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MINLP Model for Reliability Optimization of System Design and Maintenance Based on Markov Chain Representation

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Abstract

This paper proposes an MINLP model that represents the stochastic process of system failures and repairs as a continuous-time Markov chain, based on which it optimizes the selection of redundancy and the frequency of inspection and maintenance tasks for maximum profit. The model explicitly accounts for every possible state of the system. A decomposition method and a scenario reduction method are applied to this example to drastically reduce the computational effort. We show by an example that the proposed model and algorithms are capable of solving a practical problem based on the air separation process example that motivated our work, which features multiple stages, potential units and failure modes.

Keywords: reliability design, maintenance, optimization, Markov Chain, MINLP

1. Introduction

Plant availability has been a critical consideration for the design and operation of chemical processes as it represents the expected fraction of normal operating time, which directly impacts the ability of making profits. In practice, discrete-event simulation tools are used to examine the availability of a few selected designs of different redundancy levels under various maintenance and spare parts inventory policies (Sharda and Bury, 2008). However, the best plan selected through simulation is usually suboptimal because the list of design alternatives is often not exhaustive. Thus, there is a strong motivation for systematic optimization tools of redundancy design considering operational factors.

Several works have been reported regarding reliability considerations at the design phase (Kuo and Wan (2007)). In order to obtain a more comprehensive optimal design, it is important to consider the impact of operational factors such as maintenance on plant availability and their costs (Ding and Kamaruddin (2015)). Alaswad and Xiang (2017) provide a review for condition-based maintenance optimization models for stochastically deteriorating system with either discrete or continuous states. Pistikopoulos et al. (2001) and Goel et al. (2003) formulate MILP models for the selection of units with different reliability and the corresponding production and maintenance planning for a fixed system configuration.

Markov chain is a powerful mathematical tool that is extensively used to capture the stochastic process of systems transitioning among different states. Shin and Lee (2016) formulate the planning level problem of a procurement system as an Markov Decision Process to account for exogenous uncertainties coming from lead time and demand. Lin et al. (2012) model a utility system using Markov chain and carry out RAM (reliability, availability & maintainability) analysis iteratively to decide the optimal reliability design.

Terrazas-Moreno et al. (2010) use Markov chain as an uncertainty modeling tool for the optimal design of production site network considering reliability and flexibility. Kim (2017) presents a reliability model for k-out-of-n systems using a structured continuous-time Markov chain, which is solved with a parallel genetic algorithm.

Given the aforementioned research gaps and knowledge basis, this work extends our recent mixed-integer framework (Ye et al., 2017) and introduces a systematic approach to model the stochastic failure and repair process of the superstructure system as a continuous-time Markov chain. The new framework explicitly accounts for the long term property of each possible reliability scenario. Therefore, it is able to incorporate various kinds of decision making processes. Especially, comparing to Terrazas-Moreno et al. (2010) and Kim (2017), corrective maintenance and condition-based maintenance are incorporated in order to find the overall optimal selection of parallel units.

2. Motivating Example and Problem Statement

Consider an air separation unit (ASU) shown in Figure 1 as a motivating example. Critical processing units include the main air compressor, the pre-purifier, the booster air compressor, and the liquid O_2 pump.

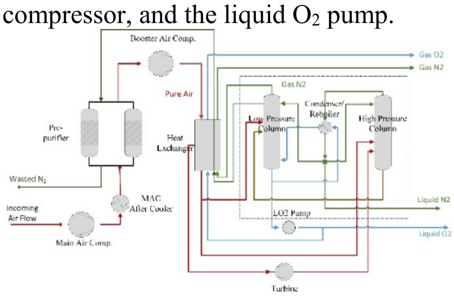

Figure 1. Typical flowsheet of an ASU

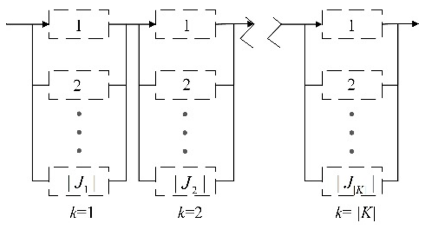

Figure 2. System topology and notations

The failure of any one of these processing stages can result in the failure of the entire system, which will compromise its ability to meet customer demands. In order to effectively increase the system availability, two strategies are considered.

2.1. Redundancy selection

The first strategy is to install parallel units for the critical stages. In Figure 2, the availability superstructure is formulated as a serial system of sequential stages, where each stage k has several potential design alternatives. The number and selection of parallel units have impacts on system reliability as well as capital costs.

2.2. Inspection Interval and Condition-based Maintenance

The second strategy is to carry out condition-based maintenance (CBM). Specifically, the units go through periodic inspections, and follow-up maintenance if the inspection result indicates that the equipment is going to fail shortly. The intervals between the periodic inspections impact individual unit failure rates and operational costs.

The ultimate goal is to achieve the optimal overall net present value for the system.

3. Mathematical Formulation

3.1. Markov Chain Representation

As mentioned in section 2.1, for each processing stage k, there is a set of potential units indexed by j. It is assumed that the time to failure and time to repair of each potential unit j follow exponential distributions with respective rate parameters $\lambda_{k,j}^{0}$ and $\mu_{k,j}$, which constitutes the transition matrix Q_k of the continuous-time Markov Chain of stage k. The system transition matrix W is then calculated as follows.

$$W = I_{(n_{|K|}n_{|K|-1}\cdots n_2)} \otimes Q_1 + I_{(n_{|K|}n_{|K|-1}\cdots n_3)} \otimes Q_2 \otimes I_{n_1} + \cdots + Q_{|K|} \otimes I_{(n_{|K|-1}n_{|K|-2}\cdots n_1)} \quad (1)$$

Element $W(\bar{s},\bar{r})$of matrix $W$is the transition rate from state $\bar{s}$ to state $\bar{r}$, which belongs to the system state space $\bar{S}$.

3.2. Logical constraints

Binary variable $y_{k,j}$ indicates whether unit *j* in stage *k* is selected.

$$\sum_{j \in J_k} y_{k,j} \geq N_k \quad (2)$$

Each possible combination of the potential units of one stage is called a stage design alternative indexed by h. Binary variable $z_{k,h}$ indicates the selection of stage design *h* for stage k. Set D contains the tuples of (j,k,h) where unit j is selected in design h of stage k, based on which $z_{k,h}$ and $y_{k,j}$are connected as shown in (3), (4) and (5).

$$z_{k,h} \leq y_{k,j}, (j,k,h) \in D \quad (3)$$

$$z_{k,h} \leq 1 - y_{k,j}, (j,k,h) \notin D \quad (4)$$

$$z_{k,h} \geq \sum_{(j,k,h) \in D} y_{k,j} + \sum_{(j,k,h) \notin D} (1 - y_{k,j}) - |J_k| + 1, k \in K, h \in H_k \quad (5)$$

Equation (6) requires that one and only one design is selected for each stage.

$$\sum_{h \in H_k} z_{k,h} = 1, \forall k \in K \quad (6)$$

The combination of certain stage designs of each stage k is called a system design, which is indexed with $\bar{h}$. Set HC contains the tuples of $(k,h,\bar{h})$ where system design $\bar{h}$ contains stage design h of stage k. Binary variable $\bar{z}_{\bar{h}}$ indicates the existence of system design $\bar{h}$, and is related to the values of $z_{k,h}$.

$$\bar{z}_{\bar{h}} \leq z_{k,h}, \forall (k,h,\bar{h}) \in HC \quad (7)$$

$$\bar{z}_{\bar{h}} \geq \sum_{k \in K} \sum_{h \in H_k, (k,h,\bar{h}) \in HC} z_{k,h} - |K| + 1, \forall \bar{h} \in \bar{H} \quad (8)$$

Binary variable $\overline{zz}_{\bar{s}}$ indicates the existence of system state $\bar{s}$, which is required to be equal to $\bar{z}_{\bar{h}}$, if $\bar{s} \in \bar{T}_{\bar{h}}$, the subspace supported by system design $\bar{h}$.

$$\overline{zz}_{\bar{s}} = \bar{z}_{\bar{h}}, \forall \bar{s} \in \bar{T}_{\bar{h}} \quad (9)$$

$\pi_{\bar{s}}$ is the stationary probability of state $\bar{s}$, which has an upper bound of 1, and equals zero if state $\bar{s}$ does not exist.

$$\pi_{\bar{s}} \leq \overline{zz}_{\bar{s}}, \forall \bar{s} \in \bar{S} \quad (10)$$

The stationary probability distribution satisfies (11) and (12), which are based on the system transition matrix W and the existence of each system state.

$$\sum_{\bar{s} \in \bar{S}} \pi_{\bar{s}} \mathbf{W}(\bar{s},\bar{r}) \leq M(1 - \overline{zz}_{\bar{r}}), \forall \bar{r} \in \bar{S} \quad (11)$$

$$\sum_{\bar{s} \in \bar{S}} \pi_{\bar{s}} \mathbf{W}(\bar{s},\bar{r}) \geq M(\overline{zz}_{\bar{r}} - 1), \forall \bar{r} \in \bar{S} \quad (12)$$

Finally, the availability of the system is one minus the sum of the stationary probability of all failed states:

$$A = 1 - \sum_{\bar{s} \in \bar{S}^f} \pi_{\bar{s}} \quad (13)$$

3.3. Consideration of Inspections

The range of possible inspection intervals t_k^{insp} is discretized into a finite set of choices, T_l^{insp}. The selection of inspection intervals for each stage k is represented with binary variables $x_{k,l}$, where $x_{k,l} = 1$ when time length T_l^{insp} is selected for stage k.

$$\sum_{l \in L} x_{k,l} = 1, \forall k \in K \tag{14}$$

$$t_k^{insp} = \sum_{l \in L} x_{k,l} T_l^{insp}, \forall k \in K \tag{15}$$

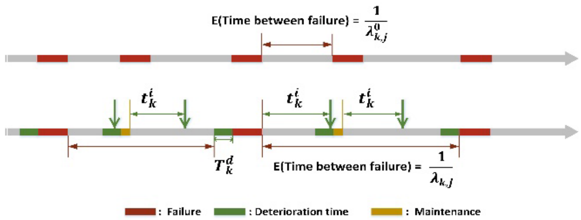

Figure 3. The impacts of inspections and maintenance

The equivalent failure rate $\lambda_{k,j}$with inspections and maintenance is calculated in (16). Figure 3 shows a sketch of how $\lambda_{k,j}$can be different from $\lambda_{k,j}^0$.

$$\lambda_{k,j}^0 - \lambda_{k,j} = \sum_{l \in L} x_{k,l} (e^{-\lambda_{k,j}^0 T_l^{insp}} - e^{-\lambda_{k,j}^0 (T_l^{insp} + T_k^d)}) / T_l^{insp}, \forall k \in K, j \in J_k \tag{16}$$

Each stage k has a inspection cost c_insp_k. Equation (17) enforces that the inspection cost for stage k is proportional to its inspection frequency and cost rate.

$$inspCost = \sum_{k \in K} c\_insp_k \sum_{l \in L} x_{k,l} \frac{T}{T_l^{insp}} \tag{17}$$

The repair cost is calculated according to the failure states. The repair cost in each state $\bar{s} \in \bar{S}$ is equal to the frequency of $\bar{s}$: $-W(\bar{s},\bar{s})\pi_{\bar{s}}$, times the summation of the repair costs of all the units that are failed in state $\bar{s}$.

$$repaCost = -T \sum_{\bar{s} \in \bar{S}} W(\bar{s},\bar{s}) \pi_{\bar{s}} \sum_{(k,j) \in KJ_{\bar{s}}^f} c\_repa_k \tag{18}$$

The number of times for follow-up maintenance to take place in a single unit relative to its number of repairs is calculated by the relative difference between the equivalent failure rate and the original failure rate.

$$mainRatio_k \geq y_{k,j} (\lambda_{k,j}^0 - \lambda_{k,j}) / \lambda_{k,j}, \forall j \in J_k \tag{19}$$

Equation (20) follows the same logic as in (18) to calculate costs according to failure states. Here, $c_repa_k$is replaced by c_main_k times $mainRatio_k$, which is the number of follow-up maintenance relative to the number of repairs.

$$mainCost = -T \sum_{\bar{s} \in \bar{S}} W(\bar{s},\bar{s}) \pi_{\bar{s}} \sum_{k \in K_{\bar{s}}^f} mainRatio_k c\_main_k \tag{20}$$

In addition to the costs, maintenance also causes downtime, which will result in the decrease of availability. Equation (21) calculates the downtime caused by maintenance in terms of the failure states $\bar{s} \in \bar{S}^f$ and those stages that fail in $\bar{s}$. The net system availability A is calculated as A minus the ratio of downtime caused by maintenance to the entire time horizon T.

$$mainTime = -T \sum_{\bar{s} \in \bar{S}^f} W(\bar{s},\bar{s}) \pi_{\bar{s}} \sum_{k \in K_{\bar{s}}^f} mainRatio_k T\_main_k \tag{21}$$

$$A^{net} = A - \frac{mainTime}{T} \tag{22}$$

3.4. Objective Function

The objective to be maximized is the Net Present Value NPV, the present value of net cash flow minus the investment costs. The yearly net cash flow is equal to revenue RV minus all the operational costs, and divided by number of years T. It is discounted by $\sum_{i=1}^{T}\frac{1}{(1+r)^i}$, where r is the rate of return (RoR) of cash flow. $instCost$ is the investment cost for installing the units depending on binary variables $y_{k,j}$. The revenue RV is proportional to system availability.

$$\max NPV = \frac{1}{T}\left(\frac{1-(1+r)^{-T}}{r}\right)(RV - repaCost - inspCost - mainCost) - instCost \tag{23}$$

$$inst\,\mathrm{Cos}\,t = \sum_{k\in K}\sum_{j\in J_k} y_{k,j} c\_inst_{k,j} \tag{24}$$

$$RV = A^{net}\cdot rv \tag{25}$$

4. Example

The model is applied to the motivating example of ASU (air separation unit) introduced in section 2, where the compressors have six failure modes and at least 2 units are needed for the pre-purifier. A time horizon of 10 years is considered. The exact numbers of relevant cost and reliability parameters of the units are proprietary information. Mean time between failures (MTBF) range from 5-25 years. Mean time to repair (MTTR) range from 8 - 1080hours. Capital cost of each unit range from \$85k - \$800k. Repair costs range from \$2k - \$20k per time. Inspection costs range from \$0.05k - \$0.5k per time. Maintenance costs range from \$1k -\$10k per time. Maintenance times range between 1 and 2 days. Inspection window lengths range between 5 and 6 day.

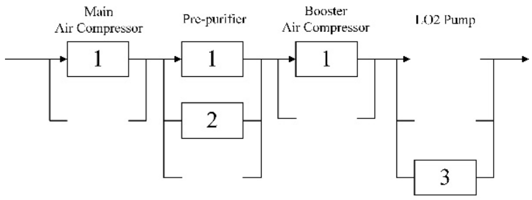

Figure 4. The optimal design

We propose an algorithm (Ye et al., 2019) that solves upper bounding MILPs and lower bounding MINLPs iteratively until the gap is closed. The algorithm converges in 7 rounds (ϵ= 1.7%) to the flowsheet shown in Figure 4. Only the least number of units are selected for each stage. For the main air compressor and the booster air compressor, more reliable and expensive units are selected, while for the LO2 pump, the solution selects the cheapest one. The expected system availability is 0.9866. The expected net present value is \$15,649.4k, with a revenue of \$29,597.8k and a penalty of \$421.9k. \$2,083k is spent on unit investment, \$262.7k is spent on inspections, \$7.4k is spent on maintenance, and \$48.9k on repair. Qualitatively speaking, this solution tends to spend more effort on reducing the failure rates for those failure modes with longer repair time.

The MILP models are solved with Xpress 29.01(748.8 CPUs), and the MINLP models are solved with SBB 25.1.1(492.0 CPUs).

5. Conclusions

In this paper, the stochastic process of system failures and repairs is modeled as a continuous-time Markov chain. Moreover, the impact of maintenance is incorporated.

With a general air separation unit as the motivating example, two strategies are considered to increase the availability of the system. The first strategy is to install parallel units for certain processing stages. The second strategy is to carry out periodic inspections, and condition-based maintenance if the inspection results indicate that the equipment will fail shortly. A non-convex MINLP model is proposed accordingly.

The non-convex MINLP model does not scale well, and has a large number of bilinear and multilinear terms. A decomposition scheme was proposed to reduce the size of the model and the computational time (Ye et al., 2019). The motivating example of the air separation unit is solved with the proposed specialized solution method.

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