A Branch and Cut Search for the Deterministic Optimization of the Thermal Unit Commitment Problem. Part II: Computational Results

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Abstract**-- This paper analyzes the computational performance of the deterministic optimization approach for the Security Constrained Unit Commitment problem (SCUC) addressing thermal generating units, that was developed in its companion paper. The technique proposed in the first part is based on suitable integer cutting planes, specific for the SCUC problem, and a Branch and Bound search which incorporates the cuts and results in a particular Branch and Cut algorithm for the SCUC problem. In this paper, the performance of the proposed cuts is properly illustrated. Then, the application of the proposed Branch and Cut algorithm is exemplified with two case studies and their variants, with the aim of showing the performance of the approach for problems of different dimensions and characteristics. Comparisons with other deterministic solvers is also carried out.**

Index Terms**-- Energy optimization, Unit Commitment problem, deterministic optimization, Branch and Cut algorithm, Computational performance**

I. Nomenclature

For a list of the symbols used in this paper, we refer to the first part of this two-paper series.

II. Introduction

In this paper, the performance of the deterministic optimization approach presented in the companion paper [1] is illustrated by solving problems of different dimensions, up to real-size cases.

In Section III, the performance of the proposed integer cutting planes is illustrated, by showing the reduction in the relaxation gap that can be obtained by introducing the cuts.

In Section IV two application examples over a 24-h horizon are addressed with the proposed technique. In Section IV.A, a case study widely used as test problem is solved, considering 10 different instances by increasing the number of generating units. In Section IV.B, a 38-unit system is addressed, which is particularly hard to solve. For both examples, two variants are analyzed by considering and excluding the ramp rate constraints. For each case, three relative tolerances for global optimality are set: 0.3, 0.2 and 0.1% and comparisons with other deterministic solvers, SBB, DICOPT and CPLEX, are also showed.

Finally, Section V outlines the conclusions of the results presented.

III. Performance of the Proposed integer Cutting Planes

The algorithm and solution of the test problems and case studies were implemented in GAMS ([2]) on a laptop with Intel Core i7 Q740 1.73GHz and 8 GB RAM memory. The optimization problems for updating the integer cutting planes, i.e. problems P1 and P2, the original MIQP problem and the relaxed problem were all solved with CPLEX 12.2 solver (For the definition of these problems, the reader is referred to the companion paper [1]).

In order to illustrate the performance of the proposed integer cutting planes, they are applied to reduce the relaxation gap on ten instances of a base problem that includes 10 thermal units over a scheduling time horizon of 24 hours ([3]). For this problem, a 10% of the load demand is required as spinning reserve in each time period, and no ramp rate constraints are imposed. The operating units characteristics and the hourly load distribution for the 10-units system are listed on Tables X and XI of the Appendix, respectively.

The ten instances that will be considered are obtained by scaling the basic problem from 1 to 10, resulting in 10 systems with up to 100 units. The load demand is scaled up accordingly.

For each instance, an initial solution is calculated with CPLEX, setting the relative tolerance for optimality as the default value of 1%, since a lower value slows down the solver convergence.

By using these solutions to set the upper bound $cost^{UP}$, the

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integer cutting planes are computed by solving sequentially P1 and P2 problems for each time period, in order to eliminate non integer solutions that do not improve the upper bound by at least 0.3%, i.e. ε^{abs}=0.003 $cost^{UP}$ is set in the definition of both problems (see (24) of the companion paper [1]).

The solution of the relaxed problem (lower bound) is compared for both cases: with and without the integer cutting planes. The results are reported in Table I. As it is shown, the relaxation gap (difference between upper and lower bound) is considerably reduced by introducing the proposed cuts. Also note that the relaxation gap decreases as the problem size increases.

TABLE I
PERFORMANCE OF INTEGER CUTTING PLANES

N° units	Upper bound	Lower bound without cuts	Lower bound with cuts	% gap reduction
10	565303	558128	562316	58.4%
20	1126936	1116256	1119493	30.3%
30	1687841	1674383	1677407	22.5%
40	2248242	2232511	2235365	18.1%
50	2812193	2790639	2791250	2.8%
60	3372820	3348767	3349599	3.5%
70	3933142	3906894	3909173	8.7%
80	4493726	4465022	4467456	8.5%
90	5054788	5023150	5025844	8.5%
100	5621543	5581278	5581784	1.3%

These examples illustrate how the proposed cuts can tighten the relaxed problem by eliminating non-integer solutions. As a consequence, the process of reducing the relaxation gap is accelerated along with the assurance that the desired optimality tolerance has been reached.

For all instances Table I shows that the lower bound after incorporating the proposed integer cutting planes will be higher for better initial solutions. This is not surprising since a better upper bound will produce a more restricted relaxed problem (see (24) of the companion paper [1]), and consequently more non integer solutions will be eliminated. Moreover, significant reductions were obtained for the smaller instances, while smaller reductions were obtained for the larger cases, which nevertheless still proved to be useful to reduce the number of nodes in the tree search.

IV. Application Examples

A. Case study 1

The proposed optimization approach detailed in the first part of this two-paper series [1], hereafter denoted by B&C, was applied to solve a case study widely used as a test problem ([3]). The case study is based on a 10-unit system and a planning time horizon of 24 hours with time periods of one hour. The elementary problem is scaled up 10-fold on the generating units and load demand, resulting in a real-size case study of 100 thermal units. The spinning reserve requirement to be met is set as a 10% of the load demand for each time period. The generating units data and the hourly load distribution for the original 10-unit system are given in Tables X and XI of the Appendix, respectively. The original problem does not include ramp rate constraints.

Two variants of each instance are examined. The first does not consider ramp rate constraints, similarly to the original problem. In the second case study, ramp rate constraints are included: ramp-up and ramp-down rates of each unit are equal and estimated at 20% of the unit maximum power output per time period, whereas the startup and shutdown ramp rates of each unit are set at its maximum generation output ([4], [5]).

The 10 instances for both cases were solved with the proposed optimization approach for three relative tolerances for global optimality: 0.3, 0.2 and 0.1%. The MIQP problem for the base 10-unit system contains 240 binary and 480 continuous variables, while the largest system with 100 units involves 2400 binary and 4800 continuous variables. The number of constraints ranges from 2445 for the former up to 24009 for the latter, when no ramp rate constraints are considered. If ramp rate constraints are included, the number of constraints ranges respectively from 2905 up to 28609. The computational results for the instances of the original system are reported in Table II, while Table III reports the computational results for the second case, where ramp rate constraints are taken into account. For all instances solved, Tables II and III report the total CPU time required to solve the problem, the total number of nodes analyzed in the branch and cut tree, the maximum number of nodes that had been stored in memory during the search and the objective value of the obtained optimal solution satisfying the specified relative tolerance, i.e. the optimal operation cost.

As it can be seen, instances with ramp rate constraints are computationally more demanding than those without these constraints. In all cases solved, the required tolerance for global optimality was achieved within reasonable CPU times. The optimal production schedules for the most challenging case (100-unit system) for both instances, with and without ramp rate constraints, are presented respectively on Tables XIV and XV of the Appendix section.

This case study has been widely used in order to test different methodologies for solving the UC problem efficiently. Most of the proposed techniques are heuristic, and therefore, they cannot determine how close the solution is from the optimum. Reference [6] reports the objective values of the solutions found for different authors addressing the problem through various approaches.

Fewer deterministic approaches have been proposed to address the UC problem. Reference [7] solved the 10 instances of the original problem, without ramp rate constraints. They proposed a linearization of the quadratic objective function and solved the resulting Mixed Integer Linear Programming (MILP) with the CPLEX solver in the GAMS environment, for an optimality relative tolerance of 0.5%. After that, the economic dispatch is run to evaluate the quadratic cost. Hence, the authors can provide good solutions in reasonable computational time. However, the exact global optimality tolerance for the original MIQP problem is not determined.

With the aim of comparing the proposed approach with other deterministic methodologies, the 10 instances for both cases were also solved with the SBB, DICOPT and CPLEX solvers with the GAMS modeling system. For all cases, a time limit of 3600 seconds was imposed. A tolerance of 0.3% was required for the three solvers. Since CPLEX and the proposed B&C show similar performances for that tolerance and in order to establish a further comparison, CPLEX was additionally tested for 0.2% of tolerance for global optimality. Table IV reports the computational performances for the cases that neglect ramp rate constraints, and Table V shows the results for the same instances when the ramp rate constraints are taken into account. For each instance solved with SBB, Tables IV and V present the total CPU time in seconds, the total number of nodes and iterations and the relative tolerance

TABLE II
COMPUTATIONAL RESULTS OF THE PROPOSED B&C: CASE STUDY 1, WITHOUT RAMP RATE CONSTRAINTS

	Relative tolerance: 0.3%				Relative tolerance: 0.2%				Relative tolerance: 0.10%			
nº of units	CPU time, s	nº nodes	max. node	opert. cost, $	CPU time, s	nº nodes	max. node	opert. cost, $	CPU time, s	nº nodes	max. node	opert. Cost, $
10	0.4	0	0	563990	1.3	3	2	563938	1.7	3	2	563938
20	4.0	3	2	1124858	5.3	3	2	1123587	7.4	3	2	1123370
30	14.0	3	2	1683532	14.2	3	2	1683532	51.4	15	6	1683154
40	19.7	3	2	2243688	39.3	9	4	2243688	131.5	21	4	2242678
50	25.8	3	2	2801238	26.7	3	2	2800617	144.2	15	4	2800717
60	37.9	3	2	3361951	35.5	3	2	3360939	233.8	21	6	3360492
70	52.1	3	2	3921228	118.3	9	4	3921228	958.3	63	14	3921101
80	22.4	0	0	4480798	249.0	33	4	4480798	1555.5	105	12	4480798
90	23.4	0	0	5040234	378.6	51	4	5039429	1463.5	111	20	5039429
100	28.6	0	0	5597993	360.0	39	4	5597962	1723.2	117	20	5597770

TABLE III
COMPUTATIONAL RESULTS OF THE PROPOSED B&C: CASE STUDY 1, WITH RAMP RATE CONSTRAINTS

	Relative tolerance: 0.3%				Relative tolerance: 0.2%				Relative tolerance: 0.10%			
nº of units	CPU time, s	nº nodes	max. node	opert. cost, $	CPU time, s	nº nodes	max. node	opert. cost, $	CPU time, s	nº nodes	max. node	opert. Cost, $
10	1.9	3	2	565243	2.3	3	2	565243	2.4	3	2	565186
20	12.0	3	2	1125950	14.6	3	2	1125674	32.5	9	4	1125577
30	27.2	3	2	1686916	67.6	15	4	1686319	305.9	51	14	1686079
40	33.7	3	2	2246556	108.8	15	6	2246486	479.8	57	12	2246120
50	49.5	3	2	2805295	183.0	21	4	2804758	812.0	45	14	2804669
60	72.2	3	2	3365466	334.3	27	4	3365320	931.2	39	12	3365260
70	97.1	3	2	3928622	410.7	39	4	3927604	1391.4	45	10	3927602
80	102.4	3	2	4488591	324.0	21	8	4487787	1823.0	57	12	4487504
90	95.2	3	2	5047651	699.7	33	12	5047531	1917.2	51	14	5047461
100	91.0	3	2	5606181	507.1	27	8	5606181	2066.8	45	4	5606011

TABLE IV
COMPUTATIONAL PERFORMANCE FOR SBB, DICOPT AND CPLEX: CASE STUDY 1, WITHOUT RAMP RATE CONSTRAINTS

	SBB				DICOPT			CLPEX						
	Required tolerance: 0.3%				Required tolerance:0.3%			Required tolerance: 0.3%			Required tolerance: 0.2%			
nº of units	CPU time, s	nodes	Iters.	reached relat. tol.	CPU time, s	Maj. iter	Iters.	CPU time, s	nodes	Iters.	CPU time, s	nodes	Iters.	reached relat.tol.
10	3600	163235	1567855	2,02%	0.8	2	604	0.2	0	469	0.2	0	565	0,20%
20	3600	47967	1068975	0,74%	6.3	2	3154	6.8	569	24905	8.3	1026	45387	0,20%
30	3600	20268	806797	0,82%	9.7	2	8426	11.3	340	22658	50.7	12478	694182	0,20%
40	3600	13633	567940	0,73%	20.7	2	8712	17.6	869	51252	291.1	44993	3214228	0,20%
50	3600	11186	416629	0,51%	27.5	2	8792	31.3	2878	110876	1739.8	265092	13394497	0,20%
60	3600	9673	341854	0,49%	44.4	2	19464	29.3	1535	68216	184.7	22695	1048141	0,20%
70	3600	6040	344539	0,43%	63.6	2	20280	56.5	4037	152677	3600	285112	17227997	0,22%
80	3600	5251	254781	0,41%	103.4	2	32515	44.9	2064	85696	3600	275143	14755943	0,22%
90	3600	5334	226694	0,45%	103.6	2	17427	76.9	3363	147288	3600	241514	12819135	0,21%
100	3600	4054	193907	0,42%	163.5	2	36754	97.3	4826	200139	3600	289794	11516833	0,22%

TABLE V
COMPUTATIONAL PERFORMANCE FOR SBB, DICOPT AND CPLEX: CASE STUDY 1, WITH RAMP RATE CONSTRAINTS

	SBB				DICOPT			CLPEX						
	Required tolerance: 0.3%				Required tolerance:0.3%			Required tolerance: 0.3%			Required tolerance: 0.2%			
nº of units	CPU time, s	nodes	Iters.	reached relat. tol.	CPU time, s	Maj. iter	Iters.	CPU time, s	nodes	Iters.	CPU time, s	nodes	Iters.	reached relat.tol.
10	3600	100584	1754445	1,69%	1.1	2	850	0.3	0	816	0.4	0	929	0,20%
20	3600	23507	941594	0,82%	6.7	2	7332	14.8	1582	86859	28.4	5257	314495	0,20%
30	3600	13487	565400	0,81%	16.7	2	8368	27.0	2579	121244	840.8	132087	7757818	0,20%
40	3600	9152	423349	0,63%	27.5	2	13263	32.0	1447	77729	2401.9	194909	13574196	0,20%
50	3600	6953	316920	0,55%	47.9	2	18514	58.3	3741	186281	3600	253103	16069490	0,22%
60	3600	4842	270369	0,48%	68.9	2	23390	76.0	3644	173321	3600	173429	13005408	0,22%
70	3600	3817	212520	0,53%	105.6	2	23049	83.7	3555	168484	3600	216789	9859178	0,25%
80	3600	3018	176432	0,50%	138.6	2	26803	98.6	3666	168269	3600	210029	8639819	0,25%
90	3600	2658	140462	0,39%	209.1	2	34231	128.7	4557	181909	3600	164886	7031254	0,24%
100	3600	2390	109878	0,40%	242.1	2	30474	149.7	4308	165027	3600	163063	6014342	0,24%

TABLE VI
BEST OBJECTIVE VALUES (OPERATIVE COST) OBTAINED FOR EACH METHOD FOR THE CASE STUDY 1

Best solutions: Operative cost, $								
	Without ramp rate constraints				With ramp rate constaints			
nº of units	B&C	SBB	DICOPT	CPLEX	B&C	SBB	DICOPT	CPLEX
10	563938	572468	563938	564189	565186	570678	565217	565186
20	1123370	1125845	1124927	1123329	1125577	1127537	1125929	1125577
30	1683154	1688954	1684232	1683067	1686079	1690395	1686717	1686075
40	2242678	2249518	2245261	2242596	2246120	2249624	2248111	2246120
50	2800717	2805663	2803892	2800495	2804669	2809871	2806322	2804741
60	3360492	3365694	3361457	3360027	3365260	3369413	3366985	3365260
70	3921101	3924225	3924405	3921031	3927602	3932786	3931052	3927256
80	4480798	4483632	4483871	4480379	4487504	4493194	4489337	4487516
90	5039429	5045894	5045587	5039349	5047461	5049532	5052681	5047313
100	5597770	5605045	5602364	5597843	5606011	5611067	5612237	5606011

that was achieved for global optimality after finishing within the time limit imposed. For DICOPT solver, these Tables indicate the CPU time in seconds, the number of major iterations and the total number of iterations. Finally, for CPLEX executions, Tables IV and V report CPU time in seconds, total number of nodes and iterations, and the achieved relative tolerance for global optimality after finishing when the required tolerance is not reached within the time limit imposed.

From the results presented the superiority of the proposed B&C search is evident. In all instances, the proposed method was able to guarantee that the required optimality tolerances was reached within the time limit imposed. Furthermore, compared to CPLEX and SBB the proposed algorithm required a much smaller number of nodes. SBB and DICOPT exhibit different performances. SBB could not achieve the required tolerance within 3600 seconds in any of the instances. On the other hand, DICOPT has very good performances for small systems, but for systems with more than 50 units, the executions times reported by the proposed B&C are faster. Among the 3 solvers tested for 0.3% of tolerance for global optimality, CPLEX was the one showing best performance. The execution times of CPLEX are evenly comparable with the ones corresponding to the proposed B&C, showing that CPLEX solver is a competitive tool. Next, CPLEX was also tested by requiring 0.2% of tolerance. In this case, for systems with more than 50 and 70 units for the instances with and without ramp rate constraints, respectively, CPLEX could not reach the required global optimality tolerance within the imposed time limit.

In order to compare the quality of solutions provided by each method, the objective values of the best solutions obtained are listed in Table VI for both cases, with and without ramp rate constraints.

The objective values of the solutions provided by B&C, CPLEX and DICOPT are very similar. In most cases, CPLEX provides the best solutions followed by the proposed B&C. In fact, from Table VI it can be seen that the best solution for each instance is found by: CPLEX in 16 cases, the proposed B&C in 9 cases and DICOPT in 1 case. However, considering the small difference in the objective values, the comparison between CPLEX, B&C and DICOPT solutions is hardly relevant. On the other hand, SBB does not provide good solutions, and its convergence is the slowest. Although CPLEX is highly efficient in finding good feasible solutions, for nontrivial instances and tight tolerances it is not able to prove that those solutions effectively satisfy the tolerance for global optimality.

B. *Case study 2*

The proposed optimization approach was also applied to a test case study based in a Taipower 38-unit system, taken from [8]. Even though the number of units is smaller than that for the largest instance of the case study 1, the problem is particularly hard to solve. The planning time horizon is 24 hours with time periods of one hour. In order to reproduce the same conditions employed in other works, the spinning reserve requirement to be met is assumed to be 11% of the load demand for each time period. Hot and cold start up costs are equally set to a constant value (start-up cost). The 38-unit data and the hourly load distribution are given in Tables XII and XIII of the Appendix, respectively.

Again, two instances of the problem are considered: with and without ramp rate constraints. As in case study 1, three relative tolerances for global optimality were applied: 0.3%, 0.2% and 0.1%. The MIQP problem contains: 912 binary, 1824 continuous variables, and 13633 constraints with no ramp rate constraints. When ramp rate constraints are taken into account, the number of constraints increases to 15381. The computational results are reported in Table VII.

TABLE VII
COMPUTATIONAL RESULTS OF THE PROPOSED B&C: CASE STUDY 2

	Without ramp rate constraints				With ramp rate constraints			
Rel. Tol.	CPU time, s	nº nodes	Max. nodes	operat. cost, $	CPU time, s	nº nodes	Max. nodes	operat. cost, $
0,3%	30.2	3	2	203346218	57.6	7	4	203402782
0,2%	108.5	15	6	203323396	154.4	21	8	203402782
0,1%	311.8	49	16	203321193	572.3	77	20	203353299

In all cases, the required tolerance for global optimality is achieved in modest computational times. The optimal production schedules of the best solutions for the cases without and with ramp rate constraints are presented in Tables XVI and XVII of the Appendix, respectively. Case study 2 also illustrates the complexity that is added to the problem by the ramping constraints.

Different heuristic techniques have been proposed in the literature to address this case study. References [9], [10] and [11] propose and apply heuristic methodologies for this example and also list and compare the objective values and computational requirements provided by other authors addressing the same problem with other approaches.

The case study 2 was also solved with SBB, DICOPT and CPLEX solvers in GAMS with the purpose of comparing the proposed B&C search with the other deterministic techniques. For all cases, a time limit of 3600 seconds was imposed. For this case study, as in the first example, a tolerance for global optimality of 0.3% was required.

Table VIII summarizes the computational performance of SBB, DICOPT and CPLEX for the case study 2 for both instances: with and without ramping constraints. Unlike example 1, CPLEX could not converge even for 0.3% of optimality tolerance.

TABLE VIII
COMPUTATIONAL PERFORMANCE FOR CPLEX, SBB AND DICOPT: CASE STUDY 2

		Without ramp rate constraints	With ramp rate constraints
SBB	CPU time, s	3600	3600
	nodes	8173	6870
Relative	Iters.	400862	350740
tolerance required: 0.3%	reached relat. tol.	1,579%	1,601%
DICOPT	CPU time, s	3600	3600
Relative	major iter	101	81
tolerance required: 0.3%	Iters.	3117730	3532377
CPLEX	CPU time, s	3600	3600
	nodes	377310	311416
Relative	Iters.	19128290	15698237
tolerance required: 0.3%	reached relat. tol.	0,474%	0,516%

For this example, the advantage of applying the proposed approach against SBB, DICOPT and CPLEX solvers is even more evident. While the B&C search could achieve for both instances global optimality tolerances within 0.1% in reasonable execution times, none of the 3 tested solvers could converge after 1 hour. Here again, CPLEX showed better performance against SBB and DICOPT, even it did not converge; i.e. CPLEX found better solutions.

Table IX lists the objective values of the best solutions obtained with each method, for both instances: with and without ramping constraints.

TABLE IX
BEST OBJECTIVE VALUES (OPERATION COST) OBTAINED FOR EACH METHOD FOR CASE STUDY 2

	Best solutions: Operative cost, $	
	Without ramp rate constraints	With ramp rate constraints
B&C	203321193	203353299
SBB	204116508	204145376
DICOPT	204128604	203936021
CPLEX	203321193	203378227

As Table IX shows, the proposed B&C search found the best solution in both cases. With CPLEX the same solution is obtained for the instance with no ramping constraints, and a very good solution in the other case. This fact reaffirms that CPLEX is more efficient than SBB and DICOPT and can effectively provide a good-quality solution. However, it fails to prove that the tolerance for global optimality is satisfied. On the other hand, for this same case study, SBB and DICOPT cannot find the global solutions.

V. CONCLUSIONS

In this paper and its companion [1], a deterministic optimization approach for solving the SCUC problem has been presented. It consists of a Branch-and-Cut search that capitalizes on new integer cutting planes developed specifically for the SCUC problem. The cuts and the Branch-and-Cut search are presented in detail in the first part of this two-series paper, whereas, the present paper analyzes the computational performances of this methodology.

Firstly, the performance of the proposed cuts was properly exemplified. The examples showed that the cuts are highly efficient for reducing the relaxation gap, which consequently, will accelerate the algorithm convergence.

Then, the performance of the proposed methodology in solving the MIQP was tested with two case studies and their variants, which are widely addressed in the literature as test problems. The approach proved to be highly efficient, since it could achieve up to 0.1% of tolerance for global optimality in all cases, within reasonable computational times. Comparisons with other deterministic solvers, SBB, DICOPT and CPLEX, were also carried out. The proposed B&C approach, while having only a small effect on the objective function value, for most of the cases requires significantly less CPU time compared to SBB, DICOPT and CPLEX solvers.

Summarizing, as illustrated by the results presented in this papers, the proposed deterministic optimization approach can effectively solve the SCUC problem for global optimality within a specified tolerance. Furthermore, the computational requirements of the B&C approach remain at an acceptable level for the case studies addressed in this paper.

VI. APPENDIX

A. *Data for case study 1*

The data for the 10-unit system of case study addressed in section IV.A are provided in Tables X and XI. ([3]).

TABLE X
10-UNIT SYSTEM DATA FOR CASE STUDY 1

Units	p_i^L (MW)	p_i^U (MW)	a_i ($/h)	b_i ($/MWh)	c_i ($/MW^{2}h)	TU_i (h)	TD_i (h)	Hsc_i ($/h)	Csc_i ($/h)	T^{cold}_i (h)	T^{ini}_i (h)
1	150	455	1000	16.19	0.00048	8	8	4500	9000	5	8
2	150	455	970	17.26	0.00031	8	8	5000	10000	5	8
3	20	130	700	16.60	0.00200	5	5	550	1100	4	-5
4	20	130	680	16.50	0.00211	5	5	560	1120	4	-5
5	25	162	450	19.70	0.00398	6	6	900	1800	4	-6
6	20	80	370	22.26	0.00712	3	3	170	340	2	-3
7	25	85	480	27.74	0.00079	3	3	260	520	2	-3
8	10	55	660	25.92	0.00413	1	1	30	60	0	-1
9	10	55	665	27.27	0.00222	1	1	30	60	0	-1
10	10	55	670	27.79	0.00173	1	1	30	60	0	-1

TABLE XI
LOAD DEMAND FOR CASE STUDY 1

Hour	1	2	3	4	5	6
Demand (MW)	700	750	850	950	1000	1100
Hour	7	8	9	10	11	12
Demand (MW)	1150	1200	1300	1400	1450	1500
Hour	13	14	15	16	17	18
Demand (MW)	1400	1300	1200	1050	1000	1100
Hour	19	20	21	22	23	24
Demand (MW)	1200	1400	1300	1100	900	800

B. Data for case study 2

The data for the 38-unit system of case study addressed in section IV.B are provided in Tables XII and XIII. ([8]).

TABLE XII
TAIPOWER 38-UNIT SYSTEM DATA FOR CASE STUDY 2

units	p_i^L (MW)	p_i^U (MW)	a_i ($/h)	b_i ($/MWh)	c_i ($/MW^{2}h)	TU_i (h)	TD_i (h)	*Startup cost*($/h)	UR_i (MW/h)	DR_i (MW/h)
1	220	550	64782	796.9	0.3133	18	8	805000	92	138
2	220	550	64782	796.9	0.3133	18	8	805000	92	138
3	200	500	64670	795.5	0.3127	18	8	805000	84	120
4	200	500	64670	795.5	0.3127	18	8	805000	84	120
5	200	500	64670	795.5	0.3127	18	8	805000	84	120
6	200	500	64670	795.5	0.3127	18	8	805000	84	120
7	200	500	64670	795.5	0.3127	18	8	805000	84	120
8	200	500	64670	795.5	0.3127	18	8	805000	84	120
9	200	500	172832	915.7	0.7075	7	7	402500	128	256
10	114	500	172832	915.7	0.7075	7	7	402500	128	256
11	114	500	176003	884.2	0.7515	7	7	402500	128	256
12	114	500	173028	884.2	0.7083	7	7	402500	128	256
13	110	500	91340	1250.1	0.4211	9	8	575000	110	170
14	90	365	63440	1298.6	0.5145	12	8	575000	92	125
15	82	365	65468	1298.6	0.5691	12	8	575000	92	125
16	120	325	72282	1290.8	0.5691	10	8	575000	82	125
17	65	315	190928	238.1	25.881	1	1	23000	320	70
18	65	315	285372	1149.5	38.734	1	1	23000	320	70
19	65	315	271376	1269.1	36.842	1	1	23000	320	70
20	120	272	39197	696.1	0.4921	9	8	575000	55	91
21	120	272	45576	660.2	0.5728	9	8	575000	55	91
22	110	260	28770	803.2	0.3572	11	8	460000	53	132
23	80	190	36902	818.2	0.9415	14	7	92000	48	98
24	10	150	105510	33.5	52.123	1	1	23000	460	20
25	60	125	22233	805.4	11.421	8	8	115000	42	60
26	55	110	30953	707.1	20.275	14	7	287500	28	56
27	35	75	17044	833.6	30.744	14	7	253000	20	38
28	20	70	81079	2188.7	16.765	1	1	5750	70	30
29	20	70	124767	1024.4	26.355	1	1	5750	70	30
30	20	70	121915	837.1	30.575	1	1	5750	70	30
31	20	70	120780	1305.2	25.098	1	1	5750	75	30
32	20	60	104441	716.6	33.722	1	1	7670	70	30
33	25	60	83224	1633.9	23.915	1	1	7670	70	30
34	18	60	111281	969.5	32.562	1	1	7670	70	20
35	8	60	64142	2625.8	18.362	1	1	7670	70	20
36	25	60	103519	1633.9	23.915	1	1	7670	75	30
37	20	38	13547	694.7	8.482	11	8	69000	10	20
38	20	38	13518	655.9	9.693	11	8	69000	10	20

TABLE XIII
LOAD DEMAND FOR CASE STUDY 2

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Demand (MW)	5700	5400	5150	4850	4950	4800	4850	5400	6700	7850	8000	8100	6900	8150	8250	8000	7800	7100	6800	7300	7100	6800	6550	6450

C. Optimal production schedules for case study 1

The optimal production schedules for the 100-unit systems neglecting and including ramp rate constraints are presented in Tables XIV and XV, respectively.

TABLE XIV
CASE STUDY 1:OPTIMAL PRODUCTION SCHEDULE (MW) FOR THE 100-UNIT SYSTEMS, WITHOUT RAMP RATE CONSTRAINTS

	Time period																							
units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-10	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
11-18	245.0	295.0	390.0	451.5	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	375.0	325.0	425.0	455.0	455.0	455.0	455.0	377.5	350.0
19-20	245.0	295.0	390.0	451.5	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	375.0	325.0	425.0	455.0	455.0	455.0	455.0	377.5	0.0
21	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
22-23	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
24-25	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
26-27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
28-30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
31	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
32	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
33-34	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
35	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
36-39	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
41-42	0.0	0.0	25.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.2	0.0	0.0
43	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.2	25.0	0.0
44-45	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.1	0.0	0.0
46-47	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	0.0	0.0	0.0
48-49	0.0	0.0	0.0	0.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.1	0.0	0.0
50	0.0	0.0	0.0	0.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	0.0	0.0	0.0
51-52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
53-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	0.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
57-58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	0.0	0.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
59-60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
61-62	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
63-64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
66	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
67-70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
71-72	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0
73-77	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
81-88	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
89-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
91-92	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
93-98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99-100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XV
CASE STUDY 1: OPTIMAL PRODUCTION SCHEDULE (MW) FOR THE 100-UNIT SYSTEMS, INCLUDING RAMP RATE CONSTRAINTS

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-10	455.0	451.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
11-18	245.0	299.0	390.0	451.5	455.0	455.0	455.0	442.2	451.2	455.0	455.0	455.0	455.0	439.2	455.0	364.0	325.0	402.9	455.0	455.0	455.0	452.9	377.5	350.0
19-20	245.0	299.0	390.0	451.5	455.0	455.0	455.0	442.2	451.2	455.0	455.0	455.0	455.0	439.2	455.0	364.0	325.0	402.9	455.0	455.0	455.0	452.9	377.5	0.0
21	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
22-23	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
24-25	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
27-28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
31-32	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
33-34	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
35	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
36-38	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
39-40	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
41	0.0	0.0	25.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
42	0.0	0.0	25.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
43-44	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
45	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	25.0	0.0
46-47	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
48	0.0	0.0	0.0	0.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
49-50	0.0	0.0	0.0	0.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
51-52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
53	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	80.0	80.0	0.0	0.0	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
54-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
56-57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	0.0	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
59	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	80.0	80.0	80.0	0.0	0.0	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	80.0	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
61	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
62-64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
66-67	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
68-69	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
71-72	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	37.0	44.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
73-78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.0	10.0	21.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
79-80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
81-82	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
83-88	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
89	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
91	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
92-98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99-100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE XVI
CASE STUDY 2: OPTIMAL PRODUCTION SCHEDULE (MW) FOR THE SYSTEM WITHOUT RAMP RATE CONSTRAINTS

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-2	550.0	486.1	476.5	442.6	453.8	437.0	442.6	495.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0
3-8	500.0	489.3	479.6	445.7	456.9	440.0	445.7	498.2	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	279.3	422.2	433.2	405.5	401.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	279.3	422.2	433.2	405.5	401.0	353.3	341.2	385.2	353.3	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	386.1	402.7	413.8	283.9	418.4	428.8	402.7	398.5	353.6	342.2	383.6	353.6	363.1	359.4	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	411.8	409.7	427.3	439.0	301.3	443.9	454.9	427.3	422.8	375.2	363.1	407.0	375.2	385.3	381.3	388.3
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	258.2	254.6	284.2	303.9	110.0	312.2	330.7	284.2	276.7	196.6	176.3	250.2	196.6	213.6	207.0	218.7
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90.0	164.2	161.2	185.5	201.6	90.0	208.4	223.5	185.5	179.4	113.8	97.1	157.6	113.8	127.7	122.3	131.9
15	82.0	82.0	82.0	82.0	82.0	82.0	82.0	82.0	148.4	145.8	167.7	182.3	82.0	188.4	202.1	167.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	184.1	166.8	0.0	0.0	0.0	0.0	0.0	0.0	237.5	236.9	241.8	245.0	0.0	246.3	249.3	241.8	240.5	227.5	0.0	236.2	227.5	230.3	0.0	231.1
18-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20-21	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0
22	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
23	190.0	150.5	147.2	136.0	139.7	134.1	136.0	153.4	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0
24	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	125.0	125.0	125.0	117.7	120.8	116.1	117.7	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
26	110.0	97.3	95.8	90.5	92.3	89.7	90.5	98.6	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0
27	58.2	43.6	42.6	39.1	40.3	38.6	39.1	44.5	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
28-34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.0	38.0	38.0	38.0	36.3	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0
38	27.6	23.0	22.7	21.6	21.9	21.4	21.6	23.3	38.0	38.0	38.0	38.0	33.8	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0

D. *Optimal production schedules for case study 2*

The optimal production schedules for the 38-unit systems neglecting and including ramp rate constraints are presented in Tables XVI and XVII, respectively

TABLE XVII
CASE STUDY 2: OPTIMAL PRODUCTION SCHEDULE (MW) FOR THE SYSTEM WITH RAMP RATE CONSTRAINTS

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-2	550.0	486.1	476.5	442.6	453.8	437.0	442.6	493.3	550.0	550.0	550.0	550.0	516.5	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0
3-8	500.0	489.3	479.6	445.7	456.9	440.0	445.7	496.5	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	302.1	430.1	433.2	405.5	401.0	353.3	341.2	385.2	353.3	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	302.1	430.1	433.2	405.5	401.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	386.1	402.7	413.8	301.6	429.6	428.8	402.7	398.5	353.6	342.2	383.6	353.6	363.1	359.4	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	411.9	409.7	427.3	439.0	323.9	451.9	454.9	427.3	422.8	375.2	363.1	407.0	375.2	385.3	381.3	388.3
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	258.3	254.6	284.2	303.9	163.0	273.0	330.7	284.2	276.7	196.6	176.3	250.2	196.6	213.6	207.0	218.7
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90.0	164.3	161.2	185.5	201.6	90.0	182.0	223.5	185.5	179.4	113.8	97.1	157.6	113.8	127.7	122.3	131.9
15	82.0	82.0	82.0	82.0	82.0	82.0	82.0	82.0	148.5	145.8	167.7	182.3	82.0	174.0	202.1	167.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	184.1	166.8	0.0	0.0	0.0	0.0	0.0	0.0	237.5	236.9	241.8	245.0	0.0	291.5	249.3	241.8	240.5	227.5	0.0	236.2	227.5	230.3	0.0	231.1
18-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20-21	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0
22	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
23	190.0	150.5	147.2	136.0	139.7	134.1	136.0	152.8	190.0	190.0	190.0	190.0	160.6	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0
24	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	125.0	125.0	125.0	117.7	120.8	116.1	117.7	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
26	110.0	97.3	95.8	90.5	92.3	89.7	90.5	98.4	110.0	110.0	110.0	110.0	102.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0
27	58.2	43.6	42.6	39.1	40.3	38.6	39.1	55.0	75.0	75.0	75.0	75.0	55.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
28-34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.0	38.0	38.0	38.0	28.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0
38	27.6	23.0	22.7	21.6	21.9	21.4	21.6	27.5	37.5	38.0	38.0	38.0	28.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0

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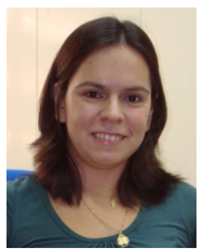

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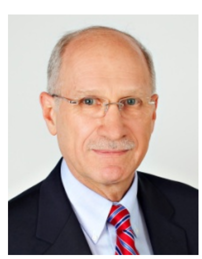

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