Modeling for Integrated Refinery Planning with Crude-oil Scheduling

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Abstract

This work aims to solve the integrated optimization for whole refinery processes, which spans from crude oil operations to the refining processes and blending operations. We define a multi-period refinery planning problem by considering the scheduling for crude oil in which the objective function is to maximize the net profit. The optimization procedure simultaneously determines the variables of crude oil scheduling, refinery planning, and blending recipes in each time period. A hierarchical hybrid continuous-discrete time representation is proposed for the integrated optimization problem. This integrated optimization problem yields a Mixed-Integer Nonlinear Programming model for the refinery-wide multi-period optimization. The main contribution of this work is the novel mathematical optimization model proposed for the entire process optimization of refinery production. Computational results with the solver DICOPT illustrate the scope of integrated optimization solutions for a real refinery plant with different horizon lengths, demonstrating the tractability, validity and capability for obtaining near-optimal solutions of the proposed integrated model.

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1. Introduction

The profit margins of refinery companies are severely affected by the international and national markets of crude oil and oil products, and it is even more severe for the refineries dependent on imported crude oil (Shah et al., 2011). COVID-19 will directly influence international crude-oil industries, transportation of imported crude oil, and oil products' market prices. Therefore, it becomes essential and necessary to optimize the entire refinery operation considering integrating its different process stages.

The entire refinery addressed in this paper can be separated into three parts: crude oil operations, refining processes, and blending operations. The crude oil operations determine the feedstocks for the refining processes, which then determine the blending components for the blending operations that finally determine the final oil products.

By performing an integrated optimization of a refinery, the supply of crude oil, the flexibility of the refinery processes, and the market demand for oil products are coordinated and optimized, increasing its operational feasibility and profit margin. However, the challenges of the entire process refinery optimization include multiple time-scales, hybrid process networks with different types of operation processes, and rigorous property requirements, leading to the difficulties for solving the corresponding large-scale model (Grossmann, 2012; Brunaud and Grossmann, 2017).

This work considers the integrated optimization of the refinery, covering the scheduling for crude oil operations, the refinery planning for refining and blending processes. For the crude oil scheduling, we consider the unloading operation from crude oil vessels to storage tanks, transfer and mixing operations from storage tanks to charging tanks, and feeding operations from charging tanks to the Crude Distillation Unit (CDU). Swing cut models for the CDU are used in Menezes et al. (2013). The flow and properties of crude oils and intermediate products are controlled and tracked using mass and properties balance equations. For the blending operations, nonlinear blending properties equations are used to determine the blending recipes for oil products that satisfy the Chinese national standards (GB 17930-2016, 2016). The energy consumption of the refining process is taken into consideration and optimized together with the properties of intermediate and final oil products. Thus, we focus on the entire refinery economic optimization and the detailed properties balance and blending recipes to guarantee the quality of oil products.

The proposed multi-period optimization model for refinery planning with crude oil scheduling maximizes the net profit of refinery production. A continuous-time representation is used for crude oil scheduling and a hybrid continuous-discrete time representation for refinery and blending planning. Considering this, we obtain an integrated optimization problem that consists of nonlinear constraints and both continuous and discrete variables. This optimization problem yields a Mixed-integer Nonlinear Program (MINLP) model for the refinery problem. Aiming at a real complex refinery process, we obtain an integrated solution for the entire process and analyze the sensitivity of the solution by using the solver DICOPT.

This paper is organized as follows: Section 2 provides related references in refinery optimization. In Section 3, we define the entire-process integrated optimization problem for the whole refinery process. The integrated mathematical model for the refinery optimization problem is formulated in Section 4. Section 5 implements computational experiments based on real refinery data provided by

China Petroleum & Chemical Corporation, Sinopec. Finally, we provide conclusions on the proposed optimization model.

2. Literature review

Most of the research on refinery optimization tackles independently each of the following operations: crude oil scheduling, refinery planning, oil product blending, and oil product scheduling (Shah and Ierapetritou, 2011). Few papers aim for the integration of these components. Grossmann (2012) surveyed Enterprise-wide Optimization (EWO) with the framework of mathematical programming, in which refinery planning is one of the critical EWO applications.

Pinto et al. (2000) addressed the planning and scheduling problems, respectively, for refinery operations with real applications divided into three problems: scheduling crude oil, planning diesel production, and on-line blending operations. Mouret et al. (2011) studied the integration problem of refinery planning and crude-oil scheduling for a single period. Taking the refinery planning as a pooling problem, they propose a nonconvex MINLP for the integration problem and develop a Lagrangean decomposition method to solve the model. In this work, no changeover time for the crude oils feeding the CDU was considered.

Jia and Ierapetritou (2004) addressed the short-term scheduling for an entire refinery process, divided into three problems: crude oil operations, production unit scheduling, and gasoline blending and distribution. They formulated each of the three problems as Mixed Integer Linear Programming (MILP) models and solved them independently. Shah and Ierapetritou (2011) addressed the integrated scheduling for refinery from production units to end-product blending units, formulated into an MILP model based on continuous-time representation. These authors assumed that the supply of raw materials is unlimited, the recipe for the CDUs was fixed, and the properties of the blending components were constant. Kelly et al. (2014) proposed a method to optimize both the component distillation curve and the blending recipes using monotonic interpolation. These actions covered the refining process and blending operation to optimize the qualities and output oil products. On the refinery planning side, the optimization problem has been studied over decades from a single period to multiple periods, ranging from the CDU model with constant yields to more sophisticated methods such as the fractionation index (Alattas et al., 2011). The models of refinery planning range from simple Linear Programming (LP) models to more detailed and complex Nonlinear Programming (NLP), MILP, and MINLP models (Klingman et al., 1987; Pinto et al., 2000; Jia and Ierapetritou, 2004; Neiro and Pinto, 2005; Bengtsson and Nonås, 2010; Mouret et al., 2011; Alattas et al., 2012; Albahri et al., 2019).

Menezes et al. (2013) formulated a swing-cut model for refinery distillation units. These authors considered the adjacent swing cuts with different properties, which improved the accuracy of the blended properties. Alhajri et al. (2008) proposed an NLP model for refinery planning and operations optimization, including a rigorous process model with operating variables and blending property models.

Alattas et al. (2011, 2012) proposed an MINLP model for the multi-period refinery production planning with a nonlinear CDU model. They introduced the fractionation index model for the CDU, integrated it with an LP planning model, and finally extended it to a multi-period operation. Zhao et al. (2014) addressed refinery planning by integrating the production and utility systems. The energy consumption and generation models of the processing unit are introduced to correlate the two systems. Zulkafli and Kopanos (2018) developed a stochastic programming method for the integrated cleaning planning of production and utility systems. Zhang and Rong (2008) studied the scheduling for fuel gas systems in a refinery to minimize the energy systems operation cost.

To the best of our knowledge, there are no published works address the integrated optimization for the whole process in a refinery considering the properties of oil products and energy consumption. Research on multi-period refinery planning primarily focuses on the problem with assumptions of simplified process and blending models. Therefore, their planning solutions usually ignore or approximate the properties of oil products. However, in real refineries, enterprises have to face pressure from both product quality and energy consumption constraints (Guo and Xu, 2004; Hua, 2009).

This paper aims to integrate multi-period optimization for refinery planning with crude oil scheduling, which is formulated into an integrated model to simultaneously coordinate the feedstocks of crude oil and the production of the refining processes to satisfy the demand of oil products. We use continuous-time representation for the crude oil scheduling, and discrete-time representation for the refinery planning problem. The properties of material flows are controlled with property balance equations for each operation. In addition, the rigorous nonlinear blending equations are embedded in the integrated optimization model to represent the oil product properties and enforce quality constraints. We also consider the changeovers of CDU feedstocks, a swing-cut model for CDU, and processing units energy consumption. Finally, based on the proposed model and the data from a real refinery plant, we report the optimization solution for the entire refinery process and illustrate the advantages of integrated optimization.

Nomenclature

Indices /Sets

$g \in CT$	charging tanks
$i \in I$	materials
$j \in J$	refining units
$k \in K$	processing sequence in a period
$n \in N$	events (transfer, changeover)
$p \in P$	properties of material
$s \in ST$	storage tanks
$t \in T$	time periods
$u \in U$	CDUs
v	crude oil vessels
IC	set of component oils
IF_j	set of feed streams of unit j
IP	set of final products
IR	feedstocks, including all types of crude oil, some additive
IS_j	product streams of unit j
JF_i	unit set of feed stream i
JP_i	unit set produced material i

Parameters

$API_{1,2}$	parameters for API properties

BX_i	upper bound of blending ratio for component $i \in IC$
BL_i	lower bound of blending ratio for component $i \in IC$
CA	cost of changeovers between charging tanks
$CD_{i,i'}$	cost of changeovers between different type of crude oil in CDU
CG_g	inventory cost of charging tank g
$CI_{i,t}$	inventory cost of material i in refinery in time period t
CL	unloading cost from vessels to storage tanks
CN_{1-5}	parameters for Cetane Number properties
CO_j	unit operation production cost of unit j
CR_i	price of feedstock i
CS_s	inventory cost of storage tank s
CU_u	cost price of utility u
CW	waiting cost of crude oil vessel v in the sea
$D_{i,t}$	demand amount of final product i in time period t
EA_i^U	upper bound of unit comprehensive energy consumption for unit j
$EI_{j,t}$	index parameter for unit comprehensive energy consumption of unit j during t
$FG_{g,u}^L$	lower bound of flowing rate from charging tank g to CDU u
$FG_{g,u}^U$	upper bound of flowing rate from charging tank g to CDU u
$FP_{1,2}$	freezing point parameters for blending operations
$FS_{s,g}^L$	lower bound of flowing rate from storage tank s to charging tank g
$FS_{s,g}^U$	upper bound of flowing rate from storage tank s to charging tank g

$FV_{v,s}^{L}$	lower bound of flowing rate from vessel v to storage tank s
$FV_{v,s}^{U}$	upper bound of flowing rate from vessel v to storage tank s
H	total production horizon
Ht	horizon length of each time period t
Ig_{g}^{0}	initial inventory amount of charging tank g at the beginning of H
Ig_{g}^{U}	upper bound of inventory amount of charging tank g
$Igp_{g,p}^{0}$	initial property p in charging tank g
$Igp_{g,p}^{L}$	lower bounds of material property p in charging tank g
$Igp_{g,p}^{U}$	upper bounds of material property p in charging tank g
$In0_{i,t}$	initial inventory amount of material i in period t
InX_{i}	inventory amount upper limitation of material i
Is_{s}^{0}	initial inventory amount of storage tank s at the beginning of H
Is_{s}^{U}	upper bound of inventory amount of storage tank s
$K_{i,p}$	property value p of feedstock $i \in IR$
$KX_{j,p}$	upper bound of property p of feedstock feeding unit j
RON_{1-4}, RON_{c}	octane number parameters for blending operations
$PL_{i,p}$	lower bound of property p for oil product i
PR_{i}	prices of final products $i \in IP$
$Ps_{s,p}^{0}$	initial property p in storage tank s
$Ps_{s,p}^{L}$	lower bounds of material property p in storage tank s
$Ps_{s,p}^{U}$	upper bounds of material property p in storage tank s

$PX_{i,p}$	upper bound of property p for oil product i
$RVP_{1,2}$	RVP parameters for blending operations
$RX_{i,j}$	upper bounds of swing amount for side-product i of unit j
$TA_{v,t}$	arriving time of vessel v in time period t
$UX_{u,t}$	upper bounds of supply utilities u during time period t
VA_i	price for oil product i
$VG_{g,u}^U$	upper bound of supply amount from charging tank g to CDU u
$VS_{s,g}^U$	upper bound of transferring volume from storage tank s to charging tank g
$VV_{v,s}^U$	upper bound of unloading volume from vessel v to storage tank s
$WL_{i,j,t}$	minimal processing amount of material i for unit j during time period t
$WX_{j,t}$	production capacities of unit j during time period t
$\alpha_{i,i',j}$	yield of side product i' from processing material i in unit j
$\beta_{i,i',j,p}$	coefficient of property p for material i' produced from unit j with feedstock i
$\gamma_{i,i'}^L$	lower bounds of proportion for component product $i \in IC$ to blend final product $i' \in IP$
$\gamma_{i,i'}^U$	upper bounds of proportion for component product $i \in IC$ to blend final product $i' \in IP$
θ_j	constant rate of energy consumption for unit j
$P_{i,j}$	yield of side-product i produced from unit j
$\gamma_{i,j,p}$	property coefficient of material i produced from unit j

Binary Variables

$X_{i,u,k,t}$	1 when material i is the kth processed in CDU u during period t, otherwise 0
$Xg_{g,u,n,t}$	1 when there is a flow from charging tank g to CDU u in event n during time period t, otherwise 0
$Xs_{s,g,n,t}$	1 when there is a flow from storage tank s to charging tank g in event n during time period t, otherwise 0
$Xv_{v,s,n,t}$	1 when there is a flow from vessel v to storage tank s in event n during time period t, otherwise 0
$Y_{i,i',u,t}$	1 when material i is processed just before i' in the same CDU u during t, otherwise 0
$YB_{i,i',u,t}$	1 when the sequence variable $Y_{i,i',u,t}$ is broken, otherwise 0
$YE_{i,u,t}$	1 when material i is finally processed in CDU u during t, otherwise 0
$YS_{i,u,t}$	1 when material i is firstly processed in CDU u during t, otherwise 0
$YT_{i,i',u,t}$	1 when material i is finally processed in t and i' is firstly processed in the following period $t+1$ for same CDU u, otherwise 0
Continuous Variables	
CP_t	total operation cost of crude oil process in time period t
$Ec_{j,t}$	unit comprehensive energy consumption of unit j in period t
$f_{i,j,t}^{API}$	API value of oil product i blended from j in time period t
$f_{i,j,t}^{CN}$	Cetane number of oil product i blended from j in time period t
$f_{i,j,t}^{FP}$	Freezing point value of oil product i blended from j in time period t
$f_{i,j,t}^{RON}$	Octane number of oil product i blended from j in time period t

$f_{i,j,t}^{RVP}$	RVP value of oil product i blended from j in time period t
$f_{i,j,t}^{SUL}$	sulfur content of oil product i blended from j in time period t
$Ig_{g,n,t}$	inventory amount of charging tank g in event n of time period t
$Igp_{g,p,n,t}$	property p of charging tank g in event n of time period t
$Inv_{i,t}$	inventory amount of material i during period t
$Is_{s,n,t}$	inventory amount of storage tank s in event n of time period t
$LP_{j,t}$	working load of unit j in period t
NPV	net profit of the integrated optimization problem over the entire horizon H
NPV'	net profit of the refinery planning over the entire horizon H
$Ps_{s,p,n,t}$	property p of storage tank s in event n of time period t
$SQ_{i,t}$	outsourcing amount of material i during time period t
$Sr_{i,t}$	upward swing amount to side product i
$T_{v,t}^{start}$	starting unload time of vessel v in time period t
$T_{v,t}^{end}$	ending unload time of vessel v in time period t
$Tg_{g,u,n,t}^{b}$	beginning time of supply operation from charging tank g to CDU in event n during time period t
$Tg_{g,u,n,t}^{e}$	ending time of supply operation from charging tank g to CDU in event n during time period t
$Ts_{s,g,n,t}^{b}$	beginning time of transferring operation from storage tank s to charging tank g in event n during time period t

$Ts^{e}_{s,g,n,t}$	ending time of transferring operation from storage tank s to charging tank g in event n during time period t
$Tv^{b}_{v,s,n,t}$	beginning time of unloading operation from vessel v to storage tank s in event n during time period t
$Tv^{e}_{v,s,n,t}$	ending time of unloading operation from vessel v to storage tank s in event n during time period t
$Vf_{i,j,t}$	consumed amount of material i in unit j during period t
$VG_{g,u,n,t}$	supply amount from charging tank g to CDU u in event n during period t
$Vo_{i,j,t}$	produced amount of material i in unit j during period t
$Vop_{i,j,p,t}$	material property p of material i produced from unit j during period t
$Vp_{i,j,p,t}$	material property p of material i feeding unit j during period t
$\tilde{V}p_{i,j,p,t}$	relative material property p of material i feeding unit j during period t
$VS_{s,g,n,t}$	transferring amount from storage tank s to charging tank g in event n during period t
$VV_{v,s,n,t}$	unloading amount from vessel v to storage tank s in event n during period t
$VVp_{v,p,t}$	unloading property p from vessel v during period t
$Wd_{j,t}$	total processing amount of unit j during period t
$Wp_{j,p,t}$	material property p feeding unit j during period t
$Wu_{i,j,t}$	processing amount of material i in unit j during period t

3. Problem statement

We define the integrated multi-period optimization for refinery planning with crude oil scheduling. The network topology of the integrated refinery processes is shown in Figure 1.

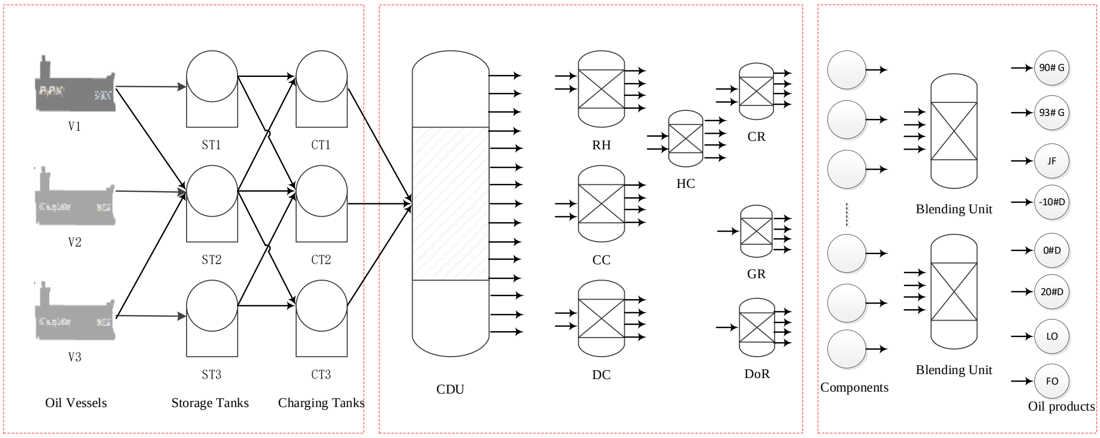


Figure 1. Flowsheet of integrated refinery processes

This problem must find all the flows in the process network, including operation timing, processing amount and properties, blending recipes for final oil products, and energy consumption in each time period to maximize the net profit. In addition, the solution must satisfy the operations rules, mass and property balances, technical recipes, limitation of energy consumption, and demand for the amount and quality of the products.

Given are the following items for the problem of the integrated multi-period refinery optimization:

1) The topology of the whole refinery process, including the primary units, tanks, and their interconnections.
2) The number of crude oil vessels, including their arrival times, types and amount of transported crude oil, and waiting cost in the port.
3) The number of processing units, including their technical models, constraints in feedstock properties, operation rules, and processing capacity.

4) The number of storage and mixing tanks, like crude oil, crude oil mix, intermediate-product, and oil product tanks, with operation rules and storage capacity.
5) The number of different types of flows, like crude oils, intermediate-products, oil products, and additives, including primary properties, properties limitations, mixing or blending recipes, yields, supply or demand amount, and prices.
6) The properties like specific gravity, API, sulfur content, Octane number, Reid Vapor Pressure (RVP), and flashpoint correlation.
7) The energy consumption of each refinery unit, maximum capacity, and costs.
8) The time horizon and number of time periods considered for production.

The optimization problem is then to determine the following variables for each time period:

1) The unloading times of crude oil from crude oil vessels.
2) The volumetric flow and start times of crude oil from crude oil vessels to storage tanks and storage tanks to charging tanks.
3) The amount of crude oil transferred from each charging tank to the CDUs.
4) The changeover sequence of feedstocks for the CDUs.
5) The amount of each feedstock processed in the CDUs.
6) The flows among the refining units.
7) The amount of each type of crude oil, intermediate product, component oil, and oil product in the inventory.
8) The properties of each mixed, produced, and blended material.
9) The mixing or blending ratios of each crude oil or component oil.
10) The energy consumption of each unit.

The integrated optimization problem must satisfy the following constraints:

1) The operation rules of the production units and tanks, e.g., the continuous processes of CDUs, and non-simultaneous input and output operations for the storage and charging tanks.
2) The mass balance of storage and charging tanks and production units.
3) The properties balance of production units and storage tanks.
4) The capacities of tanks and processing units.
5) The limitation of the mixing and blending recipes, like the bounds of blending ratios.
6) The demand for intermediate and final oil products.
7) The limitation of the oil product specification, e.g., the lower bounds of octane number for gasoline and the upper bounds of sulfur content.
8) The upper bounds of energy consumption.

The objective function of the integrated refinery optimization is to maximize the net profit over the planning horizon, which equals the total profit of oil products minus the cost of crude oil, processing production, changeovers of feedstock, inventory, energy consumption, and crude oil operations. In addition, perfect mixing and blending operations are assumed.

4. Mathematical formulation

The first part of this section covers the hybrid continous-dicrete time representation method used to model the different process stages. Next, we introduce the mathematical programming formulation of the problem, which results in a nonconvex MINLP.

4.1 Hybrid time representation method

There are two types of time-representation methods for process scheduling and planning problems, discrete-time and continuous-time (Floudas, 2004; Lee and Maravelias, 2017; Maravelias, 2021). The discrete-time model partitions the time horizon in a grid and assumes that the operations need to start at the elements of the fixed time grid (Lee et al., 1996). This model is easier to formulate, but there is a tradeoff between model size and solution quality given by how fine the time grid is. A discrete-time model can be advantageous when the time discretization matches the decision-making timing. On the other hand, a continuous-time model does not assume that the operations need to happen at fixed times at the cost of a more complicated formulation. For our model, we decided to consider a continuous-time representation for the crude oil scheduling part, motivated by increasing the computational performance of our model, and a discrete-time representation for the refinery planning model with embedding sequence-based representation for the processing sequence of crude oil in CDUs. Consider that any finite choice of time discretization leads to approximating the solution obtainable through a discrete-time representation. To match the different time-scale representation of our models within an integrated optimization of both the planning problem and scheduling problem, we propose a hybrid time representation combining the planning and scheduling depiction time representations.

Figure 2 is an illustration of such a hybrid continuous-discrete time representation for the integrated optimization problem. The unit-specific event-based continuous-time $n \in \{1,2,\dots,N\}$ is used in the crude oil scheduling for each period t (Furman et al., 2007). The processing sequence of crude oil in the refinery during each period t is represented by the sequence-based continuous-time method. The multiple periods for refinery planning are represented with discrete time periods, $t \in$

$\{1,2,\ldots,T\}$. For each discrete period t, we adopt a continuous-time representation to represent the detailed operation scheduling for crude oil and processing sequence in refining processes.

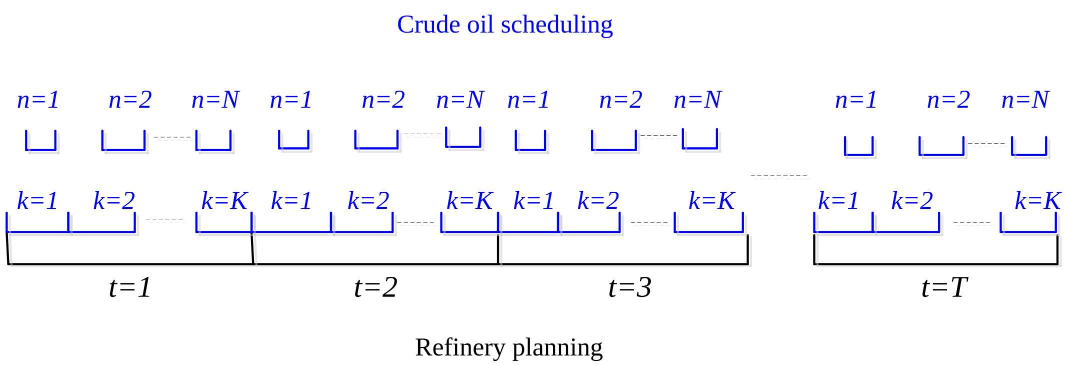


Figure 2. Illustration of hybrid continuous-discrete time-representation for the integration of refinery planning with crude oil scheduling

The choice of N is dependent on the number of crude oil types and the numbers of storage tanks, which is generally equal or greater than the number of crude oil types. Here, we assume that the number of events, N, is equal to the number of sequences K. That yields a one-to-one correspondence between the processing sequence of the CDU and supply events from the charging tanks to the CDU. This assumption guarantees that the processing sequence of crude oil in refinery CDU is consistent with the supply sequence from charging tanks to CDU.

4.2 Mathematical model description

We formulate the mathematical model in two parts: crude oil scheduling and refinery planning, according to the flow layout of the entire refinery processes.

4.2.1 Multiperiod crude oil scheduling

A transfer event-based continuous-time representation is adopted for the crude oil scheduling (Furman et al., 2007) to model the batch flow operations. This time description requires fewer events than a general unit-specific event-based method since a single event can represent both input and output flows in a tank. We extend the model presented at Furman et al. (2007) for multiple periods, and we integrate it with a refinery planning model.

For the constraints from the crude oil vessels to the storage tanks, from the storage tanks to the charging tanks, and from the charging tanks to the CDUs, given the similarity of the corresponding equations, we only give the explicit expression constraints for the flows from the crude oil vessels to the storage tanks in the following representation of the crude oil scheduling.

1) Flow transfer constraints

The flow $VV_{v,s,n,t}$ from the vessel v to the storage tank s in a particular transfer event $n \in N$during time period t is forced to zero if the binary variable $Xv_{v,s,n,t}$ is equal to zero, which represents the existence of flow for this stream in the event n, as Eq. (1). When $Xv_{v,s,n,t}$ is equal to 1, the flow is constrained by an upper bound.

$$VV_{v,s,n,t} \leq VV_{v,s}^{U} Xv_{v,s,n,t} \quad \forall v,s,n,t \tag{1}$$

The relationship between the time variables of flow events $Tv_{v,s,n,t}^{b}$, $Tv_{v,s,n,t}^{e}$, and the flows follows a big-M formulation of the following logic proposition: when a flow transfer event occurs, the flow is larger than the lower bound of flow rate times the duration of the flow event. Since the supply flows of the CDUs are continuous, the constraint only involves continuous variables, as in Eq. (4).

$$FV_{v,s}^{L}(Tv_{v,s,n,t}^{e} - Tv_{v,s,n,t}^{b}) - FV_{v,s}^{L} Ht\big(1 - Xv_{v,s,n,t}\big) \leq VV_{v,s,n,t} \quad \forall v,s,n,t \tag{2}$$

$$FV_{v,s}^{U}(Tv_{v,s,n,t}^{e} - Tv_{v,s,n,t}^{b}) + FV_{v,s}^{U} Ht\big(1 - Xv_{v,s,n,t}\big) \geq VV_{v,s,n,t} \quad \forall v,s,n,t \tag{3}$$

$$FG_{g,u}^{L}(Tg_{g,u,n,t}^{e} - Tg_{g,u,n,t}^{b}) \leq VG_{g,u,n,t} \quad \forall g,u,n,t \tag{4}$$

Where Ht is the horizon time for each time period t.

2) Timing constraints for the same arc in the network

When a transfer event n from vessel v to storage tank s takes place, the end time of event n is before the start time of event $n+1$, as specified in Eq. (5). The same applies to the events from the storage tanks to the charging tanks and from the charging tanks to the CDUs.

$$Tv_{v,s,n+1,t}^{b} \geq Tv_{v,s,n,t}^{e} - Ht\big(1 - Xv_{v,s,n,t}\big)\sum_{t' \leq t} t' \quad \forall v,\ s,\ n\ <\ |N|, t \tag{5}$$

The beginning time and ending time of transfer event n is always before the beginning time and ending time of event $n+1$ in the network, as in Eq. (6) and Eq. (7), respectively.

$$Tv_{v,s,n+1,t}^{b} \geq Tv_{v,s,n,t}^{b} \quad \forall v,\ s,\ n\ <\ |N|, t \tag{6}$$

$$Tv_{v,s,n+1,t}^{e} \geq Tv_{v,s,n,t}^{e} \quad \forall v,\ s,\ n\ <\ |N|, t \tag{7}$$

3) Timing constraints for different arcs in the network

In the transfer event from different vessels v, v' to a storage tank s, the end time of event n has to occur before the start time of event $n+1$. This condition must be enforced if the event n from vessel v to storage tank s happens and is represented in Eq. (8).

$$Tv_{v',s,n+1,t}^{b} \geq Tv_{v,s,n,t}^{e} - Ht(1 - Xv_{v,s,n,t})\sum_{t' \leq t} t' \quad \forall v' \neq v,\ s,\ n\ <\ |N|, t \tag{8}$$

If the intermediate transfer event n from storage tank s to charging tank g takes place, the end time of this event must be before the beginning time of the event $n+1$ from vessel v to storage tank s, as shown in Eq. (9). It is similar for the event n from charging tank g to refining unit j.

$$Tv_{v,s,n+1,t}^{b} \geq Ts_{s,g,n,t}^{e} - Ht(1 - Xs_{s,g,n,t})\sum_{t' \leq t} t' \quad \forall v,\ s, g,\ n\ <\ |N|, t \tag{9}$$

If the previous event n from vessel v to storage tank s takes place, the end time of this event is before the beginning time of the following event $n+1$ from storage tank s to charging tank g.

$$Ts_{s,g,n+1,t}^{b} \geq Tv_{v,s,n,t}^{e} - Ht(1 - Xv_{v,s,n,t})\sum_{t' \leq t} t' \quad \forall v,\ s, g,\ n <\ |N|, t \tag{10}$$

For the same storage tank s, the beginning time of event $n+1$ from storage tank s to charging tank g' must be after the end time of event n if the event n from storage tank s to charging tank

g takes place. This relationship also applies to the changeover events from charging tank g to CDU u.

$$Ts^{b}_{s,g',n+1,t} \geq Ts^{e}_{s,g,n,t} - Ht(1 - Xs_{s,g,n,t}) \sum_{t' \leq t} t' \quad \forall s, g' \neq g,\ n\ <\ |N|, t \tag{11}$$

Here, we assume that input operations to a storage/charging tank must finish before the output operation from the same tank when a single event represents input and output operations. This is enforced by Eq. (12).

$$Ts^{b}_{s,g,n,t} + Ht(1 - Xs_{s,g,n,t}) \sum_{t' \leq t} t' \geq Tv^{e}_{v,s,n,t} - Ht(1 - Xv_{v,s,n,t}) \sum_{t' \leq t} t' \quad \forall v, s, g,\ n, t \tag{12}$$

Where $t' \in T$ are time periods.

4) Timing constraints for unloading operations from crude oil vessels

When a crude oil vessel v arrives, the unloading operation can begin. However, the start time of unloading operation from a vessel v is not before the beginning time of a transfer event from vessel v to storage tank s if the event happens. And the end time of unloading operation from a vessel v must be after the end time of a transfer event from vessel v to storage tank s if the event takes place.

$$TA_{v,t} \leq T^{start}_{v,t} \quad \forall v, t \tag{13}$$

$$T^{start}_{v,t} \leq Tv^{b}_{v,s,n,t} + Ht(1 - Xv_{v,s,n,t}) \sum_{t' \leq t} t' \quad \forall v, s, n, t \tag{14}$$

$$T^{end}_{v,t} \geq Tv^{e}_{v,s,n,t} - Ht(1 - Xv_{v,s,n,t}) \sum_{t' \leq t} t' \quad \forall v, s, n, t \tag{15}$$

$T^{start}_{v,t}$ is used to calculate the waiting of crude oil vessels in the sea. The difference between $T^{start}_{v,t}$ and $T^{end}_{v,t}$ is the unloading time period.

5) Timing constraints for CDU operation

The operation of the refinery CDU is continuous, which means there is a supply flow from charging tank g to CDU u at every time during the horizon. Eq. (16) is used to enforce this condition, combined with Eq. (17). Eq. (17) states that if the supply event n from charging tank g to CDU u occurs, the end time of event n is equal to the start time of event $n+1$.

$$\sum_{g,n}\left[Tg^{e}_{g,u,n,t} - Tg^{b}_{g,u,n,t}\right] = Ht \quad \forall u,t \tag{16}$$

$$Tg^{b}_{g',u,n+1,t} \le Tg^{e}_{g,u,n,t} + Ht(1 - Xg_{g,u,n,t}) \quad \forall g' \neq g,u,n \ < \ |N|,t \tag{17}$$

6) Discrete time-grid constraints for crude oil operations among time periods

For all the flows in the crude oil supply network, the start times of the first events in each time period t are greater than the beginning time of this time period, and the end times of the last events in each period t are less than the end time of the period.

$$Tv^{b}_{v,s,n,t} \ge Ht(\sum_{t' \le t} t' - 1) \quad \forall v,\ s,\ n = 1,t \tag{18}$$

$$Tv^{e}_{v,s,n,t} \le Ht\sum_{t' \le t} t' \quad \forall v,\ s,\ n = |N|,t \tag{19}$$

7) Material balance constraints

For each storage and charging tank, the inventory amount at the end of event n is equal to the inventory amount at the end of event $n-1$, plus the receiving amount from vessels or storage tanks at event n, and minus the amount sent to charging tanks or CDUs. Eq. (21) is the inventory balance equation between time period $t-1$ and t. The inventory amount $Is_{s,n,t}$ of crude oil in storage tanks/ charging tanks at each event n and time period t is limited by the tank storage capacity, as in Eq. (22).

$$Is_{s,n-1,t} + \sum_{v} VV_{v,s,n,t} = Is_{s,n,t} + \sum_{g} VS_{s,g,n,t} \quad \forall s,n > 1,t \tag{20}$$

$$Is_{s,n',t-1} + \sum_{v} VV_{v,s,n,t} = Is_{s,n,t} + \sum_{g} VS_{s,g,n,t} \quad \forall s,n' = N,n = 1,t > 1 \tag{21}$$

$$Is_{s,n-1,t} + \sum_{v} VV_{v,s,n,t} \le Is^{U}_{s} \quad \forall s,n > 1,t \tag{22}$$

8) Material properties balance constraints

For each storage and charging tank, the material properties $Ps_{s,p,n,t}$ at the end of event n are equal to the material properties at the end of event $n-1$, plus the receiving material properties from vessels/storage tanks at event n, as seen in Eq. (23). The material properties balance equation between time period $t-1$ and t is represented in Eq. (24). The material properties of crude oil in storage tanks and charging tanks need to lie between given bounds, which are the component requirements of CDU for the feedstocks, as in Eq. (25).

$$Is_{s,n-1,t}Ps_{s,p,n-1,t} + \sum_{v} VV_{v,s,n,t}VVp_{v,p,t} = (Is_{s,n,t} + \sum_{g} VS_{s,g,n,t})Ps_{s,p,n,t} \quad \forall s,p,n>1,t \tag{23}$$

$$Is_{s,n',t-1}Ps_{s,p,n',t-1} + \sum_{v} VV_{v,s,n,t}VVp_{v,p,t} = (Is_{s,n,t} + \sum_{g} VS_{s,g,n,t})Ps_{s,p,n,t} \quad \forall s,n'=N,n=1,t>1 \tag{24}$$

$$Ps^{L}_{s,p} \leq Ps_{s,p,n,t} \leq Ps^{U}_{s,p} \quad \forall s,p,n,t \tag{25}$$

9) Supply-demand of CDUs

The amount supplied from charging tanks to the CDU u is equal to the processing amount of u in period t. Here, we assume that crude oil from different charging tanks is a different crude oil. This constraint connects the crude oil feedstock and refinery process. Therefore, for the independent scheduling model for crude oil operations, the processing amount of CDUs is the demand for the charging tanks, taken as a constant.

$$\sum_{n} VG_{g,u,n,t} = Wu_{i,u,t} \quad \forall g,i,u,t \tag{26}$$

10) Allocation constraints for supplying CDUs

At one event n, there is at most one charging tank g supplying CDU u. And there is at most one CDU being supplied from charging tank g at one event n.

$$\sum_{g} Xg_{g,u,n,t} \leq 1 \quad \forall u,n,t \tag{27}$$

$$\sum_{u} Xg_{g,u,n,t} \leq 1 \quad \forall g,n,t \tag{28}$$

11) Cost of crude oil operations in each period

The cost of crude oil operations in each time period t is calculated from Eq. (29) (Karuppiah, 2007). The terms involved in it are: 1) waiting cost of vessels in the sea, 2) offloading cost from the crude oil vessels to the storage tanks, 3) inventory cost of crude oil in storage and 4) charging tanks, and 5) changeover cost of charging tanks for feeding the CDU.

$$\begin{aligned} CP_t = & \sum_{v} CW\left(T_{v,t}^{start} - TA_{v,t}\right) + CL\left(T_{v,t}^{end} - T_{v,t}^{start}\right) \\ & + \frac{Ht}{2|N|+1}\left\{\sum_{s} CS_s\left(\sum_{n}\left(Is_{s,n,t} + \sum_{v} VV_{v,s,n,t}\right) + \sum_{n<|N|} Is_{s,n,t} + 2Is_{s,t}^{0}\right)\right. \\ & \left. + \sum_{g} CG_g\left(\sum_{n}\left(Ig_{g,n,t} + \sum_{s} VS_{s,g,n,t}\right) + \sum_{n<|N|} Ig_{g,n,t} + 2Ig_{g,t}^{0}\right)\right\} \\ & + CAe^{\sum_{g,u,n} Xg_{g,u,n,t} - |U|} \quad \forall t \end{aligned} \tag{29}$$

We complete the optimization model of the scheduling for crude oil operations by minimizing the summation of CP_t, as in $min\ \sum_t CP_t$, for all period t subject to the constraints in Eqs. (1)-(28).

4.2.2 Multiperiod integration for refinery planning

The integrated multi-period refinery planning model needs to represent the processing quantity and sequence of the feedstocks in the CDUs, the intermediate flows, component inventory, and blending properties of oil products in each period.

In the hybrid continuous-discrete time representation, the larger discrete-time grid is used to divide the production planning horizon, and in each time grid, a continuous sequence-based representation is used.

To describe the refinery processing units systematically, we adopt the general model of a refinery unit as in Figure 3, which represents a general unit in a refinery with an inlet mixer and output splitters (Alhajri et al., 2008). The utility flows are added to represent the comprehensive energy flows. The swing-cut model is used for the CDU, and linear yield models are used for the other units (Menezes et al., 2013). These yield coefficients were obtained through our interaction with Sinopec.

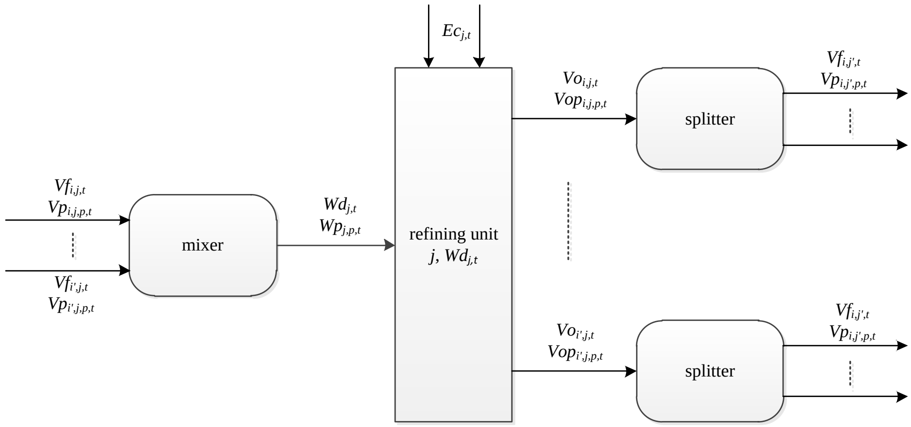


Figure 3. General production unit model of refinery process

To incorporate the allocation constraints for the CDU, we implemented constraints similar to the ones in Alattas et al. (2012), which are based on the traveling salesman to determine the sequences of products processed in each period at each CDU, as seen in Eqs (30) and (31).

1) Allocation constraints for CDU

The constraints below are similar to those presented in Alattas et al. (2012), based on the traveling salesman constraints to determine the processing sequence. For each sequence position k of CDU u, there must be one type of crude oil i assigned, shown in Eq. (30). In addition, each type of crude oil must be processed exactly once in each period.

$$\sum_{i \in IR} X_{i,u,k,t} = 1 \qquad \forall k,u,t \tag{30}$$

$$\sum_{k} X_{i,u,k,t} = 1 \qquad \forall i,u,t \tag{31}$$

Eqs. (32) and (33) are the relationships between sequence $Y_{i,i',u,t}$ and assignment $X_{i,u,k,t}$ variables, which express that if material i is processed in CDU u in period t, there must be another material i' to be processed in the same period, and vice versa.

$$\sum_{i'} Y_{i,i',u,t} = \sum_{k} X_{i,u,k,t} \quad \forall i,u,t \tag{32}$$

$$\sum_{i} Y_{i,i',u,t} = \sum_{k} X_{i',u,k,t} \quad \forall i',u,t \tag{33}$$

Eq. (34) states that the assignment variable determines the sequence variable. If there is only one type of feedstock processed in a period, Eqs. (35) and (36) are given to limit this condition.

$$Y_{i,i',u,t} \leq \sum_{k} X_{i,u,k,t} \quad \forall i,i',u,t \tag{34}$$

$$\sum_{k} X_{i',u,k,t} + Y_{i,i,u,t} \leq 1 \quad \forall i,i',u,t \tag{35}$$

$$Y_{i,i,u,t} \geq X_{i,u,k,t} - \sum_{i' \neq i} X_{i',u,k,t} \quad \forall i,u,t \tag{36}$$

A binary variable $YB_{i,i',u,t}$ is introduced to break subcycles in each time period (Alattas et al., 2012). Eq. (37) enforces that there must be a variable to break the cycles equal to 1 for a period of unit u. Eq. (38) is that the sequence variable dominates the corresponding subtour-breaking variable.

$$\sum_{i,i'} YB_{i,i',u,t} = 1 \quad \forall u,t \tag{37}$$

$$YB_{i,i',u,t} \leq Y_{i,i',u,t} \quad \forall i,i',u,t \tag{38}$$

We also define the first and last processing crude oil in a time period. Eqs. (39) and (40) are single-choice constraints. Eqs. (41) and (42) correspond to the relationships between the assignment variables and the subtour-breaking variables. If there is a subtour-breaking variable $YB_{i,i',u,t}$ equal to 1 in a time period, material i' must be firstly processed in that time period.

$$\sum_{i} YS_{i,u,t} = 1 \quad \forall u,t \tag{39}$$

$$\sum_{i} YE_{i,u,t} = 1 \quad \forall u,t \tag{40}$$

$$YS_{i',u,t} \geq \sum_{i} YB_{i,i',u,t} \quad \forall i',u,t \tag{41}$$

$$YE_{i,u,t} \geq \sum_{i'} YB_{i,i',u,t} \quad \forall i,u,t \tag{42}$$

Eqs. (43) and (44) provide the relationships between the cross sequencing variables and the assignment variables of the first place in $t+1$ and the last place in t for the sequencing variables $YT_{i,i',u,t}$ crossing time periods.

$$\sum_{i} YT_{i,i',u,t} = YS_{i',u,t+1} \quad \forall i',u,t < |T| \tag{43}$$

$$\sum_{i'} YT_{i,i',u,t} = YE_{i,u,t} \quad \forall i,u,t \tag{44}$$

The supply sequence of crude oil in the crude oil scheduling is consistent with the processing sequence of crude oil in refinery planning in each period. The consistency constraints are presented in Eq. (45). For example, when crude oil i is the k^{th} processed in CDU u, the supplying sequence must be assigned at the same event point.

$$X_{i,u,k,t} \leq Xg_{g,u,n,t} \quad \forall i,g,k=n,u,t \tag{45}$$

2) Mass balance for processing units

The mass balance of the mixer before the processing unit j is that the summation of the input amount $Vf_{i,j,t}$ is equal to the processing amount $Wd_{j,t}$ of unit j during period t as in Figure 3.

$$\sum_{i \in IF_j} Vf_{i,j,t} = Wd_{j,t} \quad \forall j,t \tag{46}$$

In the same way, the summation of produced products $Vo_{i,j,t}$ is equal to the processed amount. Thus, the mass balance of the splitter after the processing unit j is given in Eq. (49).

$$Wd_{j,t} = \sum_{i \in IS_j} Vo_{i,j,t} \quad \forall j,t \tag{47}$$

$$Vo_{i,j,t} = \rho_{i,j} Wd_{j,t} \quad \forall i \in IS_j, j, t \tag{48}$$

$$Vo_{i,j,t} = \sum_{j' \in JS_i} Vf_{i,j',t} \quad \forall i \in IS_j, j, t \tag{49}$$

3) Processing capacity of each unit

The processing amount of unit j during period t cannot exceed the production capacity of the unit.

$$Wd_{j,t} \le WX_{j,t} \quad \forall j,t \tag{50}$$

However, in order to guarantee smooth production and avoid frequent changeovers, the minimal processing amount of each feedstock is required.

$$WL_{i,j,t}X_{i,j,k,t} \le Wu_{i,j,t} \quad \forall i,k,j,t \tag{51}$$

4) Mass balance of CDU

For the swing cut products of the CDU, the produced amount is the sum of the amounts predicted by the swing cut model as shown in Eqs. (52) and (53). Here $Sr_{i,t}$ is the upward swing amount to side product i in period t.

$$Vo_{i-1,j,t} = \rho_{i-1,j}Wd_{j,t} + Sr_{i,t} \quad \forall i-1, i \in IS_j, j = 'CDU', t \tag{52}$$

$$Vo_{i+1,j,t} = \rho_{i+1,j}Wd_{j,t} + \rho_{i,j}Wd_{j,t} - Sr_{i,t} \quad \forall i, i+1 \in IS_j, j = 'CDU', t \tag{53}$$

5) Material properties balance of CDU

The property balance for the general processing unit is as follows.

$$\sum_{i \in IS_j} Vf_{i,j,t}Vp_{i,j,p,t} = Wd_{j,t}Wp_{j,p,t} \quad \forall j,p,t \tag{54}$$

$$Wp_{j,p,t}\gamma_{i,j,p} = Vp_{i,j,p,t} \quad \forall i \in IF_j, j, p, t \tag{55}$$

$$Vop_{i,j,p,t} = Vp_{i,j',p,t} \quad \forall i \in IS_j, j, j' \in JF_i, t \tag{56}$$

6) Material balance in each period

The inventory amount $Inv_{i,t}$ of material i in time period t is equal to the inventory amount of period $t-1$, plus the produced amount and the supplying amount minus the consumed amount and demand amount of time period t.

$$Inv_{i,t} = In0_i + \sum_{j \in JP_i} Vo_{i,j,t} + SQ_{i,t} - \sum_{j' \in JS_i} Vf_{i,j',t} - D_{i,t} \quad \forall i, t = 1 \tag{57}$$

$$Inv_{i,t} = Inv_{i,t-1} + \sum_{j \in JP_i} Vo_{i,j,t-1} + SQ_{i,t-1} - \sum_{j' \in JS_i} Vf_{i,j',t-1} - D_{i,t-1} \quad \forall i, t > 1 \tag{58}$$

7) Inventory capacity of the material

The inventory amount of material i is limited by its storage capacity.

$$Inv_{i,t} \leq InX_i \quad \forall i,t \tag{59}$$

8) Blending properties constraints

The nonlinear blending equations are considered to accurately predict the properties of final oil products.

$$Vp_{i',j,p,t} = f\left(Vf_{i,j,t}, Vp_{i,j,p,t}\right) \quad \forall i \in IC_j, i' \in IP_j, j = \text{'}blending\ unit\text{'}, p, t \tag{60}$$

The blending properties include Octane number (RON), API, Sulfur content, Reid vapor pressure (RVP), freezing point (FP), and Cetane index. The analytical equations of the blending properties are adopted from Riazi (2005). Here, we assume that the blending tanks are oil product specific. Eqs. (61) and (61a) show the correlation for RON between the component oils $Vp_{i,j,p,t}$ and the oil product $f_{i',j,t}^{RON}$.

$$f_{i',j,t}^{RON} = \frac{\sum_{i \in IC_j} Vf_{i,j,t}\left(RON_3 \cdot \bar{V}p_{i,j,p,t}^3 - RON_2 \cdot \bar{V}p_{i,j,p,t}^2 + RON_1 \cdot \bar{V}p_{i,j,p,t} - RON_c\right)}{Vo_{i',j,t}} \quad \forall i' \in IP_j, j, p = \text{'}RON\text{'}, t \tag{61}$$

$$\bar{V}p_{i,j,p,t} = Vp_{i,j,p,t}/RON_4 \quad \forall i \in IC_j, j, p = \text{'}RON\text{'}, t \tag{61a}$$

The API $f_{i',j,t}^{API}$ representing the density of the oil products is calculated by

$$f_{i',j,t}^{API} = \frac{Vo_{i',j,t}}{\sum_{i \in IC_j} Vf_{i,j,t} Vp_{i,j,p,t}} API_1 - API_2 \quad \forall i' \in IP_j, j, p = \text{'}SG\text{'}, t \tag{62}$$

Here, the Specific Gravity (SG) of component oil is used to calculate the API value of blended oil products.

The sulfur content $f_{i',j,t}^{SUL}$ of the oil product is calculated using a material property balance as follows

$$f_{i',j,t}^{SUL} = \frac{\sum_{i \in IC_j} Vf_{i,j,t} Vp_{i,j,p,t}}{Vo_{i',j,t}} \quad \forall i' \in IP_j, j, p = \text{'}Sulfur\ content\text{'}, t \tag{63}$$

The equation of RVP is defined as

$$f_{i',j,t}^{RVP} = \left(\frac{\sum_{i \in IC_j} Vf_{i,j,t} Vp_{i,j,p,t}^{RVP_1}}{Vo_{i',j,t}}\right)^{RVP_2} \quad \forall i' \in IP_j, j, p = 'RVP', t \tag{64}$$

The equation of freezing point is given as

$$f_{i',j,t}^{FP} = \sum_{i \in IC_j} Vf_{i,j,t} \, e^{FP_1 \cdot ln \frac{Vp_{i,j,p,t} + FP_2}{FP_3}} \quad \forall i' \in IP_j, j, p = 'FP', t \tag{65}$$

The following equation predicts the Cetane Number $f_{i',j,t}^{CN}$ for diesel products,

$$f_{i',j,t}^{CN} = \sum_{i \in IC_j} \frac{Vf_{i,j,t}}{Vo_{i',j,t}} \left[CN_1 \left(\frac{f_{i,j,t}^{API} (CN_2 AP_i + CN_3)}{CN_4} \right) + CN_5 \right] \quad \forall i' \in IP_j, j, t \tag{66}$$

The blending proportions of component oils and properties of final oil products must lie within their bounds.

$$\gamma L_{i,i'} Vo_{i',j,t} \leq Vf_{i,j,t} \leq \gamma U_{i,i'} Vo_{i',j,t} \quad \forall i \in IC_j, i' \in IP_j, j = 'blending\ units', t \tag{67}$$

$$PL_{i,p} \leq Vp_{i,j,p,t} \leq PX_{i,p} \quad \forall i \in IP_j, j, p, t \tag{68}$$

9) Energy consumption

We adopt the energy factor method to obtain the unit comprehensive energy consumption $Ec_{j,t}$ of unit j in period t, which is a function of the working load (Guo and Xu, 2004). Moreover, the unit energy consumption $Ec_{j,t}$ of unit j is limited with an upper bound for this type of unit.

$$Ec_{j,t} = EI_{j,t} \left[1 + \theta_j \left(\frac{1}{Ld_{j,t}} - 1 \right) \right] \quad \forall j, t \tag{69}$$

$$Ld_{j,t} = \frac{Wd_{j,t}}{WX_{j,t}} \quad \forall j, t \tag{69a}$$

$$Ec_{j,t} \leq EA_j^U \quad \forall j, t \tag{70}$$

10) Objective function for refinery integrated optimization

The net profit of the refinery on the production horizon is the total value of oil products minus the cost of crude oils, processing production of units, inventory of all materials, changeover cost of crude oil in CDUs, energy consumption, and crude oil operation. Here, the total changeover times of feedstocks in the CDU are equal to the cycle sequencing variables minus the breaking variable in one period plus the changeover between periods (Alattas et al., 2012).

$$\begin{aligned} max \quad NPV = & \sum_{i \in IP,t} VA_i Inv_{i,t} - \sum_{i \in IR,u,t} (CR_i + CO_u^{CDU}) Vf_{i,u,t} - \sum_{j \in J \setminus u,t} CO_j \, Wd_{j,t} \\ & - \sum_{i \in I} CI_{i,t} Inv_{i,t} - \sum_{i,i' \in IR,u,t} CD_{i,i'} \big(Y_{i,i',u,t} - YB_{i,i',u,t} + YT_{i,i',u,t} \big) \\ & - \sum_{j,u,t} CU_u Ec_{j,t} Wd_{j,t} - \sum_{t} CP_t \end{aligned} \tag{71}$$

The proposed formulation for the optimization of the integrated crude oil and refinery processes includes Eqs. (1)-(71) with the objective function in Eq. (71), which is a nonconvex MINLP with nonlinear equations, namely Eqs. (29), (60)-(66), and (69).

For the refinery planning problem, the objective function is presented in Eq. (71'), which does not consider the sum of crude oil operations cost over time and is subject to constraints in Eqs. (30)-(70).

$$\begin{aligned} max \quad NPV' = & \sum_{i \in IP,t} VA_i Inv_{i,t} - \sum_{i \in IR,u,t} (CR_i + CO_u^{CDU}) Vf_{i,u,t} - \sum_{j \in J \setminus u,t} CO_j \, Wd_{j,t} \\ & - \sum_{i \in I} CI_{i,t} Inv_{i,t} - \sum_{i,i' \in IR,u,t} CD_{i,i'} \big(Y_{i,i',u,t} - YB_{i,i',u,t} + YT_{i,i',u,t} \big) \\ & - \sum_{j,u,t} CU_u Ec_{j,t} Wd_{j,t} \end{aligned} \tag{71'}$$

The proposed model has the following assumptions: 1) the number of storage tanks and charging tanks are assumed to be equal to the number of types of crude oil, 2) the model for the CDU is based on a simplified swing-cut model, 3) the other units in the refinery planning side are modeled using a fixed yield, and 4) each type of crude oil processed once in each time period.

5. Computational results

The proposed model is implemented in GAMS 26.1.0 (Brook et al., 1988). The operating system is Windows 10 with a Processor Intel Core i7-6820HQ, CPU 2.70GHz, and 16 GB of RAM. The solutions to the MINLP problems are obtained using the solvers BARON 18.11.12 and DICOPT 2 (Tawarmalani and Sahinidis, 2005; Grossmann et al., 2002; Bernal et al., 2020). The NLP and MILP subproblems solution are obtained using CONOPT 3.17 and CPLEX 12.8.0, respectively (Drud, 1994; IBM CPLEX, 2018).

5.1 Test problem description

We use the data from a coastal refinery plant of Sinopec, one of the largest refining companies in the world (Long and Chen, 2010). The network topology of the integrated refinery process is shown in Figure 1. It includes three vessels (V1, V2, V3), three storage tanks (ST1, ST2, ST3), and three charging tanks (CT1, CT2, CT3) in the crude oil transfer stage. Here, the tanks storing or charging the same type of crude oil are regarded as one tank. In the oil refining stage, it includes eight refinery units, including a CDU, a vacuum column (VC), a reforming hydrogeneration (RH) unit, a catalytic cracker (CC), a delayed coker (DC), a hydrogen cracker (HC), a catalytic reformer (CR), a gasoline refiner (GR) and a diesel oil refiner (DoR). As a result, three types of crude oil are available, denoted by CR1, CR2, and CR3. In addition, there are fifty-six intermediate products, including nineteen component oils. The component oils are used to blend eight final oil products according to the National IV standard for oil products in China (GB 17930-2016, 2016). The proposed model parameters are listed in Appendix A. Considering different lengths of planning periods, we perform numerical computations to show the results of the proposed model.

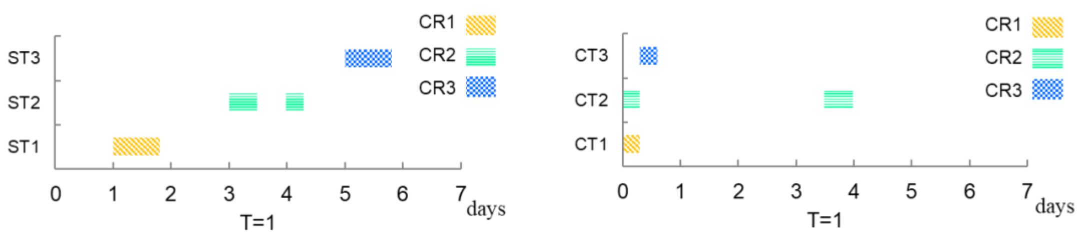
We address eight cases with horizon lengths from 7 to 84 days. Considering seven days in one period, we have Case 1 having a single period, Case 2 with two periods, and twelve periods are studied in Case 8.

5.2 Example - Case 1

This is an example case with single period of one week for the integrated optimization refinery problem. The optimal net profit of this case is 25.27 million dollars. The total cost in the solution is mainly composed of the cost of raw materials and the cost associated with the energy consumption, which are 92.1% and 5.81%, respectively.

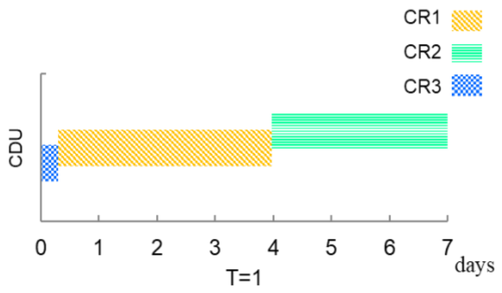
The solution of one period for refinery production is shown in Figure 4. Figure 4 (a)-(c) are the Gantt charts of the continuous-time scheduling solutions from vessels to storage tanks, from storage tanks to charging tanks, and from charging tanks to CDU, respectively. Figure 4 (d) is the resulting Gantt chart of the continuous-time sequence for CDU processing operations with changeovers in one time period.

We find that the transfer operations of crude oil flows are smooth, and the supply for CDU is continuous with reasonable changeovers. The solution maximizes the total profit by modifying the production ratios of the final oil products. The properties of the final oil products also satisfy the quality criteria of the oil products.

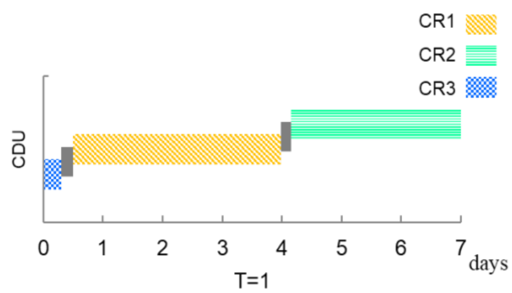


(a) Scheduling for crude oil from vessels to storage tanks

(b) Scheduling for crude oil from storage tanks to charging tanks



(c) Scheduling for crude oil from storage tanks to charging tanks



(d) Scheduling for crude oil in CDU with changeovers

Figure 4. Scheduling results of integrated optimization for refinery process with one period of one week

To compare the proposed integrated model with the sequential models, we sequentially solve the separate models in a hierarchical approach, first the refinery planning problem and then crude oil scheduling after fixing the solution of the previous problem. We consider this approach since it is usually used in industry, where the refinery planning problem is prioritized. First, we solve the refinery planning model with consideration of the supply capacity of crude oil. Then, using the obtained processing amount of crude oil as the demand amount of CDU for charging tanks, the crude oil scheduling model is solved by minimizing the operating cost.

The net profit of the sequentially optimized models is equal to the objective value of refinery planning minus the objective value of crude oil scheduling. Thus, we obtain the net profit of the sequential solution to be 22.54 million dollars, which is 10% lower than the integrated model solution of 25.27 million dollars.

5.3 Computational results for eight study instances

We evaluate the commercial solvers, BARON and DICOPT, on the eight different problem-size instances generated from our proposed model with 7-84 days as a time horizon. In addition, BARON is used as a global solver, and DICOPT is used to obtain an optimal/suboptimal solution within an acceptable solution time. We set 1% optimality relative tolerance and a time limit of 3600 seconds for each instance. The model details on each of these test cases can be found in Table 1.

Table 1. Number of variables and constraints for Cases 1-8

Case	Horizon (days)	Number of Cons.	Number of Vars.	Number of Bin. Vars.	Number of nonlinear non-zeroes elements
1	7	1929	1119	105	193
2	14	3848	2245	210	414
3	21	5771	3350	315	621
4	28	7718	4465	420	828
5	35	9637	5575	525	965
6	42	11564	6689	630	1158
7	56	15418	8917	840	1544
8	84	23126	13373	1260	2316

Table 2 reports the CPU time spent by every solver in each instance, and the corresponding best solution found for the objective function of net profit. It also reports the number of OA iterations performed by DICOPT, the objective function upper bound (UB) predicted by BARON, and the optimality gap calculated for both solvers with respect to the best solution found provided by BARON.

Notice that the solver BARON provides optimized feasible solutions for the former 5 tested instances within the time limit of 3600 seconds, whose average relative gap is 39.31%. For Cases 6, 7 and 8, BARON fails to obtain feable solutions. DICOPT is able to sucessfuly solve instances 1-6 within 500 seconds. The average relative gap between the best solutions of DICOPT and the best solutions of BARON for Cases 1-5 is 2.58%. For the most challenging seventh and eighth instances with a time horizon of 56 and 84 days, DICOPT reaches the time limit of 3600 seconds, although it still provides good quality feasible solutions. When solving the instances, DICOPT spent most of the time in the MILP subproblems. The default version of DICOPT implements for general nonconvex MINLP problems, for which bounds of the master MILP problem are not guanateed to be rigorous. A heuristic stopping criterion of worsening is used, where the solver is stopped if the NLP subproblem obtains a worse objective function than the previous iteration (Viswanathan and Grossmann, 1990). This heuristic stopping criterion may make the solver to terminate prematurely.

Table 2. Solution results and times for Cases 1 - 8

		BARON				DICOPT			
Case	Horizon (days)	Net Profit (10^6\$)	UB over Net Profit (10^6\$)	CPU time (sec.)	Gap (%)	Net Profit (10^6\$)	CPU time (sec.)	OA Iters.	Gap (%)
1	7	25.27	27.28	3600	7.95	25.07	2.60	6	0.79
2	14	29.91	42.88	3600	43.36	29.91	7.08	5	0.00
3	21	29.56	39.09	3600	32.24	30.32	140.44	6	2.57
4	28	34.84	51.47	3600	47.73	38.11	98.54	8	9.39
5	35	89.63	148.11	3600	65.25	89.77	228.37	3	0.16
6	42	-	177.91	3600	-	94.97	447.34	3	-
7	56	-	237.08	3600	-	119.65	3600	2	-
8	84	-	353.15	3600	-	186.89	3600	2	-

The integrated optimization solutions, including the refinery planning and the crude oil scheduling, can be obtained for all eight instances. As expected, the feedstocks of crude oil coordinate with the crude oil processing operations in the refinery planning. The output of the oil products satisfies the demand of the oil products, and the properties of oil products are within the required thresholds. The crude oil with high price-performance is preferred for processing. When the crude oil with high price-performance are insufficient, the scheduling solution will arrange processing the crude oil with secondary price-performance. The price-performance of crude oil is dependent on the price, ingredients, side-product yields of crude oil, and final demand. The proposed integrated model can evaluate the price-performance for each type of crude oil, then obtain the rational and valid scheduling solutions. Furthermore, for Instances 8 and with 12 time periods, the near cyclic operations for processing crude oils are between periods, convenient from the operation implementation perspective.

To examine the robustness of the proposed model, we generate nine scenarios based on the Cases 1, 4 and 8 mentioned above, considering the fluctuation of the given parameters. Cases 1, 4 and 8 represent small-scale, medium-scale and large-scale instances, respectively. The variations of input data involve the arrival times of crude oil vessels, the inventory amount of crude oil in storage tanks, and the demand for oil products. The details of the scenarios can be found in Appendix A.

We choose DICOPT as the solver for these instances. In addition to the heuristic stopping criterion for DICOPT that is based on the worsening of the NLP subproblem, we consider as an alternative to perform a maximum of 20 cycles or iterations to possibly avoid stopping prematurely the search. The computational results are given in Table 3. Cases 1.1, 4.1 and 8.1 are the cases with the variation in the vessel arrival times, Cases 1.2, 4.2 and 8.2 are the cases with different values for the initial inventory of crude oil, and Cases 1.3, 4.3 and 8.3 are the cases with different scenarios changing the demand of oil products.

Table 3. Solution results and times for Cases 1, 4 and 8 evaluated in different scenarios

Case / Scenario	DICOPT							
	NVP (10^6\$)		CPU times (sec.)		OA Iters.		Number of feasible solutions found	
	Default*	Max cycles*	Default	Max cycles*	Default	Max cycles*	Default	Max cycles*
1	25.07	25.07	5.97	9.49	6	20	1	3
1.1	25.07	25.07	2.89	12.68	3	20	1	3
1.2	23.45	23.45	4.28	15.14	4	20	2	4
1.3	24.92	24.92	11.79	13.45	11	20	1	2
4	38.11	38.11	29.28	253.55	3	20	1	3
4.1	38.02	38.07	145.49	260.61	4	20	2	2
4.2	35.59	35.60	66.92	143.81	6	20	3	3
4.3	37.56	37.57	21.71	240.11	3	20	1	4
8	186.89	186.90	3600	3600	2	2	1	1
8.1	186.84	186.84	3600	3600	2	2	1	1
8.2	180.08	180.08	3600	3600	2	2	1	1
8.3	189.07	189.07	3600	3600	2	2	1	1

*Default stops when no improvement; *Max cycles = 20 iterations

As seen in Table 3, the scenarios 1.2, 4.2, 8.2 with one empty storage tank (Appendix A, Table A.5) have the most significant impact on the objective function of the proposed model. This is due to the storage tank's lack of initial inventory, leading to an overall economic loss. Regarding the solution times, the demand for oil products has a minimal effect on it. In contrast, the simultaneous arrival time scenarios are more demanding when solving them, particularly the MILP subproblems. If the arrival times of oil tanks are asynchronous, although there are variations of demand for oil products, the proposed integrated optimization model can be solved efficiently. Finally, note that the choice of setting the maximum OA iterations to 20 only led to very marginal improvements to obtain several alternative feasible solutions, in instances 4.1, 4.2, 4.3, and 8, order of 0.1% improvement or much less in the profit. Thus, we can conclude that using DICOPT with the default termination provides very good solutions, that might be slightly improved in few cases by setting the maximum of 20 iterations.

6. Conclusions

This paper has addressed the integrated multi-period optimization problem for refineries, covering the refinery production planning integrated with crude oil scheduling. The objective of the optimization problem is to obtain optimal sequences of operations following the model of the refinery and provide feasible plans and schedules for practical production by satisfying the constraints of production consistency and feedstock of crude oils. An integrated MINLP model is formulated based on hybrid continuous-discrete time representation, including nonlinear equations for mixing operations and blending processes. We show that the continuous-time representation of the scheduling leads to an exact and more efficient detail formulation. Furthermore, the proposal of the hybrid-time representation allows for it to be integrated with a discrete-time represented planning problem.

The computational tests were implemented based on the data from a Sinopec refinery plant. We proposed MINLP problems to maximize the net profit of the entire process, which were successfully solved with the solver DICOPT obtaining optimal or suboptimal solutions within one hour in a commercial laptop. The values of the optimization model were shown by having the three main components of the refinery process optimized simultaneously using real data and including rigorous property equations while keeping the model tractable for the current MINLP solvers. Future work will focus on the more complex models and explore the decomposition techniques to solve this optimization problem.

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Appendix A. Data for Whole Refinery Processes

Table A.1. Data of crude oils from a refinery plant

Crude oil	Supply capacity (10^4ton/day)	Price ($/ton)	API	Sulfur content (wt%)
CR1	0.93-1.14	459.81	34.00	0.09
CR2	1.00-1.14	413.10	31.57	0.15
CR3	0.96-1.14	374.25	27.90	0.63

Table A.2. Capacity of refinery units

Units	CDU	VC	RH	CR	CC	HC	DC	GR	DoR
Capacity (10^4 ton/y)	800.00	241.00	145.14	140.14	252.00	130.00	145.14	218.57	41.29

Table A.3. Yields of side products for crude oils

Yields (%)	CR1	CR2	CR3
Dry gas	0.0001	0.0000	0.0008
Liquid gas	0.0004	0.0001	0.0076
Light naphtha	0.0029	0.0004	0.0185
Heavy naphtha	0.0370	0.0286	0.1227
Naphtha/kerosene	0.0175	0.0049	0.0313
Kerosene1	0.0624	0.0552	0.0879
Jet fuel/Diesel	0.0240	0.0235	0.0224
Atmosphere Cut 2	0.0819	0.0887	0.0983
Atmosphere Cut 3	0.1409	0.1593	0.1409
Vacuum Cut 1	0.0864	0.1070	0.0348
Vacuum Cut 2	0.1157	0.1267	0.1045
Vacuum Cut 3	0.1676	0.1567	0.1315
Vacuum Residuum	0.2580	0.2440	0.1945
Loss	0.0051	0.0050	0.0043

Table A.4. Prices for final oil products

Oil products	Gasoline 90 (G90)	Gasoline 93 (G93)	Jet Fuel (JET)	Diesel -10 (D10)	Diesel 00 (D00)	Heavy Diesel20 (D+2)	Chem. Light oil	Fuel oil250
Price ($/ton)	508.59	538.51	521.64	495.25	495.25	502.58	440.08	261.95

Table A.5. Data for different Scenarios

Case / Scenario	Arriving time of crude oil vessels (days)	Initial inventory amount of ST and CT (10^4 ton)	Demand Lower Bounds for oil products (10^4 ton/period)
1	*TA*={1 3 5}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*={1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1}
1.1	*TA*={3 3 5} Simultaneous	Is^0={3 3 3}, Ig^0={3 3 2}	*D*={1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1}
1.2	*TA*={1 3 5}	Is^0={3 0 3}, Ig^0={3 3 2} - 17.65%	*D*={1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1}
1.3	*TA*={1 3 5}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*={1.7,1.5,1.0,0.5,0.5,1.0,0.1,0.1}
4	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*= *Para(t)** {1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*={1, 0.8, 1.1, 1.2}
4.1	*TA*=[3 3 5; 8 10 12; 17 17 19; 22 24 26] Simultaneous	Is^0={3 3 3}, Ig^0={3 3 2}	*D*= *Para(t)** {1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*= {1, 0.8, 1.1, 1.2}
4.2	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26}	Is^0={3 0 3}, Ig^0={3 3 2} - 17.65%	*D*= *Para(t)**{1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*= {1, 0.8, 1.1, 1.2}
4.3	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*=*Para(t)**{1.7,1.5,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*= {1, 0.8, 1.1, 1.2}
8	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26; 29 30 33; 37 40 41; 44 44 46; 51 55 54; 58 58 60; 65 67 67; 72 74 76 ; 79 81 82}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*= *Para(t)**{1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*= {1.0, 0.8, 1.1, 1.2, 1.3, 0.9, 1.3, 0.9, 1.0, 1.2, 0.9, 0.8}
8.1	*TA*={3 3 5; 8 10 12; 17 17 19; 22 24 26; 30 30 33; 37 40 41; 44 44 46; 51 55 54; 58 58 60; 65 67 69; 74 74 76 ; 79 81 82}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*= *Para(t)**{1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*={1.0, 0.8, 1.1, 1.2, 1.3, 0.9, 1.3, 0.9, 1.0, 1.2, 0.9, 0.8}
8.2	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26; 29 30 33; 37 40 41; 44 44 46; 51 55 54; 58 58 60; 65 67 67; 72 74 76 ; 79 81 82}	Is^0={3 0 3}, Ig^0={3 3 2} - 17.65%	*D*= *Para(t)**{1.2,1.0,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*={1.0, 0.8, 1.1, 1.2, 1.3, 0.9, 1.3, 0.9, 1.0, 1.2, 0.9, 0.8}
8.3	*TA*={1 3 5; 8 10 12; 15 17 19; 22 24 26; 29 30 33; 37 40 41; 44 44 46; 51 55 54; 58 58 60; 65 67 67; 72 74 76 ; 79 81 82}	Is^0={3 3 3}, Ig^0={3 3 2}	*D*= *Para(t)**{1.7,1.5,1.0,0.5,0.5,1.0,0.1,0.1} *Para(t)*={1.0, 0.8, 1.1, 1.2, 1.3, 0.9, 1.3, 0.9, 1.0, 1.2, 0.9, 0.8}