A General Continuous State Task Network Formulation for Short Term Scheduling of Multipurpose Batch Plants with Due Dates

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Abstract: A new continuous-time MILP model for the short term scheduling of multipurpose batch plants with due dates is presented. The proposed model is a general State Task Network (STN) formulation that accounts for variable batch sizes and processing times, various storage policies (UIS/FIS/NIS/ZW), utility constraints (other than units), and allows for batch mixing and splitting. Its key features are: (a) a continuous, common for all units time partitioning, (b) assignment constraints are expressed using only the binary variables defined for tasks, (c) start times of tasks are eliminated, (d) a new class of tightening valid inequalities is added to the MILP formulation, and (e) a new disjunctive programming formulation is used for the matching of due dates with time points. The proposed model is more general than the previously reported models and is computationally efficient.

Keywords: Scheduling, Multipurpose batch plants, Scheduling with due dates

1. INTRODUCTION

The problem of short-term scheduling of multipurpose batch plants with release dates for the raw materials and due dates for the final products is a very challenging and practically important problem. Due to its complexity, the models that have been proposed [1,2] do not account for all the features of the general problem. Three commonly made assumptions, for example, are that there are no utility requirements, that each order comprises a single batch and that orders can be pre-assigned to time points/events. In this work we propose a model that addresses the general problem, i.e. it accounts for complex plant configurations (batch splitting and mixing, recycle streams), various storage policies (UIS/FIS/NIS/ZW), variable batch sizes, processing times and utility requirements, and multiple release and due dates. A State Task Network [3] MILP model is proposed with continuous-time representation [4].

2. PROBLEM STATEMENT

We assume that we are given:

(i) a fixed or variable time horizon
(ii) the available equipment units and storage tanks, and their capacities
(iii) the available utilities and their upper limits
(iv) the production recipe for all tasks (mass balance coefficients, utility requirements)
(v) the initial amounts and prices of all states
(vi) the deliveries of raw materials and orders of final products (amounts and time)

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The goal is to determine:

(i) the assignment, sequence and timing of tasks taking place in each unit

(ii) the batch size of tasks (i.e. the processing time and the required utilities)

(iii) the amount of states purchased and sold

Various objective functions such as the maximization of additional production, or the final inventory of intermediate states can be accommodated within the proposed model.

3. MATHEMATICAL FORMULATION

The general continuous STN MILP model of Maravelias and Grossmann [5] is used as basis. In the proposed model the time horizon is divided into N time intervals of unequal and unknown duration. Also, tasks that can be assigned to different units are treated as individual tasks for each unit assignment. In this model, assignment constraints are expressed through task binaries Ws_{in} and Wf_{in}. Binary Ws_{in} (Wf_{in}) is 1 if task i starts at (finishes at or before) time point n, T_n. The batch size of task i that starts at, is being processed at, and finishes at or before time point n is denoted by Bs_{in}, Bp_{in} and Bf_{in}, respectively, and the amount of state s consumed (produced) by task i at time point n is benoted by B^I_{isn} (B^O_{isn}). The amount of state s at time point n is denoted by S_{sn}, and the amount of utility r consumed by various tasks at time point n is denoted by R_{rn}. The start, processing and finish time of task i that starts at time point n is denoted by Ts_{in}, Tp_{in}, and Tf_{in}, respectively. The parameters of the model include the time horizon H, the minimum/maximum batch size B_i^{MIN}/B_i^{MAX}, the storage capacity C_s, the mass fractions ρ^I_{is}/ρ^O_{is}, and the coefficients for the fixed and variable term of processing time and utility requirements (α_i, β_i, γ_{ir}, δ_{ir}). The basic constraints of the model of Maravelias and Grossmann are the following:

3.1. Assignment constraints

$$\sum_{i\in I(j)}\sum_{n'\le n}(Ws_{in'}-Wf_{in'})\le 1 \quad \forall j,\forall n \tag{1}$$

$$\sum_n Ws_{in}=\sum_n Wf_{in} \quad \forall i \tag{2}$$

$$\sum_{i\in I(j)} Ws_{in}\le 1 \quad \forall j,\forall n \tag{3}$$

$$\sum_{i\in I(j)} Wf_{in}\le 1 \quad \forall j,\forall n \tag{4}$$

3.2. Calculation of start, processing and finish time

$$Tp_{in}=\alpha_i Ws_{in}+\beta_i Bs_{in} \quad \forall i,\forall n \tag{5}$$

$$Tf_{in}\le Ts_{in}+Tp_{in}+H(1-Ws_{in}) \quad \forall i,\forall n \tag{6}$$

$$Tf_{in}\ge Ts_{in}+Tp_{in}-H(1-Ws_{in}) \quad \forall i,\forall n \tag{7}$$

$$Ts_{in}=T_n \quad \forall i,\forall n \tag{8}$$

3.3. Time matching constraints

$$Tf_{in-1}\le T_n+H(1-Wf_{in}) \quad \forall i,\forall n \tag{9}$$

$$Tf_{in-1}\ge T_n-H(1-Wf_{in}) \quad \forall i\in ZW(i),\forall n \tag{10}$$

3.4. Batch size constraints and material balances

$$B_i^{MIN} Ws_{in} \le Bs_{in} \le B_i^{MAX} Ws_{in} \quad \forall i, \forall n \tag{11}$$

$$B_i^{MIN} Wf_{in} \le Bf_{in} \le B_i^{MAX} Wf_{in} \quad \forall i, \forall n \tag{12}$$

$$Bs_{in-1} + Bp_{in-1} = Bp_{in} + Bf_{in} \quad \forall i, \forall n \tag{13}$$

$$B_{isn}^{I} = \rho_{is} Bs_{in} \quad \forall i, \forall n, \forall s \in SI(i) \tag{14}$$

$$B_{isn}^{O} = \rho_{is} Bf_{in} \quad \forall i, \forall n, \forall s \in SO(i) \tag{15}$$

$$S_{sn} = S_{s,n-1} + \sum_{i \in O(s)} B_{isn}^{O} - \sum_{i \in I(s)} B_{isn}^{I} \quad \forall s, \forall n > 1 \tag{16}$$

$$S_{sn} \le C_s \quad \forall s, \forall n \tag{17}$$

3.5. Utility constraints

$$R_{irn}^{I} = \gamma_{ir} Ws_{in} + \delta_{irs} Bs_{in} \quad \forall i, \forall r, \forall n \tag{18}$$

$$R_{irn}^{O} = \gamma_{ir} Wf_{in} + \delta_{irs} Bf_{in} \quad \forall i, \forall r, \forall n \tag{19}$$

$$R_{rn} = R_{rn-1} - \sum_{i} R_{irn-1}^{O} + \sum_{i} R_{irn}^{I} \quad \forall r, \forall n \tag{20}$$

$$R_{rn} \le R_r^{MAX} \quad \forall r, \forall n \tag{21}$$

3.6. Time ordering constraints

$$T_{n=1} = 0 \tag{22}$$

$$T_{n=|N|} = H \tag{23}$$

$$T_{n+1} \ge T_n \quad \forall n \tag{24}$$

3.7. Tightening constraints

$$\sum_{i \in I(j)} \sum_{n} Tp_{in} \le H \quad \forall j \tag{25}$$

$$\sum_{i \in I(j)} \sum_{n' \ge n} Tp_{in'} \le H - T_n \quad \forall j, \forall n \tag{26}$$

$$\sum_{i \in I(j)} \sum_{n' \le n} (\alpha_i Wf_{in'} + \beta_i Bf_{in'}) \le T_n \quad \forall j, \forall n \tag{27}$$

3.8. Release and due date constraints

Let K be the set of orders that must be met, or deliveries of materials. The due date (release date) of order (delivery) $k \in K$ is TD_k and the amount due (delivered) is AD_k. The set of orders that correspond to state s is denoted by $K(s)$ and the set of deliveries that correspond to s is denoted by $L(s)$. Since, in the general case, we do not know at which time point n of the model each order (delivery) takes place, we use binaries Y_{kn} to relate order (delivery) k with time point n; i.e. Y_{kn} is *1* if order (delivery) k takes place at time point n. The following disjunction is used to relate each order (delivery) k with a time point n,

$$\begin{pmatrix} Y_{kn} \\ TD_k = T_n \end{pmatrix} \vee \begin{pmatrix} \neg Y_{kn} \\ T_n \ge 0 \end{pmatrix} \quad \forall k \in K, \forall n \tag{28}$$

The convex hull reformulation of disjunction in (28), after the elimination and aggregation of variables, consists of equations (29) – (31):

$$T_n^k = TD_k Y_{kn} \quad \forall k \in K, \forall n \tag{29}$$

$$T_n = T_n^k + \overline{T}_n^k \quad \forall k \in K, \forall n \tag{30}$$

$$\overline{T}_n^k \le H(1-Y_{kn}) \quad \forall k \in K, \forall n \tag{31}$$

In addition, each order (delivery) must coincide with one time point *n*,

$$\sum_n Y_{kn} = 1 \quad \forall k \in K \tag{32}$$

The material balance constraint (16) of the model of Maravelias and Grossmann (2002) is modified as follows:

$$S_{sn} = S_{s,n-1} + \sum_{k \in L(s)} AD_{kn} - \sum_{k \in K(s)} AD_{kn} + \sum_{i \in O(s)} B_{isn}^O - \sum_{i \in I(s)} B_{isn}^I \quad \forall s, \forall n > 1 \tag{33}$$

where AD_{kn} is the amount of order (delivery) *k* at time point *n*; i.e. it is non-zero if order (delivery) *k* takes place at time point *n* and zero otherwise, and it is calculated as follows:

$$AD_{kn} = AD_k Y_{kn} \quad \forall k \in K, \forall n \tag{34}$$

3.9. Objective function

If in addition to meeting the due dates the objective is to maximize the revenue from the sales of extra production, the objective function is given by (35) where *FP* is the set of final products and ζ_s is the price of state *s*,

$$\max Z = \sum_{s \in FP} \zeta_s S_{s|N|} \tag{35}$$

If there are orders for which there is no due date, the objective is to minimize the makespan and the objective is given by (36), where MS is the makespan.

$$\min MS \tag{36}$$

In this case, *H* is an upper bound on the makespan and it is replaced by *MS* in constraints (23), (25) and (26).

Finally, the model can be used to minimize the inventory level of final products over time (Eq. (37)) or the final inventory of intermediate states (Eq. (38)), where *INT* is the set of intermediate states:

$$\min \sum_n \sum_{s \in FP} S_{sn} \tag{37}$$

$$\min \sum_{s \in INT} S_{s|N|} \tag{38}$$

The proposed MILP model (M) consists of equations (1) – (15), (17) – (27), (29) – (34), and one of (35), (36), (37) and (38), where Ws_{in}, Wf_{in}, $Y_{kn} \in \{0,1\}$, and all the continuous variables are non-negative.

3.10. Remarks

In the case where the release and due dates are distinct, constraints (31) and (32) can be simplified into (39) and (40); i.e. fewer constraints and variables.

$$T_n = \sum_k T_n^k + \overline{T}_n \quad \forall n \tag{39}$$

$$\overline{T}_n \le H(1-\sum_k Y_{kn}) \quad \forall n \tag{40}$$

The timing and relative order of deliveries and due dates can be used to fix some of the Y_{kn} binaries, and derive valid inequalities that reduce the feasible space. For the latest order, for example, we can fix $Y_{k|N|} = 1$, or if $TD_k < TD_{k'}$, inequality (41) is valid,

$$\sum_{n' \le n} Y_{kn'} \ge \sum_{n' \le n} Y_{k'n'} \quad \forall n \tag{41}$$

The binary variables Y_{kn} can be modelled as Special Ordered Sets of type 1 (SOS1) variables as well. The number of time points, finally, is determined through an iterative procedure in which we increase the time points until there is no improvement in the objective function.

4. EXAMPLE

The proposed model (M) was implemented in the example shown in Figure 1, whose data are given in Table 1. There are six units (U1, U2, …U6) available for the ten tasks. Unlimited storage is available for states F1, F2, INT1, INT2, P1, P2, P3 and WS; finite intermediate storage is available for states S3 (15 kg) and S4 (40kg); no intermediate storage is available for states S2 and S6, while zero-wait policy applies for states S1 and S5. States F1, F2, and S4 are initially available in sufficient amounts. Furthermore, each task requires one of the three following utilities: cooling water (CW), low pressure steam (LPS), and high pressure steam (HPS). The maximum availability for CW, LPS and HPS is 25, 40 and 20 kg/min, respectively. Constant processing times are assumed. The orders to be met are as follows: two orders of 2 and 4 tons for product P1 at t=8 and t=12, one order of 2 tons for product P2 at t=10, and one order of 3 tons of product P3 at t= 11. The minimization of the inventory level of final products over the entire time horizon is used as objective function.

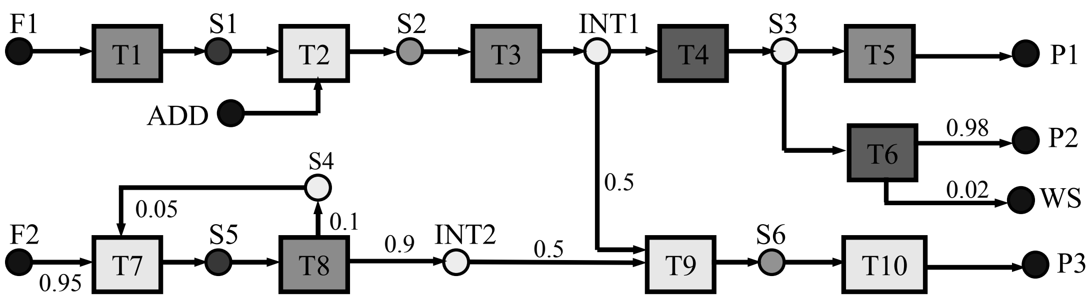

Figure 1: Sate Task Network of Example

The optimal solution yields the equipment Gantt chart in Figure 2, where the batch size of tasks is shown in parentheses, and optimal value of zero; i.e. the batches that produce the final products finish exactly at the due dates. The optimal solution is found when the 12-hour time horizon is divided into 10 intervals. The MILP problem consists of 3,382 constraints, 240 binary and 1,711 continuous variables. The optimal solution was found in 690 nodes and 22.7 CPU sec on a PIII at 1GHz, using GAMS 20.7/CPLEX 7.5. As can be seen in Figure 2 the orders for all products are delivered exactly on time. Note that tasks T5 and T6 are delayed by two hours in order to minimize the earliness (and thus the inventory) of the orders of products P1 and P2.

Table 1: Example data (B^{MAX} in tons, α in hr, γ in kg/min, δ in kg/min per ton).

Task	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
Unit	U1	U2	U3	U1	U4	U4	U5	U6	U5	U6
B^{MAX}	5	8	6	5	8	8	3	4	3	4
Dur (α)	2	1	1	2	2	2	4	2	2	3
Utility	LPS	CW	LPS	HPS	LPS	HPS	CW	LPS	CW	CW
γ	3	4	4	3	8	4	5	5	5	3
δ	2	2	3	2	4	3	4	3	3	3

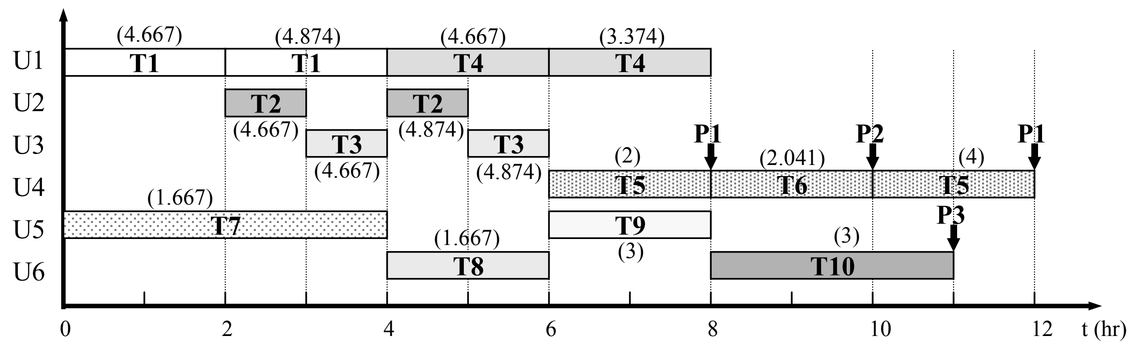

Figure 2: Equipment Gantt chart of Example

5. CONCLUSIONS

The proposed model is, to our knowledge, the first continuous-time STN model that addresses the general scheduling problem of multipurpose batch plants with multiple due and release dates. Previously proposed approaches address simplifications of the general problem by either considering less general plant configurations (e.g. no utility constraints or no batch splitting/mixing is allowed for each order) or by pre-assigning orders to time points/events.

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