Optimal Synthesis of Heat Exchanger Networks

Involving Isothermal Process Streams

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Abstract

This paper proposes a new MINLP model for heat exchanger network synthesis that includes streams with phase change. The model considers every possible combination of process streams that may arise within a chemical process: streams with sensible heat, streams with latent heat, and streams with both latent and sensible heat. As part of the optimization strategy, the superstructure is modeled with logical conditions that are used for the proper placement of heat integration for streams with change of phase. The proposed MINLP model provides the network structure that minimizes the total yearly cost. Several examples are presented to illustrate the capabilities of the proposed model.

Keywords: Heat exchanger networks; Optimization; MINLP; Isothermal streams; Latent heat.

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1. Introduction

Heat exchanger networks have been the subject of numerous investigations in the past decades because of their impact in the energy recovery of industrial plants. Two major methodologies haven been proposed for the synthesis of heat exchanger network problem, namely the sequential and the simultaneous approach. One of the most-widely known sequential approaches is the pinch point design method (Linnhoff and Hindmarsh, 1983), in which targets for the minimum utility requirement, the minimum number of exchanger units and the minimum capital cost of the network are obtained sequentially. Several heuristic rules are then used to synthesize a network that approaches these targets. Other methodologies based on mathematical programming techniques to predict the minimum utility requirements include the transportation problem formulation by Cerda *et al.* (1983) and the transshipment problem approach by Papoulias and Grossmann (1983).

Works based on the simultaneous approach solve the problem without any decomposition, and can explicitly handle the trade-offs between the capital and operational costs of the network. As part of this approach, the work by Yee and Grossmann (1990) has provided a basic framework through the use of a staged-superstructure that was formulated as a MINLP model with the objective of minimizing simultaneously the utilities and capital costs of the network. Some extensions to such framework include the works by Verheyen and Zhang (2006), Chen and Hung (2004) and Konukman *et al*. (2002) that incorporated some flexibility aspects into the design of heat exchanger networks. The works by Serna *et al*. (2004), Mizutani *et al*. (2003) and Frausto *et al.* (2003) have also extended the work of Yee and Grossmann (1990) to incorporate detailed exchanger design models as well as pressure drop equations. Sorsak and Kravanja (2004) and Ma *et al*. (2000) have formulated MINLP models for the retrofit of heat exchanger networks. For a recent review on methodologies for the synthesis of heat exchanger networks, see the paper by Furman and Sahinidis (2002).

Most of the methodologies proposed to solve the heat exchangers network problem have not considered isothermal streams with phase change (i.e. streams that transfer their latent heat). Isothermal streams arise for instance as part of separation processes and refrigeration sections. They also arise as an approximation of multicomponent streams that undergo phase change in the coolers and reboilers in distillation columns. Furthermore, streams with phase change include the subcooling and partial condensation of outlet of reactor streams. Methodologies reported for the synthesis of heat exchanger networks for isothermal streams, and more generally with phase change, have typically oversimplified the problem, for instance by assuming 1 K drop or increase in the temperature of these streams. Hence, no rigorous model has been reported to handle this kind of problem.

As part of an optimization model for simultaneous flowsheet optimization and heat integration, Grossmann *et al*. (1988) modeled the pinch location strategy by Duran and Grossmann (1986) for non-isothermal and isothermal streams using disjunctive programming. However, this method does not synthesize network structures. Castier and Queiroz (2002) reported a methodology based on the pinch point method to solve the energy targeting problem considering streams with phase changes. Liporace *et al.* (2004) presented an alternative simplified procedure in which the streams with phase change are split into sub-streams. The last two methodologies are sequential in nature, such that the trade-off between capital and energy costs may not be properly achieved.

To provide some insight into the issue of streams with phase change, Figure 1 shows the composites curves (Linnhoff and Flower, 1978) for a problem that includes both isothermal and non isothermal streams. The horizontal segments represent the isothermal streams. The problem poses an interesting modeling challenge for a proper mathematical programming formulation. For instance, one could in principle decompose the streams that undergo a change of phase into substreams for the superheated, saturated and subcooled parts. Aside from leaving open the question on how best to handle the isothermal streams for the saturated section, this approach would lead to solutions with one exchanger for each part of the heat content of the stream, which would most likely not provide the optimal solution to the original problem.

We present in this paper a rigorous model for the synthesis of heat exchanger networks that includes isothermal streams. The proposed model, which uses as a basis the MINLP model by Yee and

Grossmann (1990), is extended with appropriate constraints that allow the handling of isothermal streams with only phase change and streams that also involve sensible heat effects.

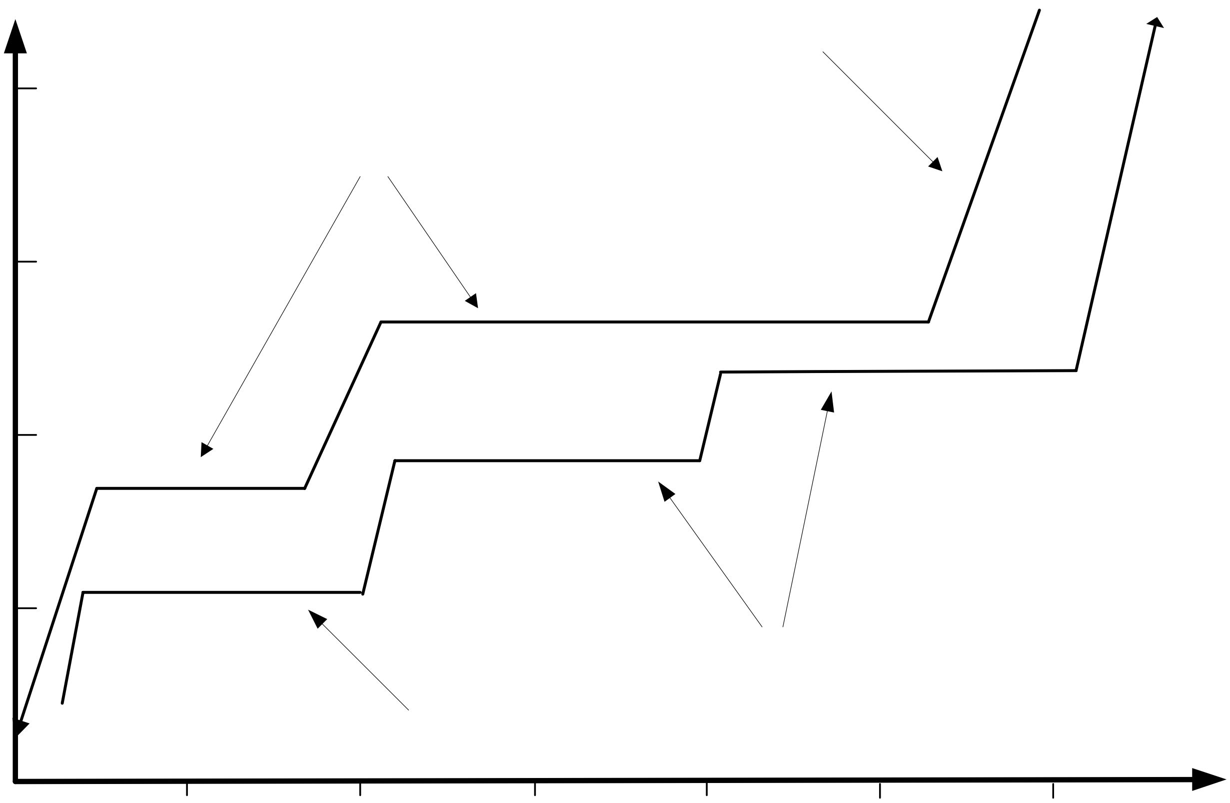

Figure 1. Composite curves for a problem that includes isothermal and non isothermal streams

2. Outline of the proposed model

If one considers that process streams can exchange sensible and latent heats, three types of situations can occur for heat transfer between any hot and any cold stream within a process. Figure 2 shows the temperature profiles for the three different types of hot and cold streams that will be considered. The proposed model can handle any type of exchange between these streams. It is worth mentioning that virtually all methods reported for the synthesis of heat exchanger networks have been based on heat exchanges for only cases (a) and (d) in Figure 2.

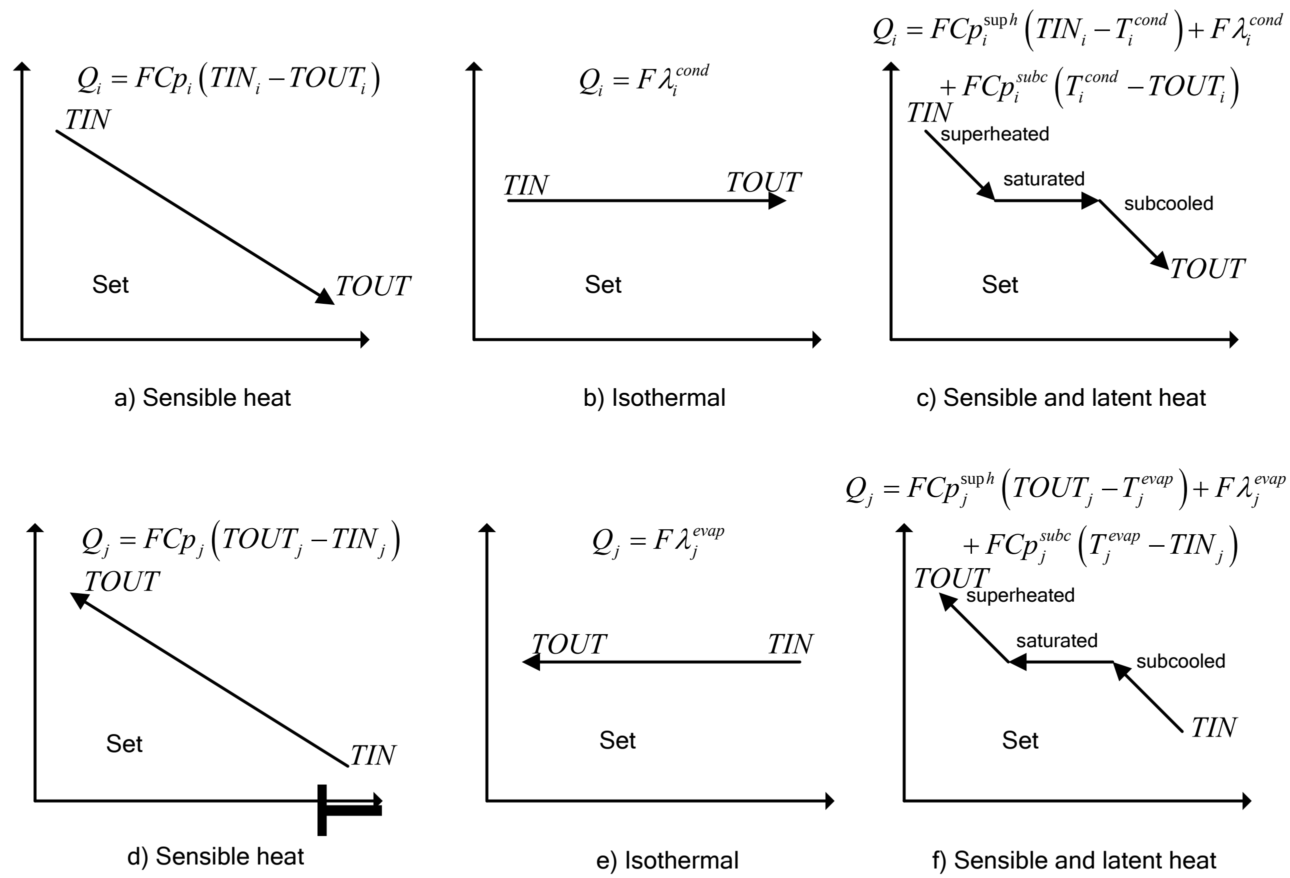

Figure 2. Types of streams considered in the model

The proposed MINLP model that includes isothermal and non-isothermal streams is based on the superstructure formulation by Yee and Grossman (1990). Figure 3 shows a superstructure involving two hot and two cold process streams. The number of stages in the superstructure is commonly specified as max{N_H, N_C}. In each stage of the superstructure, stream splitting is allowed to provide the possibility of heat exchange between hot and cold streams. Within the formulation, isothermal mixing is assumed with which it is only necessary to consider inlet and outlet temperatures at each stage of the superstructure, and no variables for the flows are required. These intermediate temperatures in the superstructure are treated as optimization variables, and the utility exchangers are placed in the extremes of the superstructure.

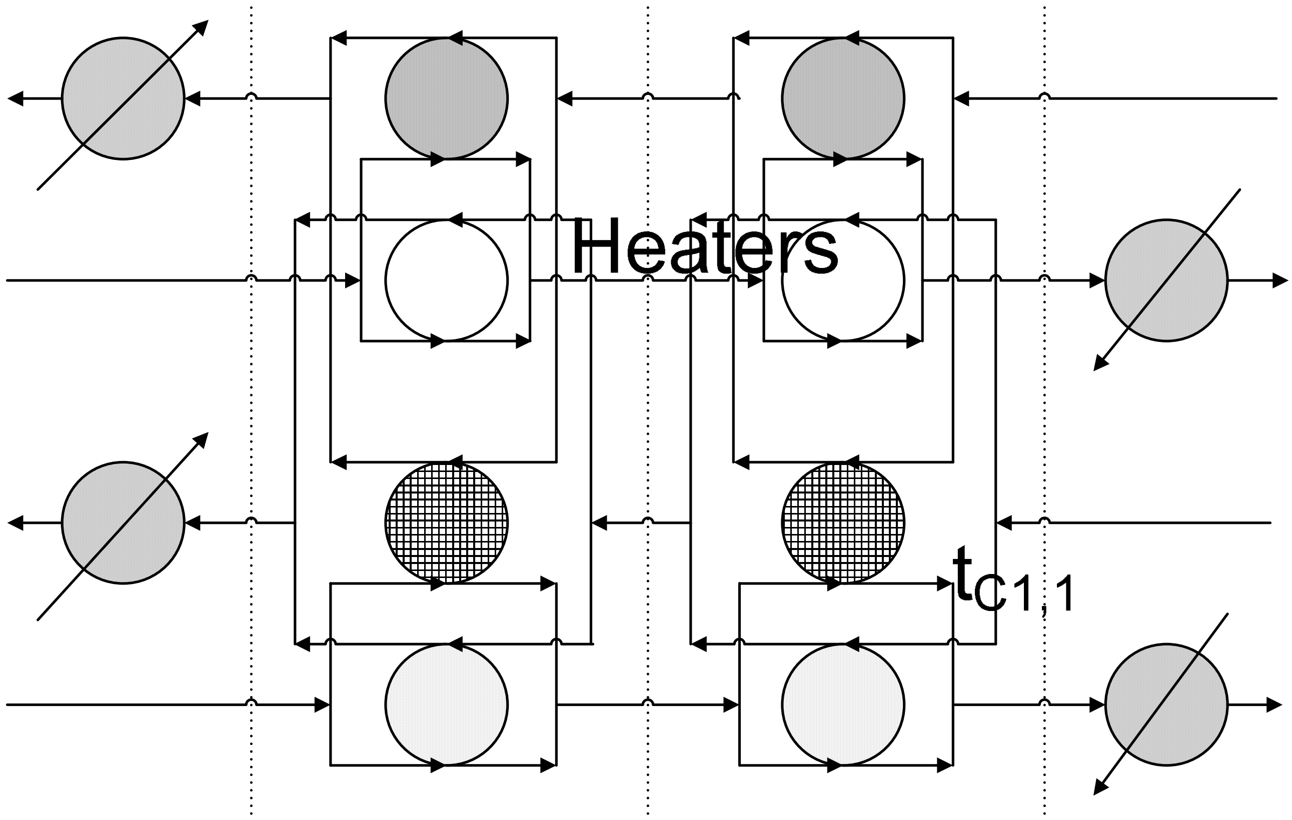

Figure 3. Superstructure for two hot and two cold streams

3. Model formulation

To formulate the MINLP model, the following sets are defined. The set *HPS* contains the total number of hot process streams; of these, the set *HPS1* contains the hot process streams that exchange sensible heat in the network (Figure 2a), *HPS2* contains the hot process isothermal streams (i.e., they exchange only latent heats and their temperatures remain constant, Figure 2b), and *HPS3* contains the hot process streams that exchange both latent and sensible heat (Figure 2c). Similarly, *CPS* is the set for cold process streams, and *CPS1*, *CPS2* and *CPS3* are the subsets for cold streams that are non isothermal, isothermal and that have a combination of latent and sensible heat, as depicted in Figures 2d, 2e and 2f, respectively. It is therefore assumed in the formulation that the dew and bubble points are the same, which is strictly correct only for pure components. This is, however, a reasonable approximation for many multicomponent mixtures, since very often the differences between dew and bubble points are small. Also, a detailed model for the dependence of temperature with latent heat is not considered in this work.

The MINLP model can then be written as follows.

Overall Energy Balance for Each Stream. The total heat transfer balance for each stream is given by the equations,

$$\left(TIN_i - TOUT_i\right) FCp_i = \sum_{k \in ST} \sum_{j \in CPS} q_{ijk} + qcu_i, \quad i \in HPS1 \tag{1}$$

$$F\lambda_i^{cond} = \sum_{k \in ST} \sum_{j \in CPS} q_{ijk} + qcu_i, \quad i \in HPS2 \tag{2}$$

$$\left(TIN_i - T_i^{cond}\right) FCp_i + F\lambda_i^{cond} + \left(T_i^{cond} - TOUT_i\right) FCp_i = \sum_{k \in ST} \sum_{j \in CPS} q_{ijk} + qcu_i, \quad i \in HPS3 \tag{3}$$

$$\left(TOUT_j - TIN_j\right) FCp_j = \sum_{k \in ST} \sum_{i \in HPS} q_{ijk} + qhu_j, \quad j \in CPS1 \tag{4}$$

$$F\lambda_j^{evap} = \sum_{k \in ST} \sum_{i \in HPS} q_{ijk} + qhu_j, \quad j \in CPS2 \tag{5}$$

$$\left(TOUT_j - T_j^{evap}\right) FCp_j + F\lambda_j^{evap} + \left(T_j^{evap} - TIN_j\right) FCp_j = \sum_{k \in ST} \sum_{i \in HPS} q_{ijk} + qhu_j, \quad j \in CPS3 \tag{6}$$

Here, $F\lambda_i^{cond}$ and $F\lambda_j^{evap}$ are the total latent heats for condensation and evaporation for streams i and j, respectively.

For non-isothermal streams, only their sensible heat is considered, equations (1) and (4). For the isothermal streams, equations (2) and (5), only their latent heat is considered. For the cases in which both latent heat and sensible heat are involved, equations (3) and (6) apply. Notice that in Yee and Grossmann (1990) only equations (1) and (4) were used.

Energy Balance for Each Stage. The energy balance for each stage of the superstructure is only required for non-isothermal streams; the balance provides the intermediate temperatures in the superstructure (for isothermal streams, the temperature is the same in all stages of the superstructure).

Therefore, the equations are applied only for the streams that undergo either total or partial sensible heat transfer,

$$\left(t_{i,k} - t_{i,k+1}\right) FCp_i = \sum_{j \in CPS} q_{ijk}, \quad k \in ST, i \in HPS1 \tag{7}$$

$$\left(t_{i,k} - t_{i,k+1}\right) FCp_i + q^{\Lambda}_{i,k} = \sum_{j \in CPS} q_{ijk}, \quad k \in ST, i \in HPS3 \tag{8}$$

$$\left(t_{j,k} - t_{j,k+1}\right) FCp_j = \sum_{i \in HPS} q_{ijk}, \quad k \in ST, j \in CPS1 \tag{9}$$

$$\left(t_{j,k} - t_{j,k+1}\right) FCp_j + q^{\Lambda}_{j,k} = \sum_{i \in HPS} q_{ijk}, \quad k \in ST, j \in CPS3 \tag{10}$$

Here, $q^{\Lambda}_{i,k}$ and $q^{\Lambda}_{j,k}$ are the total latent heats for condensation and evaporation that are exchanged in stage *k* for streams *i* and *j*, respectively. Notice that any exchanger in the superstructure can exchange only sensible heat, only latent heat, or both sensible and latent heat.

Assignment of Inlet Temperatures to the Superstructure. These equations are written as in Yee and Grossmann (1990).

$$TIN_i = t_{i,1}, \quad i \in HPS \tag{11}$$

$$TIN_i = t_{j,NOK+1}, \quad j \in CPS \tag{12}$$

Temperature Feasibility. Equations are needed to ensure a monotonic decrease of temperature for non isothermal streams throughout the superstructure,

$$t_{i,k} \geq t_{i,k+1}, \quad k \in ST, i \in HPS1 \; or \; i \in HPS3 \tag{13}$$

$$t_{i,k} = TIN_i, \quad k \in ST, i \in HPS2 \tag{14}$$

$$t_{j,k} \geq t_{j,k+1}, \quad k \in ST, j \in CPS1 \; or \; j \in CPS3 \tag{15}$$

$$t_{j,k} = TIN_j, \quad k \in ST, j \in CPS2 \tag{16}$$

$$TOUT_i \leq t_{i,NOK+1}, \quad i \in HPS1 \; or \; i \in HPS3 \tag{17}$$

$$TOUT_j \leq t_{j,1}, \quad j \in CPS1 \; or \; j \in CPS3 \tag{18}$$

Equations (17) and (18) are not necessary for isothermal streams because of the specifications given in equations (14) and (16).

Heating and Cooling Duties. Heat loads for hot and cold utilities are calculated based on the temperatures for the first and the last stage of the superstructure, respectively. These equations are valid for non-isothermal streams; for isothermal streams, total heat balances (equations (2) and (4)) have already considered the hot and cold utilities.

$$\left(t_{i,NOK+1} - TOUT_i\right) FCp_i = qcu_i, \quad i \in HPS1 \tag{19}$$

$$\left(t_{i,NOK+1} - TOUT_i\right) FCp_i + q_i^{\Lambda,cu} = qcu_i, \quad i \in HPS3 \tag{20}$$

$$\left(TOUT_j - t_{j,1}\right) FCp_j = qhu_j, \quad j \in CPS1 \tag{21}$$

$$\left(TOUT_j - t_{j,1}\right) FCp_j + q_j^{\Lambda,hu} = qhu_j, \quad j \in CPS3 \tag{22}$$

where $q_i^{\Lambda,cu}$ and $q_j^{\Lambda,hu}$ are the condensation and evaporation heats processed with utilities.

Latent Heat Balance. For streams that exchange their latent heats, the following heat balances are needed,

$$F\lambda_i^{cond} = \sum_{k \in ST} q_{i,k}^{\Lambda} + q_i^{\Lambda,cu}, \quad i \in HPS2 \; or \; i \in HPS3 \tag{23}$$

$$F\lambda_j^{evap} = \sum_{k \in ST} q_{j,k}^{\Lambda} + q_j^{\Lambda,hu}, \quad j \in CPS2 \; or \; j \in CPS3 \tag{24}$$

With this formulation latent heats can be exchanged anywhere in the superstructure. Note that the latent heat load for any stream can be distributed across several units. Limits are imposed to ensure that the latent heat can only be exchanged if the temperature for the phase change falls between the inlet and outlet temperatures of a given stage in the superstructure.

Feasibility of the Latent Heat Exchange. One of the major parts of the modeling task is to set the constraints that ensure a proper transfer of the latent heat of the isothermal streams within the superstructure. The exchange of the latent heat of a hot process stream undergoing cooling and/or phase change should be allowed only if the condensation temperature lies between the inlet and outlet temperature of a given stage. Therefore, the formulation should reflect that if the outlet temperature of stage k is higher than the condensation temperature, the latent heat exchanged in such stage, $q_{i,k}^{\Lambda}$, must be zero. Also, if the inlet temperature for stage k is lower than the condensation temperature for the hot stream i, then it is not possible to exchange the latent heat in this stage because it has already been transferred in previous stages. The following diagrams and disjunctions illustrate these situations.

For the case where the inlet temperature of the hot stream is above the saturation temperature, the following disjunction applies (see Figure 4).

$$\begin{pmatrix} Y_{i,k}^{1} \\ t_{i,k+1} \geq T_i^{cond} + \delta \\ q_{i,k}^{\Lambda} = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y_{i,k}^{1} \\ t_{i,k+1} \leq T_i^{cond} \\ q_{i,k}^{\Lambda} \geq 0 \end{pmatrix}$$

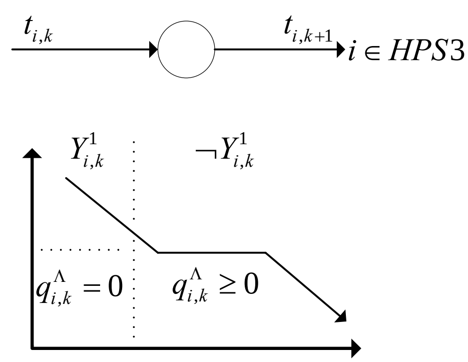

Figure 4. Inlet temperature above condensation

If the inlet temperature is lower than the saturation temperature, then the following disjunction can be written (see Figure 5).

$$\begin{pmatrix} Y_{i,k}^{2} \\ t_{i,k} \leq T_i^{cond} - \delta \\ q_{i,k}^{\Lambda} = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y_{i,k}^{2} \\ t_{i,k} \geq T_i^{cond} \\ q_{i,k}^{\Lambda} \geq 0 \end{pmatrix}$$

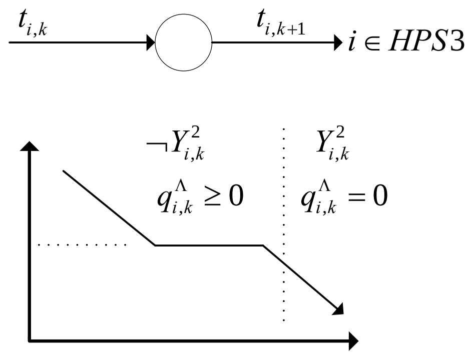

Figure 5. Outlet temperature below condensation

where $Y_{i,k}^{1}$ and $Y_{i,k}^{2}$ are logical variables that are true when $t_{i,k+1} > T_i^{cond}$ and $t_{i,k} < T_i^{cond}$, respectively.

The same analysis can be performed for latent heat loads of hot streams processed with cold utilities. In these cases the cooler is placed at the end of the superstructure, and if the temperature of the last stage of the superstructure (*NOK* +1) is lower than the condensation temperature, then the latent heat load has been exchanged in previous stages; otherwise the cooler heat load might include some condensation of the hot stream. The following disjunction can then be formulated (see Figure 6),

$$\begin{pmatrix} Y_i^3 \\ t_{i,NOK+1} \leq T_i^{cond} - \delta \\ q_i^{\Lambda,cu} = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y_i^3 \\ t_{i,NOK+1} \geq T_i^{cond} \\ q_i^{\Lambda,cu} \geq 0 \end{pmatrix}$$

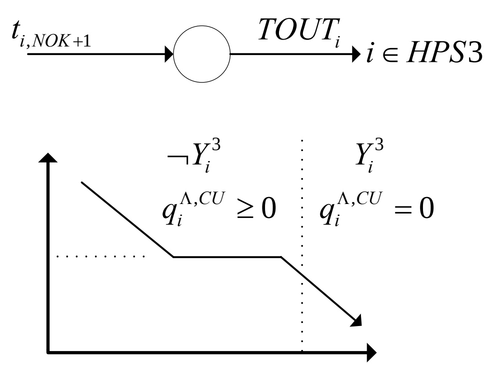

Figure 6. Outlet temperature below condensation for cooler

where Y_i^3 is a logical variable that is true when $t_{i,NOK+1} < T_i^{cond}$.

If we reformulate the disjunctions using the convex hull transformation (Raman and Grossmann, 1994), the following sets of constraints arise. For the first disjunction,

$$t_{i,k+1} = t_{i,k+1}^1 + t_{i,k+1}^2, \quad i \in HPS3, k \in ST \tag{25}$$

$$t_{i,k+1}^1 \geq \left(T_i^{cond} + \delta\right) y_{i,k}^1, \quad i \in HPS3, k \in ST \tag{26}$$

$$t_{i,k+1}^2 \leq \left(T_i^{cond}\right)\left(1 - y_{i,k}^1\right), \quad i \in HPS3, k \in ST \tag{27}$$

$$t_{i,k+1}^1 \leq \left(TIN_i\right) y_{i,k}^1, \quad i \in HPS3, k \in ST \tag{28}$$

$$q_{i,k}^{\Lambda} \leq \left(F\lambda_i^{cond}\right)\left(1 - y_{i,k}^1\right), \quad i \in HPS3, k \in ST \tag{29}$$

For the second disjunction,

$$t_{i,k} = t_{i,k}^3 + t_{i,k}^4, \quad i \in HPS3, k \in ST \tag{30}$$

$$t_{i,k}^3 \leq \left(T_i^{cond} - \delta\right) y_{i,k}^2, \quad i \in HPS3, k \in ST \tag{31}$$

$$t_{i,k}^4 \geq \left(T_i^{cond}\right)\left(1 - y_{i,k}^2\right), \quad i \in HPS3, k \in ST \tag{32}$$

$$t_{i,k}^4 \leq \left(TIN_i\right)\left(1 - y_{i,k}^2\right), \quad i \in HPS3, k \in ST \tag{33}$$

$$q_{i,k}^{\Lambda} \leq \left(F\lambda_i^{cond}\right)\left(1 - y_{i,k}^2\right), \quad i \in HPS3, k \in ST \tag{34}$$

For the disjunction of the cold utility,

$$t_{i,NOK+1} = t_i^5 + t_i^6, \quad i \in HPS3 \tag{35}$$

$$t_i^5 \leq \left(T_i^{cond} - \delta\right) y_i^3, \quad i \in HPS3 \tag{36}$$

$$t_i^6 \geq \left(T_i^{cond}\right)\left(1 - y_i^3\right), \quad i \in HPS3 \tag{37}$$

$$t_i^6 \leq \left(TIN_i\right)\left(1 - y_i^3\right), \quad i \in HPS3 \tag{38}$$

$$q_i^{\Lambda,cu} \le \left(F\lambda_i^{cond}\right)\left(1-y_i^3\right), \quad i \in HPS3 \tag{39}$$

In equations (25) to (39), y is a set of binary variables used to model the disjunctions, and δ is a small parameter (typically δ was set as $1\text{x}10^{-2}$).

For the cold process streams, the analysis is similar. For a cold process stream j that contains both latent and sensible heat, the latent heat can be exchanged in a process-process exchanger only if its evaporation temperature falls between the inlet and the outlet temperature of the stage. Therefore, if the outlet temperature for stage k is lower than the stream evaporation temperature, the latent heat that can be exchanged in stage k is zero, as illustrated in Figure 7,

$$\begin{pmatrix} Y_{j,k}^4 \\ t_{j,k} \le T_j^{evap} - \delta \\ q_{j,k}^{\Lambda} = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y_{j,k}^4 \\ t_{j,k} \ge T_j^{evap} \\ q_{j,k}^{\Lambda} \ge 0 \end{pmatrix}$$

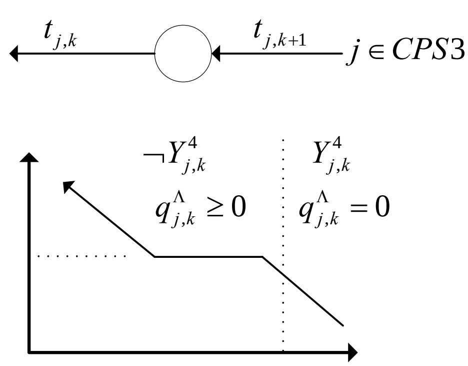

Figure 7. Outlet temperature below evaporation

where the logical variable $Y_{j,k}^4$ is true when $t_{j,k} < T_j^{evap}$. Similarly, if the inlet temperature into stage k is higher than the evaporation temperature, the latent heat exchanged in this stage is set to zero. The following disjunction can be written (see Figure 8),

$$\begin{pmatrix} Y_{j,k}^{5} \\ t_{j,k+1} \geq T_j^{evap} + \delta \\ q_{j,k}^{\Lambda} = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y_{j,k}^{5} \\ t_{j,k+1} \leq T_j^{evap} \\ q_{j,k}^{\Lambda} \geq 0 \end{pmatrix}$$

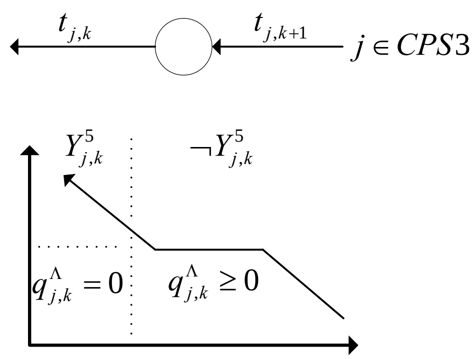

Figure 8. Inlet temperature above evaporation

where $Y_{j,k}^{5}$ is true when $t_{j,k+1} > T_j^{evap}$.

In terms of the convex hull formulation, the first disjunction for the cold streams can be written as,

$$t_{j,k} = t_{j,k}^{1} + t_{i,k}^{2}, \quad j \in CPS3, k \in ST \tag{40}$$

$$t_{j,k}^{1} \leq \left(T_j^{evap} - \delta\right) y_{j,k}^{4}, \quad j \in CPS3, k \in ST \tag{41}$$

$$t_{j,k}^{2} \geq \left(T_j^{evap}\right)\left(1 - y_{j,k}^{4}\right), \quad j \in CPS3, k \in ST \tag{42}$$

$$t_{j,k}^{2} \leq \left(TOUT_j\right)\left(1 - y_{j,k}^{4}\right), \quad j \in CPS3, k \in ST \tag{43}$$

$$q_{j,k}^{\Lambda} \leq \left(F\lambda_j^{evap}\right)\left(1 - y_{j,k}^{4}\right), \quad j \in CPS3, k \in ST \tag{44}$$

For the second disjunction, we have,

$$t_{j,k+1} = t_{j,k+1}^{3} + t_{j,k+1}^{4}, \quad j \in CPS3, k \in ST \tag{45}$$

$$t_{j,k+1}^{3} \geq \left(T_j^{evap} + \delta\right) y_{j,k}^{5}, \quad j \in CPS3, k \in ST \tag{46}$$

$$t^4_{j,k+1} \le \left(T^{evap}_j\right)\left(1 - y^5_{j,k}\right), \quad j \in CPS3, k \in ST \tag{47}$$

$$t^3_{j,k+1} \le \left(TOUT_j\right) y^5_{j,k}, \quad j \in CPS3, k \in ST \tag{48}$$

$$q^{\Lambda}_{j,k} \le \left(F\lambda^{evap}_j\right)\left(1 - y^5_{j,k}\right), \quad j \in CPS3, k \in ST \tag{49}$$

For the use of hot utilities, the latent heat of a cold process stream can be exchanged only with the hot utility if the inlet temperature to the heater is lower than the evaporation temperature of the stream. The heater is placed at the extreme of the superstructure (before stage 1), and the following disjunction is used to model this situation (see Figure 9),

$$\begin{pmatrix} Y^6_j \\ t_{j,1} \ge T^{evap}_j + \delta \\ q^{\Lambda,hu}_j = 0 \end{pmatrix} \vee \begin{pmatrix} \neg Y^6_j \\ t_{j,1} \le T^{evap}_j \\ q^{\Lambda,hu}_j \ge 0 \end{pmatrix}$$

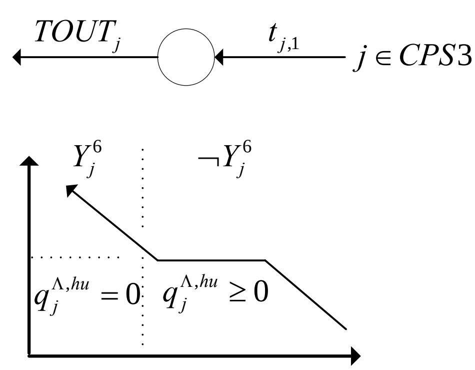

Figure 9. Allocation of latent heat for heaters

where Y^6_j is a logical variable that is true when $t_{j,1} > T^{evap}_j$. The convex hull leads to the following set of constraints,

$$t_{j,1} = t^5_j + t^6_j, \quad j \in CPS3 \tag{50}$$

$$t^5_j \ge \left(T^{evap}_j + \delta\right) y^6_j, \quad j \in CPS3 \tag{51}$$

$$t^6_j \le \left(T^{evap}_j\right)\left(1 - y^6_j\right), \quad j \in CPS3 \tag{52}$$

$$t_j^5 \le \left(TOUT_j\right) y_j^6, \quad j \in CPS3 \tag{53}$$

$$q_j^{\Lambda,hu} \le \left(F\lambda_j^{evap}\right)\left(1 - y_j^6\right), \quad j \in CPS3 \tag{54}$$

Finally, it must be noted that an additional constraint that is needed to allow the exchange of latent heat between any hot process stream *i* of the type *HPS3* and any cold process stream *j* of the type *CPS3* is that the difference between the condensation temperature of the hot stream (T_i^{cond}) and the evaporation temperature (T_j^{evap}) of the cold stream be higher than a given value of ΔT_{MIN}. This constraint is implemented directly from the original design data for the problem.

Upper Bound Constraints. Upper bound constraints are used to determine the existence of a heat exchanger, which occurs only if the heat load is higher than zero; otherwise, the heat exchanger does not exist. These constraints are modeled as follows:

$$q_{ijk} - Q_{i,j}^{\max} z_{ijk} \le 0, \quad i \in HPS,\ j \in CPS,\ k \in ST \tag{55}$$

$$qcu_i - Q_i^{\max} zcu_i \le 0, \quad i \in HPS \tag{56}$$

$$qhu_j - Q_j^{\max} zhu_j \le 0, \quad j \in CPS \tag{57}$$

where *z, zcu, zhu* are binary variables and $Q^{\max}$ corresponds to the upper limit for heat transfer. The value of $Q_{i,j}^{\max}$ is set as the smallest heat content of the two streams involved in the match.

Temperature Differences. Temperature differences are required to calculate the heat transfer area for each heat exchanger in the network. Binary variables are used to activate or deactivate the following constraints to ensure feasible driving forces for exchangers when they are selected as part of the network structure:

$$dt_{ijk} \le t_{ik} - t_{jk} + \Delta T^{\max}\left(1 - z_{ijk}\right), \quad i \in HPS,\ j \in CPS,\ k \in ST \tag{58}$$

$$dt_{i,j,k+1} \le t_{i,k+1} - t_{j,k+1} + \Delta T^{\max}\left(1 - z_{i,j,k+1}\right), \quad i \in HPS,\ j \in CPS,\ k \in ST \tag{59}$$

$$dtcu_i \le t_{i,NOK+1} - TOUT_{cu} + \Delta T^{\max}\left(1 - zcu_i\right), \quad i \in HPS \tag{60}$$

$$dthu_j \le TOUT_{hu} - t_{j,1} + \Delta T^{\max}\left(1 - zhu_j\right), \quad j \in CPS \tag{61}$$

where $\Delta T^{\max}$ is an upper limit for the temperature difference. These equations are written as inequalities since these constraints will be active whenever the exchanger exists because the cost of the exchanger decreases with increasing *dt*. The binary variables are set to one to enforce positive driving forces when a heat exchanger exists. A proper estimation of the upper limit for temperature differences, $\Delta T^{\max}$, is carried out through the following condition,

$$\begin{aligned}
&\text{if} \quad TIIN_i - TJIN_j < \Delta T_{MIN} \\
&\qquad \Delta T_{i,j}^{\max} = \text{abs}\left[TIIN_i \text{ - } TJIN_j \right] + \Delta T_{MIN} \\
&\text{else} \\
&\Delta T_{i,j}^{\max} = \max\left\{0, TIIN_i - TJIN_j, TJOUT_j - TIOUT_i\right\}
\end{aligned} \tag{62}$$

where a minimum approach temperature is included to avoid infinite areas (or extremely large values).

Logarithmic Mean Temperature Differences. To avoid singularities in the calculation of the log mean temperature difference (LMTD), Chen (1987) proposed an approximation that is suitable for cases without change of phase or with only isothermal change of phase (Figures 2a, 2b, 2d, 2e). Such estimation of LMTD is based on the temperatures at both ends of the exchanger. When streams change phase with superheating or subcooling (Figures 2c, 2f), however, such calculation may not reflect a proper average since the temperature difference inside the exchanger might be either lower or higher than the differences at both ends of the exchanger. A more accurate estimation of LMTD is needed for those cases. Therefore, the following disjunction can be conceptually written for any match between a hot process stream $i \in HPS3$ and a cold process stream $j \in CPS3$,

$$\begin{pmatrix} Y_{i,j,k}^{7} \\ lmtd_{i,j,k} = f\left(dt_{i,j,k}, dt_{i,j,k+1}, dt_{i,j}^{sat}\right) \\ q_{i,k}^{\Lambda} \geq \delta \\ q_{j,k}^{\Lambda} \geq \delta \end{pmatrix} \vee \begin{pmatrix} \neg Y_{i,j,k}^{7} \\ lmtd_{i,j,k} = f\left(dt_{i,j,k}, dt_{i,j,k+1}\right) \end{pmatrix}$$

where the logic variable $Y_{i,j,k}^{7}$ is true if both streams *i* and *j* exchange latent heat in match *k*.

To include the temperature difference between boiling and condensing temperatures, a type of geometric mean based on three points of the exchanger is used. The following constraints are then written to model the LMTD disjunction as a big-M formulation,

$$lmtd_{i,j,k} \le \left\{ \left(dt_{i,j,k}\right)\left(dt_{i,j,k+1}\right)\left(dt_{i,j}^{sat}\right)\left(\frac{dt_{i,j,k}+dt_{i,j,k+1}+dt_{i,j}^{sat}}{3}\right) \right\}^{1/4} + lmtd_{i,j}^{\max}\left(1-y_{i,j,k}^{7}\right), i \in HPS3,\ j \in CPS3, k \in ST \tag{63}$$

$$lmtd_{i,j,k} \le \left\{ \left(dt_{i,j,k}\right)\left(dt_{i,j,k+1}\right)\left(\frac{dt_{i,j,k}+dt_{i,j,k+1}}{2}\right) \right\}^{1/3} + lmtd_{i,j}^{\max} y_{i,j,k}^{7}, i \in HPS,\ j \in CPS, k \in ST \tag{64}$$

where Equation (64) is Chen's approximation for LMTD (Chen, 1987), and Equation (63) includes the difference between the saturation temperatures for the hot and cold process streams in an extended form of Chen's equation in order to approximate the profiles in figures 2c and 2f.

The 0-1 variable $y_{i,j,k}^{7}$ can be related to the other binary variables through the following linear inequalities,

$$\left.\begin{array}{l} y_{i,k}^{1} + y_{i,j,k}^{7} \le 1 \\ y_{i,k}^{2} + y_{i,j,k}^{7} \le 1 \\ y_{i,k}^{4} + y_{i,j,k}^{7} \le 1 \\ y_{i,k}^{5} + y_{i,j,k}^{7} \le 1 \end{array}\right\} \quad i \in HPS3,\ j \in CPS3, k \in ST \tag{65}$$

An additional relationship is needed to ensure that $y_{i,j,k}^{7}$ must be equal to 1 when all other *y's* are equal to zero,

$$y_{i,k}^{1} + y_{i,k}^{2} + y_{j,k}^{4} + y_{j,k}^{5} + y_{i,j,k}^{7} \ge 1, i \in HPS3,\ j \in CPS3, k \in ST \tag{66}$$

Heat Transfer Coefficients. For streams than only exchange either latent heat of sensible heat (i.e., streams in the sets *HPS1*, *HPS2*, *CPS1* and *CPS2*) the heat transfer coefficients are given as part of the design data. For streams with heat contents that include both types, an equation for the use of the proper value can be related to the logical variables used in the model formulation. Thus, for a hot stream $i \in HPS3$ in any stage of the superstructure, the film heat transfer coefficient is given by,

$$h_{i,k} = h_i^{suph} y_{i,k}^1 + h_i^{subc} y_{i,k}^2 + h_i^{mean} \left(1 - y_{i,k}^1 - y_{i,k}^2\right), i \in HPS3, k \in ST \tag{67}$$

whereas for a match of a hot stream with a cold utility,

$$h_i^{cu} = h_i^{subc} y_i^3 + h_i^{mean} \left(1 - y_i^3\right), i \in HPS3 \tag{68}$$

where

$$h_i^{mean} = \frac{h_i^{suph} FCP_i \left(TIN_i - T_i^{cond}\right) + h_i^{subc} FCP_i \left(T_i^{cond} - TOUT_i\right) + h_i^{cond} F \lambda_i^{cond}}{FCP_i \left(TIN_i - T_i^{cond}\right) + FCP_i \left(T_i^{cond} - TOUT_i\right) + F \lambda_i^{cond}}, i \in HPS3 \tag{69}$$

Note from (67) that for streams that are either above or below saturation the inlet heat transfer coefficient reduces to either $h_i^{\sup h}$ or h_i^{subc}. For the case where phase change is included h_i^{mean} represents an average heat transfer coefficient weighted by corresponding heat content contributions.

Similarly, for a cold stream $j \in CPS3$,

$$h_{j,k} = h_j^{suph} y_{j,k}^5 + h_j^{subc} y_{j,k}^4 + h_j^{mean} \left(1 - y_{j,k}^4 - y_{j,k}^5\right), j \in CPS3, k \in ST \tag{70}$$

and for a match with a hot utility,

$$h_j^{hu} = h_j^{suph} y_j^6 + h_j^{mean} \left(1 - y_j^6\right), j \in CPS3 \tag{71}$$

where,

$$h_j^{mean} = \frac{h_j^{suph} FCP_j \left(TOUT_j - T_j^{evap}\right) + h_j^{subc} FCP_j \left(T_j^{evap} - TIN_j\right) + h_j^{evap} F \lambda_j^{evap}}{FCP_j \left(TOUT_j - T_j^{evap}\right) + FCP_j \left(T_j^{evap} - TIN_j\right) + F \lambda_j^{evap}}, j \in CPS3 \tag{72}$$

Objective Function. The objective function is defined as the minimization of the total yearly costs, which includes the cost of utilities and the fixed and variables costs of the exchangers. Chen's approximation is used again for the LMTD for heaters and coolers. Therefore, the objective function is written as follows:

$$
\begin{aligned}
&\min \sum_{i\in HPS} CCUqcu_i + \sum_{j\in CPS} CHUqhu_j \\
&+ \sum_{i\in HPS}\sum_{j\in CPS}\sum_{k\in ST} CF_{i,j} z_{,j,k} + \sum_{i\in HPS} CF_{i,cu} zcu_i + \sum_{j\in CPS} CF_{cu,j} zhu_j \\
&+ \sum_{i\in HPS}\sum_{j\in CPS}\sum_{k\in ST} C_{i,j} \left\{ \frac{q_{i,j,k}\left(\dfrac{1}{h_{i,k}} + \dfrac{1}{h_{j,k}}\right)}{lmtd_{i,j,k} + \delta} \right\}^{\beta} \\
&+ \sum_{i\in HPS} C_{i,cu} \left\{ \frac{qcu_i \left(\dfrac{1}{h_i^{cu}} + \dfrac{1}{h_{cu}}\right)}{\left[\left(dt_{i,cu}\right)\left(TOUT_i - TIN_{cu}\right)\left(\dfrac{dt_{i,cu} + TOUT_i - TIN_{cu}}{2}\right) + \delta\right]^{1/3}} \right\}^{\beta} \\
&+ \sum_{j\in CPS} C_{hu,j} \left\{ \frac{q_{hu,j} \left(\dfrac{1}{h_{hu}} + \dfrac{1}{h_j^{hu}}\right)}{\left[\left(dt_{hu,j}\right)\left(TIN_{hu} - TOUT_j\right)\left(\dfrac{dt_{hu,j} + TIN_{hu} - TOUT_j}{2}\right) + \delta\right]^{1/3}} \right\}^{\beta}
\end{aligned}
\tag{73}
$$

In summary, the proposed MINLP model for the HEN synthesis including isothermal and non-isothermal streams consists of the minimization of equation (73), subject to the constraints given by equations (1-72). The continuous variables (t, q, dt, $lmtd$, h, q^{Λ}) are nonnegative, and z and y are sets of binary variables.

Remarks

1) In the MINLP model the objective function is nonlinear and all the constraints are linear, except for equations (63) and (64) for the logarithmic mean temperature differences.

2) Notice that the overall energy balance for streams that transfer only sensible heat (Equations (1) and (4)) are linearly dependent, because they can be obtained by combining the energy balances for each stage (Equations (7) and (9)) and the energy balances for utilities (Equations (19) and (21)). Although they have been written here for the sake of completeness, Equations (1) and (4) can be therefore removed of the model. A similar observation applies for Equations (3) and (6). The overall energy balance for pure isothermal streams (i.e., streams that only exchange latent heat), however, are needed in the model because energy balances for each stage are not written for these types of streams.

3) For nonisothermal streams that require splitting, isothermal mixing at the end of a stage is used as a simplifying assumption in the model (as in Yee and Grossmann, 1990). On the other hand, isothermal mixing is a natural consequence for a split stream with change of phase, and in that case the heat exchangers can be placed either in series or in parallel. For the general case one can use the two-stage procedure by Yee and Grossmann to obtain unequal outlet temperatures of split streams.

4) The model for streams with change of phase is strictly correct for pure components. For multicomponent mixtures, an approximation such that the bubble and dew temperatures are the same is made.

4. Examples

Five case studies are presented to show the application of the proposed algorithm. In the examples the capital cost function for the heat exchangers was given by \$1,650 $(A)^{0.65}$ (A in m^2), and an annuity factor of 0.23 yr^{-1} was used. The unit costs for the heating and cooling utilities were assumed as 100 \$/kW year and 10 \$/kW year, respectively. A value of ΔT_{MIN} of 5°K was used for all the examples. The solver DICOPT included in the general algebraic modeling system GAMS (Brooke *et al.*, 2006) was used for the solution of the problems.

Example 1

This is a fairly simple problem that serves as a motivational example, in which only isothermal streams are considered. Table 1 gives the stream data for two hot and two cold process streams, along with heating and cooling utilities. Figure 10 shows the composite curves for this problem. Notice that for these types of examples the composite curves may overlap even when the minimum temperature difference is not reached.

For this simple problem, the original *Synheat* model proposed by Yee and Grossmann (1990) was unable to get a feasible solution. That model approximates isothermal streams using a one degree temperature change with a suitable (generally large) value of the heat capacity that matches the total heat content of that stream. The main difficulty of this procedure has to do with scaling problems. Another difficulty of the *Synheat* model for these types of problems is the proposed value for the parameter $\Delta T_{i,j}^{\max}$; such a model uses the following equation to calculate the upper limit for the temperature difference,

$$\Delta T_{i,j}^{\max} = \max\left\{0, TIIN_i - TJIN_j, TJOUT_j - TIOUT_i\right\} \tag{74}$$

Equation (74) works properly for non isothermal streams, but for the case of isothermal streams it can lead to errors. For example, considering the data given in Table 1 and a match between hot stream *H1* and cold stream *C1*, the parameter $\Delta T_{i,j}^{\max}$ given by Equation (74) is equal to 10. From the original design data, the difference $t_{ik} - t_{jk}$ is -10, such that the match between streams *H1* and *C1* should not be allowed by the model. If the binary variable is zero, then the right hand side of Equation (58), $t_{1,k} - t_{1,k} + \Delta T_{1,1}^{\max}$, becomes zero, which violates the value of ΔT_{MIN} and therefore leads to an infeasible solution from the original *Synheat* model even when the exchanger is not selected. Through the use of equation (62) a proper bound for $\Delta T_{i,j}^{\max}$ is estimated, and the model here presented provides an optimal solution to this problem.

Table 1. Data for Example 1

stream	*TIN* [K]	*TOUT* [K]	$F\lambda$ [KW]	Type	T_{change}^{phase} [K]	*h* [KW/(m^2 K)]
H1	400	400	4,000	*HPS2*	400	1.80
H2	425	425	3,000	*HPS2*	425	1.90
HU	627	627	-	-	-	2.50
C1	410	410	4,000	*CPS2*	410	1.70
C2	390	390	3,000	*CPS2*	390	1.85
CU	303	315	-	-	-	1.00

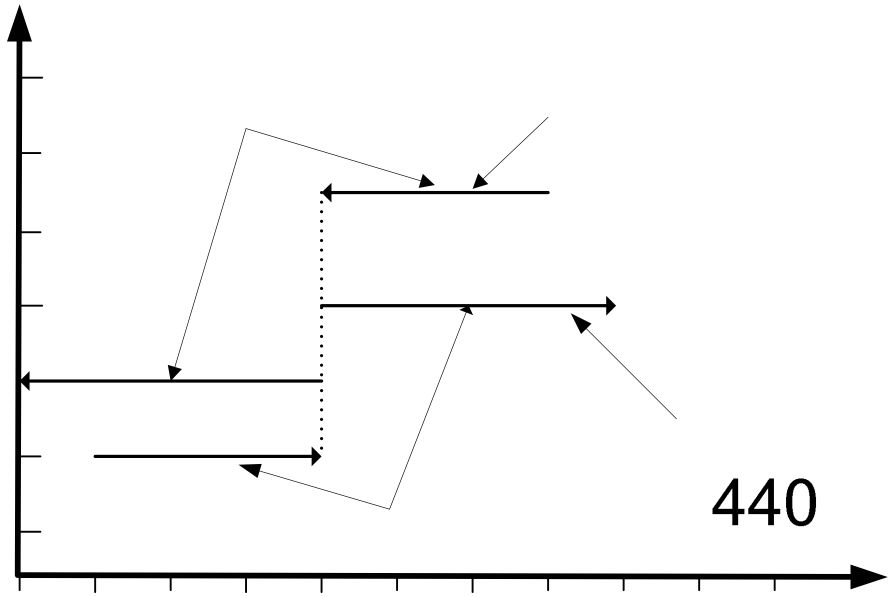

Figure 10. Composite curves for Example 1

Figure 11 shows the network obtained using the proposed model, which has a total yearly cost of $142,629/year, with utility and annualized investment costs of $110,000/year and $32,629/year, respectively. Note that this network achieves maximum energy recovery as shown in Figure 10.

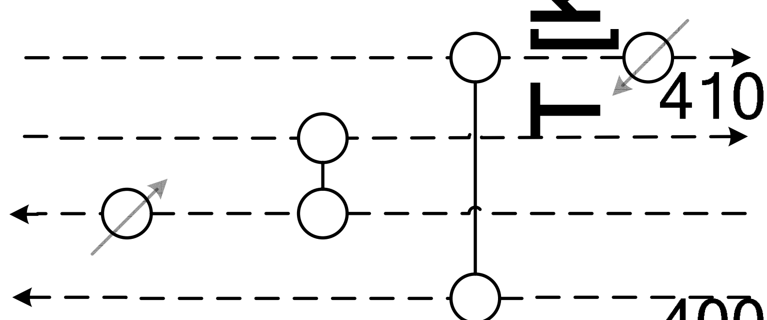

Figure 11. Network obtained for Example 1

Example 2

The stream data for Example 2 are given in Table 2. The composite curves for this problem are shown in Figure 1 for a HRAT of 5°K. This example consists of a set of streams with latent heat only, and another set of streams with sensible heat only. Therefore, the model for this case does not include the constraints for sets *HPS3* and *CPS3*.

Table 2. Stream data for Example 2

stream	*TIN* [K]	*TOUT* [K]	*F* [Kg/s]	*Cp* [KJ/(Kg K)] *or* λ [KJ/Kg]	*FCp* [KW/K] or $F\lambda$ [KW]	Type	T^{phase}_{change} [K]	*h* [KW/(m^2 K)]
H1	503	308	15.44	4.3	66.4	*HPS1*	-	0.81
H2	425	425	13.00	2,540.0	33,020.0	*HPS2*	425	1.78
H3	381	381	6.50	1,980.0	12,870.0	*HPS2*	381	1.62
HU	627	627	-	-	-	-	-	2.5
C1	323	503	11.69	4.2	49.1	*CPS1*	-	0.72
C2	408	408	9.98	1,845.0	18,413.1	*CPS2*	408	1.91
C3	391	391	7.97	2,321.0	18,498.4	*CPS2*	391	1.76
C4	353	353	7.24	2,258.0	16,347.9	*CPS2*	353	1.84
CU	303	315	-	-	-	-	-	1.00

Figure 12 shows the optimal network obtained using the proposed model, while Table 3 shows the details for the heat exchangers of the network. The dashed lines in Figure 12 are used to denote the isothermal streams. Nine units are required, out of which seven correspond to process-process exchangers. The hot and cold utility costs are \$510,639/year and \$18,470/year, respectively, and the investment cost for the heat exchangers is \$157,905/year. The total yearly cost for the network is \$687,014/year.

Figure 12 shows a schematic representation of the temperature profiles for each of the heat exchangers of the network. Notice that exchangers 2, 3 and 4 may be placed either in series or in parallel for the hot process stream *H2* (an isothermal stream). The same observation applies for exchangers 1 and 4 for the cold process stream *C3,* and exchangers 5 and 6 for cold process stream *C4*.

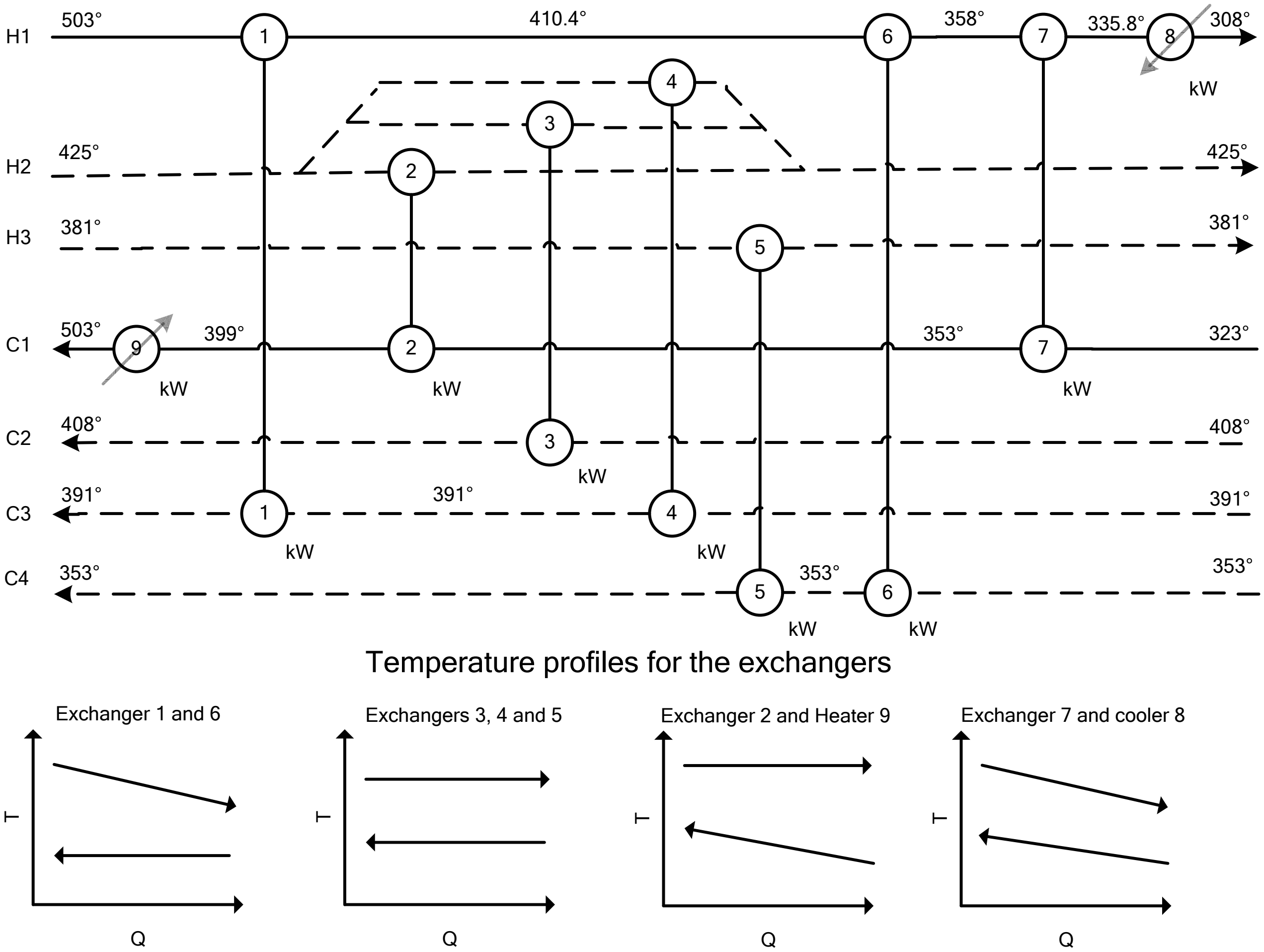

Figure 12. Optimal network for Example 2

Table 3. Details for the units of Figure 12

Exchanger	q [kW]	A [m^2]
1	6,150.0	212.2
2	2,258.6	97.6
3	18,413.1	1,175.6
4	12,348.3	410.[illegible]
5	12,870.0	533.5
6	3,477.9	297.8
7	1,473.0	465.8
8	1,847.0	374.0
9	5,106.4	53.5

5,106.4

6,150.0

Example 3

Table 4 shows the stream data for Example 3, which consists of four hot and three cold process streams. All process streams in this example are isothermal. Figure 13 shows the composite curves for this case. It can be noticed that there is no pinch temperature if one takes the value of 5 °K as a reference. For this example, the equations for the subsets *HPS2* and *CPS2* apply. The optimal network obtained is shown in Figure 14. The lowest value of temperature differences among streams is 15 °K, which is observed in exchanger 3. All streams are used for energy integration in the network, except for stream *H1*, which must be fully processed with cooling utilities. Table 5 shows the data for the exchangers of the network. The optimal solution shows utility and investment costs of $125,870/year and $30,104/year, respectively, which yields a total annual cost of $155,974/year.

Table 4. Stream data for Example 3

stream	*TIN* [K]	*TOUT* [K]	*F* [Kg/s]	λ [KJ/Kg]	$F\lambda$ [KW]	Type	T_{change}^{phase} [K]	*h* [KW/(m^2 K)]
H1	340	340	0.80	2,375	1,900.0	*HPS2*	340	1.52
H2	390	390	0.79	1,890	1,493.1	*HPS2*	390	1.63
H3	420	420	1.20	2,162	2,594.4	*HPS2*	420	1.75
H4	475	475	0.82	2,438	1,999.1	*HPS2*	475	1.58
HU	627	627	-	-	-	-	-	2.50
C1	350	350	0.50	1,985	992.5	*CPS2*	350	1.81
C2	375	375	0.79	2,280	1,801.2	*CPS2*	375	1.72
C3	400	400	1.88	2,320	4,361.6	*CPS2*	400	1.64
CU	303	315	-	-	-	-	-	1.00

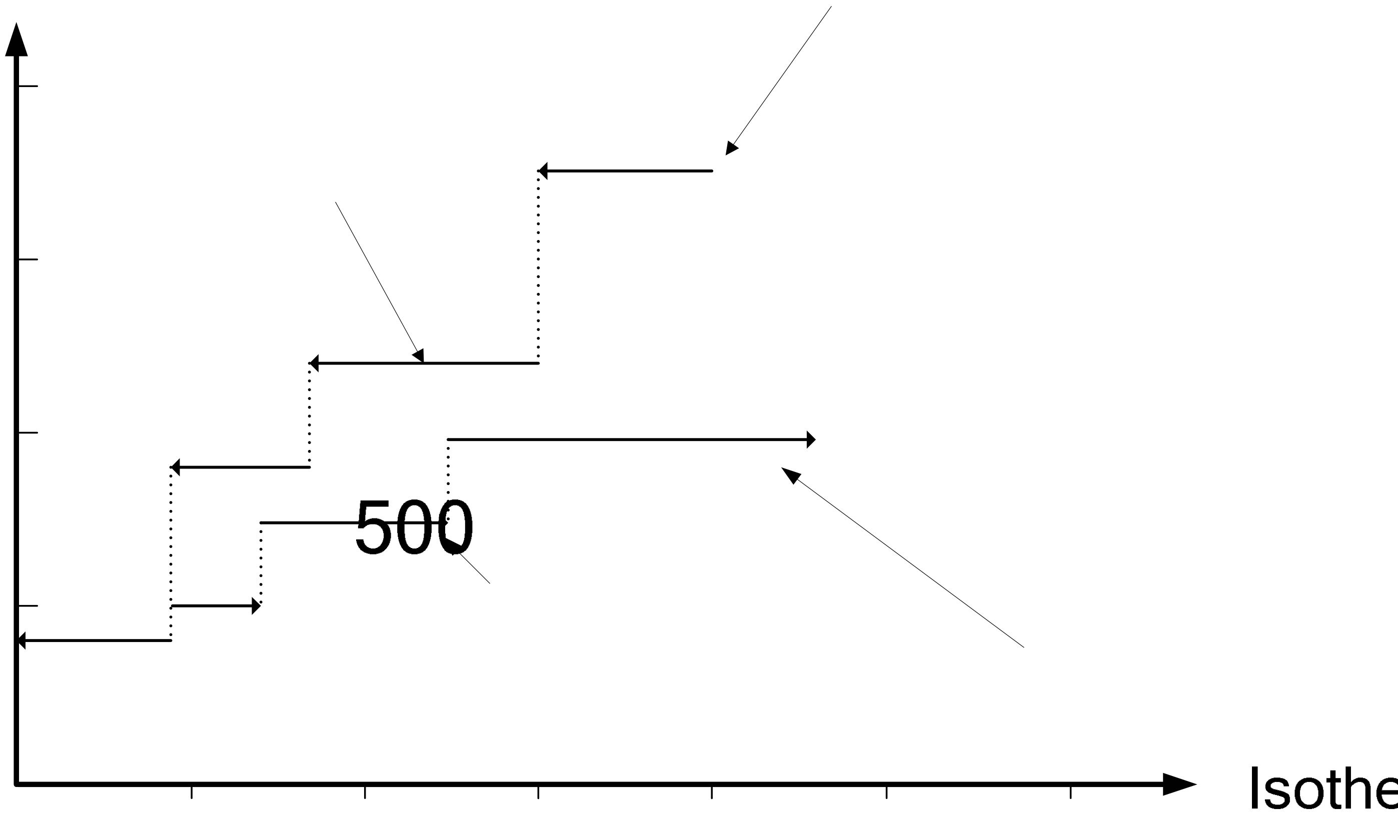

Figure 13. Composite curves for Example 3

450

T [K]

400

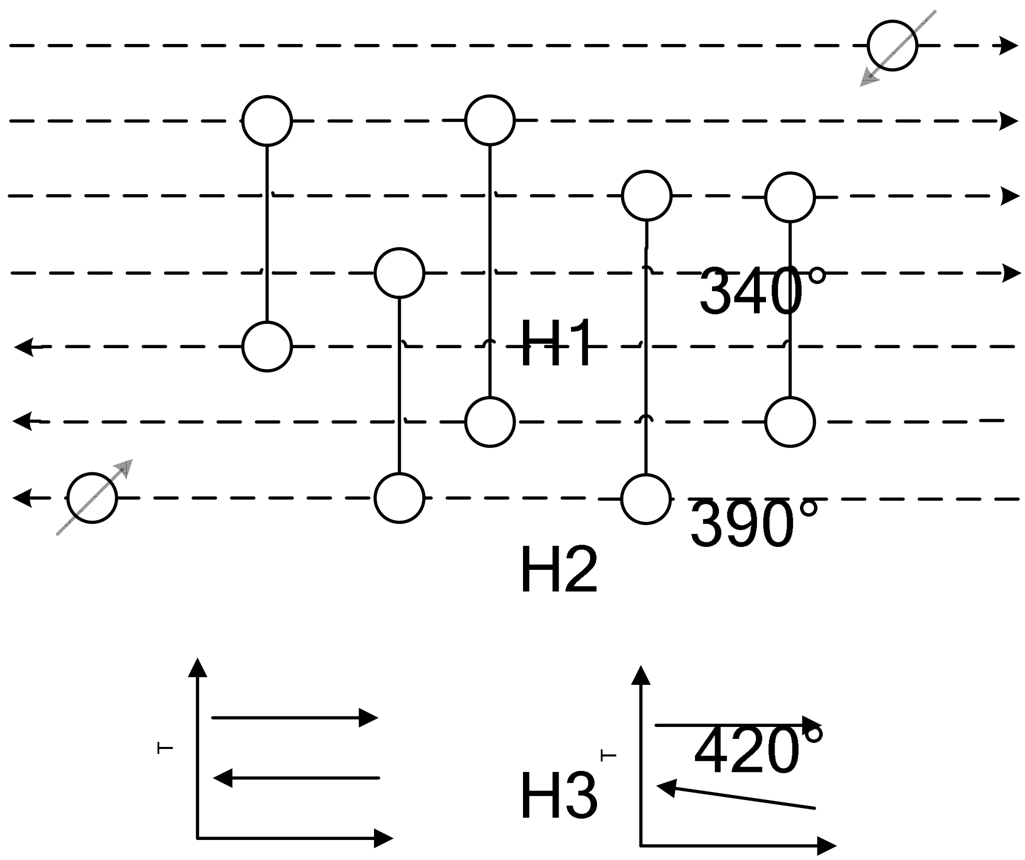

Figure 14. Optimal network for Example 3

Table 5. Details for the exchangers of Example 3

Exchanger	*q* [kW]	*A* [m^2]
1	992.5	28.93
2	1,999.1	33.12
3	500.6	39.87
4	1,293.8	76.41
5	1,300.6	33.32
6	1,900.0	102.92
7	1,068.7	4.75

Example 4

This example consists of finding a network for three hot and three cold process streams. Streams *H1* and *C1* exchange only latent heat, streams *H2* and *C2* exchange only sensible heat, and streams *H3* and *C3* exchange both latent and sensible heat. In this problem, streams *H3* and *C3* have sensible heat segments for both superheated and subcooled conditions, and film heat transfer coefficients for each segment are included in the stream data (see Table 6).

Table 6. Data for Example 4

stream	*TIN* [K]	*TOUT* [K]	*F* [Kg/s]	*Cp* [KJ/(Kg K)]	λ [KJ/Kg]	*FCp* [KW/K]	$F\lambda$ [KW]	Type	T^{phase}_{change} [K]	*h* [KW/(m^2 K)]
H1	590	380	11.31	3.9	-	44.109	-	*HPS1*	-	0.53
H2	480	480	7.98	-	2,130	-	16,997.4	*HPS2*	480	1.80
H3	500	320	8.16	4.2	1,881	34.272	15,348.9	*HPS3*	400	[0.52;0.71;2.1]
HU	627	627	-	-	-	-	-	-	-	2.5
C1	450	600	8.66	4.5	-	38.970	-	*CPS1*	-	0.54
C2	452	452	5.33	-	2,251	-	11,997.8	*CPS2*	452	2.3
C3	310	550	6.42	3.7	1,725	23.754	11,074.5	*CPS3*	380	[0.62;0.80;1.9]
CU	303	315	-	-	-	-	-	-	-	1.00

Figure 15 shows the optimum network obtained using the proposed model. One can observe how in exchanger 4 both process streams *H2* and *C3* exchange sensible and latent heats; stream *C3* enters the exchanger at its supplied temperature (subcooled conditions) and exists as a superheated stream, while stream *H3* enters the exchanger at superheated conditions and exits with partial condensation. A cooler (exchanger 8) is then needed to bring stream *H3* from a mixture of vapor-liquid to a subcooled liquid. Process stream *H2* fully integrates stream *C2* (exchanger *5*) and requires the use of a cold utility to reach its target conditions. In this problem the estimation of film heat transfer coefficients for streams with multiple segments of heat transfer must be made. For stream *H3,* the mean coefficient in exchanger 4 and in cooler 8 (where both latent and sensible heats are processed) is 0.857 kW/m^2 K, while for stream *C3* the film coefficient is 1.483 in exchanger 4 (both latent and sensible heat), and 0.62 in exchanger 2 and heater 10 (in which only sensible heat is exchanged).

Exchanger 4 integrates sensible and latent heats from both the hot and cold streams. The calculated area with the proposed approximation of LMTD for those cases (Equation 63) is 1,175 m^2, which shows an error of 3.0 percent with respect to a more accurate calculation of the area that takes into account the different segments of heat transferred by each stream (1,212 m^2). With the use of a single LMTD given by Chen's approximation (Equation 64), the estimated area is 1,026 m^2, which represents an error of 15.3 percent. Thus, it is clear that Equation 64 provides very good approximations to exchangers with sensible and latent heats.

One characteristic of the network is the many matches that are observed with the minimum temperature difference. The optimal network for this example has heating and cooling costs of \$142,851/year and \$145,885/year, respectively, and an investment cost of \$167,411/year, yielding a total yearly cost of \$456,147/year.

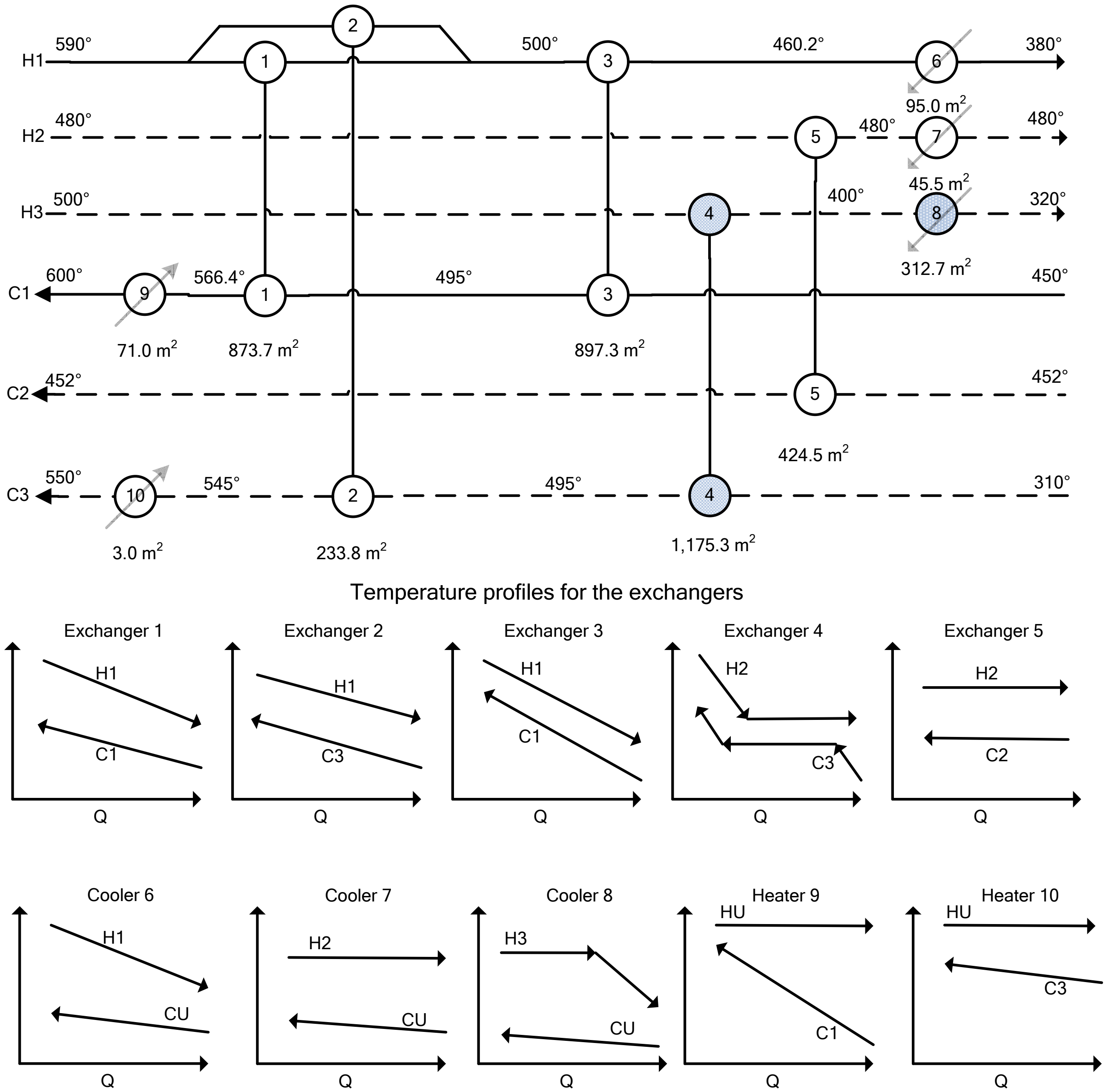

Figure 15. Optimal network for Example 4

1,309.7 kW 2,782.1 kW

Example 5

The stream data for this example are shown in Table 7, in which process streams *H1* and *C1* must exchange both latent and sensible heat. This example also includes streams that exchange only latent or sensible heat. Figure 16 shows the optimal network obtained for this case. Six internal exchangers, two heaters and two coolers comprise the network. From the network structure and the temperature profiles of Figure 16, one can see that in exchanger 4 both sensible and latent heats are processed for stream *H1*, which integrates completely the heat content of stream *C4*. Exchanger 6 is used to complete the integration of *H1* through the sensible heat and part of the latent heat of stream *C1*. In exchanger 3, the remaining of the latent heat and part of the sensible heat (superheated) for C1 is integrated with *H4*, a nonisothermal stream. Notice how the model accomplishes the placement of both sensible and latent heats in single units whenever possible, as opposed to the use of one unit for each heat transfer segment. Values for film heat transfer coefficients for streams that transfer both latent and sensible heats are as follows. For the hot process stream *H1,* the film coefficient is 0.672 kW/m^2 K in exchangers 4 and 6, and for the cold process stream *C1* it is 1.878 in exchangers 3 and 6, and 0.56 in heater 9. Exchanger 6 performs energy integration between two streams that belong to the sets *HSP3* and *CPS3*. The calculated area for this exchanger using the proposed equation for LMTD in Equation 64 is 302.3 m^2, while the application of Chen's approximation only gives 289.8 m^2. A detailed calculation provides a value of 327 m^2. The optimal network shows hot and cold utility costs of $42,030/year and $49,824/year, respectively. With an annualized investment cost of $80,201/year, the total annual cost is $172,055/year.

Table 7. Data for Example 5

stream	*TIN* [K]	*TOUT* [K]	*F* [Kg/s]	*Cp* [KJ/(Kg K)]	*λ* [KJ/Kg]	*FCp* [KW/K]	*Fλ* [KW]	Type	T^{phase}_{change} [K]	*h* [KW/(m^2 K)]
H1	620	440	9.5	4.0	1,895	38.00	18,002.5	*HPS3*	440	[0.52;0.73;1.5]
H2	590	450	7.8	3.9	-	30.42	-	*HPS1*	-	0.58
H3	420	420	8.1	-	2,185	-	17,698.5	*HPS2*	420	1.98
H4	598	355	6.9	4.2	-	28.98	-	*HPS1*	-	0.54
HU	627	627	-	-	-	-	-	-	-	2.5
C1	350	420	7.1	4.8	1,862	34.08	13,220.2	*CPS3*	370	[0.56;0.87;2.1]
C2	380	380	6.6	-	2,310	-	15,246.0	*CPS2*	380	1.61
C3	350	450	11.9	4.2	-	49.98	-	*CPS1*	-	0.83
C4	418	418	6.8	-	1,975	-	13,430.0	*CPS2*	418	1.72
CU	303	315	-	-	-	-	-	-	-	1.00

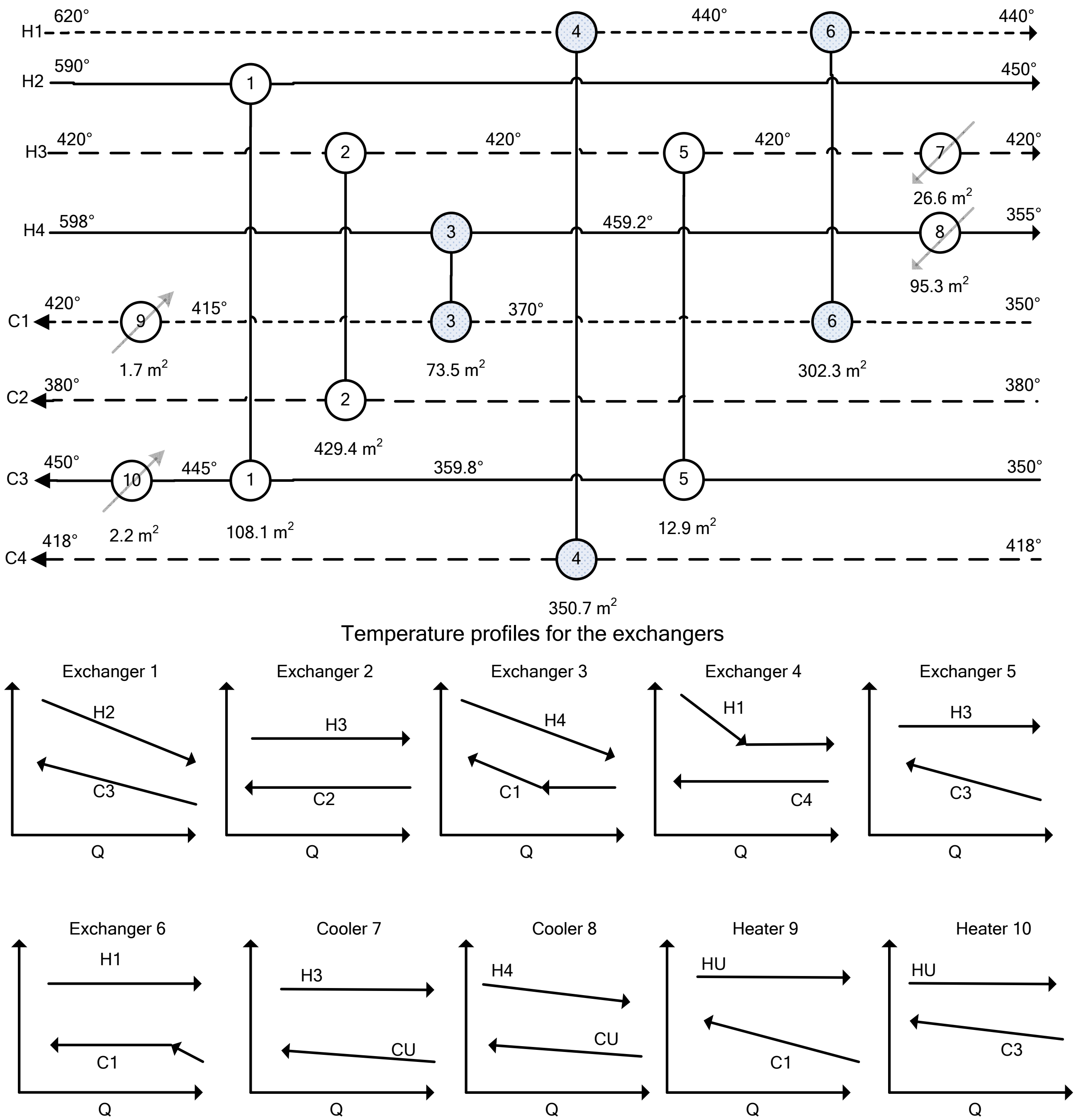

Figure 16. Optimal networks for Example 5

170.4 kW

Finally, we report the sizes and computer times needed to obtain the optimal solutions. Table 8 shows the CPU time for the example problems, run with DICOPT under a Pentium 4 system with 2.66 GHz. It can be observed how the solution to the cases of study here presented was readily obtained using the proposed model.

Table 8. CPU time for the example problems

Example	Number of binary variables	Number of continuous variables	Number of constraints	CPU time [s]
1	12	41	49	0.29
2	55	158	213	0.54
3	55	158	201	0.31
4	72	316	425	0.57
5	109	435	577	0.95

4. Conclusions

A new model has been proposed that can handle isothermal streams in the synthesis of heat exchanger networks. Although these types of streams are very common in industrial practice, a proper synthesis methodology for handling these streams had not been reported before. The model is based on a MINLP formulation and provides the network with the minimum total annual cost, which includes the capital cost of the exchangers and the utility costs. The model consists of a nonlinear objective function, with almost all of the constraints being linear. The nonlinearities in the objective function and in two of the model constraints arise because of the need for the logarithmic mean term on temperature differences for the exchangers design equation for the various cases. The model has been formulated to include streams that are to exchange both types of latent and sensible heat within the network. Five examples that reflect different types of problems with isothermal streams have been solved to illustrate the application of the proposed model. No convergence problems were observed for any of the case studies that were analyzed, and the computational requirements were small.

Acknowlegements

This work was performed while A. Jiménez was a Fulbright Scholar at Carnegie Mellon University. J. Ponce was supported through a scholarship from CONACYT, Mexico.

Nomenclature

A heat transfer area

C	area cost coefficient
CCU	unit cost of cold utility
CHU	unit cost of hot utility
CF	fixed charge for exchangers
Cp	specific heat capacity
CPS	$\{j \mid j$ is a cold process stream$\}$
$CPS1$	$\{j \mid j$ is a non isothermal cold process stream$\}$
$CPS2$	$\{j \mid j$ is an isothermal cold process stream$\}$
$CPS3$	$\{j \mid j$ is a stream that exchanges latent and sensible heat$\}$
CU	cold utility
$dt_{i,j,k}$	temperature approach difference for match (i,j) at temperature location k
$dtcu_i$	temperature approach difference for match between hot stream i and a cold utility
$dthu_j$	temperature approach difference for match between cold stream j and a hot utility
$dt_{i,j}^{sat}$	temperature difference between saturated streams
F	flow rate
FCp	heat capacity flow rate
h	fouling heat transfer coefficient
h^{cond}	fouling heat transfer coefficient for condensation
h^{evap}	fouling heat transfer coefficient for evaporation
h^{mean}	mean fouling heat transfer coefficient
h^{suph}	fouling heat transfer coefficient for superheated part of a stream
h^{subc}	fouling heat transfer coefficient for subcooled part of a stream
HPS	$\{i \mid i$ is a hot process stream$\}$
$HPS1$	$\{i \mid i$ is a non isothermal hot process stream$\}$
$HPS2$	$\{i \mid i$ is an isothermal hot process stream$\}$

$HPS3$	$\{i \mid i$ is a stream that exchange latent and sensible heat$\}$
HU	hot utility
$lmtd_{i,j,k}$	log-mean temperature difference
$lmtd_{i,j}^{\max}$	upper limit for the *lmtd* for match i,j
NOK	total number of stages
$q_{i,j,k}$	heat exchanged between hot process stream i and cold process stream j in stage k
qcu_i	heat exchanged between cold utility and hot stream i
qhu_j	heat exchanged between hot utility and cold stream j
$Q^{\max}$	upper bound for heat exchange
ST	$\{k \mid k$ is a stage in the superstructure, $k = 1, \ldots, NOK\}$
$t_{i,k}$	temperature of hot stream i at the hot end of stage k
$t_{j,k}$	temperature of cold stream j at the hot end of stage k
$t_{i,k}^{n}$	disaggregated variables used to model disjunctions
$t_{j,k}^{n}$	disaggregated variables used to model disjunctions
T_i^{cond}	condensation temperature for hot stream i
T_j^{evap}	evaporation temperature for cold stream j
T_{change}^{phase}	phase change temperature
TIN_i	inlet temperature of stream i
$TOUT_i$	outlet temperature of stream i
$\Delta T^{\max}$	upper bound for temperature difference
ΔT_{MIN}	minimum approach temperature difference
Y	boolean variables used to model disjunctions
y	binary variables used to model disjunctions
$z_{i,j,k}$	binary variables for match (i,j) in stage k
zcu_i	binary variables for match between cold utility and hot stream i

zhu_j	binary variables for match between hot utility and cold stream j

Greek Symbols

λ	unit latent heat
$F\lambda_i^{cond}$	condensation heat load for hot stream i
$F\lambda_j^{evap}$	evaporation heat load for cold stream j
$q_{i,k}^{\Lambda}$	condensation heat load of hot stream i exchanged at stage k
$q_{j,k}^{\Lambda}$	evaporation heat load of cold stream j exchanged at stage k
$q_i^{\Lambda,cu}$	condensation heat load that is exchanged with the cold utility
$q_j^{\Lambda,hu}$	evaporation heat load exchanged with a hot utility
β	exponent for area in cost equation
δ	small number

Subscripts

i	hot process stream
j	cold process stream
k	index for stage (1, …, *NOK*) and temperature location (1, …, *NOK*+1)

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