A Branch and Cut Search for the Deterministic Optimization of the Thermal Unit Commitment Problem. Part I: Methodology

Marian G. Marcovecchio, Augusto Q. Novais, Ignacio E. Grossmann

***Abstract*-- This two-paper series proposes a novel deterministic optimization approach for the Unit Commitment problem (UC), more specifically for the Security Constrained Unit Commitment problem (SCUC) addressing thermal generating units. In the first part, a suitable mathematical programming model is presented, which contemplates all the inherent constraints and a single set of binary variables, i.e. the on/off status of each generator at each time period, leading to a convex Mixed Integer Quadratic Programming (MIQP) formulation. This gives rise to a very difficult optimization problem, hard to solve through deterministic approaches for high-dimensional instances. To overcome this challenge, an effective solution methodology based on valid integer cutting planes is proposed, and implemented through a Branch and Cut search for finding the optimal solution. In the second of this two-paper series, the application of the model and proposed approach is illustrated with several examples of different dimensions and characteristics.**

***Index Terms*-- Energy optimization, Unit Commitment problem, deterministic optimization, Branch and Cut algorithm**

I. NOMENCLATURE

Indexes

i unit index

t time period index

Constants

I total number of thermal generating units

T length of the planning time horizon

a_i, b_i, c_i coefficients of the fuel cost function of unit i

D_t power load demand for time period t

R_t spinning reserve required at time period t

p_i^L minimum power generation of unit i

p_i^U maximum power generation of unit i

TU_i minimum uptime of unit i

TD_i minimum downtime of unit i

T^{ini}_i initial status of unit i

DR_i ramp-down limit of unit i

UR_i ramp-up limit of unit i

Hsc_i hot start cost of unit i

Csc_i cold start costs of unit i

T^{cold}_i cold start hours of unit i

Dc_i shut-down cost of unit i

$cost^{UP}$ upper bound for the objective function

ε^{abs} absolute tolerance for global optimality

ε^{rel} relative tolerance for global optimality

A_t^{opt1} objective value of the optimal solution of problem P1 for time period t

A_t^{opt2} objective value of the optimal solution of problem P2 for time period t

A_t^{LO} lower bound for the number of committed units at time period t

A_t^{UP} upper bound for the number of committed units at time period t

Variables

$u_{i,t}$ binary variable representing the on/off status of unit i at period t

$p_{i,t}$ power output of unit i in period t

$cu_{i,t}$ start-up cost of unit i in period t

$cd_{i,t}$ shut-down cost of unit i in period t

A_t auxiliary variable for computing integer cutting planes

II. INTRODUCTION

RELIABLE and secure production of electric power is fundamental for both suppliers and users. The increasing electricity demand raises the need to study carefully the possible alternatives when it is planned to install new generation plants or to expand existing ones. However, besides the target of profitability pursued by the supplier companies, the greatest concern currently focuses on reducing the fossil fuels consumed for power generation, while at the same time mitigating CO2 emissions. This trend has led to the study of combined methods of production, through the integration of conventional generation sources with other, preferably from renewables, while still ensuring profitability

Manuscript received December 12, 2011. This work was supported by the Fundação para a Ciência e a Tecnologia (FCT), Program "Ciência 2008", Portugal.

M. G. Marcovecchio is with the Unidade de Modelação e Optimização de Sistemas Energéticos, Laboratório Nacional de Energia e Geologia, Estrada do Paço do Lumiar 22, 1649-038, Lisbon, Portugal; on leave from INGAR/CONICET, Instituto de Desarrollo y Diseño and UNL, Universidad Nacional del Litoral, Santa Fe, Argentina (e-mail: marian.marcovecchio@lneg.pt)

A. Q. Novais is with the Unidade de Modelação e Optimização de Sistemas Energéticos, Laboratório Nacional de Energia e Geologia, Estrada do Paço do Lumiar 22, 1649-038, Lisbon, Portugal (e-mail: augusto.novais@lneg.pt)

I. E. Grossmann is with Chemical Engineering Department, Carnegie Mellon University, Pittsburgh, PA 15213,USA (e-mail: grossmann@cmu.edu).

([1]-[2]).

Electricity supply systems are admittedly very complex. Diverse factors are involved, like fluctuations and uncertainties in load and prices. In addition, the contingencies that may arise in the generation process must be considered in order to prevent or reduce the risks of blackouts or insufficient provision ([3]-[4]).

Traditionally, the power network was vertically integrated and governed by a utility system operator who was familiar with the system characteristics and costs, and decided the production plan in order to meet demand while minimizing the generation costs.

In the past decade, this business model has changed from a highly regulated and vertically integrated industry to one that is deregulated and horizontally integrated where a larger number of issues come into play. Thus, the Generation Companies (GENCOs) seek to maximize their own profit and present their bids to the Independent System Operator (ISO), which in turn analyzes the cost of the offers and decides the production plan, minimizing the generation cost. Various market structures can emerge depending on the specific relations and exchanges of information between the GENCOs, and the characteristics of the ISO ([5]).

Additionally, the planning of the generation and distribution of electric power is typically based on three different classes of decisions defined according to the length of the planning time horizon: long-term decisions (capacity, type and number of power generators); medium term decisions (scheduling of the existing units for the planning horizon); short-term decisions (programming of the power that each committed unit must produce to meet the real-time electricity demand). These three levels of decision are usually referred to as Power Expansion, Unit Commitment (UC) and Economic Dispatch, respectively. In particular, the Unit Commitment problem has been more widely studied due to its practical importance. ([6]-[7]).

The UC problem can be formulated as a mathematical programming problem for the generation and distribution of electric power under either of two alternative models: Security Constrained Unit Commitment (SCUC) for a highly regulated industrial setup, and Price-Based Unit Commitment (PBUC) for a deregulated one. In SCUC, the on/off status and production levels for given power generators are determined in order to meet a time-varying expected demand of electricity over a given time horizon, while operating constraints and system reserve are satisfied and the cost of production is minimized. On the other hand, in the PBUC the decisions are taken according to financial risks under no obligation of satisfying the expected demand; that is, a PBUC is established by each GENCO and the objective is to maximize its own profit.

An optimal solution for the UC problem can have a significant economic impact. However, solving the UC problem is very difficult since it is NP hard due to the combinatorial nature of the set of feasible solutions. For real-world instances the underlying optimization problem can be very expensive to solve.

A large effort has therefore been spent over the last decades to develop efficient methods capable of solving the UC problem for real instances, or at least for obtaining good solutions in reasonable computational times.

This two-paper series focuses on thermal UC problem under security constraints (SCUC). The methods proposed in the literature to solve this problem are both deterministic and heuristic. Approaches based on deterministic methods include: priority list ([8]-[9]), integer mathematical programming (linear and nonlinear) ([10]-[11]), Branch-and-Bound search ([12]), dynamic programming ([13]), Lagrangian relaxation ([14]-[16]), decomposition techniques ([17]-[18]) and others. However, few deterministic methods have been proposed that guarantee global optimality. Reference [12] developed a Branch-and-Bound algorithm for solving the UC problem. The method does not assume priority ordering, but it is based on the assumption that a unit with nonzero start-up cost will cycle at most once in a period of 24 hours. The algorithm is used to solve a 19-unit system, for a time horizon of 24 hours, guaranteeing global optimality within a tolerance of 0.7%. Reference [17] employed Benders decomposing method to address the hydrothermal power system: the master problem contains only integer variables and determines the Unit Commitment of thermal plants; the sub-problem includes only continuous variables for the economic dispatch. The sub-problem is further decomposed in thermal and hydro systems. A system consisting of 30 hydro and 20 thermal plants is solved, guaranteeing global optimality within a 2% tolerance. Reference [10] proposed a mixed-integer linear formulation for the thermal Unit Commitment, which requires a single set of binary variables. The quadratic cost function is approximated by a set of piecewise linear functions with four segments. Once, the MILP problem is solved, the economic dispatch is run to evaluate exactly the quadratic cost. A base 10-unit system and its variants with up to 100 units are solved, within 0.5% of global optimality for the MILP problem. However, the actual global optimality tolerance for the original integer quadratic programming problem is neither determined nor analyzed. Also, the case studies addressed do not include ramp constraints.

As, for heuristic approaches, the most widely used are: artificial neural networks ([19]), genetic algorithms ([20]-[21]), evolutionary programming ([22]-[23]), simulated annealing ([24]-[25]), fuzzy systems ([26]), particle swarm optimization ([27]-[28]), tabu search ([29]) and hybrid methods ([30]).

References [6], [7] and [31] give complete reviews for contributions on deterministic and heuristic methodologies for solving the UC problem. Nevertheless, the methods proposed so far are not sufficiently flexible for solving to optimality real world problems in acceptable computational times.

In the first of this two-paper series a deterministic optimization approach is proposed for the thermal SCUC problem. The problem can be stated as follows. Given is a number of thermal power generators (each with different operating and production characteristics) and a specified time-variant demand over the planning time horizon. The problem is then to determine the start-up and shut-down schedules and power production for each unit in order to minimize the operational cost while meeting demand.

The mathematical model addressed is a convex mixed-integer quadratic programming problem (MIQP) having a single set of binary variables representing the status of the each generator at each time period. The continuous relaxation of the MIQP for the SCUC problem is convex, since all constraints are linear and the objective function is quadratic and convex. However, due to the presence of the integer variables, the problem might become computationally expensive to solve as the problem size increases.

A computational strategy that takes advantage of the particular characteristics of the UC problem is proposed for its solution in this paper. It consists of valid integer cutting planes and a Branch and Bound search incorporating the proposed cuts, which results in a particular Branch and Cut algorithm for the UC problem.

In this paper and its companion [32], the proposed deterministic approach is described and implemented to solve several published case studies. Although the UC problem is NP-hard, the results show that the proposed technique is capable of finding the optimal solution for real-world sized instances in reasonable computational time ([32]).

The paper is organized as follows. Section III presents a detailed description of the mathematical formulation for the SCUC problem. Section IV describes the proposed deterministic optimization approach. The steps to construct the proposed integer cutting planes are firstly outlined in Section IV.A. Then, in Section IV.B a particular implementation of the general Branch-and-Bound framework is described. Finally, Section V outlines the general conclusions of the present work. The second part of this two-paper series ([32]) presents the computational results that illustrate the performance of the proposed technique.

III. Mathematical Problem Formulation

The thermal SCUC problem is formulated as the following MIQP model.

Consider a set of I thermal generating units and a specified time-varying demand over T time periods defining the planning time horizon, with the units being indexed with i=1,...,I and the time periods with t=1,...,T. The definition of the parameters and variables can be found in the Nomenclature Section.

The objective function to be minimized is the operating cost, which includes fuel consumption calculated by a quadratic function with fixed charges, and fixed start-up and shut-down costs:

$$\min \quad cost=\sum_{i=1}^{I}\sum_{t=1}^{T}\left[\left(a_i u_{i,t}+b_i p_{i,t}+c_i p_{i,t}^2\right)+cu_{i,t}+cd_{i,t}\right] \tag{1}$$

The constraints to be satisfied are given by (2) to (20).

System power balance for each period:

$$\mathrm{D_t} \le \sum_{i=1}^{I} \mathrm{p_{i,t}} \qquad t=1,\dots,T \tag{2}$$

Spinning reserve is guaranteed by the available capacity of active units:

$$\mathrm{D_t}+\mathrm{R_t} \le \sum_{i=1}^{I} \mathrm{p_i^U u_{i,t}} \qquad t=1,\dots,T \tag{3}$$

The generation power limits of each unit at each time period are imposed by:

$$\mathrm{u_{i,t} p_i^L} \le \mathrm{p_{i,t}} \le \mathrm{p_i^U u_{i,t}} \qquad i=1,\dots,I;\ t=1,\dots,T \tag{4}$$

Note that when the unit is offline ($\mathrm{u_{i,t}}=0$), the previous constraint forces the corresponding power output to be 0. Therefore, generation limits are only imposed when the unit is online.

Requirements of minimum up and down times are mathematically modelled by the sets of constraints (5)-(10).

The on-off status of a unit i in its earliest periods of operation might be determined by its initial status and its minimum up and down times (TD_i and TU_i, respectively), according to the following conditions:

$$\mathrm{u_{i,t}}=0 \qquad \forall i:\ T^{ini}_i<0;\ t=1,\dots,(TD_i+T^{ini}_i) \tag{5}$$

$$\mathrm{u_{i,t}}=1 \qquad \forall i:\ T^{ini}_i>0;\ t=1,\dots,(TU_i-T^{ini}_i) \tag{6}$$

where T^{ini}_i is an integer number indicating the initial status of unit i, i.e. the number of periods that unit i has been switched off (T^{ini}_i<0) or turned on (T^{ini}_i>0).

Constraints (7) model the unit minimum-up time for the general case. If unit i is started-up at time period t (t ≥ 2), then (7) imposes that the unit must remain online for the following (TU_i-1) time periods. Equation (8) considers the case in which the unit is started-up at the first time period.

$$\mathrm{u_{i,t}-u_{i,t-1} \le u_{i,t+j}} \qquad i=1,\dots,I;\ t=2,\dots,T;\ j=1,\dots,(TU_i-1) \tag{7}$$

$$\mathrm{u_{i,1} \le u_{i,1+j}} \qquad \forall i:\ T^{ini}_i<0;\ j=1,\dots,(TU_i-1) \tag{8}$$

Constraints (9) and (10) model the unit minimum down-time. Equations (9) consider the general case: if unit i is shut down at time period t (t ≥ 2), then this constraint enforces unit i to stay offline for the following (TD_i-1) time periods. If the unit i is shut down at the first time period, (10) ensures that its minimum down-time is satisfied.

$$\mathrm{u_{i,t+j} \le u_{i,t}-u_{i,t-1}+1} \qquad i=1,\dots,I;\ t=2,\dots,T;\ j=1,\dots,(TD_i-1) \tag{9}$$

$$\mathrm{u_{i,1+j} \le u_{i,1}} \qquad \forall i\ /\ T^{ini}_i>0;\ j=1,\dots,(TD_i-1) \tag{10}$$

The ramp rate limits are modelled by (11) and (12).

$$\mathrm{p_{i,t-1}-DR_i u_{i,t-1}-p_i^U(1-u_{i,t}) \le p_{i,t}} \qquad i=1,\dots,I;\ t=2,\dots,T \tag{11}$$

$$\mathrm{p_{i,t} \le p_{i,t-1}+UR_i u_{i,t-1}+p_i^U(1-u_{i,t-1})} \qquad i=1,\dots,I;\ t=2,\dots,T \tag{12}$$

The ramp rate limits must be applied over unit i at time period t only if the unit is online at that period and was online also at time period (t-1). Equations (11) and (12) ensure that the ramp rate limits are imposed only in that case. Otherwise, they guarantee that power output is not incorrectly limited.

The start-up cost for each unit is time-dependent and this dependence is typically modelled as an exponential function. When the time span is discretized, evaluating the exponential function on the discrete points is equivalent to evaluate a piecewise constant function. Usually, this start-up cost dependence is simplified by assuming a low cost if the generator was down for a short period of time and a higher value otherwise. In practice this assumption is acceptable to represent real cases.

Specifically, the start-up cost function is usually defined as: $cu_{i,t}=Hsc_i$ if downtime≤(TD_i+T^{cold}_i), and $cu_{i,t}=Csc_i$ otherwise.

This cost function can be modelled by equations (13)-(17).

For the general case, (13) constrains variable $cu_{i,t}$ to be greater or equal to Hsc_i if unit i is started-up at time period t ($t \geq 2$). Equation (14) considers the case in which the unit is turned on at the first time period.

$$(\mathrm{u_{i,t}} - \mathrm{u_{i,t-1}})\mathrm{Hsc_i} \leq \mathrm{cu_{i,t}} \qquad i=1,\ldots,I;\ t=2,\ldots,T \qquad (13)$$

$$\mathrm{u_{i,1}}\mathrm{Hsc_i} \leq \mathrm{cu_{i,1}} \qquad \forall i\ /\ T^{ini}_i<0 \qquad (14)$$

If instead the unit i is turned on at the time period t and the downtime at that moment is greater than $(TD_i+T^{cold}_i)$, (15) and (16) impose $cu_{i,t}$ to be greater or equal than Csc_i value. Equation (15) models this requirement for $t>(TD_i+T^{cold}_i)$, while (16) takes into account the initial status for $t\leq(TD_i+T^{cold}_i)$:

$$\left(\mathrm{u_{i,t}} - \sum_{j\leq TD_i+T^{cold}_i+1} \mathrm{u_{i,t-j}}\right)\mathrm{Csc_i} \leq \mathrm{cu_{i,t}} \qquad i=1,\ldots,I;\ (TD_i+T^{cold}_i) < t \leq T \qquad (15)$$

$$\left(\mathrm{u_{i,t}} - \sum_{j<t} \mathrm{u_{i,t-j}}\right)\mathrm{Csc_i} \leq \mathrm{cu_{i,t}} \qquad \forall i\ /\ T^{ini}_i<0;\ (TD_i+T^{cold}_i+T^{ini}_i+1)<t\leq(TD_i+T^{cold}_i) \qquad (16)$$

Equation (17) is necessary to ensure that variable $cu_{i,t}$ takes value 0 when the unit i is not turned on at the time period t:

$$0 \leq \mathrm{cu_{i,t}} \qquad i=1,\ldots,I;\ t=1,\ldots,T \qquad (17)$$

Thus, (13) and (14) ensure that if the unit i is turned on at the time period t, then at least the cost Hsc_i is incurred. If the downtime is greater than $(TDi+T^{cold}_i)$, constraints (15) and (16) raise the minimum value for the startup cost to Csc_i. Equation (17) is necessary to prevent that variable $cu_{i,t}$ takes negative values when the unit i is not turned on at the time period t. Finally, since variables $cu_{i,t}$ are only involved in equations (13)-(17), the optimization procedure will ensure that the cost assumed by variable $cu_{i,t}$ will be exactly 0, Hsc_i or Csc_i, for each case.

Some units can incur in a shut-down cost when they are turned off. This is modeled by (18) for the general case ($t \geq 2$); (19) computes the cost for shutting down unit i at the first time period of the planning time horizon considering the initial status of the units. Constraint (20) prevents variable $cd_{i,t}$ taking negative values if the unit i is not shut down at the time period t.

$$(\mathrm{u_{i,t-1}} - \mathrm{u_{i,t}})\mathrm{Dc_i} \leq \mathrm{cd_{i,t}} \qquad i=1,\ldots,I;\ t=2,\ldots,T \qquad (18)$$

$$(1 - \mathrm{u_{i,1}})\mathrm{Dc_i} \leq \mathrm{cd_{i,1}} \qquad \forall i\ /\ T^{ini}_i>0 \qquad (19)$$

$$0 \leq \mathrm{cd_{i,t}} \qquad i=1,\ldots,I;\ t=1,\ldots,T \qquad (20)$$

Analogously to the start-up cost and since variables $cd_{i,t}$ only appear in equations (18)-(20), after optimization $cd_{i,t}$ will assume exactly either of the values 0 or Dc_i.

Finally, the specification on the variables is as follows:

$$0 \leq \mathrm{p_{i,t}} \qquad i=1,\ldots,I;\ t=1,\ldots,T \qquad (21)$$

$$\mathrm{u_{i,t}} \in \{0,1\} \qquad i=1,\ldots,I;\ t=1,\ldots,T \qquad (22)$$

The thermal SCUC problem can be mathematically represented by the MIQP model given by equations (1)-(22). The mathematical programming model for I thermal generating units and T time periods for the planning time horizon involves: IxT binary and 3(IxT) continuous variables. All constraints are linear, and the objective function is convex since it is quadratic with positive coefficients. The main difficulty for solving this problem is due to the presence of binary variables. In fact, solving this problem becomes very hard when the number of units and time periods attain non trivial values. Thus, numerous methods have been proposed for addressing the SCUC problem, mainly heuristic techniques. In this two-paper series, the thermal SCUC will be tackled by a deterministic optimization approach, which is described in detail in the following sections.

IV. Deterministic Optimization Approach

The solution of the SCUC problem will be addressed through a deterministic optimization approach in which the global optimal solution is found within a specified tolerance for global optimality. The proposed methodology consists of a Branch and Cut implementation: appropriate integer cutting planes are defined and a branch and cut search is developed that exploits the characteristics of the proposed cuts.

A. Integer cutting planes

The proposed integer cutting planes are quite simple but highly efficient. They are based on the idea of finding valid lower and upper bounds for the number of committed units at each time period. By implementing them, the relaxation gap of the MIQP is considerably reduced, and consequently the global optimality for adjusted tolerance can be reached in reasonable computational times.

Next, the steps to construct the cuts are described. An upper bound for the objective function, $cost^{UP}$, is assumed to be available. If a feasible solution of the problem is known, then its objective value is an appropriate upper bound; otherwise, a large enough value is chosen. The absolute and relative tolerances for global optimality are denoted as: ε^{abs} and ε^{rel}, respectively.

Step 1:

For each time period t, the two following NLP problems are solved:

P1: $\min \mathrm{A_t}$ P2: $\max A_t$

s.t.: Equations (2) to (21)

$$\mathrm{A_t} = \sum_{i=1}^{I} \mathrm{u_{i,t}} \qquad t=1,\ldots,T \qquad (23)$$

$$\mathrm{cost} = \sum_{i=1}^{I}\sum_{t=1}^{T}\left[\left(\mathrm{a_i u_{i,t}} + \mathrm{b_i p_{i,t}} + \mathrm{c_i p_{i,t}^2}\right) + \mathrm{cu_{i,t}} + \mathrm{cd_{i,t}}\right] \leq \mathrm{cost^{UP}}\text{-}\varepsilon^{abs} \qquad (24)$$

$$0 \leq \mathrm{u_{i,t}} \leq 1 \qquad i=1,\ldots,I;\ t=1,\ldots,T \qquad (25)$$

$$\mathrm{A_t} \in \left[\mathrm{A_t^{LO}}, \mathrm{A_t^{UP}}\right] \qquad t=1,\ldots,T \qquad (26)$$

Equation (23) defines the variable A_t that is minimized (P1) and maximized (P2); it computes the sum of the states of the units for each time period. Equation (24) imposes an upper bound, $(\mathrm{cost^{UP}}\text{-}\varepsilon^{abs})$, for the operating cost. Equation (25) relaxes the discrete requirements of the variables u.

Initially: $A_t^{LO}=0$ and $A_t^{UP}=I$, both values being adjusted as the cuts are computed.

Step 2:

For each time period t, set:

$$\mathrm{A_t^{LO}} = \left\lceil \mathrm{A_t^{opt1}} \right\rceil \quad \text{and} \quad \mathrm{A_t^{UP}} = \left\lfloor \mathrm{A_t^{opt2}} \right\rfloor$$

where A_t^{opt1} and A_t^{opt2} are the objective value of the optimal solutions for problems P1 and P2, respectively.

Note that the global optimal solutions can be found for P1 and P2 since they are convex NLP problems where the integer variables have been relaxed to be continuous.

Step 3:

The inequalities:

$$A_t^{LO} \leq \sum_{i=1}^{I} u_{i,t} \quad (27)$$

$$\sum_{i=1}^{I} u_{i,t} \leq A_t^{UP} \quad (28)$$

are valid integer cutting planes for the relaxed problem, since they only exclude feasible points of the relaxed problem with some variables $u_{i,t}$ at non-integer values. At the same time, the cuts eliminate feasible solutions for the original MIQP that do not improve the upper bound $cost^{UP}$ at least by ε^{abs}.

If the binary variables were not relaxed to be continuous, the variable A_t would represent the number of units that are committed at time period t, and P1 and P2 would find the minimum and maximum number of units that are required at time period t to satisfy all the original constraints, while the cost is decreased at least by ε^{abs}. However, as the original binary variables $u_{i,t}$ are considered continuous, the optimal values of A_t, i.e. A_t^{opt1} and A_t^{opt2}, are not necessarily integer for the general case, but the real number of online units must indeed be integer. Therefore, A_t^{opt1} and A_t^{opt2} are bounds for A_t, but they are not as tight as possible. After the rounding in Step 2, A_t^{LO} and A_t^{UP} represent valid inequalities that eliminate non integer solutions.

The proposed integer cutting planes lead to 2 linear constraints for each time period, (27) and (28). However, for practical purposes, the cuts are implemented by adding (23) to the relaxed problem and adjusting the bounds A_t^{LO} and A_t^{UP} as the cuts are being computed and updated.

In practice the cuts are generated sequentially. That means that the cuts are calculated and the bounds A_t^{LO} and A_t^{UP} are updated at the relaxed problem, as well as at problems P1 and P2. Therefore, they modify the feasible region for the computation of the subsequent cuts. In fact, the cuts become progressively tighter as the feasible region of the relaxed problem is being reduced.

Clearly, $A_t^{LO} \leq A_t^{UP}$ will be true for each period of time, since A_t^{LO} and A_t^{UP} are lower and upper bounds for the number of committed units, respectively, provided that the relaxed problem is feasible to reduce the upper bound of the objective function by the required tolerance. On the other hand, if the global optimal solution of the MIQP problem is found, and the integer cutting planes are sequentially updated using its objective value as upper bound $cost^{UP}$, then as long as problems P1 and P2 are feasible, the value for each pair of parameters: A_t^{LO} and A_t^{UP} should become identical, i.e. $A_t^{LO} = A_t^{UP}$. Furthermore, once the global optimum is found and an improvement on the objective function of ε^{abs} is required, the MIQP will be infeasible, which will then provide the termination criterion. The same conclusion can also be achieved while sequentially updating the cuts, i.e. P1 and P2 will eventually become infeasible, since the relaxed problem and P1 and P2 have the same feasible region.

For integer programming techniques that implement effective searches for good solutions, the crucial point for reaching convergence is proving that the found solution is actually the optimal or, otherwise, that it satisfies a specified tolerance for global optimality. For an algorithm based on branch and bound search that means that improving the lower bound is a major issue. The proposed integer cutting planes were designed with the aim of tightening the lower bound produced by the relaxed problem, which will reduce the relaxation gap, and consequently will accelerate the algorithm convergence. Their efficacy will be illustrated in the companion paper [32].

The proposed cuts will be combined with an appropriated branch-and-bound search, as it is described in the following subsection.

B. Branch and Cut search

The cuts proposed in the previous section have demonstrated to be highly efficient in reducing the relaxation gap, as will be shown in the companion paper [32]. To take advantage of this reduction technique, an appropriate Branch-and-Bound search is defined by incorporating at each node the cuts for improving the lower bounds. Hence, a particular Branch-and-Cut is developed for the SCUC problem.

The standard Branch-and-Bound search ([33]) can be applied without either linearization or under estimation, since the relaxed problem of the SCUC model is convex. Therefore, at each node of the tree, the lower bound is obtained by solving the relaxed problem, i.e. by letting the binary variables be continuous and solving the resulting convex quadratic programming problem (QP).

Thus, the basic procedure for a Branch-and-Bound search is adopted and properly adapted to solve the SCUC problem. Below, the proposed implementation of the standard algorithm which defines the proposed Branch-and-Cut search to solve the SCUC problem is specified.

Integer cutting plane implementation: In order to implement the proposed cuts, (23) is added to the relaxed problem and so only T variables (A_t) and T linear constraints are required to apply the cuts. The bounds for the variables A_t are modified to tighten the cut approximation. Therefore, no new constraint or variable is needed to adjust the cuts.

At the root node, the cuts will be initialized by estimating the value of the bounds A_t^{LO} and A_t^{UP} in an analytic way, in order to avoid an excessive computational cost. Specifically, A_t^{LO} is initialized as the minimum number of units that, being committed and operating at their upper level, would be sufficient to satisfy the demand plus the spinning reserve at time period t, when no other constraint is considered. That is, for each period of time the upper generation limits for each unit are considered, with the highest ones being chosen and added until the load and spinning reserve of each time period is met. The number of units that are necessary to consider during this simple procedure constitutes a valid lower bound for the committed unit at each time period. A similar procedure is followed to initialize the upper bounds A_t^{UP}, by considering the lower generation limits for each unit instead, and choosing the lowest among them. Only the allowed operating levels for each unit are taken into account to compute the initial values of the bounds A_t^{LO} and A_t^{UP}, while other constraints and even the cost are simply disregarded. Applying this analytic procedure, valid bounds are obtained, although they are not as tight as they would be if they were calculated by solving problems P1 and P2, which would require solving *2T* convex optimization problems.

Update of integer cutting planes: Two cuts are updated at each node of the search tree. They are selected alternatively as the ones corresponding to the time period having the largest difference (A_t^{UP} -A_t^{LO}). This selection criterion is adopted since these two cuts are the ones with greatest potential to provide a more accurate approximation for the feasible region of the MIQP problem. As usual, each new node inherits its parent's updated cuts. If either P1 or P2 become infeasible at a given node, then the node is fathomed since it will not contain any solution of the MIQP problem that improves the upper bound of the objective function $cost^{UP}$ at least by ε^{abs}. Even though QP or NLP solvers occasionally declare infeasible a problem that is actually feasible, P1 and P2 did not present this difficulty and their feasibility was well determined in all cases. With the purpose of avoiding incorrect eliminations, this result was confirmed with a number of additional tests not reported here. Each time infeasibility was detected, it was checked by solving a linear problem, where the nonlinear convex cost function is replaced by several linear underestimations. It was found that in all cases where the NLP problem was declared infeasible, the linear problem test confirmed it.

Branching variable selection: The branching is performed over the binary variables, and as usual, two child nodes are generated by fixing the selected branching variable at their integer values: 0 and 1. The new nodes are added to the waiting node list. The branching variable selection is carried out by following a priority order that is established for each node, according to the current values of the binary variables at the relaxed problem optimal solution and the fixed charge costs. Firstly, the time period to perform the branching is chosen as the first having at least one unit with a non integer value. Next, the unit is selected among the ones having non-integer value for the chosen time period at the relaxed solution, as the one with largest fixed charge cost: a_i. The criterion adopted to select the time period of the branching variable among the first ones of the time horizon is based on the observation that this choice could allow subsequent periods for the chosen unit to be determined. In fact, if the branching is performed on a variable whose value of the associated unit for the previous period was already fixed at the current node, then in one of the two new child nodes, not only the state of the unit in the branching period will be fixed, but also the states of the unit in as many subsequent periods as the unit minimum up or down time dictates. On the other hand, the unit for the branching is chosen according to the values of the charge cost. This is due to the significant influence these terms exert on the final operating cost, considering that the branching might determine the startup of the unit at the chosen period in one of the child nodes.

Lower bounding: The lower bound of the global objective function is computed at each node by solving the relaxed problem. In case of branching, each new node will inherit its parent's lower bound until the node itself is analyzed. The natural criterion for choosing the next node to be analyzed from the waiting list is also adopted here, namely the node with lowest lower bound. However, the relaxed problem is solved requiring that the cost be decreased by at least ε^{abs}, i.e. the upper bound ($cost^{UP}$- ε^{abs}) is imposed on the objective variable. Even though this bound does not affect the minimum reached by the relaxed problem, this problem could become infeasible because the current node might not contain any solution of the MIQP problem that meets such requirement and the node will be eliminated. This is equivalent to eliminating all nodes whose lower bound is greater than ($cost^{UP}$- ε^{abs}).

It should be noted that for the technique proposed here it is not necessary to solve the relaxed problem at each node. In fact, since the feasible region of the relaxed problem is the same as that of P1 and P2, the feasibility of the relaxed problem can be determined when the cuts are updated. In this way, solving the relaxed problem can be avoided, which saves on execution time. In this case, neither the lower bound for each node would be known, nor the global one. This does not affect the stopping criterion of the algorithm nor its convergence. In effect, the tolerance for global optimality is achieved when there are no nodes on the waiting list, i.e., after all generated nodes have been discarded ensuring that the relaxed problem is infeasible to improve the upper bound at least by ε^{abs}. In this case, the criterion for selecting a node from the waiting list to continue the search should be different, since the lower bound for each node is no longer available. However, since the algorithm has shown to have modest execution times even when the relaxed problem is solved at each iteration, this variant was not implemented.

Upper bounding: The initial upper bound is calculated after solving the relaxed problem. According to the obtained solution, the binary variables that are close to 0 or 1 are fixed at the corresponding value. Thus, the number of binary variables to optimize is considerably reduced. Then, a local search is carried out by optimizing the binary variables that are still free and the continuous ones. By implementing this procedure, good initial solutions can be obtained with a low computational requirement.

With the aim of improving the upper bound in the branch-and-cut tree, a local search is carried out in those nodes whose depth is multiple of a pre-specified integer number. In order to prevent spending too much computational time, a reasonable limit of time is imposed for the local search. As it was mentioned before, in case that a better solution of the original MIQP problem is found, all cuts are updated using the new upper bound.

The general steps of the proposed branch are cut are shown in Fig. 1, where RP denotes the relaxed problem, and *sd* is the search depth, i.e. an integer number for choosing the nodes where a restricted local search will be performed.

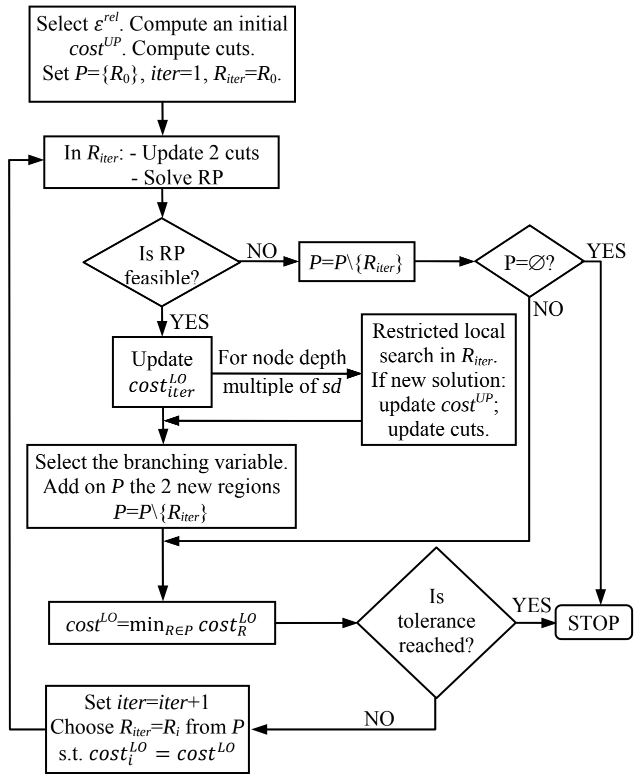

Fig.1. Branch and Cut scheme.

In the second part of this two-paper series [32], the efficiency of the proposed integer cutting planes will be illustrated. Then, the proposed optimization approach will be implemented to solve two thermal unit commitment case studies widely used in the literature as a test bed for different methodologies.

V. Conclusions

In this two-series paper, a deterministic optimization approach for solving the SCUC problem is proposed. Firstly, in this first part, the MIQP model representing the generation and distribution of electric power with thermal generating units is formulated. The model takes into account all the constraints of the system, units and demand. The objective function to be minimized is the operational cost. The model includes binary variables associated to the status of each unit at each time period.

The proposed optimization approach consists of a Branch-and-Cut search. Suitable integer cutting planes, specific for the SCUC problem, were developed. Unlike other integer cutting planes, the ones proposed here can be implemented by initially adding only T variables and T linear constraints to the relaxed problem. In order to tighten the cut approximation, the bounds of the added variables are suitably adjusted by solving two Quadratic Programming models for each time period. The performance of the proposed cuts are properly exemplified in the companion paper [32]. The examples showed that the cuts are highly efficient for reducing the relaxation gap.

A particular implementation of the general Branch-and-Bound framework was proposed for the SCUC problem. Specifically, criteria for updating the integer cutting planes and selection of branching variables were defined.

In the second part of this two-paper series, the performance of the proposed methodology in solving the MIQP are tested with two case studies and their variants, which are widely addressed in the literature as test problems.

The proposed B&C search tackles a very complex problem and unlike most of the methodologies presented in the literature that address the SCUC problem, it offers guarantee of global optimality within a specified tolerance. Furthermore, as it can be seen from the results of the companion paper [32], the computational requirements of the B&C approach remain at an acceptable level for the case studies addressed.

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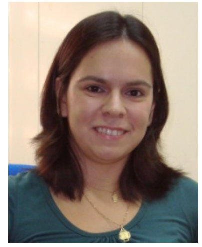
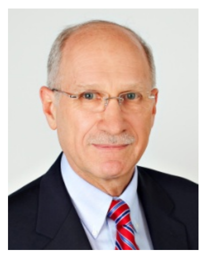


Marian G. Marcovecchio received the B. S. degree in Applied Mathematics in 2000, and the PhD. degree in Computational Mechanics Engineering in 2007 from Universidad Nacional del Litoral, Santa Fe, Argentina.
She is currently a researcher at the Unidade de Modelação e Optimização de Sistemas Energéticos, Laboratório Nacional de Energia e Geologia, Lisbon, Portugal. She is also a researcher at INGAR, Instituto de Desarrollo y Diseño, CONICET and an Associate Professor at Universidad Nacional del Litoral, Santa Fe, Argentina. Her research interests include mathematical modeling and global optimization methods; and its applications to the design, operation and planning of industrial processes.

Augusto Queiroz Novais received the Licentiate degree in Chemical Engineering from Instituto Superior Técnico (IST), Lisbon, Portugal in 1969, the PhD degree in Chemical Engineering/Process Systems from the University of Leeds, U.K. in 1978, and the Aggregation degree in Chemical Engineering from IST in 2007. He has worked for 2 years as a lecturer in Mozambique, and 5 years as a Research Fellow in the U.K.
He is a senior researcher at the Laboratório Nacional de Energia e Geologia, Lisbon, Portugal, where he acts presently as the Head of the Unidade de Modelação e Optimização de Sistemas Energéticos. His present interests include mathematical modeling and optimization methods addressing industrial processes and energy systems, and planning, scheduling and optimization of supply chains.



Ignacio Grossmann (S'99) received the B.S.degree in chemical engineering from Universidad Iberoamericana in Mexico in 1977, and the M.Sc. and Ph.D. degrees in chemical engineering from Imperial College in London.
He is the R. Dean University Professor at Carnegie Mellon University and is currently director of the Center for Advanced Process Decision-making. He is a member of the National Academy of Engineering, and his interests include mixed-integer optimization, design of energy and water systems, and planning, scheduling and supply chain optimization.