Multiperiod Optimization Model for Oilfield Production Planning: Bicriterion Optimization and Two-Stage Stochastic Programming Model

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Abstract

In this work, we present different tools of mathematical modeling that can be used in Oil and Gas industry to help improve production decision making for field development, production optimization and planning. Firstly, we formulate models to compare multiperiod optimization and single period optimization for the maximization of net present value and the maximization of total oil production. This study helps to identify the importance of multiperiod optimization in Oil and Gas production planning. Further, we formulate a bicriterion optimization model to determine the ideal compromise solution between maximization of the two objective functions, the net present value (NPV) and the total oil production. To account for the importance of hedging against uncertainty in oil production, we formulate a two-stage stochastic programming model to compute an improved expected value of NPV and total oil production in the oil well network model subject to uncertainties in oil prices and productivity indices.

1. Introduction

Hydrocarbon production development is a complex process that encompasses physics of multiphase fluid flow from reservoir, wells, pipelines and processing equipment as well as infrastructure management for drilling and operating under safe conditions. In a reservoir, production is often constrained by the reservoir petrophysics and back-pressure as a result of flow characteristics of fluids in the pipelines and fluid-handling capacity of surface facilities, as well as safety and economic considerations [1]. Determination of optimal hydrocarbon production requires planning at several horizons from one year up to a specific time horizon for the lifespan of the reservoir [2]. In many oil and gas industries the planning and decision-making takes places for a single time period, but this may result in suboptimal solutions. For a better decision-making optimization over a time horizon, multiperiod optimization is a better approach. The multiperiod problem is capable to provide optimal solutions that take into account decision variables over a long-time span and are not subject to short-term changes in the parameters related to hydrocarbon production form the reservoirs.

Production and injection well positioning, planning and surface network optimization are some aspects of optimization in Oil and Gas field planning. The optimization of the aspects mentioned above impacts the capital investment and profit generation in oil production facilities. Several studies have focused on production planning of oilfields, such as the work done by Gunnerud and

Foss, 2009 [2]; Camponogara, Plucenio, Teixeira, Campos, 2010 [30]; Kosmidis, Perkins, Pistikopoulos, 2005 [20]. These studies considered the production planning optimization problem taking into account the constraints for pressure drop in the oil wells, surface networks and incorporate gas lift in the optimization problem. In this study, we address a subset of the production planning problem taking into account the optimization of oil production in the oil well production planning problem. The works mentioned above have used models of varying complexity, but they have focused on a single objective function, i.e., the maximization of total oil production for the oilfields. In our study, we aim to highlight that maximization of total oil production is not the same as the maximization of net present value and present a formulation to determine a better solution.

Isebor, Durlofsky, 2014[26]; address a general oilfield planning and production optimization problem. In their work, they consider a bi-objective optimization problem of maximization of net present value and maximization of total oil production. They present an algorithm to approximate the Pareto curve by handling the bi-objective problem as a single objective optimization problem. In their study, they have used a Single Objective Product Formulation (SOPF) and apply it to the case of optimization of two objective functions. SOPF (Audit et al. 2008 [31]) requires an identification of a reference point "r" in the objective space. Points are generated in the objective space such that the distance between the new solution points and reference point are maximized. These points identify the subsets of Pareto front and by varying the position of r different portions of the Pareto front are generated. The study applies SOPF such that it combines a two objective optimization in a way that maximizes the objective space with the reference point, and a Pareto approximation is generated. Furthermore, they approximate the compromise solution from the Pareto curve generated using the SOPF. In our study, we present a method to obtain the Pareto curve for the maximization of the two objective functions and obtain the exact compromise solution. Details of the formulation to determine the ideal compromise solution for bicriterion optimization are explained in appendix A of the paper.

In this paper, we address an integrated production planning problem from reservoir to surface. The multiperiod problem is optimized to determine the production profiles of the oil wells for the maximization of the net present value (NPV) and the maximization of the total oil production over the time horizon. In this paper, we address the multiobjective problem for oil production optimization. The maximization of two conflicting objective functions, the NPV and the total oil production, are optimized using the approach used by Grossmann and Drabbant [12] to obtain the Pareto optimal solution and the ideal compromise solution for the oil production planning problem.

The process of oil production involves uncertainty. Hence, to optimize the production from oil wells this uncertainty in prices and other parameters such as productivity indices, is to be considered for optimization. The uncertainty in the parameters affects the oil production from the oil wells and hence the revenue generation from the production. Significant work has been done to handle uncertainty in optimization. Sahinidis, 2004 [32] and Grossmann et al. 2016 [33] present an overview of modeling techniques such as stochastic programming and robust optimization, for process systems that involve uncertainty in variables. In this study, we present a two-stage stochastic programming model. The stochastic model is based on the impact of uncertainty in parameters such as oil prices and productivity indices on the optimization on the objective functions.

The outline of this paper is as follows. First, we describe the background of the research work done on oilfield planning and production modeling. Second, we present a general overview of the research work done on oilfield production planning problem and formulate a multiperiod nonlinear programming model for a production planning problem. In addition, we propose a simplified 5 oil well model and solve the problem for a time horizon of 20 years to optimize the net present value and total oil production over the time horizon. Based on the solutions of the multiperiod NLP problem we generate a Pareto curve with the ϵ-constrained method and present a direct approach to find the ideal compromise solution. Finally, we describe a two-stage stochastic programming formulation to the expected NPV and oil production by accounting for the uncertain oil prices and productivity indices.

2. Background

Oil production from a reservoir involves different steps: exploration, appraisal, development and production. At each step, decisions are made that affect the overall performance of the oil field. Gupta and Grossmann [4], proposed a multiperiod Mixed-integer Nonlinear Programming (MINLP) model for optimal planning of offshore oil and gas infrastructure. In their work, they considered the three components oil, gas and water explicitly in their formulation. The nonlinear behavior of the reservoir is approximated by third or higher order polynomials. The model is reformulated as a mixed-integer linear programming (MILP) model. We consider the model proposed by Gupta and Grossmann [4], as reference for our model development. Significant work has been done to capture the effect of pressure drop in the oil well and surface network, e.g. mechanistic correlations [17], empirical correlations [17] and Gilbert curve [17, 23]. In this study, for the sake of simplicity, we do not consider pressure drop models for the oil wells.

Several operations are associated with the oil production from a reservoir. It involves complex fluid flow with the components water, oil and gas flowing in the pipelines together. The exact prediction of the properties of this multiphase fluid flow is difficult. Hence, the fluid flow is approximated by single-phase flow or two-phase flow. Before the start of production, the reservoir has a shut-in pressure that corresponds to the maximum pressure of the reservoir. The reservoir pressure might be high such that no external assistance is required to carry the fluid to the surface. However, during the field life the reservoir pressure steadily decreases and requires external methods such as gas lift or pumping to sustain production at operating and economical conditions. The wellhead pressure controls the fluid flow in the oil well by adjusting the bottomhole pressure. The difference between the reservoir pressure and bottomhole pressure prevents the entry of sand particles from entering in the well [17].

The total liquid produced from a reservoir is determined from the Inflow Performance Relationship (IPR). IPR determines the functional relationship between the production rate from the reservoir and the Bottom Hole Flowing Pressure (BHFP). IPR is derived from an approximation of Darcy's law [17, 24, 25] for single-phase liquid flow and is used to determine the total production rate. This correlation depends on the productivity index, bottomhole pressure of well and the reservoir pressure. The approximated formulation of Darcy's law (IPR) is represented as follows,

$$J = q_o / (p_r - p_{wf}) \tag{1}$$

where, J: Productivity index,

p_r, p_{wf}: average reservoir pressure, bottom hole flowing pressure,

q_o: oil flowrate into the well.

The liquid production depends on the bottomhole pressure and the productivity index, which is the capability of a well to produce oil. The factors that affect the productivity index are reservoir drainage area, pay zone thickness, effective permeability of the formation of the oil, well length, fluid velocity and well completion method. The fluid produced in the well is directed to a separator, which is an equipment that separates the gas, oil and water from fluid produced from the reservoir. The capacity of the separator limits the production from the oil well it is a physical design constraint in the model. The oil and gas obtained from the separator is sent to the downstream processing. Further, as the oil is extracted from wells, the water oil ratio (water cut), gas oil ratio and reservoir pressure vary nonlinearly as a function of the cumulative oil recovered from the wells. These relations of the water oil ratio, gas oil ratio and pressure are obtained from on surface characterization and dynamic modeling studies. In this paper, the sets of pressure, cumulative offtake, GOR (gas oil ratio), WCT (water cut) curves have been extracted from numerical reservoir simulations because directly integrating such software adds a significant level of complexity.

3. Problem statement

Given is a reservoir with a set of oil wells, well = {well 1, well 2…}. The oil production problem is considered for a given time horizon of TH years. The pressure profiles, gas oil ratio and water cut have a linear or polynomial correlation with the cumulative oil produced from the respective wells. These correlations are obtained from actual data from oilfields. The productivity indices are fixed for the wells and do not vary with time. Further, the maximum cumulative liquid that can be produced from all the wells is limited by the capacity of the separator to handle processing of liquid into oil, gas and water. The revenue generation from the oil production depends on the selling prices of oil and gas obtained after from the oil wells. The cost of the oil production depends on the compression cost for the gas produced from the oil wells and the processing of water. The cost and price parameters are fixed for the model. Fluctuation in the oil prices, productivity indices takes place over the course of production from the oil wells. The uncertainty of the parameters is used to develop a stochastic model.

To determine the optimum oil production from the wells two objective functions can be formulated. First, the maximization of the net present value (NPV) for the oil wells results in the optimization of the oil production subject to increase in the capital generation, which is dependent on the revenue and cost associated with oil production. Second, the objective function to maximize the total oil production from the wells over the time horizon aims to increase the oil production form each well, but does not focus on the optimization of the revenue generation. The model is solved for a time horizon of TH years with the objective to maximize the net present value of production. Further, the simplified model is solved to maximize the total oil production from all the wells over the long-term time horizon of TH years.

The time horizon of TH years is discretized into intervals Δt, of one year time periods. It is assumed that the surface pipeline network is already established, and the reservoir depletes at a natural rate. The liquid flow from the oil wells is combined and transferred to the separator, which is assumed to be of fixed capacity. It is also assumed that there is no pressure drop in the oil wells and the surface network. Further, in the IPR correlation the bottomhole pressure is assumed to be zero. The constraint equations formulated for oil and gas production are nonlinear yielding a multiperiod nonlinear programming model. This multiperiod NLP model is solved using the global optimization solver BARON [13] and local NLP solvers such as CONOPT [36], SNOPT [34, 35].

3.1 Multiperiod nonlinear programming (NLP) model

The multiperiod NLP model has objective functions to either maximize the NPV or maximize total oil production in the long-term time horizon TH. The initial investment for oil filed planning is not included in the objective functions since it is constant and is paid up-front.

The cumulative oil produced $Rorc_{w,t}$ for each well w for the time period t, is computed by summing up the oil production $ro_{w,t}$ from each well until the time T.

$$Rorc_{w,t} = \sum_{t}^{T} (ro_{w,t}) \qquad \forall\, t <= time\ ;\ T,\ time \in TH\ ;\ w \in well \qquad (2)$$

The water cut $WCT_{w,t}$, gas oil ratio $GOR_{w,t}$ and pressure variation $Pr_{w,t}$ for each well at time t is computed using the given empirical correlations (linear equations or polynomial correlations) of the cumulative oil production $Rorc_{w,t}$. These correlations of pressure variation, gas oil ratio and water cut are obtained from data from geological studies of reservoir.

$$Pr_{w,t} = f(Rorc_{w,t}) \qquad \forall\, t \in TH\ ;\ w \in well \qquad (3)$$

$$WCT_{w,t} = f(Rorc_{w,t}) \qquad \forall\, t \in TH\ ;\ w \in well \qquad (4)$$

$$GOR_{w,t} = f(Rorc_{w,t}) \qquad \forall\, t \in TH\ ;\ w \in well \qquad (5)$$

The liquid produced $rL_{w,t}$ from the oil wells is determined from the productivity indices PI_w and the pressure correlation $Pr_{w,t}$ (Eq 3). Given is the inflow performance relationships mentioned earlier (eq 1) with the assumption that the bottomhole pressure (BHP) is zero,

$$rL_{w,t} = PI_w * Pr_{w,t} \qquad \forall\, t \in TH\,;\, w \in well \qquad (6)$$

The oil produced $ro_{w,t}$ is computed from the total liquid produced (Eq6) from the oil wells and the water cut correlation (Eq4).

$$ro_{w,t} = rL_{w,t} * (1\text{-}WCT_{w,t}) \qquad \forall\, t \in TH\,;\, w \in well \qquad (7)$$

The gas produced $rg_{w,t}$ is calculated from the gas oil ratio (Eq5) and the oil produced from each well $ro_{w,t}$.

$$rg_{w,t} = ro_{w,t} * GOR_{w,t} \qquad \forall\, t \in TH\,;\, w \in well \qquad (8)$$

The total liquid produced TrL_t, total oil produced Tro_t , total gas produced Trg_t and total water produced Trw_t are computed by summing the production of liquid, oil, gas and water over the wells w.

$$TrL_t = \textstyle\sum_{well}(rL_{w,t}) \qquad \forall\, t \in TH\,;\, w \in well \qquad (9)$$

$$Trg_t = \textstyle\sum_{well}(rg_{w,t}) \qquad \forall\, t \in TH\,;\, w \in well \qquad (10)$$

$$Tro_t = \textstyle\sum_{well}(ro_{w,t}) \qquad \forall\, t \in TH\,;\, w \in well \qquad (11)$$

$$Trw_t = \textstyle\sum_{well}(rw_{w,t}) \qquad \forall\, t \in TH\,;\, w \in well \qquad (12)$$

The model has constraints on the total liquid Tro_t that can be processed in the separator. In addition, there is a constraint on the total oil produced from each well (maximum oil produced MO_{well}).

$$Tro_t \leq Sep \qquad \forall\, t \in TH \qquad (13)$$

$$ro_{w,t} \leq MO_{well} \qquad \forall\, t \in TH\,;\, w \in well \qquad (14)$$

The NPV depends on the revenue CR_t and cost CC_t associated with the oil production. The revenue is generated from the price po_t of total oil produced Tro_t, and the price pg_t of total gas produced Trg_t at each time interval from the wells. The cost is calculated from cost $gcct$ for the compression of gas produced $Trgt$ and the cost of wtc_t treatment of water produced Trw_t during oil production.

$$CR_t = \Delta t * (po_t * Tro_t + pg_t * Trg_t) \qquad \forall\, t \in TH \qquad (15)$$

$$CC_t = \Delta t * (gcc_t * Trg_t + wtc_t * Trw_t) \qquad \forall\, t \in TH \qquad (16)$$

As discussed earlier, for oil production optimization two objective functions can be used for the optimization the oil production. First, the maximization of the NPV that depends on the revenue CR_t and cost CC_t associated with oil production discounted over the time. Second, the maximization of the total oil production Z that is the sum of the total oil produced Tro_t summed over the time horizon.

max. $NPV = \sum_{TH}(disc_t * (CR_t - CC_t))$ $\quad \forall\, t \in TH \quad$ (17)

max. $Z = \sum_{TH}(Tro_t)$ $\quad \forall\, t \in TH \quad$ (18)

The model can be solved using nonlinear programming local NLP solvers (CONOPT [36], SNOPT [34, 35]) or global NLP solvers (BARON [17]).

4. Numerical results

The model is solved, and three studies are performed using the simplified multiperiod NLP model:

i) The multiperiod model, Eq 2- Eq 18, formulated above is solved for maximization of the NPV simultaneously for all time periods (single period optimization) and compared to case where the model is solved sequentially for each time period (multiperiod optimization).

ii) The bicriterion optimization problem (18) is solved to determine the Pareto curve (see appendix A) between the net present value and the total oil production to determine the optimal tradeoffs between the two objective functions. Further, a model is formulated to find the ideal compromise solution between the two objective functions.

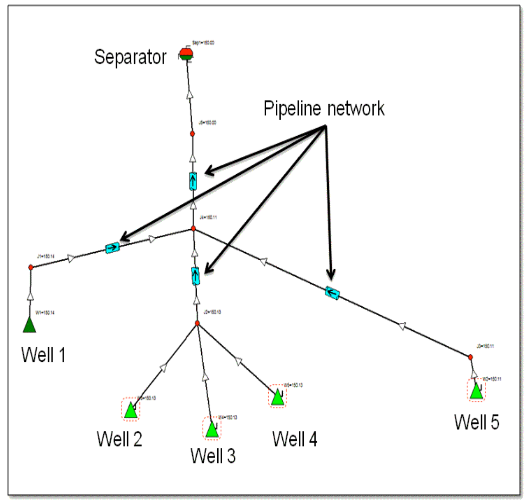

Fig 1 Oil wells network

iii) The multiperiod model formulated is assumed to have uncertainty in the oil prices and productivity index. A two-stage stochastic model is developed and solved for the two objective functions subject to uncertainty in oil prices, productivity index.

The simplified multiperiod NLP model is solved for a case of five wells, Well = {well 1, well 2, well 3, well 4, well 5} over a time horizon of 20 years. The time horizon is discretized into one year time periods. The separator capacity is fixed to 8000 stock tank barrel per day (stb /day). Fig. 1 shows the five oil wells network. The oil prices in the model range from 28 USD per barrels to 84 USD per barrels and the gas prices are in the range 0.65 USD per MMBTU to 1.3 USD per MMBTU. The rate of return to compute the NPV for the multiperiod model is 10%.

The multiperiod nonlinear programming model is solved using the global solver BARON and the local solvers CONOPT [36] and SNOPT [34, 35]. The results of the global solver and local solvers were the same, but the computation time varied.

4.1 Five well production model

a) Oil production for the maximization of NPV:

The multiperiod NLP model with an objective function to maximize the NPV yields an NPV of 6401.8 Million USD and a total oil production of 18.5 Million stock tank barrels for a time horizon of 20 years. The model consists of 1,115 equations with 1,073 variables. The global optimization solver BARON [13] is used to solve the problem with maximum time limit of 1000 CPU seconds. The model was also solved using CONOPT [36] (0.374 CPU seconds) and SNOPT [34, 35] (0.437 CPU seconds). The results of the local solvers are the same as that of BARON. The wells oil production profiles are shown in Fig. 2.

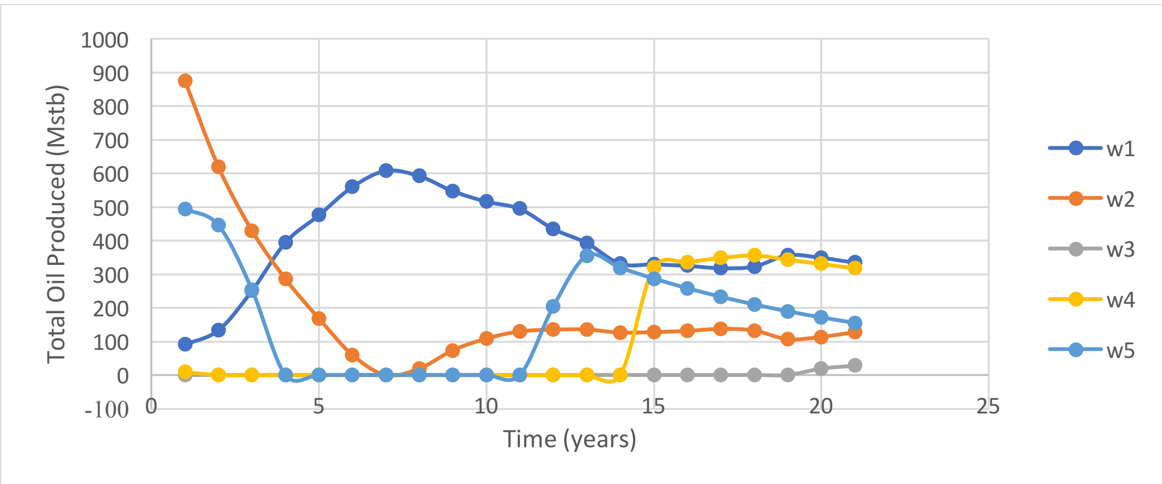

Fig. 2 Production profiles for maximization of Net present value over 20 years

b) Oil production for maximization of total oil production:

The multiperiod NLP model with an objective function to maximize the total oil production over a time horizon of 20 years yields an NPV of 6361.297 Million USD and a total oil production of 22.56 Million stock tank barrels for a time horizon of 20 years. The model consists of 1,115 equations with 1,073 variables. The global optimization solver BARON [13] is used to solve the problem with maximum time limit of 1000 CPU seconds. The model was also solved using CONOPT [36] (0.203 CPU seconds) and SNOPT [34, 35] (0.608 CPU seconds). The results of the local solvers are the same as that of BARON. The wells oil production profiles are shown in Fig. 3.

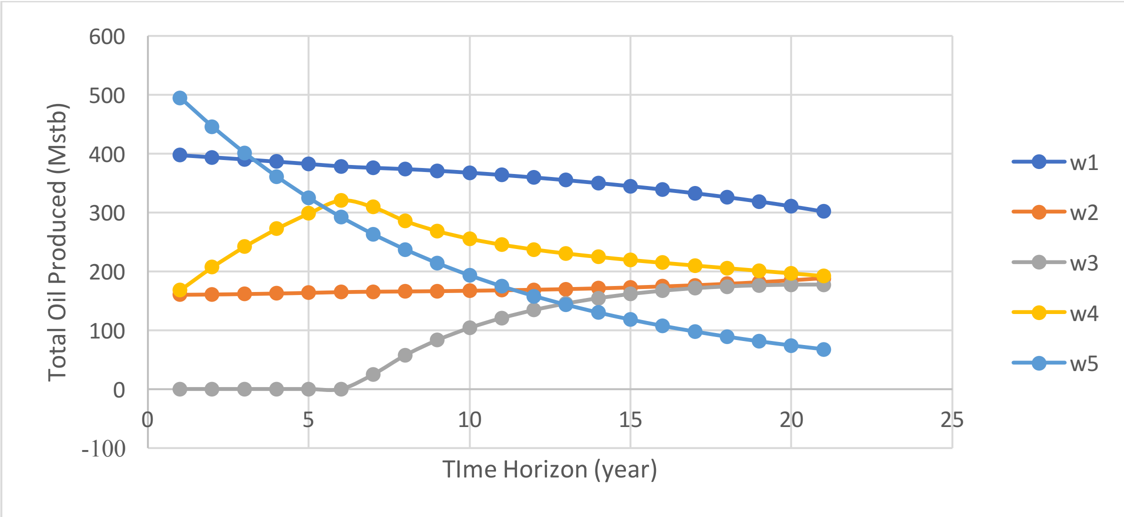

Fig. 3 Production profiles for maximization of total oil production over 20 years

4.2 Comparison of Single period optimization (SP) and Multiperiod optimization (MP)

The nonlinear programming model is solved successively for the single period optimizations over the time horizon of 20 years and compared to the multiperiod optimization for the two objective functions in eq. 17 and eq. 18.

Case a: Maximization of net present value over 20 years. The results of the multiperiod optimization model (MP) and single period model (SP) to optimize the NPV over the time horizon of 20 years yields the results shown in Table 1.

Table 1. Results of multiperiod vs single period optimization for max. NPV.

	NPV_MP (MUSD)	Total oil_MP (MMstb)	NPV_SP (MUSD)	Total oil_SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
Total	6401.8	18.504	6400.493	18.863	1.35	-0.359

The results of multiperiod and single period optimization show that the total NPV value for 20 year time period (6401.8 MUSD) is greater than the NPV summation for single period optimization for a period of 20 years (6400.493 MUSD). The model has a gain of 1.35 million USD (0.021 %) for the multiperiod optimization. In addition, the total oil produced is greater for the single period optimization by 0.359 MMstb (1.93 %). This shows that single period optimization yields 1.93 % more oil production than the multiperiod optimization.

Case b: Maximization of total oil production over 20 years. The results of multiperiod optimization model (MP) and single period model (SP) to optimize the total oil production over the time horizon of 20 years yields following results:

Table 2. Results of multiperiod vs single period optimization for max. Total oil production.

	NPV_MP (MUSD)	Total oil_MP (MMstb)	NPV_SP (MUSD)	Total oil_SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
Total	6361.3	22.563	6368.047	22.203	-6.75	0.315

The results of the multiperiod and single period cases for the objective function to maximize total oil production for a time horizon of 20 years, show that the total NPV value for multiperiod optimization (6361.3 MUSD) is less than summed NPV of single period optimization (6368.047 MUSD), which shows a gain of 6.75 million USD (0.12 %) for multiperiod over the single period optimization. Further, the total oil produced is greater for the multiperiod optimization by 0.315 MMstb, i.e., multiperiod optimization has 1.6% more oil production rate than single period.

For both cases, there are small differences in values because we are using a reduced model that does not account for depletion. Further, we perform a case study for different rates of return and oil prices. This study helps to understand the impact of different rates of return and oil prices on the NPV value, because the difference between the NPV for multiperiod and single period optimization is less for the base case solved above.

We consider five cases to study the effect rate of return and oil prices has on the two objective functions. The cases are as follow:

- BC: Base case with 10 % rate of return.
- LI: Low rate of return 5%
- HI: High rate of return 15%
- DO: Oil prices in descending order for base case
- AO: Oil price are in ascending order for base case

Table 3. Results of max NPV and max total oil production.

Case	NPV MP (MUSD)	Total oil produced MP (MMstb)	NPV SP (MUSD)	Total oil produced SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
BC	6401.84	18.5	6400.49	18.86	1.35	-0.36
LI	9254.67	18.27	9250.91	18.86	3.766	-0.6
HI	4843.16	18.65	4842.73	18.86	0.427	-0.21
DO	6664.86	18.81	6662.88	18.53	1.982	0.274
AO	6430.18	19.73	6421.27	20.48	8.91	-0.76

Table 3 shows the results for multiperiod and single period optimization for maximization of NPV for the five cases BC, LI, HI, DO, AO. The case AO in which the oil prices are in ascending order in the range from 28 USD per barrel to 84 USD per barrel yields the results with the largest difference in the NPV between SP and MP optimization (8.91 MUSD).

Table 4. Results of max NPV and max total oil production.

Case	NPV MP (MUSD)	Total oil Produced MP (MMstb)	NPV SP (MUSD)	Total oil Produced SP (MMstb)	Δ NPV (MUSD)	Δ Oil Production (MMstb)
BC	6361	22.56	6368	22.2	-6.5	0.36
LI	9199	22.56	9201	22.2	-1.43	0.36
HI	4810	22.56	4820	22.2	-9.69	0.36
DO	6627	22.56	6644	22.2	-17.4	0.36
AO	6406	22.56	6406	22.2	0.063	0.36

Table 4 shows the results for multiperiod optimization and single period optimization for the maximization of the total oil production for the cases BC, LI, HI, DO and AO. The NPV value is better for multiperiod optimization is better than the single period (0.063 MUSD) for AO. The difference in the total oil production between multiperiod and single period optimization is same for the five cases.

The results from the case study indicate the case in which the oil prices are in ascending order from 28 USD per barrel to 84 USD per barrel yield the largest differences. Single period optimization is a myopic approach; hence it computes a lower NPV for the case AO. Multiperiod optimization takes into account the overall time horizon and gives better results than single period optimization.

4.3 Bicriterion optimization model

A comparative study of the five well model was performed for maximizing NPV and maximizing total oil production for the complete time horizon of 20 years. The results for the two cases are shown in Table 5.

Table 5. Results of max NPV and max Total oil production.

Variable	Maximization of NPV	Variable	Maximization of oil
NPV (MUSD)	6401.843	NPV(MUSD)	6361.297
Total Oil(MMstb)	18.504	Total Oil(MMstb.)	22.563

The total oil production for the two cases are plotted in Fig 4.

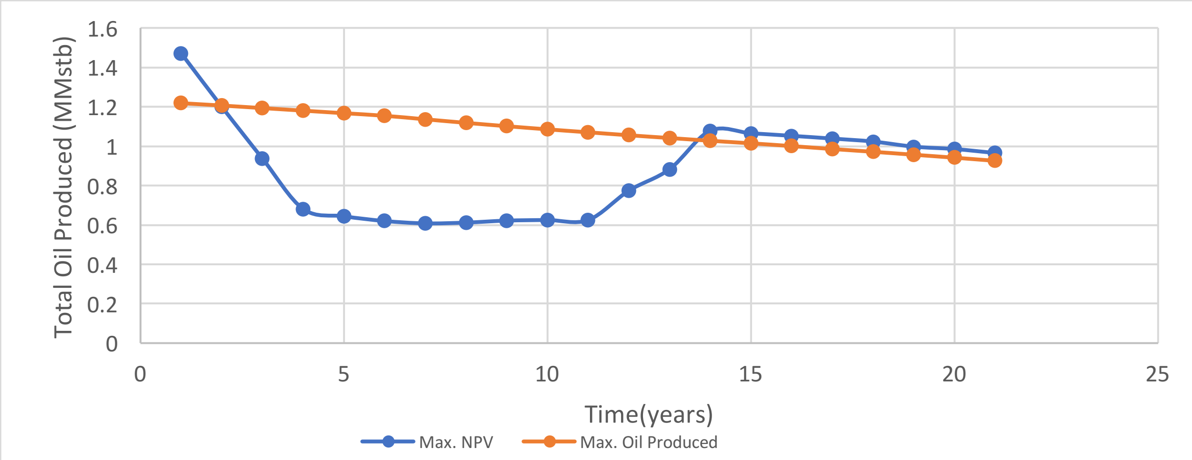

Fig. 4 Oil production for the two cases over a time horizon of 20 years

In the case of maximizing the NPV, the oil production decreases first and then increases. In contrast, for the case where the objective function is to maximize total oil production, all wells start producing from the beginning. In the case of maximizing the oil production, there is a constant decrease in the oil produced, based on the relations of total oil production and NPV maximization. A comparison was performed for different NPV values. The model was solved with the objective function to maximize total oil production with a constraint on NPV. Six different cases were formulated by imposing constraint on the maximum value of NPV. The model formulation for the bicriterion optimization is as follows.

max Total oil production

s.t. NPV ϵ

Constraints,

where, min NPV $\leq \epsilon \leq$ max NPV

Table 6. Results for Pareto analysis.

Cases	NPV(MUSD)	Total oil Produced (MMstb)
Max Oil	6361	22.563
Case1	6377	21.971
Case2	6385	21.367
Case3	6393	20.562
Case4	6397	19.978
Max NPV	6401	18.504

The results from Table 6 clearly indicate that as the NPV increases the total oil production decreases and vice versa. This behavior leads to a set of Pareto optimal solutions. Fig. 5 shows the production profile for the different values of NPV.

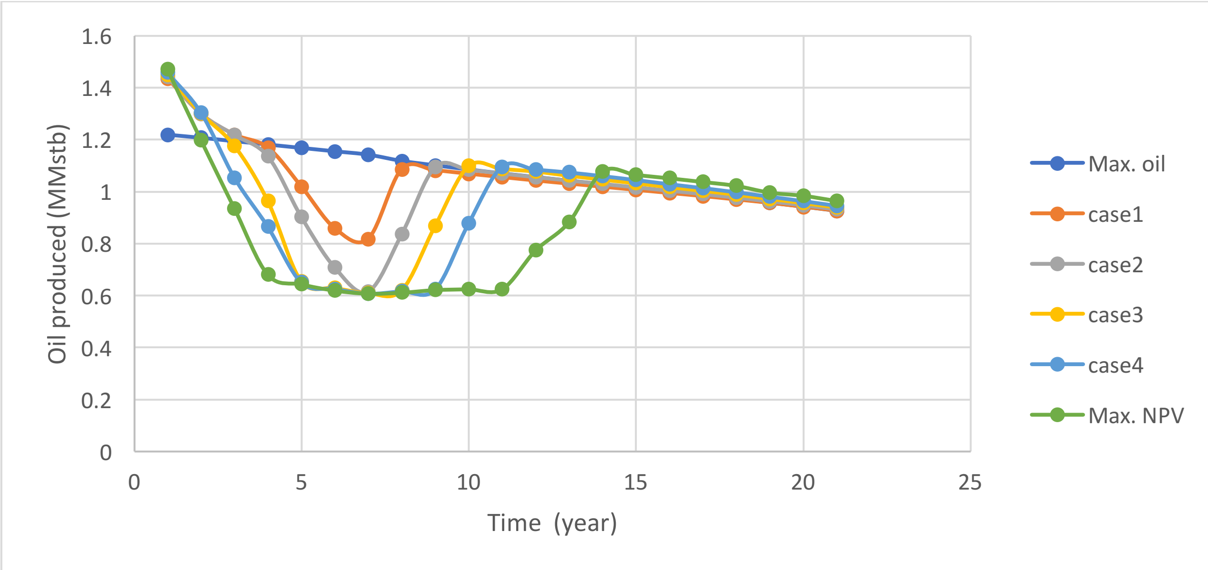

Fig. 5 Total oil production for different values of NPV

Fig. 6 shows an inverse relation between NPV and total oil production. The figure shows that changes in total oil produced have smaller variation in the NPV for low total oil production values. For higher total oil, production a small variation in the total oil produced produces a higher change in the NPV.

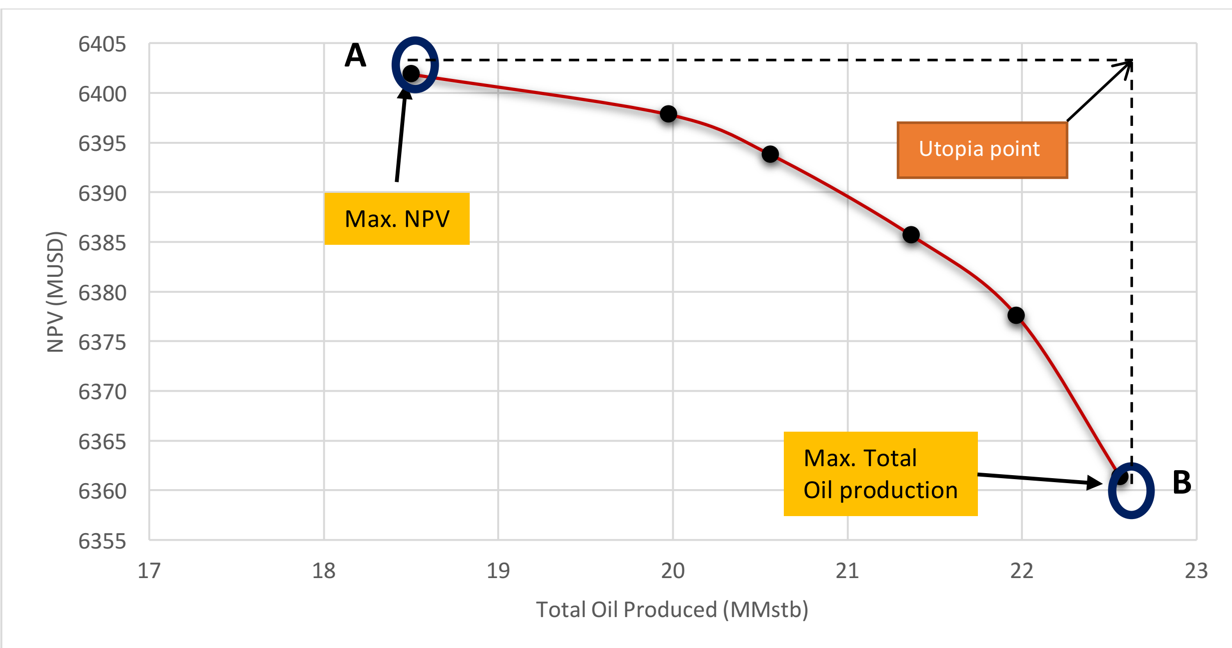

Fig. 6 Pareto curve for NPV and total oil production maximizations.

In the graph of Fig. 6, point A represents the max NPV and min oil produced, while point B represents the max Oil produced and min NPV. The Pareto curve shown in Fig. 6 corresponds to a multiobjective optimization problem with the two objective functions:

a) Maximization of NPV
b) Maximization of total oil produced.

The utopia point represents the point where we have both maximum NPV and maximum oil produced. The problem is also formulated and solved to obtain the ideal compromise solution between the NPV and the total oil production (see appendix A). The ideal compromise solution, which is the closest point from the utopia point and a Pareto optimal solution according to some norm (e.g. Euclidean norm), is given by a value 6393.759 Million USD for NPV, which is 0.13 % less than maximum NPV. The total oil production is 20.559 MMstb, which is 9 % less than the maximum value for the total oil production from the oil well. Fig. 7 shows the ideal compromise solution of the bicriterion optimization model.

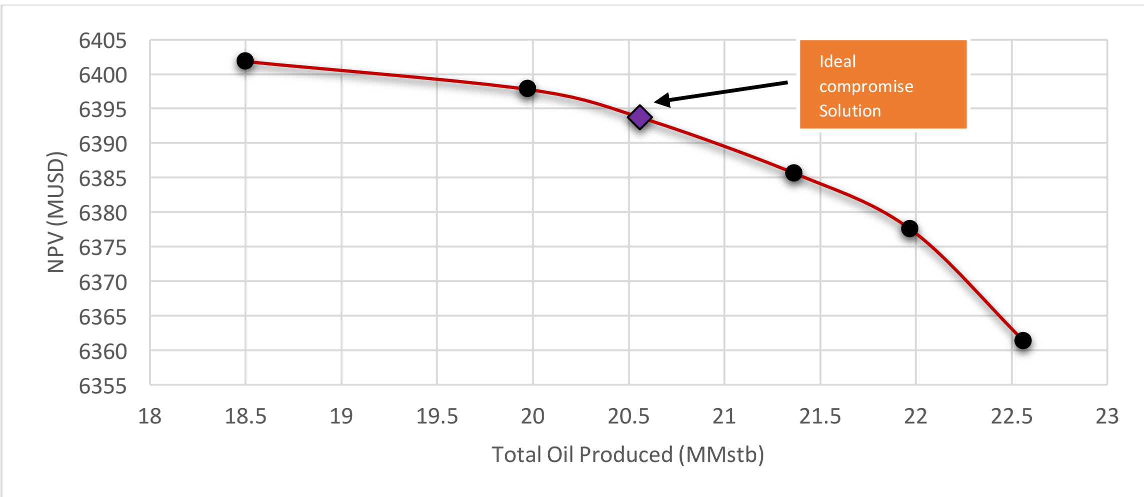

Fig. 7 Ideal compromise solution for the Pareto analysis.

Table 7. Results of bicriterion optimization.

	Maximize NPV	Ideal compromise solution	Maximize total oil
NPV(MUSD)	6401.843	6393.759	6361.297
Total Oil(MMstb)	18.504	20.559	22.563

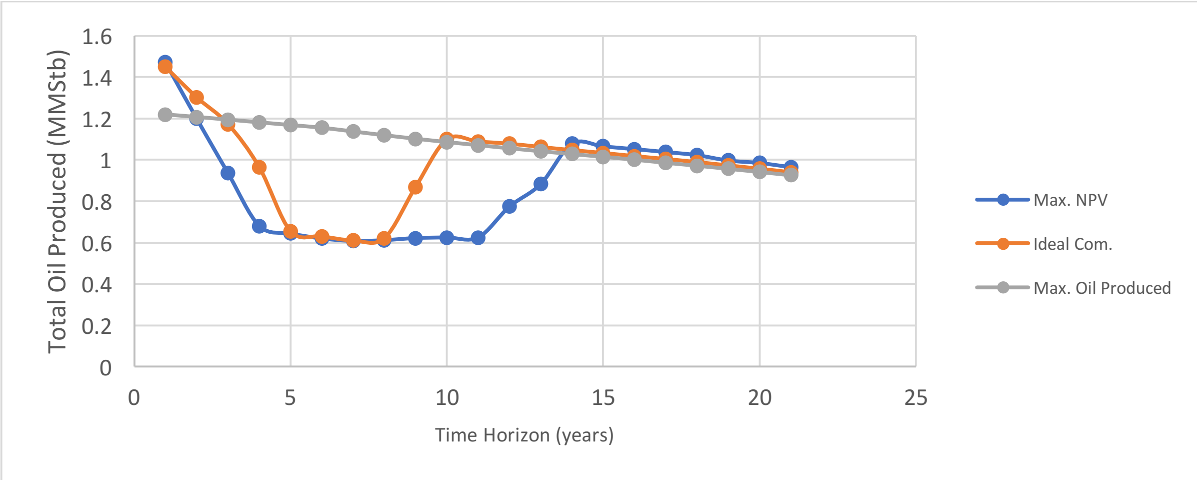

Fig. 8 Total oil produced over 20 years

The total oil production profile for the three cases of maximization of NPV, ideal compromise solution and maximization total oil production for the reduced model are shown in Fig. 9.

4.4 Two-stage stochastic model

Uncertainty in the parameters associated with oil production can be handled using stochastic programming [37], which is used for long term production planning problems. We formulate a two-stage stochastic programming model to optimize the expected value for the production model. In a two-stage stochastic programming [37] we have two sets of decisions variables, first stage variables are the here and now decision variables that are decided before the uncertainty is realized, and the second stage variables are the recourse action decisions based on the realization of uncertainty.

A two-stage stochastic programming model [37] is formulated for a deviation of ±20 % in the oil prices and productivity indices. The multiperiod NLP model is modified for the formulation of stochastic model. The first stage variables for the stochastic model are taken to be the selection of 3 oil wells form a total set of 5 oil wells. Therefore, binary variables for the selection of wells are added to the model as the first stage decision variables. Three scenarios are considered for both oil prices and productivity indices, low, medium and high. This Mixed-integer Nonlinear programming (MINLP) model is solved for the case of maximization of NPV and the case of maximization of total oil production. The probability of the scenarios for oil prices and productivity indices are {0.25, 0.5, 0.25} for the pessimistic, nominal and optimistic cases respectively. The model is solved using the SBB solver [38] and the results are shown in Table 8.

To compare the deterministic and the stochastic solutions, the value of stochastic solution (VSS) is computed [37]. VSS is the difference between the optimal solution of the two-stage stochastic model to the solution obtained by solving the two-stage model with the first stage variables fixed to the values at the optimal solution of the deterministic problem.

Table 8. Values of the deterministic and stochastic model

	Max. NPV		Max. Total Oil Produced	
	Deterministic	Stochastic	Deterministic	Stochastic
NPV (MUSD)	6455.437	6511.343	6186.86	5932.748
Total Oil Produced (MMstb)	15.774	16.283	19.46	18.794

Table 9. VSS results

Objective function	Stochastic	Expected stage 1	VSS	VSS %
Max. NPV (MUSD)	6511.343	6456.356	54.987	0.85
Max. Total Oil Produced (MMstb)	19.435	18.779	0.656	3.5

Based on the results mentioned in Table 9, the VSS for case of maximization of NPV is 54.99 MUSD (0.85 %), while for maximization of total oil production is 0.66 MMstb (3.5%). Hence, for the scenarios considered for the oil well model, the two-stage stochastic model provides an improved solution and gives a better NPV (for Max. NPV) and total oil production (for Max. Total Oil Produced) values compared to the deterministic solution.

5. Conclusion

This paper has described a multiperiod NLP model to determine the optimal solutions for oil production planning. In this study, we solve a simplified model for production from oil wells. The results of the multiperiod NLP model determines the production profiles of the oil wells as shown in Fig.2 and 3. We formulate a case study for different rates of return and oil prices to compare the objective function values for multiperiod and single period optimization (Table 3 and 4). The study mentioned in section 4.3, clearly shows that maximization of net present value or maximization of total oil production does not determine the best tradeoff between these objectives. The best tradeoff solution is an ideal compromise solution that is closest to the utopia point. This is an important finding, as in industries maximization of either of the objective functions mentioned above is considered as the best solution. It is also shown that the multiperiod model provides significantly improved solutions compared to the case where successive single-period problems are solved. Finally, the model is solved to optimize the oil production for uncertainty in oil prices and results of the VSS for the two-stage stochastic model are tabulated in Table 9. The values of stochastic solution show that the results of the two-stage stochastic model are better than the deterministic solution. Hence, considering the two-stage stochastic model improves the solution for both objective functions.

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Appendix A

Bicriterion optimization

The Pareto analysis of conflicting objective functions results in the formulation of a bicriterion optimization problem. The bicriterion optimization yields tradeoff solutions between two objective functions such as the net present value and the total oil production. To generate the Pareto cure the ϵ - constrained method [27] is used. In this method one of the objective functions is optimized subject to a constraint ϵ on the other objective function. Further, after obtaining the Pareto curve one can determine the ideal compromise solution. In this case, the resulting Pareto front clearly shows that as the NPV value is increased the total oil production is decreased as shown in Fig A1.

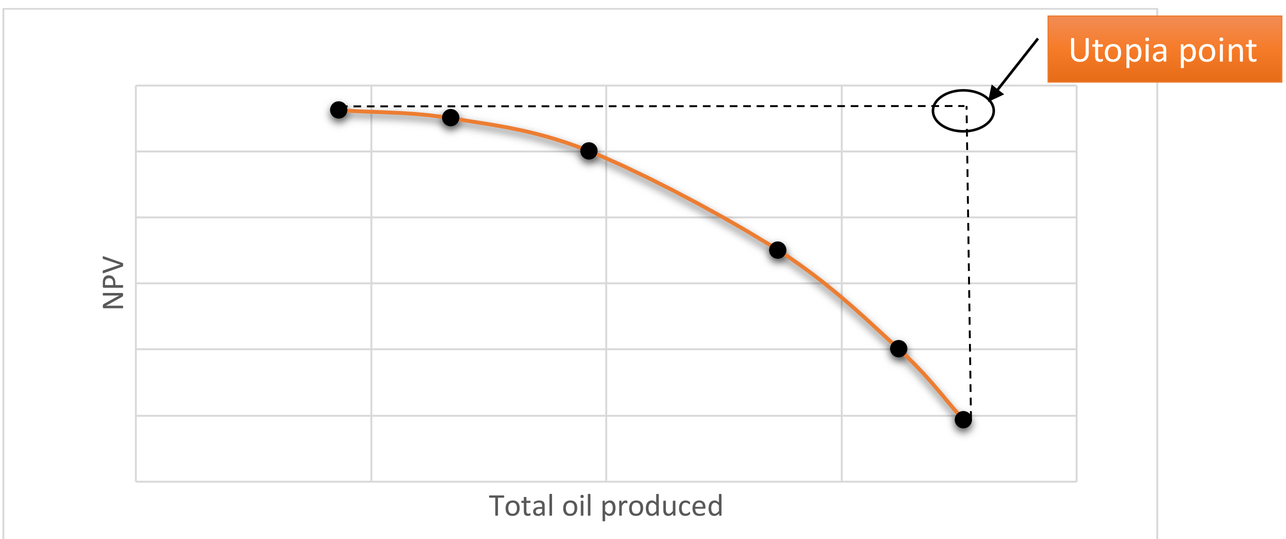

Fig. A1 Pareto curve of NPV vs Total oil production.

The ideal solution would be the one in which we obtain the maximum NPV and maximum total oil production. This point is denoted as the utopia point [28, 29]. The ideal compromise solution corresponds to the point in the Pareto curve that has the shortest distance to the Utopia point.

Model

Let f1: Net present value, f2: total oil production. [12]

- The utopia point corresponds to $[f1^U, f2^U]$. The maximum of both variables. Where superscript U represent upper bound.

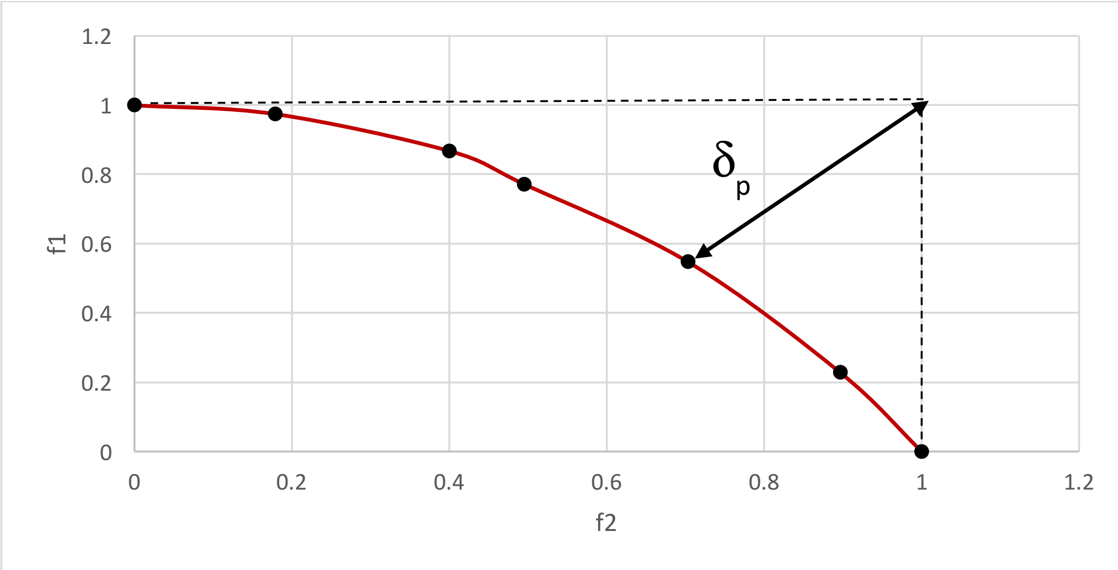

Fig. A2 Scaled Net present value (f1) vs Total oil produced(f2)

- An ideal compromise solution can be obtained by finding the point on the curve closest to utopia point i.e. minimizing the distance (δ_p) for a norm p, where:

  $_p = [(f1^U - f1)^p + (f2^U - f2)^p]^{1/p} \quad 1\ p$ *(19)*
- The variables f1 and f2 are scaled from zero to one.
- After scaling of the functions to $f1^{'}$ and $f2^{'}$. The utopia point for scaled variables is (1, 1). Norm p=2 is considered for minimizing the fractional deviations $1- f1^{'}$ and $1- f2^{'}$.
- To obtain the ideal compromise solution. For p=2, solve,

  min $((1- f1^{'})^2 + (1- f2^{'})^2)^{1/2}$, *(20)*

  s.t. Constraints