

relatively large optimality gaps are obtained for the subproblems 5 and 6, indicating that the solver cannot close the gap between the linear relaxation and the integral solution within the specified maximum time. This is explained by the fact that the linear relaxation is able to meet the safety stocks levels while the integral solution cannot.

## 6.2.5 Case 5

In this case, the best profit over two years is 3.626m.u. obtained in the third instance. This value is smaller than the previous results reported, and the maximum optimality gaps obtained are also larger, 24.2%, 193.2%, and 32.8% for the three instances studied, respectively (see Table 7). These

Table 7: Size of the models, and results for each subproblem and instances for Case 5.

Instances	SP	Equations	Total	Variables		Gap (%)	RMIP <sup>†</sup>	Profit (m.u.)	CPU (s)
				Binary	Slots				
1	Gap* = 5%								
	1	7,367	5,467	2,520	-	19.2	4.229	-0.380	3,600
	2	4,768	3,700	1,322	14	4.9	-0.289	-0.379	415
	3	8,181	6,272	2,520	14	24.2	1.612	1.261	3,600
	4	5,434	4,377	1,315	27	5.0	1.568	1.291	363
	5	8,849	6,964	2,520	27	4.0	3.750	3.527	1,028
2	Gap* = 1%								
	1	7,367	5,467	2,520	-	17.1	4.229	-0.372	3,600
	2	4,849	3,788	1,336	16	6.5	-0.288	-0.372	3,600
	3	8,210	6,284	2,520	16	193.2	1.617	0.536	3,600
	4	5,435	4,357	1,310	28	1.0	1.570	0.540	179
	5	8,874	6,974	2,520	28	10.2	3.775	3.345	3,600
3	Gap* = 1%								
	1	7,367	5,467	2,520	-	16.7	4.229	-0.375	3,600
	2	4,809	3,742	1,329	15	4.2	-0.290	-0.375	3,600
	3	8,196	6,280	2,520	15	32.8	1.615	1.187	3,600
	4	5,409	4,336	1,308	27	1.0	1.570	1.187	246
	5	8,871	6,972	2,520	27	2.2	3.762	3.626	10,800
	6	6,085	5,032	1,308	39	0.4	3.719	3.626	2,055

Minimum optimality gap and maximum CPU time: Instance 1 - Gap = 5%, CPU=3,600s, Instance 2 - Gap =1%, CPU = 3,600s, Instance 3 - Gap = 1%, CPU = 3,600s for SP = {1,2,3}, 10,800s for SP= {4,5,6}. SP - Subproblem. † - Linear relaxation in the root node.

large gaps demonstrate that this model, without considering minimum run lengths and changeovers across the due dates, does not have enough flexibility to satisfy the safety stocks and terminal constraints. The model has to fit the minimum run lengths and changeovers within the time periods as can be seen in Figure 16. These results show that the cost of adding the minimum run lengths and transition across the due dates (instead of 7,367 equations, and 5,467 total variables and 2,520 binary variables in Case 5, 9,095 equations, 8,115 continuous variables, and 3,860 binary variables in Case 1, for the first subproblem) is compensated by the gain in flexibility of the model.

### 6.2.6 Case 6

In this case, the best profit over two years is 8.069m.u. obtained in the third instance. The maximum optimality gaps obtained are 4.9%, 8.9%, and 2.8% for the three instances studied. Comparing the results in terms of computational times and profit obtained with Case 1 and this case, there is no clear evidence that one planning model is superior to the other, both in terms of objective function and CPU times.

## 7 Conclusions

A long-term scheduling approach has been presented for the single stage continuous process for manufacturing value added glass products. The proposed models represent extensions of the models proposed by Erdirik-Dogan and Grossmann (2006) and Erdirik-Dogan and Grossmann (2008) motivated by the specific features of the process under study. The extensions involve changeovers and minimum run lengths across due dates, an aggregation strategy to handle the case of changeovers without transition times between a subset of products, and the consideration of terminal constraints for the inventory levels at the end of the time horizon. The challenge posed by the long-term scheduling is addressed by using rolling horizon algorithms integrated with the aggregation and disaggregation of the time, which reduce the size of the subproblems to solve.

The results have shown that the new extensions improve the flexibility and applicability of the models by providing feasible solutions that are not possible with previous models. Problems with time horizons of 24 months with 18 months of detailed scheduling have been solved with optimality gaps below 5% with reasonable computational times of less than six hours. The resolution of the time periods has a clear impact on the size of the models, and on the quality of the results obtained. For the models with time periods of one month, the three instances studied are able to achieve results with optimality gaps below 5% for the final subproblem with moderate computational efforts. The asynchronous rolling horizon algorithm leads to more accurate inventory balances, without increasing considerably the computational efforts. From the performance of the planning and the scheduling models, within the respective rolling horizon algorithms in this problem, there is no clear evidence that the planning model is computationally more efficient. Therefore, the detailed rolling horizon algorithm used in Case 3 has the best performance. For all cases, the instance with maximum optimality gap of 5% and maximum time of 3,600s shows a good compromise between the quality of the final solution and the computational resources used.

Future work will involve the integration of a second production line, and the management of the waste glass produced and used in the furnace, with the production scheduling.

### Acknowledgments

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## Nomenclature

### Indices

$i, i', k$	products
$j, n, p$	products
$t, t', t^*$	time periods
$l$	slots

### Sets

$A_{i,t}$	subset of periods of time after the time period $t$ that cover the minimum run length
$B_{i,t}$	subset of periods of time before the time period $t$ that cover the minimum run length
$CT1$	set of products with coating 1
$CT2$	set of products with coating 2
$P$	set of products
$IM$	set of pseudo-products
$L$	set of slots
$LM_t$	subset of slots active for the time period $t$
$LMF$	singleton with the first element of the set $LM$
$LML$	singleton with the last element of the set $LM$
$T$	set of time periods
$TF$	singleton with the first element of the set of time periods
$TFP$	singleton with the first element of the set of time periods $TF$
$TL$	singleton with the last element of the set of time periods
$TP$	subset of time periods with only planning
$TS$	subset of time periods with only detailed scheduling
$TSL$	singleton with the last element of the set $TS$

### Parameters

$BCKLI_p$	Initial backlog of product $p$
$cin_v$	inventory cost in a year basis
$coper_{p,t}$	operating cost for product $p$ in period $t$
$ctran_{p,k}$	transition cost from product $p$ to product $k$
$d_{p,t}$	demand of product $p$ in period $t$
$H_t$	duration of the $t$ th time period
$HT_t$	time in terms of day at the end of the $t$ th time period
$MRT_i$	minimum run length of product $i$
$INVI_p$	initial inventory of product $p$ (ton)

$INVMAX$	maximum capacity for storage (ton)
$INVMIN_p$	minimum inventory (ton)
$r_p$	gross production rates (ton/day)
$p_{p,t}$	selling price of product $p$ in period $t$
$\eta_p$	production yield of product $p$
$\tau_{i,k}$	transition time from product $i$ to product $k$
$\eta_p$	production yield of product $p$

### Variables

$BCKL_{p,t}$	backlog of product $p$ in the time period $t$
$Area_{p,t}$	area below the inventory time graph for product $p$ at period $t$
$INVO_{p,t}$	final inventory of product $p$ at time $t$ after the demands are satisfied
$NY_{i,t}$	number of slots that product $i$ is assigned in period $t$
$O_{i,t}$	order of the assignment of product $i$ in the time period $t$
$q_t$	slack variable for maximum storage capacity violation in time period $t$
$S_{p,t}$	sales of product $p$ at period $t$
$s_{p,t}$	slack variable for the safety stock violation of product $p$ during time period $t$
$slth_p$	slack variable for the violation of the terminal inventory constraint of product $p$
$TPA_{i,t}$	production time of product $i$ in $t$ that is added to the production time of $i$ in $t - 1$
$TPB_{i,t}$	production time of product $i$ in $t$ that is added to the production time of $i$ in $t + 1$
$Te_{l,t}$	end time of slot $l$ during period $t$
$Ts_{l,t}$	start time of slot $l$ during period $t$
$T_d$	total minimum inventory level at the end of the time horizon enforced by the terminal constraints
$\eta_p$	yield of production of the product $p$
$\theta_{i,l,t}$	production time of product $i$ in slot $l$ during period $t$
$\tilde{\theta}_{i,t}$	production time of pseudo-product (aggregated) $i \in IM$ during period $t$
$\tilde{\theta}_{2p,t}$	production time of product $p \in P$ during period $t$
$\tilde{\theta}a1_{i,t}$	disaggregated variable of $\tilde{\theta}_{i,t}$
$\tilde{\theta}a2_{i,t}$	disaggregated variable of $\tilde{\theta}_{i,t}$
$\tilde{\theta}b1_{i,t}$	disaggregated variable of $\tilde{\theta}_{i,t}$
$\tilde{\theta}b2_{i,t}$	disaggregated variable of $\tilde{\theta}_{i,t}$
$\tau e_{i,k,t}$	part of the transition time between time periods assigned in the end of period $t$
$\tau s_{i,k,t}$	part of the transition time between time periods assigned in the start of period $t$

### Binary variables

$TRT_{i,k,t}$	denotes if product $i$ is followed by product $k$ at the end of period $t$
$W_{i,l,t}$	denotes if product $i$ is assigned to slot $l$ during period $t$
$XF_{i,t}$	denotes if product $i$ is the first product in period $t$
$XL_{i,t}$	denotes if product $i$ is the last product in period $t$
$YF_{i,t}$	denotes if product $i$ is assigned to more than one slot
$YOP_{i,t}$	denotes if product $i$ is assigned during period $t$
$YP_{i,t}$	denotes if product $i$ is assigned to period $t$
$Z_{i,k,l,t}$	denotes if product $i$ is followed by product $k$ in slot $l$ during period $t$
$ZP_{i,k,t}$	denotes if product $i$ precedes product $k$ in period $t$
$ZZP_{i,k,t}$	denotes if the link between products $i$ and $k$ are broken
$ZZZ_{i,k,t}$	denotes if product $i$ is followed by product $k$ at the end of period $t$

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