Integrated scheduling of rolling sector in steel production with consideration of energy consumption under time-of-use electricity prices

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Abstract

Due to increasing load and penetration of renewables, the electric grid is using time-of-use pricing for industrial customers. Involving energy-intensive processes, steel companies can reduce their production cost by accounting for changes in electricity pricing. In particular, steel companies can take advantage of processing flexibility to make better use of electric power, and thus reduce the energy cost. In this paper, we address a new integrated scheduling problem of multi-stage production derived from the rolling sector of steel production, with consideration of batching decisions and demand-side management. The problem is formulated as a continuous time mixed-integer nonlinear programming (MINLP) model with generalized disjunctive programming (GDP) constraints, which is then reformulated as a mixed-integer linear programming (MILP) model. Numerical results are presented to demonstrate that the model is computationally efficient and compact.

***Keyword**:* Integrated scheduling, continuous time modeling, MINLP, GDP

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Introduction

Nomenclature

Indices

i, j	Coils
s, s'	Units
k	Slots
t_s	The last slot of unit *s*
tp	Constant electricity price period

Sets and Parameters

S	Set of total units, $S = \{1, 2, ..., \|S\|\}$
AC	Set of units in acid rolling section
S'	Set of units that have downstream units
L_s	Set of units that are downstream units of *s*
N	Set of total coils
N_s	Set of coils that need to be processed at unit *s*, $N_s = \{1, 2, ..., n_s\}, s \in S \setminus AC$
$N_{ac}^{'}$	Set of coils that need to be processed in acid rolling section, $N_{ac}^{'} = \{1, 2, ..., ac\}$
$N_{s,s'}$	Set of coils that released from unit *s* to unit *s'*
T_s	Set of slots at unit *s*, $T_s = \{1, 2, ..., t_s\}$
TP	Set of electricity pricing periods, $TP = \{1, 2, \ldots, tp_{\max}\}$
τ_i^s	Processing time of coil *i* at unit *s*
m_1	Coefficient of total changeover costs in objective function
m_2	Coefficient of total costs of rolling facilities in objective function
m_3	Coefficient of total costs of electricity consumption in objective function
$F_{i,j}^s$	Changeover cost if coil *i* is processed followed by coil *j* at unit *s*
mc_i^s	Minimum changeover cost of coil *i* with adjacent coil at unit *s*
T_{tp}'	Length of time period *tp*
pwh^s	Electricity amount that unit *s* will consume per hour
$setup^s$	Setup time between batches at unit *s*
$nB_{\max}$	Maximum capacity of each batch
c_{roller}	A constant cost of each rolling facility (roller) in acid rolling production
ce_{tp}^{buy}	The price of electricity during time period *tp*
cp_{tp}^L	Lower bound of time period *tp*
cp_{tp}^U	Upper bound of time period *tp*

UB_s	Upper bound of timing variables of unit *s*
Discrete variable	
$nbatch_{s,k}$	Capacity of slot (batch) *k* at unit *s*
Binary variables	
$y_{i,k}^s$	If coil *i* is assigned to slot *k* at unit *s*
$z_{i,j,k}^s$	If coil *i* is assigned to slot *k* and coil *j* is assigned to slot *k*+1 at unit *s*
$r_{s,k}$	If slot *k* at unit *s* is selected (*k* is not idle)
Continuous variables	
$\widehat{tb}_i^s$	Beginning time of coil *i* at unit *s*
$\widehat{tf}_i^s$	Completion time of coil *i* at unit *s*
$\overline{tb}_k^s$	Beginning time of slot *k* at unit *s*
$\overline{tf}_k^s$	Completion time of slot *k* at unit *s*
$tb_{i,k}^s$	Beginning time of coil *i* in slot *k* at unit *s*
$tf_{i,k}^s$	Completion time of coil *i* in slot *k* at unit *s*
$cd_{i,k}^s$	Processing time of coil *i* in slot *k* at unit *s*, $s \in S \setminus AC$
sd_k^s	Duration of slot *k* at unit *s*, $s \in AC$
p_{tp}	Amount of electricity that is consumed during time period *tp*
$\Delta T_{s,k,tp}$	Time fractions of a selected slot *k* during a constant time period *tp* at unit *s*

The rolling sector is the major and most profitable sector in steel production, where raw products are further processed to fine steel products with high added-value. It is becoming increasingly important to the steel enterprise, especially when facing supply-front reform which aims at promoting lean production. Nowadays, many steel enterprises are encountering decreasing profit margins due to the rising prices of electric power and raw material, which makes it critical to control costs to remain competitive. From the various methods to reduce production costs, optimizing production management is an effective approach, for which no additional investment is required.

Usually, a rolling sector consists of acid rolling, annealing, rewinding and a series of metal coating sections. Fig. 1 shows a simplified distribution of the rolling sector, which includes some typical sections. Each section uses its own criterion for scheduling the production and most of time they organize the production independently without coordination with other sections. Usually, this mode leads to some undesired situations such as unbalanced material flow, shortage of feedstock at downstream section and losses in production efficiency, which often yields extra production cost and decreased profit. This implies the need for proper integrated scheduling over all sections in the rolling sector to guarantee optimal production and to reduce production costs.

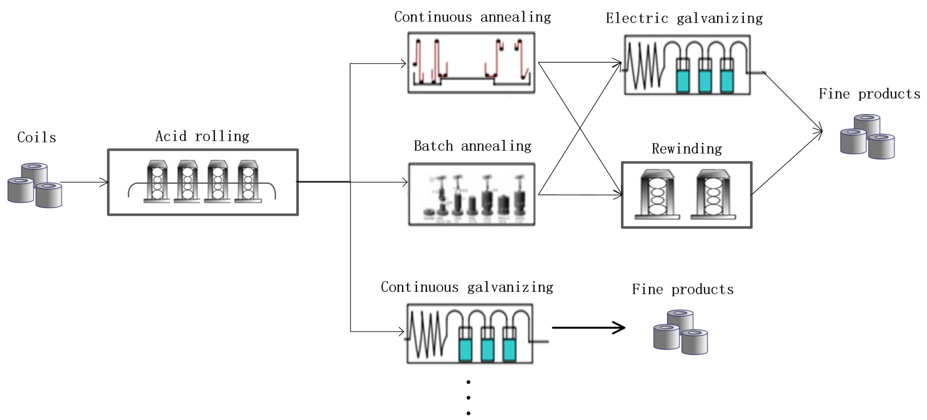

Fig. 1 Typical distribution of rolling sector

Another factor that represents a rather considerable part of production cost is electricity consumption. Steel production involves several energy-intensive processes, acid rolling for example. The main production step in acid rolling section is to roll the thick steel strip into a much thinner one. All the rolling mills are driven by electricity and it consumes a large amount of electricity to generate rolling pressure during production. Due to the rapid increase of power demand, the power grid is now using time-of-use pricing strategy on industry customers to improve the utilization of electricity power on the demand side and keep stability of the power supply. Pricing of electricity can significantly affect the production mode and profitability of steel production. In this context, the steel enterprise can take advantage of production flexibility and pricing scheme to gain potential benefits, and reduce energy cost by organizing production on proper pricing time period.

Production scheduling has become a major optimization problem of industrial significance. Harjunkoski et al. (2014), Maravelias and Sung (2009), Méndez et al. (2006) reviewed the scope for industrial applications of scheduling models and solution methods. As for the steel industry, most of the literature focuses on the upstream process of steel production involving steel making and casting process. Harjunkoski and Grossmann (2001) addressed a decomposition approach to solve complex scheduling problem in steel making process. Tang et al. (2000) proposed a multiple traveling salesman model for hot rolling scheduling. Pacciarelli and Pranzo (2004) developed a model of steelmaking-continuous casting production based on alternative graph formulation with detailed constraints that are relevant for the scheduling problem. Tang et al. (2001) gave a comprehensive analysis of scheduling of integrated steel production. Compared with much research on steelmaking scheduling, not much attention has been paid to the integrated scheduling of the rolling sector, which includes multiple finishing sections of steel production. The rolling sector is characterized by different production structure from steel making and continuous casting process. Production in the rolling sector has features of low-volume and high-variety, and more complex supply networks between the associated sections. Therefore, it is challenging to schedule the processing tasks for all sections in the rolling sector with respect of technological constraints while seeking overall optimal profit.

From the viewpoint of time representation, models can be classified into continuous-time model and discrete-time model. Compared with discrete time, continuous time models can account for timings more accurately and are closer to the real production (Floudas and Lin, 2004). The approaches to formulate continuous time models include the time-slot concept (Erdirik-Dogan and Grossmann, 2008; Jose M Pinto and Grossmann, 1996a, 1996b; Pinto and Grossmann, 1994), order precedence and event based concept. The slot concept and order precedence methods are suitable for sequential processes, while the event based concept is more suitable for network processes (Floudas and Lin, 2004). According to the characteristics of the considered scheduling problem, the slot concept is adopted in this paper.

The management of power consumption in industrial processes has recently received much attention. Mitra et al. (2012) investigated production planning of continuous process with respect of time-sensitive electricity prices based on discrete time representation. Castro et al. (2011) addressed optimal scheduling of continuous plants with energy constraints based on discrete time representation. Castro et al. (2013) proposed RTN formulations for industrial demand side management of a steel plant based on discrete-time representation. Zhang et al. (2017) proposed scheduling models based on RTN formulations and discrete time representation that incorporate the EAFs' flexibilities to reduce the electricity cost. For the models with discrete time representation, it is easier to calculate the electricity consumption due to the fixed time interval for both electricity price and processing task. In contrast, for continuous time models it is more difficult since the relevant times of event points or time slots are variables to be determined. Castro et al. (2009) proposed a new continuous time representation for handling variable electricity cost based on a resource-task network (RTN) representation. Nolde and Morari (2010) addressed the electrical load tracking scheduling of a steel section and proposed a general approach to represent task-time interaction relationships for continuous time formulations, which has been used and extended by other researchers. Hadera et al. (2015) addressed the scheduling of the melt shop section of a stainless steel production plant. They applied the general approach in Nolde and Morari (2010) to deal with complex time-sensitive electricity cost. Apart from steel production, Castro et al. (2014) applied the concept in Nolde and Morari (2010) to optimize the maintenance scheduling in a gas engine plant. They formulated the concept as GDP constraints and found a tighter model for accounting for electricity consumption. In this paper, we improve these GDP constraints in Castro et al. (2014) to deal with electricity consumption. While in Castro et al. (2014), the number of time slots is known a-priori, in this paper the number of time slots is a variable to be determined that results from the batching decisions in the acid rolling section of the steel plant.

In this paper, we address a new practical integrated scheduling problem with demand-side management consideration, which is derived from the rolling sector of steel production. The main contribution of this paper is the first attempt to determine an integrated scheduling of multiple finishing sections in steel production, and to achieve coordination of the scheduling with energy consumption. Due to the new features and distinctions with other scheduling problems in steel production, former formulations cannot be easily adapted to the problem considered in this paper. A new hybrid MINLP/GDP model is proposed in this paper to address the scheduling to the problem based on the continuous time concept. To take advantage of MILP solvers, the MINLP model is transformed into an MILP model by linearization and hull reformulation.

The remaining paper is organized as follows. Section 2 gives a brief description of the production in the rolling sector and the statement of the integrated scheduling problem under consideration. In Section 3, an MINLP model is established with GDP constraints. Section 4 presents the reformulation of the MINLP model, where the non-linear constraints are linearized, and the GDP constraints are reformulated as MILP constraints. Next, in Section 5, numerical tests are conducted and analyses of the results are made. Section 6 draws the conclusion of this paper and describes future work.

1. Problem Statement

To explain the considered scheduling problem, a brief description of the production in rolling sector is introduced first. In this paper, a series of typical sections are considered, including acid rolling, continuous annealing, galvanizing, tin plating, and rewinding as shown in Fig. 2. There is intermediate storage between the associated sections. These sections give rise to a multi-stage process. The acid rolling section is the bottleneck process of the rolling sector, which usually has parallel production units to ensure sufficient supply to downstream sections. Steel coil, the feedstock in rolling sector, starts processing with acid rolling section (C1) and then goes through several of the sections according to the processing route. These include continuous annealing (C2) or galvanizing (C3) and tin plating (C4) or rewinding (C5). The processing route of each coil is predetermined based on its final use and the sequence of processing cannot be changed. The processing mode differs for sections in the rolling sector according to various technical requirements. In general, serial-batch processing mode is preferred in acid rolling production and sequential processing mode is preferred in other sections.

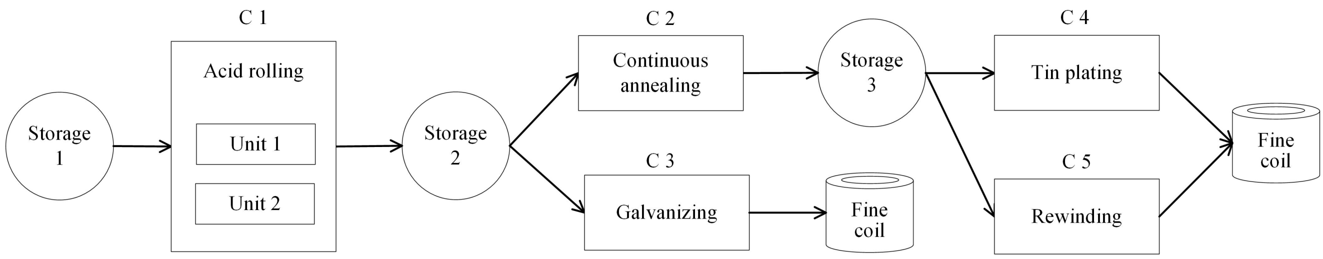

Fig. 2 Considered distribution of rolling sector

In acid rolling production, each batch consists of several coils. Each batch uses a new roller, which has a fixed cost. There is a constant setup time between two serial batches for adjusting production conditions, such as replacing the rollers. The production is continuously conducted within a batch, so that the processing time of a batch is equal to the summation of the processing times of the batched coils. To simplify the problem, the completion time of a coil in acid rolling production is roughly considered to be the same as the completion time of its batch. The capacity of each batch can vary within a maximum volume. The number of batches and the capacity of each batch need to be determined. In general, full utilization of the batch capacity is the ideal case since the cost of the roller is very expensive. However, it changes when considering the energy consumption as well. For example, if a batch is designed too large, and the production is partially conducted during the high electricity price period, then one has to determine whether the batch should be reduced to a smaller size for the sake of decreased energy cost, while this will lead to insufficient utilization of roller and waste of money. Hence, some trade-offs should be made between energy cost and facility cost. Without loss of generality, we assume that the parallel units of acid rolling section are heterogeneous, with unit-dependent energy consumption and setup time.

In the other sections, coils are characterized by many properties, such as width, thickness, annealing temperature and composition of coating metal. When processing coils that are different in characteristics, the equipment will be adjusted to meet varied processing condition and thus cause a changeover cost. The larger the difference is, the higher the changeover cost will be. Therefore, adjusting the equipment frequently should be avoided to reduce changeover costs. The difference of any adjacent processed coils is represented by sequence-dependent changeover costs in this paper.

The integrated scheduling problem of the rolling sector with consideration of batching decision and demand-side management can be stated as follows.

Given:

1) A set of units $s \in AC$ in acid rolling section and a set of units $s \in S \setminus AC$ in other sections in rolling sector;
2) Processing route of each coil;
3) Set of total coils $i \in N$, set of coils $i \in N_{ac}^{'}$ that are processed at acid rolling section and set of coils $i \in N_s$, $s \in S \setminus AC$ that are processed at unit *s*;
4) A set of time slots $k \in T_s$ at unit *s*;
5) A set of units $s' \in L_s$ that are downstream units of *s*;
6) A set of coils $i \in N_{s,s'}$ that are supplied to unit *s'* from unit *s*;
7) Processing time of coil *i* at unit *s*, τ_i^s;
8) Setup time between series batches at unit *s*, $setup^s$, $s \in AC$;
9) Maximum capacity of each batch, $nB_{\max}$;
10) A set of electricity pricing periods, $tp \in TP$,with constant electricity price of each period ce_{tp}^{buy};
11) Time boundaries of each price period, cp_{tp}^L and cp_{tp}^U.

Determine:

1) The assignment of the coils in the acid rolling section;

2) Batch composition, batch capacity, number of batches at each unit of acid rolling section and timetable of batches with consideration of electricity pricing;
3) Coil sequence and timetable in the sections after acid rolling;
4) Electricity consumption of each electricity pricing period;
5) Completion time of the entire production.

The objective is to minimize the total weighted production costs that consist of changeover costs, rolling facility costs and energy consumption costs. The following assumptions are made in the optimization problem.

1) There are coils at each intermediate storage at the beginning of the schedule;
2) To simplify the problem, we only observe the electricity consumption in one electricity pricing cycle and we assume that the coils which need to be processed in the acid rolling section can all be finished within one cycle;
3) To simplify the problem, it is assumed that there are two units in acid rolling section and one unit in each of the rest sections, which is also a base case in practical production.

2. Mathematical Formulation

The proposed MINLP/GDP formulation addresses a multi-stage process in coil production with consideration of various operational constraints and electricity consumption management. The formulation is a hybrid of the slot-based continuous-time concept (see Fig. 3), the immediate precedence concept and GDP. The immediate precedence formulation is known to represent sequence-dependent relationships, which are used to account for changeover costs in this paper. Since the model is based on the continuous-time concept, one of the challenges is to determine the location of the time slots in the discrete time periods of electricity pricing when calculating the electricity consumption. Inspired by recent work of Castro et al. (2014), we extend the GDP constraints in this paper to deal with the electricity consumption calculation. GDP (Raman and Grossmann, 1994) is a logic-based modeling method which is known for representing complex logic constraints and has been applied to many scheduling problems (Castro and Grossmann, 2012). In the context of the continuous time representation, as for time-slot based model, the number of slots needs to be specified a-priori. For the problem under consideration, the slots can be classified into two categories. For the acid rolling section, the production is batch oriented. As a consequence of the variable batch capacity, the number of slots is unknown and needs to be estimated. While for the remaining sections, the production is coil oriented, there is no need to estimate the number of slots since the number of slots is equal to the number of coils at each section, which is given. More details about the estimation of the number of slots are given in Section 3.4.

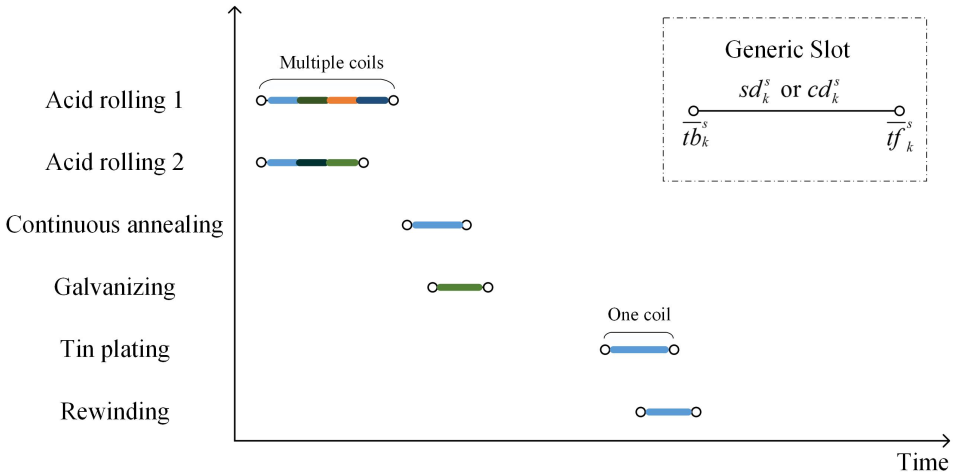

Fig. 3 Representation of time slots for each processing stage

2.1 Objective function

The objective function of this integrated scheduling problem includes three items. The first one is sequence-dependent changeover cost. The goal is to minimize the costs caused by sharp variation in characteristics of adjacent processed coils and to implicitly ensure production smoothness. The second item is the cost of rolling facilities (rollers), which aims at increasing the utilization of rollers to reduce production costs. The last item is the cost of electricity consumption by acid rolling production. The objective of the model is to minimize the total production costs. These cost items are the typical economic targets in real production. We only consider profit based objective in this paper, but other related objectives can also be defined in the objective function.

$$obj = m1 \cdot \sum_{s \in S/AC} \sum_{i \in N_s} \sum_{j \in N_s} \sum_{k \in T_s} F^s_{ij} z^s_{i,j,k} + m2 \cdot \sum_{s \in AC} \sum_{k \in T_s} c_{roller} \cdot r_{s,k} + m3 \cdot \sum_{tp \in TP} ce^{buy}_{tp} \cdot p_{tp} \quad (1)$$

2.2 Acid rolling related constraints

Since the production mode in the acid rolling section is different to other sections, the constraints related to acid rolling production are listed individually. The production in the acid rolling section is batch-oriented and one time slot corresponds to one batch which may include multiple coils. The number of batches, the schedule of batches and the compositions of each batch are determined in the model, but the coil sequencing within a given batch is not considered because in practice it can be easily determined by the schedulers. Following are the constraints that apply to the production in the acid rolling section.

- Assignment and batching related constraints:

Every coil i of set $N^{'}_{ac}$ should be exactly assigned to one batch (slot) k of one unit s in the acid rolling section.

$$\sum_{s \in AC} \sum_{k \in T_s} y_{s,k,i} = 1, \forall i \in N^{'}_{ac} \quad (2)$$

As for each slot k at unit s, the batch capacity $nbatch_{s,k}$ equals to the number of coils that are assigned to slot k.

$$nbatch_{s,k} = \sum_{i \in N^{'}_{ac}} y_{s,k,i}, \forall s \in AC, k \in T_s \quad (3)$$

For time-slot based models with variable number of slots, there are redundant slots due to the over estimation of the number of slots. In this paper, the binary variable $r_{s,k}$=1 indicates the slot k at unit s is selected; otherwise the slot is idle. Normally, idle slots are located at the end of the slots sequence. In this paper, due to the characteristics of the problem under consideration, the idle slots are located at the beginning of the slot sequence as defined by the symmetry breaking constraints (4). Therefore, the corresponding timings of idle slots are enforced to be zero by constraints (4), (7) and (8), which satisfy the logic requirements of the disjunctions in the GDP constraints (10).

$$r_{s,k} \le r_{s,k+1}, \forall s \in AC, k \in T_s \setminus \{t_s\} \qquad (4)$$

As for the selected slot *k* of unit *s*, its batch capacity $nbatch_{s,k}$ is smaller than the maximum batch capacity $B_{\max}$. Otherwise, the batch capacity is equal to zero.

$$r_{s,k} \le nbatch_{s,k} \le nB_{\max} \cdot r_{s,k}, \forall s \in AC, k \in T_s \qquad (5)$$

- Timing constraints

As for serial batch production, the duration of a batch sd_k^s is equal to the summation of the processing times τ_i^s of the batched coils (see Fig. 3).

$$sd_k^s = \sum_{i \in N'_{ac}} \tau_i^s \cdot y_{s,k,i}, \forall s \in AC, k \in T_s \qquad (6)$$

For each slot *k* at unit *s*, the relationships of beginning time $\overline{tb}_k^s$ and completion time $\overline{tf}_k^s$ are described in constraints (7).

$$\overline{tf}_k^s = \overline{tb}_k^s + sd_k^s, \forall s \in AC, k \in T_s \qquad (7)$$

There is a constant setup time $setup^s$ between adjacent batches, given by equation (8). Therefore, the beginning time of a slot is greater than or equal to its predecessor's completion time plus a setup time, as long as its predecessor is a selected slot. There is no setup time between any two idle slots.

$$\overline{tf}_k^s + setup^s \cdot r_{s,k} \le \overline{tb}_{k+1}^s, \forall s \in AC, k \in T_s \setminus \{t_s\} \qquad (8)$$

It is assumed that the completion time of a coil $\widehat{tf}_i^{s'}$ is equal to its batch completion time as shown in constraints (9), which introduce non-linearity in the model. The non-linearity is reformulated as a set of linear constraints in Section 4 since it introduces the product of the continuous variable $\overline{tf}_k^s$ and binary variable $y_{s,k,i}$.

$$\widehat{tf}_i^{s'} = \sum_{s \in AC} \sum_{k \in T_s} \overline{tf}_k^s \cdot y_{s,k,i}, i \in N_{ac}^{'}, s' \in AC \tag{9}$$

- Electricity consumption constraints

Computing the actual electricity consumption, accounting for the interactions between a production batch and the varying electricity pricing periods is the most challenging part of the model. We extend the method by Castro et al. (2014), who have represented the possible locations of a slot over a constant electricity pricing period as a GDP formulation.

Constraints (10) are the updated GDP formulations representing the interactions. Similar, but not the same as in (Castro et al., 2014), there are seven interaction possibilities of a slot (*s,k*) with respect of a constant electricity pricing period *tp* in this paper as shown in Fig 4. Constraints (10) are the updated GDP formulations. Any idle slot, indicated by the negation of the Boolean variable $\neg R_{s,k}$, has no interaction with electricity pricing period. As mentioned before, all related timings of idle slots are enforced to be equal to zero. These idle slots will not take part in the computation of energy consumption in the reformulations of constraints (10), which can speed up the convergence of the model. For any selected slot indicated by the Boolean variable $R_{s,k}$, its location over electricity pricing period falls into the remaining six possibilities, which are indicated by Boolean variables $A_{s,k,tp}$ to $F_{s,k,tp}$ in constraints (10). The time factor variable $\Delta T_{s,k,tp}$ represents time fractions of a selected slot in each pricing period. For example, for any selected slot that falls into case *A* (see Fig. 4), its beginning time is greater than or equal to the lower bound of time period *tp*, and its completion time is less than or equal to the upper bound of time period *tp,* which means that the duration of the slot is entirely contained by period *tp*. Therefore, for case *A*, the fraction $\Delta T_{s,k,tp}$ is equal to the duration of slot *s*. As for case *B*, the beginning time of slot *s* is less than or equal to the lower bound of time period *tp*, and the completion time of slot *s* is between the lower bound and upper bound of the time period *tp*. In this case, the slot *s* is partially located at the time period *tp*-1 and *tp*, and the fraction $\Delta T_{s,k,tp}$ is equal to the completion time of slot *s* minus the lower bound of time period *tp* as shown in Fig 4. All the remaining cases can be formulated in a similar way. Note that in constraints (10) a selected slot can satisfy the conditions for more than one cases, e.g. when the beginning time of slot *s* is equal to the lower bound of time period *tp* and the completion time of slot *s* is equal to the upper bound of time period *tp*, slot *s* can satisfy both the conditions of cases of *A* and *B*. This overlap has no effect on the final result of the model, since the GDP model is based on exclusive disjunctions, which means that each slot can fall into only one of the cases in the disjunctions, and the final result will satisfy the real timing logic.

$$\begin{bmatrix} R_{s,k} \\ \begin{bmatrix} A_{s,k,tp} \\ \overline{tb}_k^s \geq cp_{tp}^L \\ \overline{tf}_k^s \leq cp_{tp}^U \\ \Delta T_{s,k,tp} = sd_k^s \end{bmatrix} \veebar \begin{bmatrix} B_{s,k,tp} \\ \overline{tb}_k^s \leq cp_{tp}^L \\ \overline{tf}_k^s \geq cp_{tp}^L \\ \overline{tf}_k^s \leq cp_{tp}^U \\ \Delta T_{s,k,tp} = \overline{tf}_k^s - cp_{tp}^L \end{bmatrix} \veebar \begin{bmatrix} C_{s,k,tp} \\ \overline{tb}_k^s \geq cp_{tp}^L \\ \overline{tb}_k^s \leq cp_{tp}^U \\ \overline{tf}_k^s \geq cp_{tp}^U \\ \Delta T_{s,k,tp} = cp_{tp}^U - \overline{tb}_k^s \end{bmatrix} \\ \veebar \begin{bmatrix} D_{s,k,tp} \\ \overline{tb}_k^s \leq cp_{tp}^L \\ \overline{tf}_k^s \geq cp_{tp}^U \\ \Delta T_{s,k,tp} = cp_{tp}^U - cp_{tp}^L \end{bmatrix} \veebar \begin{bmatrix} E_{s,k,tp} \\ \overline{tb}_k^s \leq cp_{tp}^L \\ \overline{tf}_k^s \leq cp_{tp}^L \\ \Delta T_{s,k,tp} = 0 \end{bmatrix} \veebar \begin{bmatrix} F_{s,k,tp} \\ \overline{tb}_k^s \geq cp_{tp}^U \\ \overline{tf}_k^s \geq cp_{tp}^U \\ \Delta T_{s,k,tp} = 0 \end{bmatrix}, \forall tp \in TP \end{bmatrix} \veebar \begin{bmatrix} \neg R_{s,k} \\ \Delta T_{s,k,tp} = 0 \\ \overline{tb}_k^s = 0 \\ \overline{tf}_k^s = 0 \end{bmatrix}, \forall s \in AC, k \in T_s \tag{10}$$

$$R_{s,k} \veebar \neg R_{s,k}, \forall s \in AC, k \in T_s$$

$$R_{s,k} \Leftrightarrow A_{s,k,tp} \veebar B_{s,k,tp} \veebar C_{s,k,tp} \veebar D_{s,k,tp} \veebar E_{s,k,tp} \veebar F_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP$$

$$R_{s,k}, A_{s,k,tp}, B_{s,k,tp}, C_{s,k,tp}, D_{s,k,tp}, E_{s,k,tp}, F_{s,k,tp} \in \{True, False\}$$

Equation (11) calculates the total amount of electricity that is consumed during time period *tp*.

$$p_{tp} = \sum_{s \in AC} \sum_{k \in T_s} pwh_s \cdot \Delta T_{s,k,tp},\ tp \in TP \qquad (11)$$

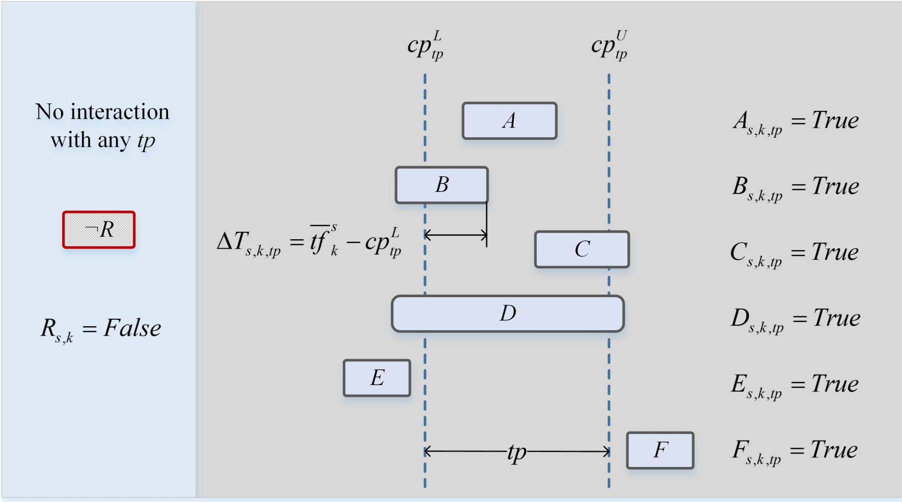

Fig. 4 Possible interactions of a slot with a constant time period

The summation of the time factors over all pricing periods is equal to the duration of the slot, given by constraints (12). It is a constraint that is obtained from the production point of view. Although redundant, it can help to decrease the integrality gap. An observation obtained from the computational result is that when solving the model, the binary variables $A_{s,k,tp}$ to $D_{s,k,tp}$ in equation (51) are relaxed as continuous variables in the range of [0, 1], and each term in equation (51) takes a value of zero. Therefore, the value of variable $\Delta T_{s,k,tp}$ is zero. This will result in poor relaxation of the model and increase the difficulty in computation. Constraints (12) ensure that the summation of $\Delta T_{s,k,tp}$ is equal to the duration of slot (s,k), and therefore the fraction variables $\Delta T_{s,k,tp}$ will not be all zero. Hence, constraints (12) help to improve the relaxation and speed up the computation of the model.

$$sd_{s,k} = \sum_{tp \in TP} \Delta T_{s,k,tp}, \forall s \in AC, k \in T_s \qquad (12)$$

2.3 General constraints

Following are some general constraints that apply to the processing tasks in the sections after the acid rolling section (see Fig. 3).

- Allocation constraints

Each coil should be exactly assigned to one slot of a corresponding unit.

$$\sum_{k \in T_s} y_{s,k,i} = 1, \ \forall s \in S \setminus AC, i \in N_s \tag{13}$$

Each slot should be exactly assigned to one coil.

$$\sum_{i \in N_s} y_{s,k,i} = 1, \ \forall s \in S \setminus AC, k \in T_s \tag{14}$$

- Timing constraints

In this paper, we need to calculate the beginning and completion times of both slots ($\overline{tb}_k^s$ and $\overline{tf}_k^s$) and coils ($\widehat{tb}_i^s$ and $\widehat{tf}_i^s$). To link up the timings of slots with the timings of coils, two set of disaggregated variables $tb_{i,k}^s$ and $tf_{i,k}^s$ are introduced in the model. $tb_{i,k}^s$ and $tf_{i,k}^s$ are the beginning time and completion time of coil i in slot k at unit s, respectively. As shown in constraints (15)-(16), $tb_{i,k}^s$ and $tf_{i,k}^s$ will make sense only when coil i is exactly assigned to slot k at unit s; otherwise, they will be equal to zero due to constraints (13) and (14).

$$tb_{i,k}^s \le UB_s \cdot y_{s,k,i}, \ \forall s \in S \setminus AC, k \in T_s, i \in N_s \tag{15}$$

$$tf_{i,k}^s \le UB_s \cdot y_{s,k,i}, \ \forall s \in S \setminus AC, k \in T_s, i \in N_s \tag{16}$$

As for slot k of unit s that is assigned to coil i, the slot duration $cd_{i,k}^s$ is equal to the processing time of i; otherwise, $cd_{i,k}^s$ is zero.

$$cd_{i,k}^s = \tau_i^s \cdot y_{s,k,i}, \forall s \in S \setminus AC, k \in T_s, i \in N_s \tag{17}$$

Equation (18) states the relationships of the disaggregated variables $tb_{i,k}^s$ and $tf_{i,k}^s$.

$$tf_{i,k}^s = tb_{i,k}^s + cd_{i,k}^s, \forall s \in S / AC, k \in T_s, i \in N_s \tag{18}$$

Constraints (19) and (20) are to calculate the beginning and completion time of slot k at unit s, respectively.

$$\overline{tb}_k^s = \sum_{i \in N_s} tb_{i,k}^s, \forall s \in S \setminus AC, k \in T_s \tag{19}$$

$$\overline{tf}_k^s = \sum_{i \in N_s} tf_{i,k}^s, \forall s \in S \setminus AC, k \in T_s \tag{20}$$

Constraints (21) state that there is no crossover between adjacent slots.

$$\overline{tf}_k^s \le \overline{tb}_{k+1}^s, \forall s \in S \setminus AC, k \in T_s \setminus \{t_s\}, i \in N_s \tag{21}$$

Beginning and completion time of coils are calculated by constraints (22) and (23).

$$\widehat{tb}_i^s = \sum_{k \in T_s} tb_{i,k}^s, \quad \forall s \in S \setminus AC, i \in N_s \tag{22}$$

$$\widehat{tf}_i^s = \sum_{k \in T_s} tf_{i,k}^s, \quad \forall s \in S \setminus AC, i \in N_s \tag{23}$$

Constraints (24) ensure that any coil can begin processing at downstream section only when it has been finished at upstream section.

$$\widehat{tf}_i^s \le \widehat{tb}_i^{s'}, \forall s \in S', s' \in L_s, i \in N_{s'} \tag{24}$$

- Immediate precedence constraints

In this paper, we apply the compact constraints proposed by Erdirik-Dogan and Grossmann (2008) to account for immediate precedence relationships of coils. For any coil i that is located at slot k of unit s, as long as k is not the last one in the slot sequence, there must be a coil j that is located at slot k+1. Similarly, for any coil j that is located at slot k+1 of unit s, there must be a coil i that is located at slot k of unit s.

$$y_{s,k,i} = \sum_{j \in N_s} z_{i,j,k}^s, \forall s \in S \setminus AC, i \in N_s, k \in T_s \setminus \{t_s\} \tag{25}$$

$$y_{s,k+1,j} = \sum_{i \in N_s} z_{i,j,k}^s, \forall s \in S \setminus AC, j \in N_s, k \in T_s \setminus \{t_s\} \tag{26}$$

2.4 Slot estimation and variable bounding

- Estimation of the number of slots

For continuous time models with slot representation, one of the big issues is the estimation of the number of slots (Floudas and Lin, 2004). On one hand, under-estimation of slots will lead to suboptimal or even infeasibility of the model; on the other hand, over-estimation of slots will increase the difficulty in solving model. So it is very important to make a proper estimation of the number of slots. As mentioned before, only the slots that are associated with acid rolling section need to be estimated. In this paper, we use a simple rule to estimate the number of slots, as given by equation (27). The number of slots sn is equal to NB_{least}, which is the minimum number of batches that can process all the ac coils at a single unit of acid rolling. NB_{least} is calculated as equation (27). Therefore, even for the extreme case where all coils are processed at the same unit, it also guarantees that the number of slots is enough to process all the coils and the production is feasible. For general cases where coils are processed at both units, the slots are also adequate and provide certain flexibility. Although we cannot guarantee that the proposed rule to estimate the number of slots will lead to the optimal solution, it still makes sense for real production.

$$sn = NB_{least} = \left\lceil \frac{ac}{nB_{\max}} \right\rceil \qquad (27)$$

- Variable bounding

The timing related constraints such as (15)-(16), (33), (43), (45), (48)-(49) need information from upper bounds of the timing variables. As mentioned before, we only observe the electricity consumption in one cycle of the electricity pricing. Therefore, the upper bound of timing variables corresponding to acid rolling section is set to be the same as the length of the cycle time $cp^U_{tp_{\max}}$. The upper bounds of timing variables in the remaining sections are calculated by equation (28).

$$UB_{s'} = UB_s + \sum_{i \in N_{s'}} \tau_i^{s'}, s \in S', s' \in L_s \qquad (28)$$

- Lower bound of total changeover costs

To obtain a tighter relaxation of the scheduling model, we specify a lower bound of the total sequence-dependent changeover costs in the objective function (1) by the underlying cost structure. The lower bound of the total changeover costs is calculated by equations (29)-(30).

$$mc_i^s = \min_{j \neq i} \left\{ F_{i,j}^s \right\}, s \in S \setminus AC, i \in N_s \qquad (29)$$

$$\sum_{s \in S \setminus AC} \sum_{i \in N_s} \sum_{j \in N_s} \sum_{k \in T_s} F_{ij}^s z_{i,j,k}^s \geq BestC = \sum_{s \in S \setminus AC} \left(\sum_{i \in N_s} mc_i^s - \max_{i \in N_s} \left\{ mc_i^s \right\} \right) \qquad (30)$$

The sequence-dependent costs are represented by a two dimensional matrix as shown in Fig. 5. We can see that each coil i has a potential minimum changeover cost mc_i^s with other coils, which is highlighted in red color in Fig.5. In the best case all the coils can be processed adjacently with the coil that has the minimum changeover cost. Let $BestC$ be the total changeover costs for the best case which is calculated by equation (30). Hence, the total changeover costs must be greater than or equal to $BestC$ as given by equation (30).

	i_1	i_2	i_3	i_4	$\cdots$	i_{n_s}
i_1	0	$F_{1,2}$	$F_{1,3}$	$F_{1,4}$	$\cdots$	F_{1,n_s}
i_2	$F_{2,1}$	0	$F_{2,3}$	$F_{2,4}$	$\cdots$	F_{2,n_s}
i_3	$F_{3,1}$	$F_{3,2}$	0	$F_{3,4}$	$\cdots$	F_{3,n_s}
i_4	$F_{4,1}$	$F_{4,2}$	$F_{4,3}$	0	$\cdots$	F_{4,n_s}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
i_{n_s}	$F_{n_s,1}$	$F_{n_s,2}$	$F_{n_s,3}$	$F_{n_s,4}$	$\cdots$	0

Fig. 5 A matrix of sequence-dependent changeover costs

3. Reformulation

The scheduling model given by equations (1)-(26), (29)-(30) includes non-linear constraints and GDP constraints. We reformulate the model as an MILP by the following steps.

- Linearization

To linearize equation (9), a set of auxiliary continuous variables $\lambda_{i,k}^{s}$ (Glover, 1975) are introduced in this paper. Let $\lambda_{i,k}^{s} = \overline{tf}_{k}^{s} \cdot y_{s,k,i}$, $s \in AC, k \in T_s, i \in N'_{ac}$, then we obtain equations (31)-(35). When binary variable $y_{s,k,i} = 0$, then equations (32)-(33) are activated and enforce $\lambda_{i,k}^{s} = 0$, which is consistent with $\overline{tf}_{k}^{s} \cdot y_{s,k,i} = 0$. Equations (34)-(35) are relaxed in this case. When $y_{s,k,i} = 1$, equations (34)-(35) are activated and enforce $\lambda_{i,k}^{s} = \overline{tf}_{k}^{s}$. Equations (32)-(33) are relaxed in this case. In this way, equation (9) is reformulated as a set of MILP constraints.

$$\widehat{tf}_{i}^{s'} = \sum_{s \in AC} \sum_{k \in T_s} \lambda_{i,k}^{s}, \forall i \in I, s' \in AC \qquad (31)$$

$$\lambda_{i,k}^{s} \geq 0 \qquad (32)$$

$$\lambda_{i,k}^{s} \leq UB_s \cdot y_{s,k,i} \qquad (33)$$

$$\lambda_{i,k}^{s} \geq \overline{tf}_{k}^{s} - UB_s \cdot (1 - y_{s,k,i}) \qquad (34)$$

$$\lambda_{i,k}^{s} \leq \overline{tf}_{k}^{s} \qquad (35)$$

- Hull reformulation of GDP constraints

The GDP constraints can be reformulated as MILP constraints in different ways, including the alternative big-M reformulation and the hull reformulation (Grossmann and Lee, 2003). The relaxed feasible region of the hull reformulation is proven to be at least as tight, if not tighter, than the one from big-M reformulation (Grossmann and Lee, 2003), although the size of hull reformulation is larger than the big-M reformulation. In this paper, the hull reformulation is adopted for reformulating the GDP constraints (10) of the model. In the reformulation, the logic variables $A_{s,k,tp}$ to $F_{s,k,tp}$ are replaced by the binary variables $a_{s,k,tp}$ to $f_{k,s,tp}$, respectively. To obtain the hull reformulation of equation (10), the timing variables of slot (*s*,*k*) in the disjunctions are disaggregated by equations (36)-(37).

$$\overline{tb}_k^s = \overline{tb}_{s,k,tp}^A + \overline{tb}_{s,k,tp}^B + \overline{tb}_{s,k,tp}^C + \overline{tb}_{s,k,tp}^D + \overline{tb}_{s,k,tp}^E + \overline{tb}_{s,k,tp}^F, \forall s \in AC, k \in T_s, tp \in TP \quad (36)$$

$$\overline{tf}_k^s = \overline{tf}_{s,k,tp}^A + \overline{tf}_{s,k,tp}^B + \overline{tf}_{s,k,tp}^C + \overline{tf}_{s,k,tp}^D + \overline{tf}_{s,k,tp}^E + \overline{tf}_{s,k,tp}^F, \forall s \in AC, k \in T_s, tp \in TP \quad (37)$$

The inequalities in the disjunctions are rewritten as equations (38)-(49) in terms of disaggregated variables and the new binary variables with respect of the logical expressions in the disjunctions. Take case *A* as an example which is given by equations (38)-(39). The disaggregated variables of beginning time and completion time of a slot are between the lower bound and upper bound of the time period *tp* when $a_{s,k,tp} = 1$; otherwise they are equal to zero. Other cases can be treated in a similar way.

$$cp_{tp}^L \cdot a_{s,k,tp} \le \overline{tb}_{s,k,tp}^A \le cp_{tp}^U \cdot a_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (38)$$

$$cp_{tp}^L \cdot a_{s,k,tp} \le \overline{tf}_{s,k,tp}^A \le cp_{tp}^U \cdot a_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (39)$$

$$0 \le \overline{tb}_{s,k,tp}^B \le cp_{tp}^L \cdot b_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (40)$$

$$cp_{tp}^L \cdot b_{s,k,tp} \le \overline{tf}_{s,k,tp}^B \le cp_{tp}^U \cdot b_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (41)$$

$$cp_{tp}^L \cdot c_{s,k,tp} \le \overline{tb}_{s,k,tp}^C \le cp_{tp}^U \cdot c_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (42)$$

$$cp_{tp}^U \cdot c_{s,k,tp} \le \overline{tf}_{s,k,tp}^C \le UB_s \cdot c_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (43)$$

$$0 \le \overline{tb}_{s,k,tp}^D \le cp_{tp}^L \cdot d_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (44)$$

$$cp_{tp}^U \cdot d_{s,k,tp} \le \overline{tf}_{s,k,tp}^D \le UB_s \cdot d_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (45)$$

$$0 \le \overline{tb}_{s,k,tp}^E \le cp_{tp}^L \cdot e_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (46)$$

$$0 \le \overline{tf}_{s,k,tp}^E \le cp_{tp}^L \cdot e_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (47)$$

$$cp_{tp}^U \cdot f_{s,k,tp} \le \overline{tb}_{s,k,tp}^F \le UB_s \cdot f_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \quad (48)$$

$$cp_{tp}^{U} \cdot f_{s,k,tp} \le \overline{tf}_{s,k,tp}^{F} \le UB_s \cdot f_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \qquad (49)$$

The logic proposition in (10) is transformed into a linear constraint as in (50), stating that exactly only one of the possible locations can happen when the corresponding slot is selected.

$$a_{s,k,tp} + b_{s,k,tp} + c_{s,k,tp} + d_{s,k,tp} + e_{s,k,tp} + f_{s,k,tp} = r_{s,k}, \forall s \in AC, k \in T_s, tp \in TP \qquad (50)$$

The time factor $\Delta T_{s,k,tp}$ is equal to the summation of the potential time fractions over all cases, as given by equation (51)

$$\begin{aligned} \Delta T_{s,k,tp} = {} & sd_k^s \cdot a_{s,k,tp} + (\overline{tf}_{s,k,tp}^{B} - cp_{tp}^{L} \cdot b_{s,k,tp}) + (cp_{tp}^{U} \cdot c_{s,k,tp} - \overline{tb}_{s,k,tp}^{C}) \\ & + (cp_{tp}^{U} - cp_{tp}^{L}) \cdot d_{s,k,tp}, \forall s \in AC, k \in T_s, tp \in TP \end{aligned} \qquad (51)$$

In summary, the proposed MILP model is given by equations (1)-(8), (11)-(26) and (29)-(51).

4. Case study and numerical experiments

To test the performance of the integrated scheduling model, numerical experiments are carried out on instances based on typical data of practical production. The models resulting from linearization and hull reformulations of the GDP constraints were implemented in GAMS 24.7.3 and solved with CPLEX 12.6.3 solver with four threads, and default options up to relative optimality tolerance = 0.0001 and 0.01, respectively. The hardware consisted on a laptop with an Intel i7-6500U (@2.50GHz) with 8GB of RAM running Windows 10 system.

A number of different-scaled instances are considered that resulted from combination of various numbers of coils in each section and the number of electricity pricing periods. The settings of coefficients and parameters are listed in Table 1. We tested the model on two scenarios of electricity pricing period with $tp_{\max} = 4$ and $tp_{\max} = 7$, respectively. The cycle time of electricity pricing is set to be 24 hours. The detailed data of electricity price is provided in Tables 2 and 3. Four groups of coils are used in the test problems with the number of coils being gradually increased.

The computational statistics of the 8 instances are given in Table 4. The size of the model has clearly a great impact on the solution time. With larger sizes of the instances, the solution time increases, especially for instance 7 and instance 8. The optimal solutions of all the instances were obtained within 3600 CPU times with a relative optimality tolerance of 0.01. Especially for instances 1 to instance 6, all optimal solutions are obtained within one minute, which can be viewed as a quick solution in practical production environment. Reducing the optimality tolerance to 0.0001, the optimal solutions for all the instances except instance 8 were obtained within 3600 CPU times. The integrality gaps in Table 4 show that the proposed model has a fairly tight relaxation.

As an illustrative example, Fig. 6 presents the Gantt chart of the optimal solution of Instance 2 obtained by CPLEX. In Instance 2, there are 4 time periods *tp*1 to *tp*4 in a cycle of electricity pricing. In the acid rolling section, there are two batches at unit 1 with 2 coils and 4 coils, respectively, and one batch at unit 2 with 4 coils. We can see that all these batches are processed in *tp*1 and *tp*2, the periods with relatively lower electricity price. These coils are then released to downstream sections and continue further processing. The completion time of the entire schedule is 47.7 hour.

Table 1. Coefficients and parameters data

$m1$	$m2$	$m3$	c_{roller} (10^3 CNY)	$B_{\max}$	$setup^1$ (hour)	$setup^2$ (hour)	pwh_1 (MW.h)	pwh_2 (MW.h)
0.3	0.2	0.5	50	4	0.8	1.2	15	17

Tabel 2 . Electricity price data of $tp_{\max} = 4$

tp	1	2	3	4
Load type	Valley-load	Flat-load	Peak-load	Flat-load
cp_{tp}^{L} (hour)	0	8	14	19
cp_{tp}^{U} (hour)	8	14	19	24
c_{tp}^{buy} (CNY/KW.h)	0.338	0.659	1.112	0.659

Tabel 3 . Electricity price data of $tp_{\max} = 7$

tp	1	2	3	4	5	6	7
Load type	Valley-load	Flat-load	Peak-load	Flat-load	Peak-load	Flat-load	Valley-load
cp_{tp}^{L} (hour)	0	6	8	11	18	21	22
cp_{tp}^{U} (hour)	6	8	11	18	21	22	24
c_{tp}^{buy} (CNY/KW.h)	0.338	0.659	1.112	0.659	1.112	0.659	0.338

Table 4. Computational statistics of the instances

Instance	Coil number					Period	Model size			Cost (10^3 CNY)	Integrality gap (%)	CPUs *
	C1	C2	C3	C4	C5		Discrete variable	Continuous variable	Constraints			
1	10	7	4	6	5	4	860	1,944	1,716	114.14	10.80	0.91 0.91
2	10	7	4	6	5	7	970	2,289	2,151	112.80	9.74	1.66 1.39
3	15	10	10	10	5	4	3,163	5,440	3,381	116.46	4.46	5.51 4.98
4	15	10	10	10	5	7	3,307	5,899	3,960	118.11	5.79	31.33 20.72
5	20	15	15	13	6	4	9,132	13,350	5,984	157.28	5.57	66.75 15.98
6	20	15	15	13	6	7	9,312	13,923	6,707	158.78	6.50	50.72 46.44
7	25	20	20	20	10	4	24,484	32,277	10,766	210.18	10.97	1511.28 1181.98
8	25	20	20	20	10	7	24,736	33,078	11,777	208.52	10.70	6172.25 1983.33

*: CPU seconds for 10^{-4} and 10^{-2} optimality tolerance, respectively.

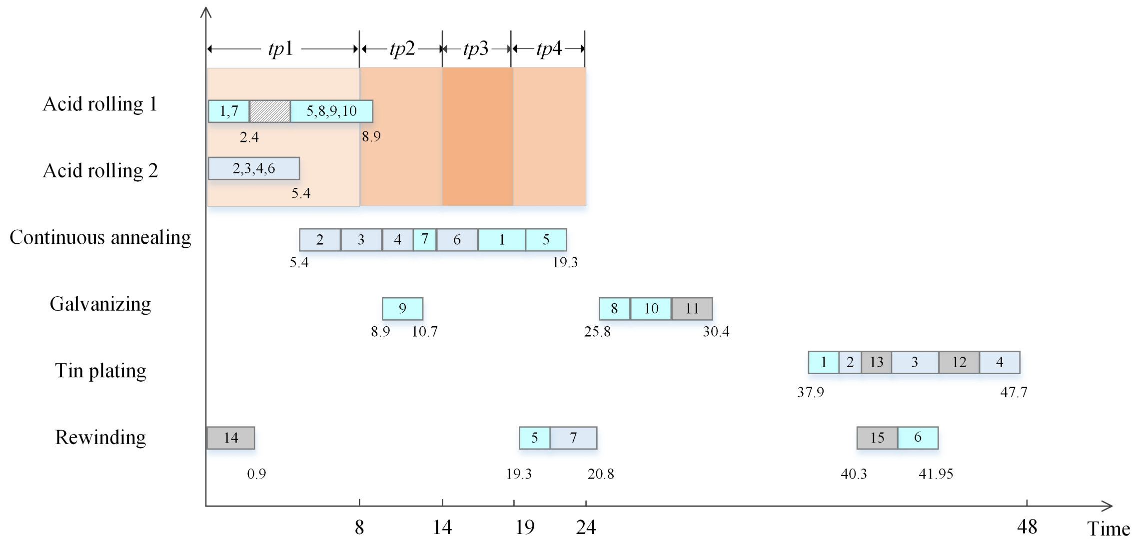

Fig. 6 Gantt chart of the optimal solution of Instance 2

5. Conclusion

In this paper, we have considered the production in the multiple finishing sections of steel industry, where the target is to obtain schedules of all these sections under various technical constraints. Especially for the energy intensive sections, electricity consumption is also optimized based on demand-side management techniques. The integrated scheduling problem of the rolling sector with consideration of energy consumption under time-of-use electricity prices was proposed to optimize the coordination of production and electricity consumption, and minimize the typical production costs.

Based on a continuous time representation, the MINLP/GDP model was formulated with nonlinear and disjunctive constraints, and then reformulated as a tight MILP model through hull reformulation and exact linearization. Also, a lower bound for the objective was specified for the model. The results of the numerical experiments demonstrated the effectiveness and tightness of the model.

The rolling sector is a multi-stage process with parallel sections at each stage. This is a typical structure in industrial production, so the proposed model can also be extended to the integrated scheduling in other similar process. As for future research, the production in the rolling sector is complicated with consumption of not only electricity but also other energy and resources such as coal and water. To achieve more efficient production, these resources should also be considered explicitly, which will make the scheduling problem more complex.

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