METHOD OF SEQUENTIAL SYNTHESIS OF OPTIMAL HEAT EXCHANGER NETWORKS WITH DIVISION OF MATERIAL AND HEAT STREAMS

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ABSTRACT

The paper introduces a new algorithmic method for the synthesis of the optimal heat exchanger network based on the assignment problem. The method considers the splitting of hot and cold streams, and uses a decomposition principle based on sequentially fixing the variables for the decomposition. The method is applied on three heat integration problems to show its performance and efficiency. It is also compared with stage-wise superstructure optimization based on mixed-integer nonlinear programming.

Keywords: synthesis of optimal heat exchanger networks, heat integration, assignment problem, superstructure, decomposition approach.

INTRODUCTION

The last decades have witnessed a number of new methods for solving the problem of optimal heat exchanger network (HEN) synthesis aimed at decreasing heat energy consumption through heat recovery by process streams [1]. These methods can be subdivided into three groups, including a thermodynamic approach, mathematical programming and stochastic optimization methods. In the thermodynamic approach [2, 3], the pinch based design method proposed by B. Linnhoff [4] has ben widely used. The economic issues of the problem, however, are not taken into account in this method. The mathematical programming methods, in their turn, convert the optimal heat exchanger network synthesis problem into a mixed-integer nonlinear programming problem [5-9], which is formulated through a superstructure optimization containing many possible options of heat exchanger network arrangements. Due to the large number of streams in the mathematical model of the heat exchanger network superstructure for real life industrial problems, this method has limited applications. Compared to the thermodynamic approach, these methods tend to give better results. However, they also have a number of disadvantages, such as multiple local optima, a large number of search variables, and computational expense due to the complex and combinatory character of the problem to be solved. These disadvantages are also typical of the integrated synthesis methods It is possible to partially avoid these disadvantages by using a sequential decomposition of the initial problem into subproblems. Papoulias and Grossmann [10] proposes to decompose this problem into three subproblems, including minimizing external hot and cold utility consumption, minimizing the number of heat exchangers, and finding the minimum capital cost. These problems can be solved by using a transshipment model [10, 11], or a transportation model [12]. The heuristic nature of this decomposition method, however, does not guarantee optimal solutions [5].

Stochastic methods, such as simulated annealing [13], cat swarm optimization [14], and genetic algorithms [15], are also used for solving HEN synthesis problems. However, taking into account that the optimization problem has large dimensionality, these methods are computationally very expensive and also cannot guarantee optimal solutions.

In paper [16], we considered a single stage HEN synthesis problem. The solutions of this problem obtained from the reference samples were compared to the results of their pinch analysis. It was shown that the single stage optimal heat integration does not allow using potential energy recovery at its full capacity due to the limitation imposed by a single heat exchange between the hot and cold streams. Paper [17] shows that energy recovery can be improved due to a multistage direct or reverse stream heat exchange between the hot and cold streams. This approach, however, may not yield the optimal solution due to a decrease in the heat exchange driving force from stage to stage of the process. Therefore, the present paper considers a different approach to extend the optimal solution search area, that is, the splitting of hot and cold streams.

THEORETICAL BACKGROUND

Assume we are given a process system with M^{h} "hot" streams S_i^{h}, (i=1,…,M^{h}) and the M^{c} "cold" streams S_j^{c}, (j=1,…,M^{c}), with their flowrates F_i^{h}, F_j^{c}, initial temperatures $T_i^{\mathrm{h,in}}$, (i=1,…,M^{h}), $T_j^{\mathrm{c,in}}$, (j=1,…,M^{c}), and specific heat capacities at constant pressure c_i^{h}, c_j^{c}, correspondingly. The heat exchanger network synthesis problem [18] consists of finding the structure of a heat exchanger network involving heat exchangers for the hot and cold streams, and, if needed coolers for the hot streams and heaters for the cold streams. Furthermore, the heat exchange areas A^{he}, the areas for heaters A^{reb} and the areas of coolers A^{col}, must be determined together with the consumption of cooling and heating utilities, F^{cu} and F^{hu} , respectively, so as to satisfy the outlet temperatures of the hot and cold streams,

$$T_i^{\mathrm{h,out}}, (i=1,\dots,M^{\mathrm{h}}), \quad T_j^{\mathrm{c,out}}, (j=1,\dots,M^{\mathrm{c}}),$$

that in turn define the heat content ΔQ_i^{h} of the hot streams, and the heat content ΔQ_j^{c} of the cold streams. The goal is to synthesize the network that minimizes the sum of annualized capital and operating costs.

The synthesis problem is a non-linear combinatorial optimization problem, which is hard to solve due to the presence of both continuous (heat loads in exchangers, consumption of utilities, areas, etc.) and discrete variables (presence or absence of potential heat exchangers as well as their interconnections).

We consider the problem of optimal heat exchanger network synthesis that takes into account a possibility of splitting the process streams. At the inlet of the hot (cold) streams, we place flow splitters or dividers D_i^{h} and D_j^{c}. They split the i th hot stream S_i^{h} into L_i^{h} of elementary streams $S_{l_i}^{\mathrm{h}}$ (i=1,…,M^{h} , $l_i = 1,...,L_i^h$) and the j th cold stream S_j^c into L_j^c of elementary streams $S_{l_j}^c$ (j=1,…,M^{c}, $l_j = 1,...,L_j^{\mathrm{c}}$), correspondingly. Every hot and cold stream of the system can have a different number of dividers. For the j^{th} cold stream, the largest number of dividers can be defined by the number of hot streams with temperatures higher than the j^{th} cold stream temperature by a value higher than the minimum permissible temperature difference. Similarly, for the i^{th} hot stream, the largest number of dividers can be defined by the number of cold streams with initial temperatures lower than the set value of the minimum permissible temperature difference. The flowrates $F_{l_i}^{\mathrm{h}}, F_{l_j}^c$ of streams $S_{l_i}^{\mathrm{h}}$ and $S_{l_j}^c$ will be equal to $\beta_{l_i}^{\mathrm{h}} F_{l_i}^{\mathrm{h}}$, $\beta_{l_j}^{\mathrm{c}} F_{l_j}^{\mathrm{c}}$,correspondingly, where $\beta_{l_i}^{\mathrm{h}}, \beta_{l_j}^{\mathrm{c}}$ are the split fractions satisfying the conditions:

$$\sum_{l_i=1}^{L_i^h} \beta_{l_i}^h = 1, \quad i = 1,...,M^h, \sum_{l_i=1}^{L_i^c} \beta_{l_j}^c = 1, \quad j = 1,...,M^c. \tag{1}$$

Clearly the stream temperatures $S_{l_i}^{\mathrm{h}}$ and $S_{l_j}^c$ ($l_i = 1,...,L_i^h, l_j = 1,...,L_j^{\mathrm{c}}$) are equal to the inlet stream temperatures S_i^h and S_j^c, correspondingly. After the heat exchanger network, the streams obtained by dividing the i th hot and the j^{th} cold streams are mixed in mixers G_i^h и G_j^c, correspondingly. Using the equation for the mixer heat and material balance, we obtain

$$\Delta Q_i^h = \sum_{l_i=1}^{L_i^h} \Delta Q_{l_i}^h, \quad F_i^{h,out} = \sum_{l_i=1}^{L_i^h} F_{l_i}^h, \quad i = 1,...,M^{\mathrm{h}}, l_i = 1,...,L_i^h \tag{2}$$

$$\Delta Q_j^c = \sum_{l_j=1}^{L_j^c} \Delta Q_{l_j}^c, \quad F_j^{c,out} = \sum_{l_j=1}^{L_j^c} F_{l_j}^c. \; j = 1,...,M^{\mathrm{c}}, l_j = 1,...,L_j^{\mathrm{c}} \tag{3}$$

We next enumerate the elementary hot and cold streams obtained after dividing the inlet streams. The numbers $M_{\mathrm{S}}^{\mathrm{h}}, M_{\mathrm{S}}^{\mathrm{c}}$, of the obtained elementary hot and cold streams will be equal, correspondingly, to:

$$M_{\mathrm{S}}^{\mathrm{h}} = \sum_{i=1}^{M^{\mathrm{h}}} L_i^{\mathrm{h}},\; M_{\mathrm{S}}^{\mathrm{c}} = \sum_{j=1}^{M^{\mathrm{c}}} L_j^{\mathrm{c}}. \quad (4)$$

We reduce the optimal heat exchanger network synthesis problem to the optimization problem of a certain global flowsheet (superstructure). All the possible heat exchanger network configurations are particular cases of a global flowsheet. Therefore, a global heat exchanger network optimization results in the optimal heat exchanger network. We construct a global flowsheet characterized by the following parameters: 1) for every total sum of the elementary l_i^{th} hot and l_j^{th} cold streams, a heat recovery exchanger can be installed; 2) for additional heating or cooling of the process streams, the HEN outlet allows for installing additional coolers and heaters (which may be eliminated in the course of the problem solution); 3) any hot and cold elementary streams can exchange heat (in a heat recovery exchanger) only one single time. The structure of the described global flowsheet is given in Fig.1.

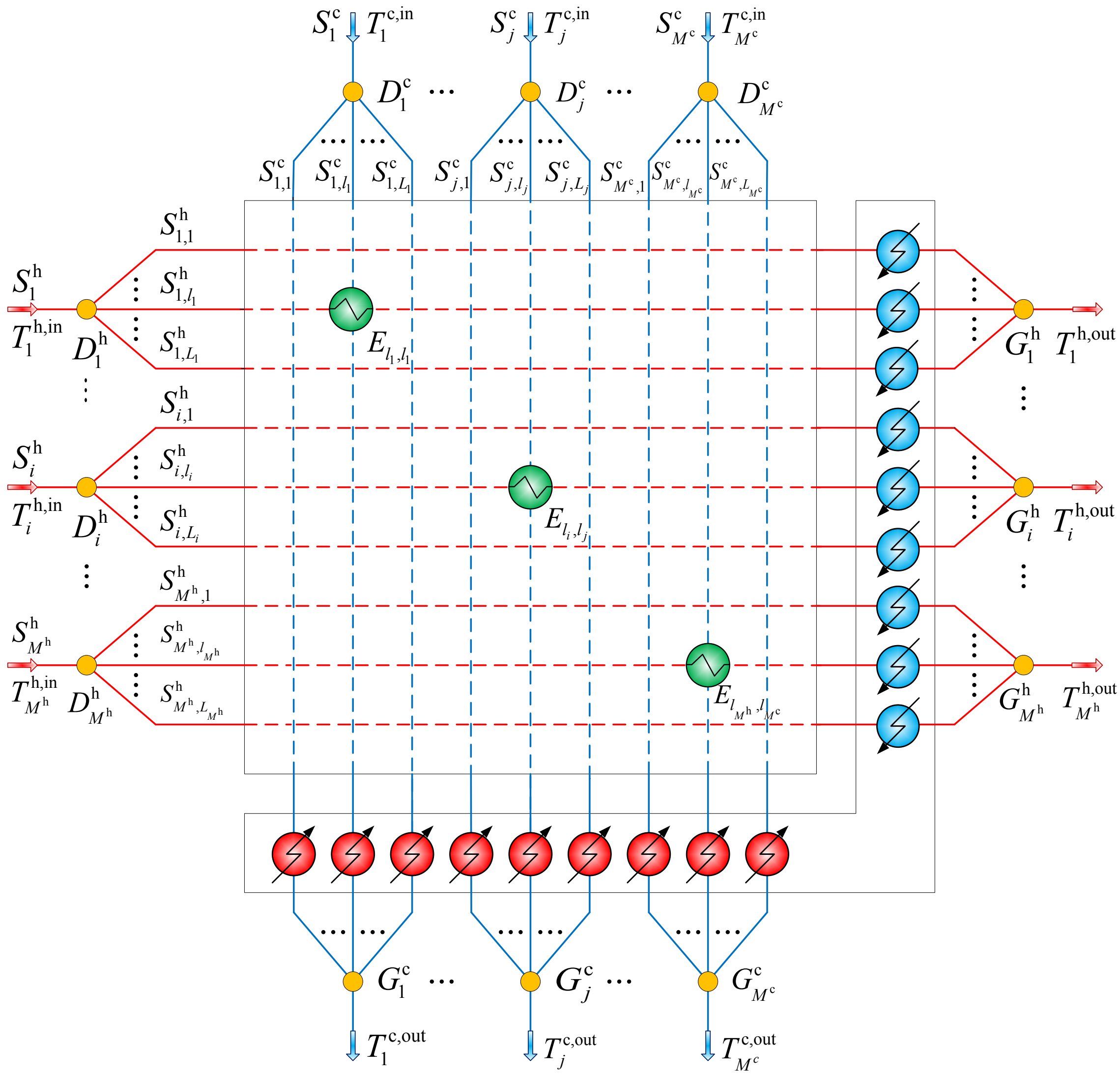

Figure 1 – Global Flowsheet of Heat Exchanger Network with Division of Material Streams

A direct solution of the set problem on the global flowsheet may require significant computational effort and may lead to local minima. Therefore, we propose to solve the problem by decomposing it. The method to solve the global flowsheet optimization problem is based on the principle of fixing the intermediate variables [19]. These variables are selected so as to eliminate the interactions between certain subsystems in a complex system when the variables are fixed. Thus, in case of known values for the fixed variables, the structures of certain subsystems in a complex system can be optimized independently.

Based on the principle of fixing the variables, the following iteration procedure can be developed. Two steps are taken at each iteration. During the first step, in case of the known values for the fixed variables, optimal structures for certain subsystems are determined. During the second step, the entire HEN is optimized using the determined structures connected by material and heat streams, whereby the values of the fixed variables are specified. Using the new values of the fixed variables, optimal structures for all subsystems are determined, and the iteration procedure starts again.

Any HEN with a possible division of process streams can be obtained using such a global flowsheet. We define $T_{l_i}^{\mathrm{h,in},(k)}, T_{l_i}^{\mathrm{h,out},(k)}$ as the temperatures of the l_i^{th} hot elementary stream at the HEN inlet and outlet, while indicating as $T_{l_j}^{\mathrm{c,in},(k)}, T_{l_j}^{\mathrm{c,out},(k)}$ the temperatures of the l_j^{th} cold stream at the HEN inlet and outlet at the k^{th} iteration. Clearly, $T_{l_i}^{\mathrm{h,in},(k)} = T_i^{\mathrm{h,in}}, T_{l_j}^{\mathrm{c,in},(k)} = T_j^{\mathrm{c,in,}}$ at all values of k. We consider the process stream division coefficients as fixed variables. At the k^{th} iteration, their values will be equal to $\beta_{l_i}^{\mathrm{h},(k)}, \beta_{l_j}^{\mathrm{c},(k)}, l_i = 1,...,(L_i^{\mathrm{h}} - 1), l_j = 1,...,(L_j^{\mathrm{c}} - 1)$, obtained at the $(k\text{-}1)^{\mathrm{th}}$ iteration. At specified values of the fixed variables, we can determine the optimal HEN structure with a possible division of process streams and operating modes of heat exchangers. In order to do so, we use the HEN synthesis method without dividing the process streams as given in papers [16, 20]. The method is based on structural decomposition of the system tested at the known values of $\Delta Q_{l_j}^{c}, \Delta Q_{l_i}^{h}$. The method implies determining the optimal economic values for the heat exchange in every pair of hot and cold streams, whereas this matrix is used to find the optimal total sum of coupled elementary streams to determine the structure and operation mode of the synthesized HEN.

For every total sum of the l_i^{th} hot and the l_j^{th} cold streams, we determine the optimal heat exchange unit structure and operating modes of heat exchangers within this unit. We define a canonical form called the superstructure of HEN elementary units. An example of such a superstructure is shown in Figure 2. We define this superstructure as heat recovery exchanger E_{l_i,l_j}, where the l_i^{th} hot and the l_j^{th} cold streams can exchange their heat, the cooler C_{l_i,l_j}, installed at the l_i^{th} hot stream at the recovery heat exchanger outlet, and the heater B_{l_i,l_j}, installed at the l_j^{th} cold stream at the heat exchanger outlet. Given the values of variables $\beta_{l_i}^{\rm h}$, $\beta_{l_j}^{\rm c}$ we can determine the optimal structure of the HEN elementary units, and operating modes of the units within, so as to minimize the total costs $f_{l_i,l_j}(\beta_{l_i}^{\rm h},\beta_{l_j}^{\rm c})$.Using the defined superstructure, the problem can be formalized as a nonlinear programming problem with equality constraints for the heat exchanger, cooler and heater mathematical models, and inequalities for the heat exchange feasibility. We choose total capital and operating costs as optimization criterion. The general problem definition for designing optimal HEN elementary units is given below in (5)-(13).

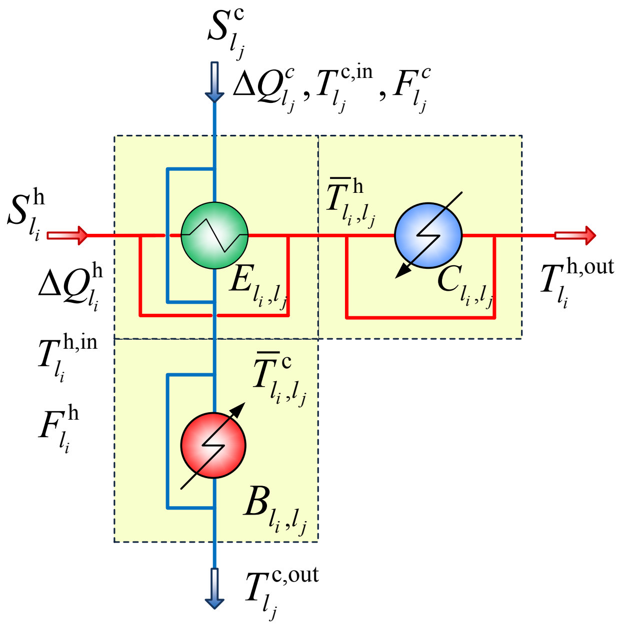

Figure 2 – Superstructure of HEN Elementary Units

$$f_{l_i,l_j}^{\rm opt} = \min_{A_{l_i,l_j}^{\rm he}, A_{l_i,l_j}^{\rm col}, A_{l_i,l_j}^{\rm reb}, F_{l_i,l_j}^{\rm cu}, F_{l_i,l_j}^{\rm hu}} f_{l_i,l_j}(A_{l_i,l_j}^{\rm he}, A_{l_i,l_j}^{\rm col}, A_{l_i,l_j}^{\rm reb}, F_{l_i,l_j}^{\rm cu}, F_{l_i,l_j}^{\rm hu}), \tag{5}$$

$$\varphi^{\rm he}(\bar{T}_{l_i,l_j}^{\rm h}, T_{l_i}^{\rm h,in}, T_{l_j}^{\rm c,in}, A_{l_i,l_j}^{\rm he}, F_{l_i}^{\rm h}, F_{l_j}^{\rm c}, U_{l_i,l_j}^{\rm he}) = 0, \tag{6}$$

$$\varphi^{col}(\bar{T}_{l_i,l_j}^{\rm h}, T_{l_i}^{\rm h,out}, T_{l_i,l_j}^{\rm cu,in}, T_{l_i,l_j}^{\rm cu,out}, A_{l_i,l_j}^{\rm col}, F_{l_i}^{\rm h}, F_{l_i,l_j}^{\rm cu}, U_{l_i,l_j}^{\rm col}) = 0, \tag{7}$$

$$\varphi^{\text{reb}}(\overline{T}^{\text{c}}_{l_i,l_j}, T^{\text{c,out}}_{l_j}, T^{\text{hu,in}}_{l_i,l_j}, T^{\text{hu,out}}_{l_i,l_j}, A^{\text{reb}}_{l_i,l_j}, F^{\text{c}}_{l_j}, F^{\text{hu}}_{l_i,l_j}, U^{\text{reb}}_{l_i,l_j}) = 0, \quad (8)$$

$$T^{\text{h,in}}_{l_i} = T^{\text{h,in}}_{i},\ T^{\text{c,in}}_{l_j} = T^{\text{c,in}}_{j}.,\ i = 1,\ldots,M^{\text{h}}, j = 1,\ldots,M^{\text{c}}, \quad (9)$$

$$l_i = 1,\ldots,L^{\text{h}}_i,\ l_j = 1,\ldots,L^{\text{c}}_j.$$

$$T^{\text{h,in}}_{l_i} - \overline{T}^{\text{c}}_{l_i,l_j} \ge \Delta T_{\min},\ \overline{T}^{\text{h}}_{l_i,l_j} - T^{\text{c,out}}_{l_j} \ge \Delta T_{\min}, \quad (10)$$

$$T^{\text{hu,out}}_{l_i,l_j} - \overline{T}^{\text{c}}_{l_i,l_j} \ge \Delta T_{\min},\ T^{\text{hu,in}}_{l_i,l_j} - T^{\text{c,out}}_{l_j} \ge \Delta T_{\min}, \quad (11)$$

$$T^{\text{h,out}}_{l_i} - T^{\text{cu,in}}_{l_i,l_j} \ge \Delta T_{\min},\ \overline{T}^{\text{h}}_{l_i,l_j} - T^{\text{cu,out}}_{l_i,l_j} \ge \Delta T_{\min}. \quad (12)$$

$$\Delta Q^{\text{he}}_{l_i,l_j} \ge 0,\ \Delta Q^{\text{col}}_{l_i,l_j} \ge 0,\ \Delta Q^{\text{reb}}_{l_i,l_j} \ge 0. \quad (13)$$

where f_{l_i,l_j} is the optimization criterion for HEN elementary units; mathematical models of the heat exchanger, cooler, and heater, correspondingly; (9) are the conditions imposed on the temperatures of the HEN inlet streams (the temperatures of the HEN inlet streams are to be kept to the set values); (10)-(13) are the limitations to the process driving force; $A^{\text{he}}_{l_i,l_j}, A^{\text{col}}_{l_i,l_j}, A^{\text{reb}}_{l_i,l_j}$ heat exchange surface areas of the recuperative heat exchanger, cooler and heater, correspondingly; $\Delta Q^{\text{he}}_{l_i,l_j}, \Delta Q^{\text{col}}_{l_i,l_j}, \Delta Q^{\text{reb}}_{l_i,l_j}$ thermal capacities of the recuperative heat exchanger, cooler and heater, correspondingly; $\Delta T_{\min}$ is the minimum allowed temperature difference. We next introduce a binary variable z_{l_i,l_j}, satisfying the condition:

$$z_{l_i,l_j} = \begin{cases} 1, \text{ heat exchange between the hot elementary flow } S^{\text{h}}_{l_i}, \\ \text{the cold elementary flow } S^{\text{c}}_{l_j}, \text{ in heat exchanger is arranged,} \\ \text{or the hot flow is cooled by a cooler, and the cold flow is heated by a heater;} \\ 0, \text{ both elementary flows } (S^{\text{h}}_{l_i}, S^{\text{c}}_{l_j}) \text{ exchange heat with other flows,} \\ \text{or one of the flows } (S^{\text{h}}_{l_i}, S^{\text{c}}_{l_j}) \text{ exchanges heat with another flow,} \\ \text{while the other flow is heated (cooled).} \end{cases}$$

Therefore, the optimal heat exchange structure arrangement problem can be reduced to the following linear integer programming problem. In an ideal case, at $M_S^h = M_S^c$, the problem of determining the optimal structure is reduced to the problem (14)–(15).

$$\min_{z_{l_i,l_j}} \sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h}\sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} f_{l_i,l_j}^{\mathrm{opt}} z_{l_i,l_j}, \quad z_{l_i,l_j} \in \{0,1\}, \tag{14}$$

$$\sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h} z_{l_i,l_j} = 1, \ \sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} z_{l_i,l_j} = 1. \tag{15}$$

$$i = 1,\ldots,M^{\mathrm{h}},\ j = 1,\ldots,M^{\mathrm{c}},$$

$$l_i = 1,\ldots,L_i^{\mathrm{h}},\ l_j^c = 1,\ldots,L_j^c.$$

The problem (14)–(15) is the well-known “assignment problem” [17], for which efficient solution methods are available.

Based on the above, we can propose the following iterative method for the optimal heat exchanger network synthesis. At the first step of each iteration, problem (14)–(15) is solved using the given values of the division coefficients for the process hot and cold streams $\beta_{l_i}^{\mathrm{h},(k)}, \beta_{l_j}^{\mathrm{c},(k)}$ obtained at the previous iteration. During the second step, the flowsheet optimization problem (16)-(23) is solved for the flowsheet structure obtained in the first step. Here, the values of the division coefficients for the hot and cold streams can be adjusted,

$$\min_{F_{l_i}^{\mathrm{c}}, F_{l_j}^{\mathrm{h}}, A_{l_i,l_j}^{\mathrm{he}}, A_{l_i,l_j}^{\mathrm{col}}, A_{l_i,l_j}^{\mathrm{reb}}, F_{l_i,l_j}^{\mathrm{cu}}, F_{l_i,l_j}^{\mathrm{hu}}} \Phi, \forall (l_i, l_j) \in Z \ , \forall (l_i, l_j) \in Z \tag{16}$$

$$\varphi^{\mathrm{he}}(\overline{T}_{l_i,l_j}^{\mathrm{h}}, T_{l_i}^{\mathrm{h,in}}, T_{l_j}^{\mathrm{c,in}}, A_{l_i,l_j}^{\mathrm{he}}, F_{l_i}^{\mathrm{h}}, F_{l_j}^{\mathrm{c}}, U_{l_i,l_j}^{\mathrm{he}}) = 0, \ \forall (l_i, l_j) \in Z \ , \tag{17}$$

$$\varphi^{col}(\overline{T}_{l_i,l_j}^{\mathrm{h}}, T_{l_i}^{\mathrm{h,out}}, T_{l_i,l_j}^{\mathrm{cu,in}}, T_{l_i,l_j}^{\mathrm{cu,out}}, A_{l_i,l_j}^{\mathrm{col}}, F_{l_i}^{\mathrm{h}}, F_{l_i,l_j}^{\mathrm{cu}}, U_{l_i,l_j}^{\mathrm{col}}) = 0, \ \forall (l_i, l_j) \in Z, \tag{18}$$

$$\varphi^{reb}(\overline{T}_{l_i,l_j}^{\mathrm{c}}, T_{l_j}^{\mathrm{c,out}}, T_{l_i,l_j}^{\mathrm{hu,in}}, T_{l_i,l_j}^{\mathrm{hu,out}}, A_{l_i,l_j}^{\mathrm{reb}}, F_{l_j}^{\mathrm{c}}, F_{l_i,l_j}^{\mathrm{hu}}, U_{l_i,l_j}^{\mathrm{reb}}) = 0, \ \forall (l_i, l_j) \in Z, \tag{19}$$

$$T_{l_i}^{\mathrm{h,in}} = T_i^{\mathrm{h,in}}, \ T_{l_j}^{\mathrm{c,in}} = T_j^{\mathrm{c,in}}, \tag{20}$$

$$T_{l_i}^{\mathrm{h,in}} - \overline{T}_{l_i,l_j}^{\mathrm{c}} \geq \Delta T_{\min}, \ \overline{T}_{l_i,l_j}^{\mathrm{h}} - T_{l_j}^{\mathrm{c,out}} \geq \Delta T_{\min}, \tag{21}$$

$$T_{l_i,l_j}^{\text{hu,out}} - \overline{T}_{l_i,l_j}^{\text{c}} \geq \Delta T_{\min},\ T_{l_i,l_j}^{\text{hu,in}} - T_{l_j}^{\text{c,out}} \geq \Delta T_{\min}, \tag{22}$$

$$T_{l_i}^{\text{h,out}} - T_{l_i,l_j}^{\text{cu,in}} \geq \Delta T_{\min},\ \overline{T}_{l_i,l_j}^{\text{h}} - T_{l_i,l_j}^{\text{cu,out}} \geq \Delta T_{\min}. \tag{23}$$

$$\Delta Q_{l_i,l_j}^{\text{he}} \geq 0,\ \Delta Q_{l_i,l_j}^{\text{col}} \geq 0,\ \Delta Q_{l_i,l_j}^{\text{reb}} \geq 0.$$

$$i = 1,...,M^{\text{h}},\ j = 1,...,M^{\text{c}},\ l_i = 1,...,L_i^{\text{h}},\ l_j = 1,...,L_j^{\text{c}}.$$

taking into account the equations for the heat and material balance of the mixer and divider (1)-(3), where Z is the total sum of the coupled elementary streams with their HEN elementary units belonging to the HEN structure at the current iteration; Φ is the total sum of the annualized capital costs in the heat recovery exchangers, together with the annualized capital and operating costs for the heaters and coolers.

We describe the components of the formulated problems (5)–(13) and (16)–(23). The optimization criterion (5) is determined as the function of the total annualized capital and operating costs for the cooler and heater and the reduced capital costs for the recuperative heat exchanger.

$$f_{l_i,l_j}^{(k)} = \left[\tilde{m}_1^{he} + \tilde{m}_2^{he}\left(A_{l_i,l_j}^{\text{he}}\right)^{\gamma^{he}}\right] + \left[\tilde{m}_1^{reb} + \tilde{m}_2^{reb}\left(A_{l_i,l_j}^{\text{reb}}\right)^{\gamma^{reb}} + \hat{m}^{reb}\Delta Q_{l_i,l_j}^{reb,(k)}\right] +$$
$$+\left[\tilde{m}_1^{col} + \tilde{m}_2^{col}\left(A_{l_i,l_j}^{\text{col}}\right)^{\gamma^{col}} + \hat{m}^{col}\Delta Q_{l_i,l_j}^{col,(k)}\right], \tag{24}$$

where $\tilde{m}_1^{\text{he}}, \tilde{m}_2^{\text{he}}, \tilde{m}_1^{\text{col}}, \tilde{m}_2^{\text{col}}, \tilde{m}_1^{\text{reb}}, \tilde{m}_2^{\text{reb}}$ are the price ratios for the recuperative heat exchanger, cooler, and heater, correspondingly, including the costs for the heat exchange equipment, its assembling and installation; $\gamma^{\text{he}}, \gamma^{\text{col}}, \gamma^{\text{reb}}$ are the correlation coefficients; $\hat{m}^{\text{col}}, \hat{m}^{\text{reb}}$ are the unit costs for the hot or cold heat carrier consumption;

The problems (5)–(13) and (16)–(23) can be simplified by the following assumptions:

1) The temperatures of the system hot and cold heat carrier outlet streams are known (therefore, the heat carrier consumption can be determined at the known thermal capacity);

2) The physical states of the heat exchange streams do not change.

Mathematical models based on calculations of the average logarithmic temperature difference are used to describe the heat exchangers, heaters and coolers. The following assumptions are introduced for the heat exchanger mathematical models:

1) The heat exchangers operate in counter current flow;

2) The hydrodynamic flow structures are characterized by an ideal displacement;

3) The total thermal resistance of the heat exchanger walls is negligible;

4) The average value of the specific heat capacity at constant pressure is used.

Therefore, the heat exchange areas can be determined from the heat transfer equation:

$$A^{\text{he}}_{l_i,l_j} = \frac{\Delta Q^{he}_{l_i,l_j}}{U^{he}_{l_i,l_j}\Delta t^{he}_{\ln}},\ A^{\text{he}}_{l_i,l_j} = \frac{\Delta Q^{reb}_{l_i,l_j}}{U^{reb}_{l_i,l_j}\Delta t^{reb}_{\ln}},\ A^{\text{col}}_{l_i,l_j} = \frac{\Delta Q^{col}_{l_i,l_j}}{U^{col}_{l_i,l_j}\Delta t^{col}_{\ln}}. \qquad (25)$$

The algorithm of the optimal HEN with the division of process streams is as follows:

Step 1. Set the initial approximations for the stream division coefficients $\beta^{\text{h},(0)}_{l_i}, \beta^{\text{c},(0)}_{l_j}, l_i = 1,...,(L^{\text{h}}_i - 1), l_j = 1,...,(L^{\text{c}}_j - 1)$.

$$\beta^{h,(0)}_{L^h_i} = 1 - \sum_{l_i=1}^{L^h_i - 1} \beta^{h,(0)}_{l_i},\ \ i = 1,...,M^h,\ \beta^{c,(0)}_{L^c_j} = 1 - \sum_{l_j=1}^{L^c_j - 1} \beta^{c,(0)}_{l_j},\ \ j = 1,...,M^c.$$

We assume k=1.

Step 2. Solve the $M^{\text{h}}_{\text{S}} \times M^{\text{c}}_{\text{S}}$ nonlinear mathematical programming problems (5)-(13) for every HEN elementary unit.

$$\min_{\Delta Q^{\text{he},(k)}_{l_i,l_j}} f^{(k)}_{l_i,l_j},$$

$$f_{l_i,l_j}^{(k)}=\left[\tilde{m}_1^{he}+\tilde{m}_2^{he}\left(\frac{\Delta Q_{l_i,l_j}^{he,(k)}}{U_{l_i,l_j}^{he}\Delta t_{\ln}^{he,(k)}}\right)^{\gamma^{he}}\right]+\left[\tilde{m}_1^{reb}+\tilde{m}_2^{reb}\left(\frac{\Delta Q_{l_i,l_j}^{reb,(k)}}{U_{l_i,l_j}^{reb}\Delta t_{\ln}^{reb,(k)}}\right)^{\gamma^{reb}}+\hat{m}^{reb}\Delta Q_{l_i,l_j}^{reb,(k)}\right]+$$
$$+\left[\tilde{m}_1^{col}+\tilde{m}_2^{col}\left(\frac{\Delta Q_{l_i,l_j}^{col,(k)}}{U_{l_i,l_j}^{col}\Delta t_{\ln}^{col,(k)}}\right)^{\gamma^{col}}+\hat{m}^{col}\Delta Q_{l_i,l_j}^{col,(k)}\right] \tag{26}$$

$$\text{where}\, U_{l_i,l_j}^{he}=\frac{1}{\dfrac{1}{\alpha_{l_i}^{h}}+\dfrac{1}{\alpha_{l_j}^{c}}},\; U_{l_i,l_j}^{col}=\frac{1}{\dfrac{1}{\alpha_{l_i}^{h}}+\dfrac{1}{\alpha^{cu}}},\; U_{l_i,l_j}^{reb}=\frac{1}{\dfrac{1}{\alpha^{hu}}+\dfrac{1}{\alpha_{l_j}^{c}}}, \tag{27}$$

$$\Delta Q_{l_i,l_j}^{reb,(k)}=\Delta Q_{l_j}^{c,(k)}-\Delta Q_{l_i,l_j}^{he,(k)},\; \Delta Q_{l_i,l_j}^{col,(k)}=\Delta Q_{l_i}^{h,(k)}-\Delta Q_{l_i,l_j}^{he,(k)}, \tag{28}$$

$$\Delta Q_{l_j}^{c,(k)}=\beta_{l_j}^{c,(k-1)}\Delta Q_j^{c},\; \Delta Q_{l_i}^{h,(k)}=\beta_{l_i}^{h,(k-1)}\Delta Q_i^{h}, \tag{29}$$

$$F_{l_i}^{h,(k)}=\beta_{l_i}^{h,(k-1)}F_i^{h},\quad F_{l_j}^{c,(k)}=\beta_{l_j}^{c,(k-1)}F_j^{c}, \tag{30}$$

$$\overline{T}_{l_i,l_j}^{c,(k)}=\frac{\Delta Q_{l_i,l_j}^{he,(k)}}{F_{l_j}^{c,(k)}c_j^{c}}+T_{l_j}^{c,in},\; \overline{T}_{l_il_j}^{h,(k)}=T_{l_i}^{h,in}-\frac{\Delta Q_{l_i,l_j}^{he,(k)}}{F_{l_i}^{h,(k)}c_i^{h}}, \tag{31}$$

$$T_{l_j}^{c,in}=T_j^{c,in},\; T_{l_i}^{h,in}=T_i^{h,in}, \tag{32}$$

$$T_{l_j}^{c,\mathrm{out}}=T_j^{c,\mathrm{out}},\; T_{l_i}^{h,\mathrm{out}}=T_i^{h,\mathrm{out}}, \tag{33}$$

$$\Delta t_{\ln}^{p,(k)}=\frac{dt_1^{p,(k)}+dt_2^{p,(k)}}{\ln\dfrac{dt_1^{p,(k)}}{dt_2^{p,(k)}}},\; \forall p\in\Omega,\ \text{if } dt_1^{p,(k)}\neq dt_2^{p,(k)}; \tag{34}$$

$$\Delta t_{\ln}^{p,(k)}=\left(dt_1^{p,(k)}dt_2^{p,(k)}\frac{dt_1^{p,(k)}dt_2^{p,(k)}}{2}\right)^{1/3},\forall p\in\Omega,\text{if}\, dt_1^{p,(k)}=dt_2^{p,(k)}; \tag{35}$$

$$dt_1^{he,(k)}=T_{l_i}^{h,in}-\overline{T}_{l_i,l_j}^{c,(k)},\; dt_2^{he,(k)}=\overline{T}_{l_i,l_j}^{h,(k)}-T_{l_j}^{c,in}, \tag{36}$$

$$dt_1^{col,(k)}=\overline{T}_{l_i,l_j}^{h,(k)}-T^{cu,out},\; dt_2^{col,(k)}=T_{l_i}^{h,out}-T^{cu,in},\ \forall(l_i,l_j)\in Z^{(k)}, \tag{37}$$

$$dt_1^{reb,(k)}=T^{hu,in}-T_{l_j}^{c,out},\; dt_2^{reb,(k)}=T^{hu,out}-\overline{T}_{l_i,l_j}^{c,(k)}, \tag{38}$$

$$dt_1^{p,(k)}-\Delta T_{\min}\geq 0,\; dt_2^{p,(k)}-\Delta T_{\min}\geq 0,\; \Delta Q_{l_i^h,l_j^c}^{p,(k)}\geq 0,\forall p\in\Omega, \tag{39}$$

$$\Omega=\{he,\ col,\ reb\}. \tag{40}$$

$\alpha_{l_i}^{h}$, $\alpha_{l_j}^{c}$ are the heat exchange coefficients of the hot and cold streams; α^{hu}, α^{cu} are the heat exchange coefficients of the hot and cold heat carriers; Ω is the set of heat exchangers in the HEN elementary unit structure.

Step 3. Solve the "assignment problem" (14). As a result, the optimal HEN structure is determined for the fixed values of the process stream division coefficients.

Step 4. Find the optimal operating mode for the HEN with a fixed structure by solving a nonlinear mathematical modeling problem(16)–(23). The design and operating parameters of the recuperative heat exchangers, coolers and heaters are used as search variables.

$$\min_{\beta_{l_i}^{h,(k)}, \beta_{l_j}^{c,(k)}, \Delta Q_{l_i,l_j}^{he,(k)}} \bar{\Phi}^{(k)}, \forall (l_i, l_j) \in Z^{(k)} \quad (41)$$

$$\bar{\Phi}^{(k)} = \sum_{l_i}^{L_i^h} \sum_{l_j}^{L_j^c} \left[\left[\tilde{m}_1^{he} + \tilde{m}_2^{he} \left(\frac{\Delta \bar{Q}_{l_i,l_j}^{he,(k)}}{U_{l_i,l_j}^{he} \Delta \bar{t}_{\ln}^{he,(k)}} \right)^{\gamma^{he}} \right] + \left[\tilde{m}_1^{reb} + \tilde{m}_2^{reb} \left(\frac{\Delta \bar{Q}_{l_i,l_j}^{reb,(k)}}{U_{l_i,l_j}^{reb} \Delta \bar{t}_{\ln}^{reb,(k)}} \right)^{\gamma^{reb}} + \hat{m}^{reb} \Delta \bar{Q}_{l_i,l_j}^{reb,(k)} \right] + \right.$$
$$\left. + \left[+ \tilde{m}_1^{col} + \tilde{m}_2^{col} \left(\frac{\Delta \bar{Q}_{l_i,l_j}^{col,(k)}}{U_{l_i,l_j}^{col} \Delta \bar{t}_{\ln}^{col,(k)}} \right)^{\gamma^{col}} + \hat{m}^{col} \Delta \bar{Q}_{l_i,l_j}^{col,(k)} \right] \right], \quad (42)$$

where $U_{l_i,l_j}^{he} = \dfrac{1}{\dfrac{1}{\alpha_{l_i}^{h}} + \dfrac{1}{\alpha_{l_j}^{c}}}, U_{l_i,l_j}^{col} = \dfrac{1}{\dfrac{1}{\alpha_{l_i}^{h}} + \dfrac{1}{\alpha^{cu}}}, U_{l_i,l_j}^{reb} = \dfrac{1}{\dfrac{1}{\alpha^{hu}} + \dfrac{1}{\alpha_{l_j}^{c}}},$ (43)

$$\Delta \bar{Q}_{l_i,l_j}^{reb,(k)} = \Delta \bar{Q}_{l_j}^{c,(k)} - \Delta \bar{Q}_{l_i,l_j}^{he,(k)}, \Delta \bar{Q}_{l_i,l_j}^{col,(k)} = \Delta \bar{Q}_{l_i}^{h,(k)} - \Delta \bar{Q}_{l_i,l_j}^{he,(k)}, \quad (44)$$

$$\Delta \bar{Q}_{l_j}^{c,(k)} = \beta_{l_j}^{c,(k)} \Delta Q_j^c, \Delta \bar{Q}_{l_i}^{h,(k)} = \beta_{l_i}^{h,(k)} \Delta Q_i^h, \quad (45)$$

$$\bar{F}_{l_i}^{h,(k)} = \beta_{l_i}^{h,(k)} F_i^h, \quad \bar{F}_{l_j}^{c,(k)} = \beta_{l_j}^{c,(k)} F_j^c, \quad (46)$$

$$\bar{\bar{T}}_{l_i,l_j}^{c,(k)} = \frac{\Delta \bar{Q}_{l_i,l_j}^{he,(k)}}{\bar{F}_{l_j}^{c,(k)} c_j^c} + T_{l_j}^{c,in}, \bar{\bar{T}}_{l_i,l_j}^{h,(k)} = T_{l_i}^{h,in} - \frac{\Delta \bar{Q}_{l_i,l_j}^{he,(k)}}{\bar{F}_{l_i}^{h,(k)} c_i^h}, \quad (47)$$

$$\Delta \bar{t}_{\ln}^{p,(k)} = \frac{d\bar{t}_1^{p,(k)} + d\bar{t}_2^{p,(k)}}{\ln \dfrac{d\bar{t}_1^{p,(k)}}{d\bar{t}_2^{p,(k)}}}, \forall p \in \Omega, \text{ if } d\bar{t}_1^{p,(k)} \neq d\bar{t}_2^{p,(k)}; \quad (48)$$

$$\Delta\overline{t}_{\ln}^{p,(k)} = \left(d\overline{t}_1^{p,(k)} d\overline{t}_2^{p,(k)} \frac{d\overline{t}_1^{p,(k)} d\overline{t}_2^{p,(k)}}{2} \right)^{1/3}, \forall p \in \Omega, \text{if}\, d\overline{t}_1^{p,(k)} = d\overline{t}_2^{p,(k)}; \tag{49}$$

$$d\overline{t}_1^{he,(k)} = T_{l_i}^{h,in} - \overline{\overline{T}}_{l_i,l_j}^{c,(k)},\ d\overline{t}_2^{he,(k)} = \overline{\overline{T}}_{l_i,l_j}^{h,(k)} - T_{l_j}^{c,in}, \tag{50}$$

$$d\overline{t}_1^{col,(k)} = \overline{\overline{T}}_{l_i,l_j}^{h,(k)} - T^{cu,out},\ d\overline{t}_2^{col,(k)} = T_{l_i}^{h,out} - T^{cu,in},\ \forall (l_i^h, l_j^c) \in Z^{(k)}, \tag{51}$$

$$d\overline{t}_1^{reb,(k)} = T^{hu,in} - T_{l_j}^{c,out},\ d\overline{t}_2^{reb,(k)} = T^{hu,out} - \overline{\overline{T}}_{l_i,l_j}^{c,(k)}, \tag{52}$$

$$d\overline{t}_1^{p,(k)} - \Delta T_{\min} \geq 0,\ d\overline{t}_2^{p,(k)} - \Delta T_{\min} \geq 0,\ \Delta\overline{Q}_{l_i^h,l_j^c}^{p,(k)} \geq 0, \forall p \in \Omega, \tag{53}$$

$$\sum_{l_j=1}^{L_j} \beta_{l_j}^{c,(k)} - 1 = 0,\ \sum_{l_i=1}^{L_i} \beta_{l_i}^{h,(k)} - 1 = 0, \tag{54}$$

$$T_{l_i}^{\mathrm{h,in}} = T_i^{\mathrm{h,in}},\ T_{l_j}^{\mathrm{c,in}} = T_j^{\mathrm{c,in}}, \tag{55}$$

$$T_{l_i}^{\mathrm{h,out}} = T_i^{\mathrm{h,out}},\ T_{l_j}^{\mathrm{c,out}} = T_j^{\mathrm{c,out}}, \tag{56}$$

$$i = 1,\ldots,M^{\mathrm{h}}, j = 1,\ldots,M^{\mathrm{c}},\ l_i = 1,\ldots,L_i^{\mathrm{h}}\ l_j = 1,\ldots,L_j^{\mathrm{c}},$$

$$\Omega = \{he,\ col,\ reb\}. \tag{57}$$

Step 5. If $\left|\Phi^{(k)} - \Phi^{(k-1)}\right| < \xi$, then the algorithm stops; otherwise set $k = k + 1$ and return to step 2.

We can make the following remarks for the given model.

1) Equations (33) and (56) define the outlet temperatures of the elementary streams. Their application simplifies the problem; in some cases, however, they require additional heaters and coolers at the outlets of the elementary streams, thus making it difficult to obtain the optimal value of the parameter. Therefore, the problem can be described more precisely by adding the search variables $T_{l_i}^{\mathrm{h,out}} (i = 1,\ldots,M^{\mathrm{h}}, l_i = 1,\ldots,L_i^{\mathrm{h}})$, $T_{l_j}^{\mathrm{c,out}}\ (j = 1,\ldots,M^{\mathrm{c}}, l_j = 1,\ldots,L_j^{\mathrm{c}})$ and limitations for the heat balance of the mixer (2)–(3).

$$T_i^{h,out} = \frac{\sum_{l_i=1}^{L_i^h} \beta_{l_i}^h F_i^{h,in} c_{l_i}^{h,out} T_{l_i}^{h,out},}{c_i^{h,out} F_i^{h,out}} \equiv \sum_{l_i=1}^{L_i^h} \beta_{l_i}^h T_{l_i}^{h,out},\ T_j^{c,out} = \frac{\sum_{l_j=1}^{L_j^c} \beta_{l_j}^c F_j^{c,in} c_{l_j}^{c,out} T_{l_j}^{c,out}}{c_j^{c,out} F_j^{c,out}} \equiv \sum_{l_j=1}^{L_j^c} \beta_{l_j}^c T_{l_j}^{c,out}, \tag{58}$$

$$T_i^{h,out} - \sum_{l_i=1}^{L_i^h} \beta_{l_i}^h T_{l_i}^{h,out} = 0,\ T_j^{c,out} - \sum_{l_j=1}^{L_j^c} \beta_{l_j}^c T_{l_j}^{c,out} = 0, \tag{59}$$

$i = 1,\ldots,M^{\mathrm{h}}, j = 1,\ldots,M^{\mathrm{c}},\ l_i = 1,\ldots,L_i^{\mathrm{h}},\ l_j = 1,\ldots,L_j^{\mathrm{c}}.$

2) Resulting from the solution of the problem (26)–(40), some heat exchangers can be excluded from the optimal HEN elementary unit structure. We introduce the Boolean variables $\delta^{he}, \delta^{reb}, \delta^{col}$, characterizing the presence or absence of units in the HEN structure:

$$\delta_{l_i,l_j}^{n,(k)} = \begin{cases} 1, \text{ the } n^{\mathrm{th}} \text{ unit for the total sum of the } l_i^{\mathrm{th}} \text{ hot and } l_j^{\mathrm{th}} \text{ cold flows is present} \\ \text{in the HEN elementary unit structure at the } k^{\mathrm{th}} \text{ iteration;} \\ 0, \text{ the } n^{\mathrm{th}} \text{ unit for the total sum of the } l_i^{\mathrm{th}} \text{ hot and } l_j^{\mathrm{th}} \text{ cold flow is absent} \\ \text{in the HEN elementary unit structure at the } k^{\mathrm{th}} \text{ iteration.} \end{cases}$$

Then the problem (41)–(57) can be simplified by adding the following conditions:

$$\left\{ \begin{array}{l} \text{if } (\delta_{l_i,l_j}^{he} = 0) \wedge (\delta_{l_i,l_j}^{reb} = 1) \wedge (\delta_{l_i,l_j}^{col} = 1), \\ \text{then } \Theta_{l_i,l_j} = \{\text{he}\}, \Delta Q_{l_i,l_j}^{he} = 0,\ \overline{T}_{l_i,l_j}^{c} = T_{l_j}^{c,in},\ \overline{T}_{l_i,l_j}^{h} = T_{l_i}^{h,in}; \\ \text{if } (\delta_{l_i,l_j}^{he} = 1) \wedge (\delta_{l_i,l_j}^{reb} = 0) \wedge (\delta_{l_i,l_j}^{col} = 1), \\ \text{then } \Theta_{l_il_j} = \{reb\},\ \Delta Q_{l_i,l_j}^{he} = \Delta Q_{l_i}^{h},\ \ \overline{T}_{l_i,l_j}^{c} = T_{l_j}^{c,out}; \\ \text{if } (\delta_{l_i,l_j}^{he} = 1) \wedge (\delta_{l_i,l_j}^{reb} = 1) \wedge (\delta_{l_i,l_j}^{col} = 0), \\ \text{then } \Theta_{l_i,l_j} = \{col\},\ \Delta Q_{l_i,l_j}^{he} = \Delta Q_{l_j}^{c},\ \overline{T}_{l_i,l_j}^{h} = T_{l_i}^{h,out}; \\ \text{if } (\delta_{l_i,l_j}^{he} = 1) \wedge (\delta_{l_i,l_j}^{reb} = 0) \wedge (\delta_{l_i,l_j}^{col} = 0), \\ \text{then } \Theta_{l_i,l_j} = \{reb, col\},\ \ \Delta Q_{l_i,l_j}^{he} = \Delta Q_{l_j}^{c} = \Delta Q_{l_i}^{h}, \\ \quad \overline{T}_{l_i,l_j}^{h} = T_{l_i}^{h,out},\ \overline{T}_{l_i,l_j}^{c} = T_{l_j}^{c,out}. \end{array} \right\} \begin{array}{l} \forall (l_i, l_j) \in Z^{(k)}, \\ i = 1,\ldots,M^{\mathrm{h}}, l_i = 1,\ldots,L_i^{\mathrm{h}}, \\ j = 1,\ldots,M^{\mathrm{c}}, l_j = 1,\ldots,L_j^{\mathrm{c}}, \end{array} \tag{60}$$

where $\delta^{he}, \delta^{reb}, \delta^{col} \in \{0,1\}$. Based on the listed conditions, the problem will be solved for every total sum of the l_i^{th} hot and the l_j^{th} cold streams for $\forall p \in \Omega / \Theta_{l_il_j}$, where Θ_{l_i,l_j} is the set of heat exchangers excluded from the HEN elementary unit structure. Keep in mind that for the described cases, the heat exchanger load $\Delta Q_{l_i,l_j}^{he}$ will be excluded from the list of search variables.

3) When solving problem (41)-(57), for the cases with the heat exchanger, cooler and heater for the total sum of the l_i^{th} hot and the l_j^{th} cold streams ($(l_i, l_j) \in Z^{(k)}$), the heat exchanger load $\Delta Q_{l_i,l_j}^{he}$ can also be excluded from the list of search variables considering that maintenance costs significantly exceed capital costs. The thermal capacity is determined based on the following conditions:

$$\begin{cases} \gamma < 1: \ \overline{T}_{l_i,l_j}^{c} = T_{l_i}^{h,in} - \Delta T_{\min}, \Delta Q_{l_i,l_j}^{he} = F_{l_j}^{c} c_j^{c} (\overline{T}_{l_i,l_j}^{c} - T_{l_j}^{c,in}); \\ \gamma > 1: \ \overline{T}_{l_i,l_j}^{h} = \Delta T_{\min} + T_{l_j}^{c,in}, \Delta Q_{l_i,l_j}^{he} = F_{l_i}^{h} c_i^{h} (T_{l_i}^{h,in} - \overline{T}_{l_i,l_j}^{h}). \end{cases} \quad (61)$$

where $\gamma = \dfrac{F_{l_j}^{c} c_j^{c}}{F_{l_i}^{h} c_i^{h}}, \ i = 1, \ldots, M^{\mathrm{h}}, j = 1, \ldots, M^{\mathrm{c}}, \ l_i = 1, \ldots, L_i^{\mathrm{h}}, \ l_j = 1, \ldots, L_j^{\mathrm{c}}.$

4) In fact, the assumption about the absence of phase transitions can be easily excluded by describing the mathematical models we use in more details. Thus, based on the heat balance equation for the heat exchanger model we derive the following:

$$\overline{T}_{l_i,l_j}^{c} = T_{l_j}^{c,in} + \frac{\Delta Q_{l_i,l_j}^{he} - rF_{l_j}^{c}}{F_{l_j}^{c} c_j^{c}}, \ \overline{T}_{l_i,l_j}^{h} = T_{l_i}^{h,in} - \frac{\Delta Q_{l_i,l_j}^{he} - rF_{l_i}^{h}}{F_{l_i}^{h} c_i^{h}}, \quad (62)$$

where r–specific heat of evaporation.

5) In the general case, when $M_{\mathrm{S}}^{\mathrm{h}} \neq M_{\mathrm{S}}^{\mathrm{c}}$, for the optimal solution, it is suggested to reduce an asymmetric assignment problem to a symmetric problem by introducing the additional rows $(M_{\mathrm{S}}^{\mathrm{c}} - M_{\mathrm{S}}^{\mathrm{h}})$ or additional $(M_{\mathrm{S}}^{\mathrm{h}} - M_{\mathrm{S}}^{\mathrm{c}})$ columns into the rating matrix. New rows contain optimal economic estimations for autonomous heating of the cold streams, while new columns will contain economic estimations for autonomous cooling of the hot streams. As the assignment problem is to be solved for every total sum of the hot and cold material and heat streams, the corresponding "imaginary" cold and hot streams are to be introduced into the system. The economic value for heating and cooling these streams is equal to zero.

The autonomous heating for the j^{th} cold stream together with the q^{th} "imaginary" hot stream f_{q,l_j}^{aut} is estimated based on the following equations:

$$f_{q,l_j}^{aut} = \tilde{m}_1^{reb} + \tilde{m}_2^{reb}\left(\frac{\Delta Q_{l_j}^{reb}}{U_{l_j}^{reb}\Delta t_{\ln}^{reb}}\right)^{\gamma^{reb}} + \hat{m}^{reb}\Delta Q_{l_j}^{reb} \quad (63)$$

$$\Delta Q_{l_j}^{reb} = \Delta Q_{l_j}^{c},\ U_{l_j}^{reb} = \frac{1}{\frac{1}{\alpha^{hu}} + \frac{1}{\alpha_{l_j}^{c}}}, \quad (64)$$

$$\Delta t_{\ln}^{reb} = \frac{dt_1^{reb} - dt_2^{reb}}{\ln\frac{dt_1^{reb}}{dt_2^{reb}}},\ \text{if}\, dt_1^{reb} \neq dt_2^{reb}; \quad (65)$$

$$\Delta t_{\ln}^{reb} = \left(dt_1^{reb} dt_2^{reb}\frac{dt_1^{reb} dt_2^{reb}}{2}\right)^{1/3},\ \text{if}\, dt_1^{reb} = dt_2^{reb};\ (66)$$

$$dt_1^{reb} = T^{hu,in} - T_{l_j}^{c,out},\ dt_2^{reb} = T^{hu,out} - T_{l_j}^{c,in}, \quad (67)$$

$i = 1,\dots,M^{\mathrm{h}}$, $j = 1,\dots,M^{\mathrm{c}}$, $l_i = 1,\dots,L_i^{\mathrm{h}}$, $l_j = 1,\dots,L_j^{\mathrm{c}}$..

In a similar way, the economic parameters $f_{l_i,w}^{aut}$ for autonomous cooling of the i^{th} cold stream together with the w^{th} "imaginary" cold stream are estimated based on the following equations:

$$f_{l_i,w}^{aut} = \tilde{m}_1^{col} + \tilde{m}_2^{col}\left(\frac{\Delta Q_{l_i}^{col}}{U_{l_i}^{col}\Delta t_{\ln}^{col}}\right)^{\gamma^{col}} + \hat{m}^{col}\Delta Q_{l_i}^{col} \quad (68)$$

$$\Delta Q_{l_i}^{col} = \Delta Q_{l_i}^{h},\ U_{l_i}^{col} = \frac{1}{\frac{1}{\alpha_{l_i}^{h}} + \frac{1}{\alpha^{cu}}}, \quad (69)$$

$$\Delta t_{\ln}^{reb} = \frac{dt_1^{col} - dt_2^{col}}{\ln\frac{dt_1^{col}}{dt_2^{col}}},\ \text{if}\ dt_1^{col} \neq dt_2^{col}; \quad (70)$$

$$\Delta t_{\ln}^{col} = \left(dt_1^{col} dt_2^{col}\frac{dt_1^{col} dt_2^{col}}{2}\right)^{1/3},\ \text{if}\ dt_1^{col} = dt_2^{col}; \quad (71)$$

$$dt_1^{col} = T_{l_i}^{h,in} - T^{cu,out},\ dt_2^{col} = T_{l_i}^{h,out} - T^{cu,in}, \quad (72)$$

$i = 1,\dots,M^{\mathrm{h}}$, $j = 1,\dots,M^{\mathrm{c}}$, $l_i = 1,\dots,L_i^{\mathrm{h}}$, $l_j = 1,\dots,L_j^{\mathrm{c}}$.

Therefore, the optimization problems for determining the optimal HEN structure are as follows:

$$R=\max\{M_S^{\mathrm{h}},M_S^{\mathrm{c}}\},\ f_{q,w}=\begin{cases} f_{l_j}^{\mathrm{aut}},\ q=(M_S^h+1)...R,\ w\in\varnothing,\ M_S^{\mathrm{h}}<M_S^{\mathrm{c}}; \\ f_{l_i}^{\mathrm{aut}},\ w=(M_S^c+1)...R,\ q\in\varnothing,\ M_S^{\mathrm{h}}>M_S^{\mathrm{c}}; \end{cases} \tag{73}$$

If $M_{\mathrm{S}}^{\mathrm{h}}<M_{\mathrm{S}}^{\mathrm{c}}$

$$\min_{z_{l_i,l_j}^{(k)},z_{q,l_j}^{(k)}}\left(\sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h}\sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} f_{l_i,l_j}^{\mathrm{opt},(k)} z_{l_i,l_j}^{(k)} + \sum_{q=(M_S^{\mathrm{c}}+1)}^{R} f_{q,l_j}^{\mathrm{aut},(k)} z_{q,l_j}^{(k)}\right), \tag{74}$$

$$\sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h} z_{l_i,l_j}^{(k)} + \sum_{q=(M_S^{\mathrm{c}}+1)}^{R} z_{q,l_j}^{(k)}=1,\ \sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} z_{l_i,l_j}^{(k)}=1,\ \sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} z_{q,l_j}^{(k)}=1. \tag{75}$$

$$i=1,...,M^{\mathrm{h}},\ j=1,...,M^{\mathrm{c}},\ l_i=1,...,L_i^{\mathrm{h}},\ l_j=1,...,L_j^{\mathrm{c}},$$

$$z_{l_i,l_j}\in\{0,1\},\ z_{q,l_j}\in\{0,1\}.$$

If $M_{\mathrm{S}}^{\mathrm{h}}>M_{\mathrm{S}}^{\mathrm{c}}$

$$\min_{z_{l_i,l_j}^{(k)},z_{q,l_j}^{(k)}}\left(\sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h}\sum_{j=1}^{M^c}\sum_{l_j=1}^{L_j^c} f_{l_i,l_j}^{\mathrm{opt},(k)} z_{l_i,l_j}^{(k)} + \sum_{w=(M_S^{\mathrm{h}}+1)}^{R} f_{l_i,w}^{\mathrm{aut},(k)} z_{l_i,w}^{(k)}\right), \tag{76}$$

$$\sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h} z_{l_i,l_j}^{(k)}=1,\ \sum_{j=1}^{M^c}\sum_{l_j^c=1}^{L_j^c} z_{l_i,l_j}^{(k)} + \sum_{w=(M_S^{\mathrm{h}}+1)}^{R} z_{l_i,w}^{(k)}=1, \sum_{i=1}^{M^h}\sum_{l_i=1}^{L_i^h} z_{l_i,w}^{(k)}=1, \tag{77}$$

$$i=1,...,M^{\mathrm{h}},\ j=1,...,M^{\mathrm{c}},\ l_i=1,...,L_i^{\mathrm{h}},\ l_j=1,...,L_j^{\mathrm{c}},$$

$$z_{l_i,l_j}\in\{0,1\},\ z_{l_i,w}^{(k)}\in\{0,1\}. \tag{78}$$

COMPUTATIONAL RESULTS

The operational performance and efficiency of the proposed method were tested with three examples from the SYNHEAT software [21, 22]. SYNHEAT is based on a mixed-integer nonlinear programming mode for optimizing a staged HEN superstructure. Two optimization algorithms were used: DICOPT code and the global optimization code BARON, when addressing the problem of heat exchange systems synthesis in the SYNHEAT program.

Example 1. There are two hot and two cold streams which exchange heat. The initial data are given in Table1. The total amounts of the heat to be collected from the hot streams is $\Delta Q^{\text{h}}_{\text{sum}} = 5000\ \text{kW}$, the total amounts of the heat energy to be transferred to the cold streams is $\Delta Q^{\text{c}}_{\text{sum}} = 4900\ \text{kW}$.

Table 1 – Input and Output Temperatures, Amounts of Heat Collected from Hot and Transferred to Cold Process Streams (Example 1).

Hot stream	$T_i^{\text{h,in}}$, K	$T_i^{\text{h,out}}$, K	ΔQ_i^{h}, kW	Cold stream	$T_j^{\text{c,in}}$, K	$T_j^{\text{c,out}}$, K	ΔQ_j^{c}, kW
H1	430	380	2000	C1	410	411	4000
H2	425	424	3000	C2	390	420	900

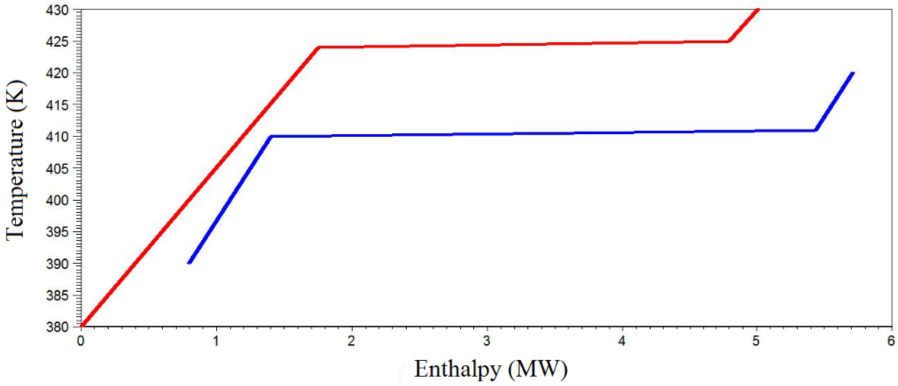

Figure 3 – Composite Curves on Temperature vs. Enthalpy Diagram at ΔT_{min}=5°C (Example 1)

Pinch analysis at the lowest permissible temperature difference ΔT_{min}=5°C gave the following results:

a) in the area where the hot and cold composite curves overlap, the maximum possible amouts of recuperated energy is $\Delta Q^{\text{he}}_{\text{max}} = 4200\ \text{kW}$;

b) the minimum amounts of supply energy is $\Delta Q^{\text{col}}_{\text{min}} = 800\ \text{kW}$;

c) the minimum amounts of removed energy is $\Delta Q^{\text{reb}}_{\text{min}} = 700\ \text{kW}$.

Table 2 gives the results of the optimal HEN iterative calculations. The problem solution resulted in a flowsheet without any division of process streams

(Figure4).The parameters to define the operating modes of the heat exchangers within HEN are given in Table3.

Table2 – Results of Optimal HEN Synthesis Using the Proposed Method (Example 1)

Iteration Number	ΔQ^{he}_{sum}, kW	ΔQ^{reb}_{sum}, kW	ΔQ^{col}_{sum}, kW	Φ, USD/year	n^{he}	n^{reb}	n^{col}
1	3,900	1,000	410	128,853	2	1	1
2	3,900	1,000	410	128,853	2	1	1

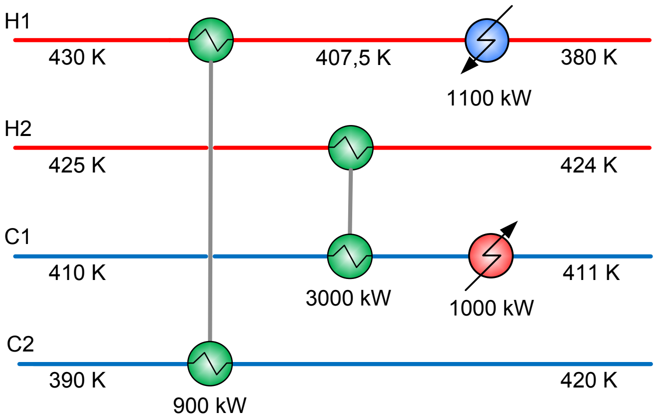

Figure 4 – HEN Diagram Obtained by Using the Proposed Algorithm (Example 1)

Table3 – Results of Solving Optimal Synthesis Problem Using the Proposed Method (Example 1)

	ΔQkW	$T_i^{h,in}$,K	$T_i^{h,out}$,K	$T_j^{c,in}$,K	$T_j^{c,out}$,K	A, m^2
Recuperative Heat Exchangers						
$E_{1_1 2_1}$	3000	430	407.5	410	411	236.7
$E_{2_1 1_1}$	900	425	424	390	420	129.3
Coolers						
$C_{2_1 1_1}$	1100	407.5	380	303	315	8.1
Heaters						
$B_{1_1 2_1}$	1000	627	626	410.9	411	4.6

The results show that HEN structure in Figure 4 is characterized by a higher value of the total cost in comparison with the structure obtained in SYNHEAT(DICOPT) software (Figure 5). Solving this problem using SYNHEAT(BARON) showed the same results as with DICOPT (Figure 6).The obtained results are presented in Table 4.

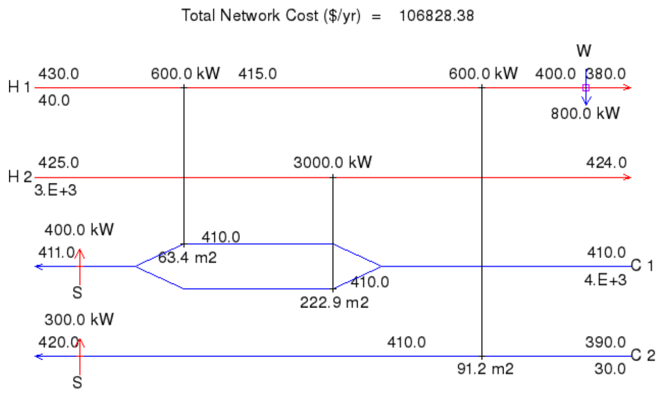

Figure 5 – HEN Diagram (Example 1) obtained in SYNHEAT(DICOPT)

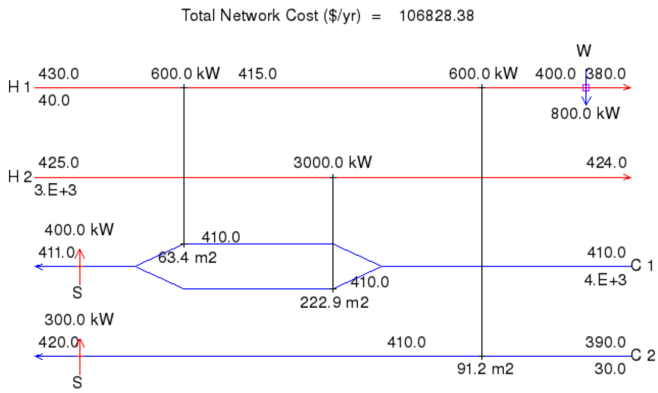

Figure 6 – HEN Diagram (Example 1) obtained in SYNHEAT Software(BARON)

Table 4 – Comparison of Results Obtained by the Proposed Method and SYNHEAT Methods(Example 1)

Name	Proposed Method	DICOPT	BARON
ΔQ^{he}_{sum},, (kW)	3,900	4,200	4,200
ΔQ^{reb}_{sum},, (kW)	1,000	700	700
ΔQ^{col}_{sum},, (kW)	1,100	800	800
n^{he}	2	3	3
n^{reb}	1	2	2
n^{col}	1	1	1
E_{sum}, USD/year	101,102	144,000	78,000
K_{sum}, USD/year	33,100	28,828.4	28,828.4
A^{he}. m^2	310.3	377.5	377.5
A^{col}, m^2	20.4	15.4	15.4
A^{reb}, m^2	4.6	3.15	3.15
Φ, USD/year TAC	134,202	106,828	106,828

Example 2. There are three hot and four cold streams exchanging heat. The initial data are given in Table 5. The composite curves on temperature vs. enthalpy diagram at ΔT_{min}=5°C are shown in Figure 7.

Table 5 –Input and Output Temperatures, amounts of heat taken from the hot flows and transfered to the cold ones (Example 2)

Hot stream	$T_i^{h,in}$, K	$T_i^{h,out}$, K	ΔQ_i^h, kW	Cold stream	$T_j^{c,in}$, K	$T_j^{c,out}$, K	ΔQ_j^c, kW
H1	503	308	12,948	C1	323	503	8,838
H2	425	425	33,020	C2	408	408	18,413.1
H3	381	381	12,870	C3	391	391	18,498.4
				C4	353	353	16,347.7

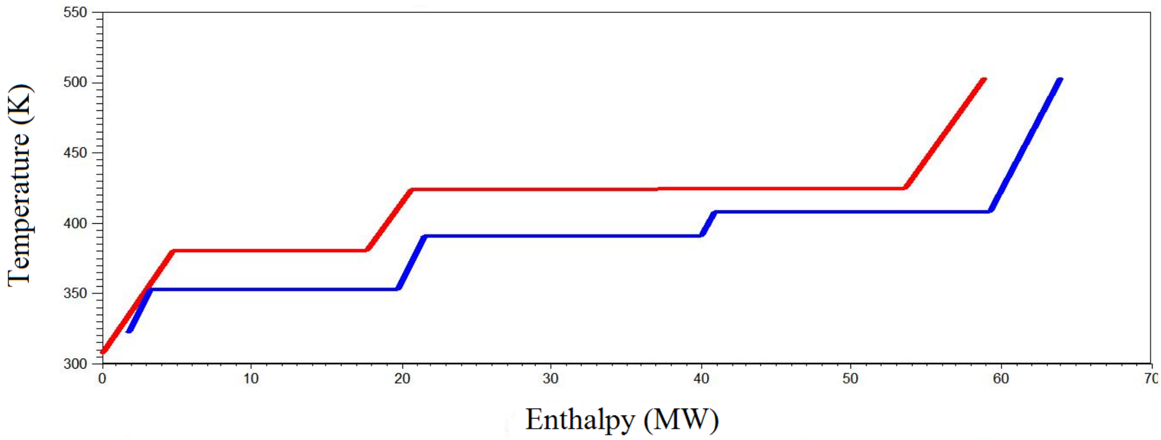

Figure 7 – Composite Curves at Temperature vs. Enthalpy Diagram at $\Delta T_{\min}$=5°C. (Example 2)

The results of pinch analysis are as follows:

$\Delta Q^{\mathrm{h}}_{\mathrm{sum}} = 58{,}838$ kW , $\Delta Q^{\mathrm{c}}_{\mathrm{sum}} = 62{,}097$ kW , $\Delta Q^{\mathrm{he}}_{\max} = 56{,}991$ kW ,

$\Delta Q^{\mathrm{col}}_{\min} = 18{,}47$ kW , $\Delta Q^{\mathrm{reb}}_{\min} = 5{,}106$ kW .

Table 6 gives the results of the optimal HEN synthesis. Figure 8 gives HEN Diagram Obtained by Using the Proposed Method.

Table 6 – Results of Optimal HEN Synthesis using the Proposed Method (Example 2)

Iteration Number	$\Delta Q^{\mathrm{he}}_{\mathrm{sum}}$, kW	$\Delta Q^{\mathrm{reb}}_{\mathrm{sum}}$, kW	$\Delta Q^{\mathrm{col}}_{\mathrm{sum}}$, kW	Φ, USD/year	n^{he}	n^{reb}	n^{col}
1	56,186	5,911.3	2,651.9	814,571.3	8	3	2
2	56,172	5,924.5	2,665.9	810,216.6	7	3	2
3	56,185	5,912.1	2,652.7	810,153.3	7	3	2
4	56,187	5,910.4	2,650.3	803,501.5	6	3	2
5	56,187	5,910	2,650.6	798,793.8	5	3	2
6	56,833.6	5,263.8	2,004.4	711,333.5	5	3	2
7	56,833.6	5,263.8	2,004.4	711,333.5	5	3	2

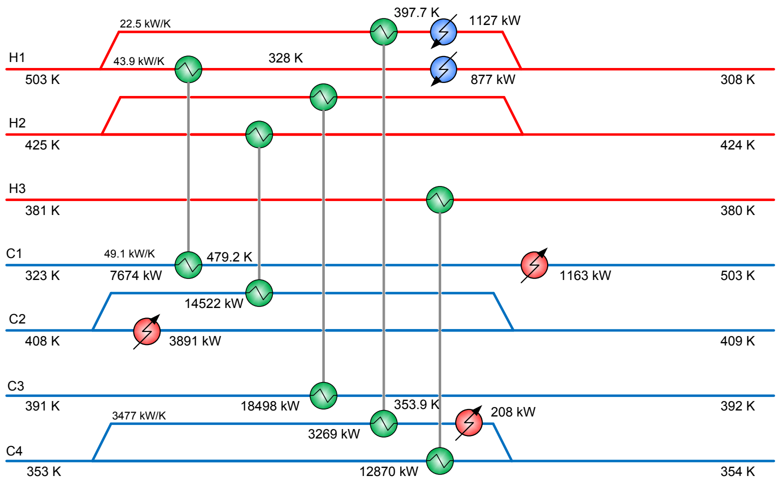

Figure 8 – HEN Diagram Obtained by Using the Proposed Method (Example 2)

Table 7 shows parameter values for the unit in case of solving the problem using the proposed algorithm.

Table7 – Results of Solving Optimal Synthesis Problem Using the Proposed Method (Example 2)

	ΔQ,kW	$T_i^{h,in}$,K	$T_i^{h,out}$,K	$T_j^{c,in}$,K	$T_j^{c,out}$, K	A, m^2
Recuperative heat exchangers						
$E_{1_1 1_1}$	7,674	503	328	323	479.2	1673
$E_{2_2 2_1}$	14,522	425	424	408	409	985
$E_{3_1 2_2}$	18,498	425	424	391	392	633.4
$E_{4_2 1_2}$	3,269	503	397.7	353	353.9	137
$E_{4_1 3_1}$	12,870	381	380	353	354	553.3
Coolers						
$C_{1_1 1_1}$	877.1	328	308	303	315	234
$C_{4_2 1_2}$	1,127.3	397.7	308	303	315	142.6
Heaters						
$B_{1_1 1_1}$	1,163.7	627	626	479.2	503	4.8
$B_{4_2 1_2}$	208.6	627	626	353.9	354	0.8
$B_{2_1 2_1}$	3,891.5	425	424	408	409	16.5

Diagrams of HEN structures in Figures 8 and 9 show that the proposed synthesis method yields lower costs than SYNHEAT (DICOPT). However, SYNHEAT (BARON) showed the best result with lowest cost as given by Fig. 10 as seen in Table 8.

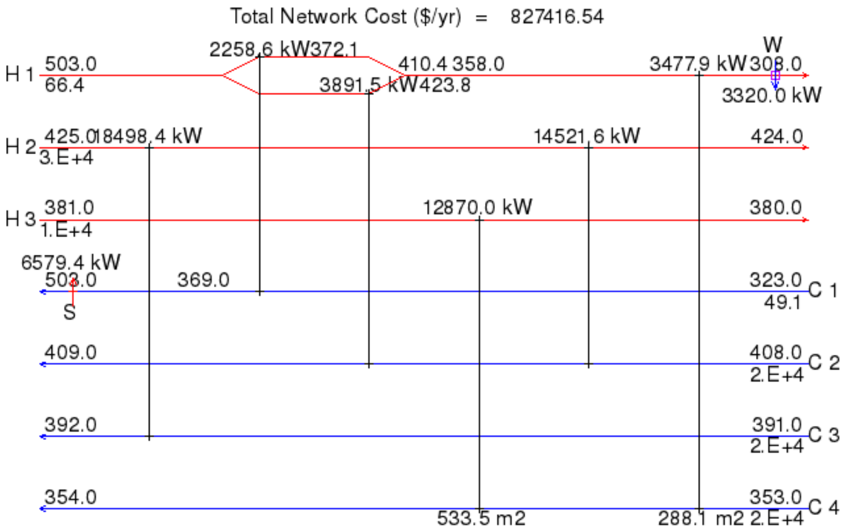

Figure 9 – HEN Diagram Obtained (Example 2) in SYNHEAT Software (DICOPT)

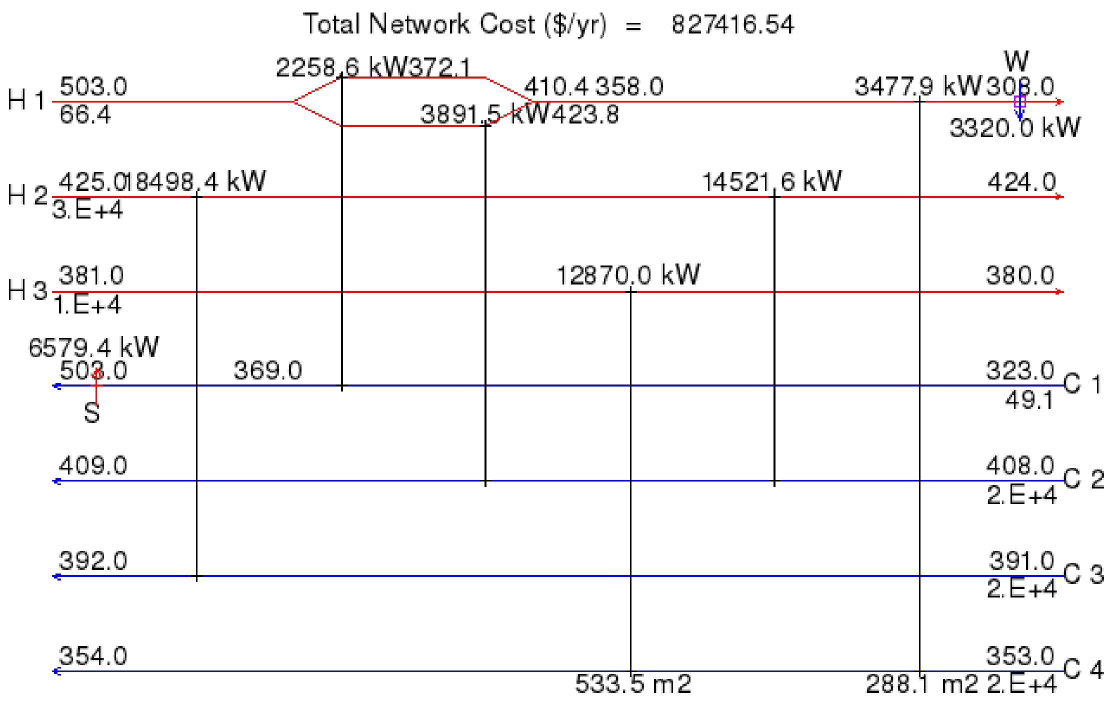

Figure 10 – HEN Diagram Obtained (Example 2) in SYNHEAT Software (BARON)

Table8 – Comparison between the Proposed Method and SYNHEAT Methods (Example 2)

Name	Proposed Method	DICOPT	BARON
ΔQ^{he}, (kW)	56,833.6	55,518	56,991
ΔQ^{reb}, (kW)	5,263.8	6,579	5,106
ΔQ^{col}, (kW)	2,004.4	3,320	1,847
n^{he}	5	6	7
n^{reb}	3	1	1
n^{col}	2	1	1
E_{sum}, USD/year	526,383	691,140	529110
K_{sum}, USD/year	184,950	136,276.5	155,614
A^{he}. m^2	3,983.6	2588.52	3189.63
A^{col}, m^2	376.7	429.33	374.05
A^{reb}, m^2	22	64.38	53.51
Φ, USD/year TAC	711,333	827,417	684,724

Example 3. There are two hot and two cold streams exchanging heat. The initial data are given in Table 9.

Table 9 – Input and Output Temperatures, amounts of Heat Collected from Hot and Transferred to Cold Process Streams (Example 3)

Hot stream	$T_i^{h,in}$, K	$T_i^{h,out}$, K	ΔQ_i^h, kW	Cold stream	$T_j^{c,in}$, K	$T_j^{c,out}$, K	ΔQ_j^c, kW
H1	650	370	2800	C1	410	650	3600
H2	590	370	4400	C2	350	500	1950

Table 10, Table 11 and Figure 11 gives the results of the optimal HEN synthesis Using the Proposed Method.

Table 10 – Results of Optimal HEN Synthesis with Using the Proposed Method (Example 3)

Iteration Number	ΔQ_{sum}^{he}, kW	ΔQ_{sum}^{reb}, kW	ΔQ_{sum}^{col}, kW	Φ, USD/year	n^{he}	n^{reb}	n^{col}
1	5,027	523	2,173	171,328	3	2	3
2	5,027	523	2,173	171,328	3	2	3

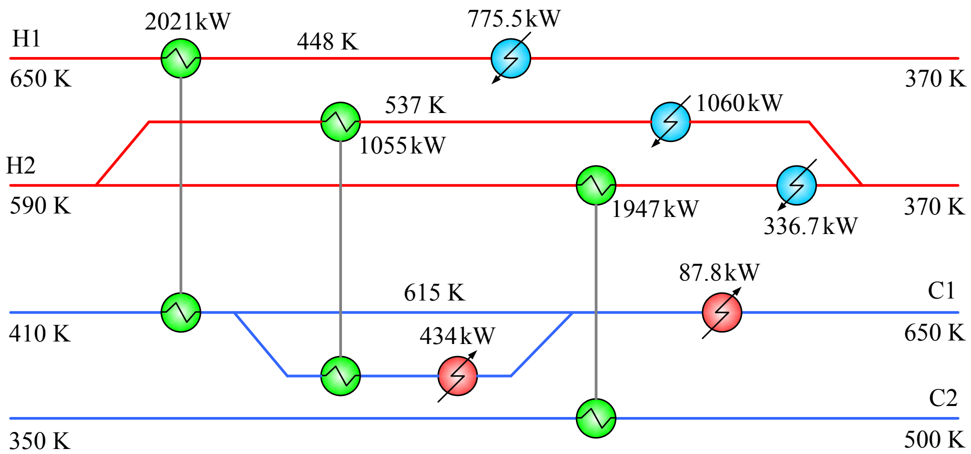

Figure 11 – HEN Diagram Obtained by Using the Proposed Method (Example 3)

Table 11 – Results of Solving Optimal Synthesis Problem Using the Proposed Method (Example 3)

	ΔQ,kW	$T_i^{\mathrm{h,in}}$,K	$T_i^{\mathrm{h,out}}$,K	$T_j^{\mathrm{c,in}}$,K	$T_j^{\mathrm{c,out}}$, K	A, m^2
Recuperative heat exchangers						
$E_{1_1 1_1}$	2,021	650	448	410	615	194
$E_{1_2 2_2}$	1,055	590	537	410	615	68.3
$E_{2_1 2_1}$	1,947	590	537	350	500	56
Coolers						
$C_{1_1 1_1}$	775	448	370	300	320	16
$C_{1_2 2_2}$	1,060	537	370	300	320	19.5
$C_{2_1 2_1}$	337	537	370	300	320	4.9
Heaters						
$B_{1_1 1_1}$	88	680	680	615	650	1.3
$B_{1_2 2_2}$	434	680	680	615	650	3.7

However, in this example, SYNHEAT (DICOPT) (Fig. 12) and SYNHEAT (BARON) (Fig. 13) obtain higher total costs than the one of the proposed method (171,328USD/year). The explanation lies on the fact that the structure of the network obtained by the proposed method in Fig. 11 is not contained in the SYNHEAT superstructure since it involves branches with two heat exchangers in series. The SYNHEAT superstructure only allows one exchanger per stream that is split (see Fig. 13). We should also note that in this case BARON takes considerably longer time than DICOPT (406.27s vs less that one second). Table 12 presents the comparison between the Proposed Method and the SYNHEAT Methods.

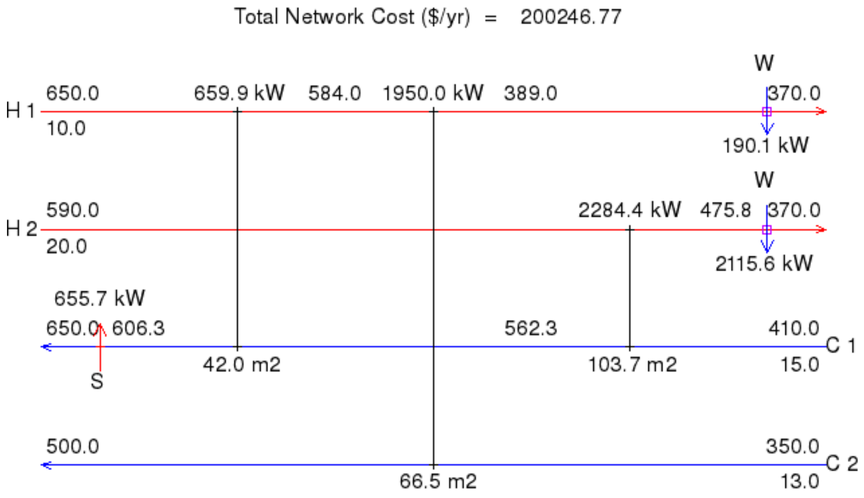

Figure 12 – HEN Diagram Obtained in SYNHEAT Software (DICOPT) (Example 3).

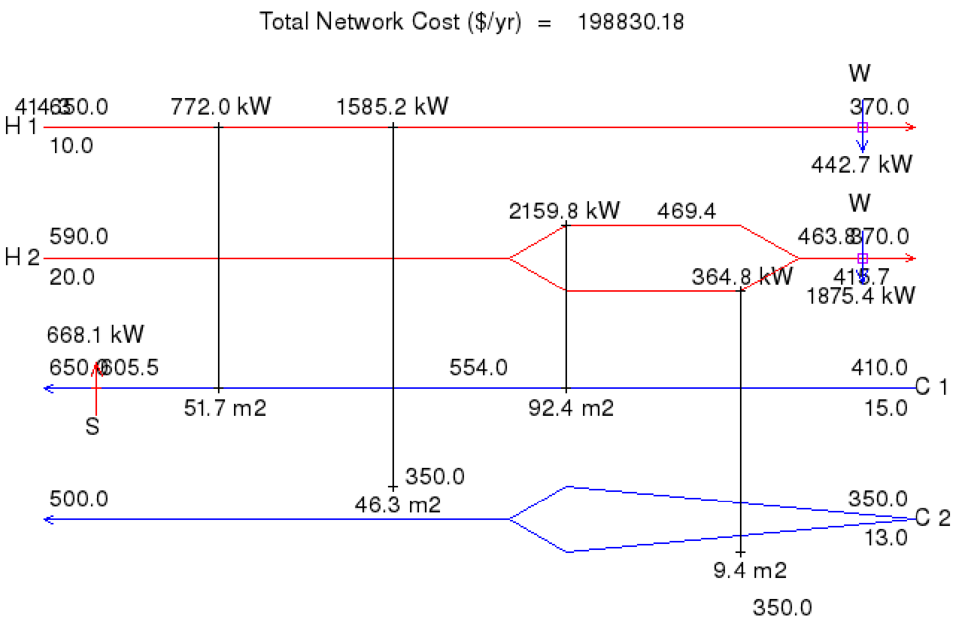

Figure 13 – HEN Diagram Obtained in SYNHEAT Software (BARON) (Example 3).

Table 12 – Comparison between the Proposed Method and SYNHEAT Methods

Name	Proposed Method	DICOPT	BARON
ΔQ^{he}, (kW)	5027.4	4894.26	4881.89
ΔQ^{reb}, (kW)	522.6	655.74	668.12
ΔQ^{col}, (kW)	2172.6	2305.74	2318.12
n^{he}	3	3	4
n^{reb}	2	1	1
n^{col}	3	2	2
E_{sum}, USD/year	72154.3	90,324.06	91561.69
K_{sum}, USD/year	99173.7	109,922.71	107268.49
A^{he}. m^2	318.2	212.2	199.94
A^{col}, m^2	44.4	44.95	47.47
A^{reb}, m^2	5	16.2	16.4
Φ, USD/year	171,328	200,247	198,830

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CONCLUSION

Synthesis methods for optimal heat exchange networks based on superstructure optimization offer a general systematic approach. These problems result from the fact that the global flowsheet embeds all the possible alternatives for the heat exchange arrangements. The use of the BARON global optimization algorithm ensures the global optimum within the selected superstructure. The proposed method addresses the computational challenges by reducing the problem to a sequence of assignment problems. In this case, the optimization problem can be solved for a number of fixed structure heat exchange networks.

It is commonly known that the accuracy in determining the optimum in solving the process system optimal design problem depends on the number of degrees of freedom in the designed system, and, therefore, the number of search variables. Thus, the proposed method has the advantage of extending the search area due to increasing the number of search variables by dividing the streams.

The basic procedure of HEN synthesis by the proposed method implies reducing the problem to a sequence of assignment problems. To apply this procedure, it is necessary to know the optimal economic estimations of the possible heat exchanges between the hot and cold streams. Therefore, the superstructure is decomposed into the elementary heat exchange units. Thus, thermal, and physical parameters of heat exchange units and heat exchanging streams (in particular, diameter, length, number of tubes, number of passes, coefficient of heat transmission, and etc.) can also be taken into consideration in calculating the optimal values. Moreover, the economic estimations for the costs of pipelines and transportation equipment for the heat exchanging streams can also be taken into account. This is important for viscous and highly corrosive components of the streams.

The computational results have shown that the proposed method is competitive compared to the global optimization of the Yee et al. [12] superstructure. As was shown in the third example problem the proposed method may yield lower cost solutions due to the fact that two or more heat exchangers may be considered in the split streams, while the Yee et al. [12] is only restricted to one exchanger per split stream.

SYMBOLS

A–heat exchange surface area of heat exchanger, m^2;

B–outlet heater;

C–outlet cooler;

c–specific heat capacity at constant pressure, kW·h/kg·K;

E–recuperative heat exchanger;

F–mass flow rate, kg/hour;

f–total reduced capital and operating costs, USD/year;

$\hat{m}_1$ –reduced price ratios of capital costs, USD/year;

$\hat{m}_2$ –reduced price ratios of capital costs, USD/year·m^2;

$\tilde{m}$ –unit cost for hot or cold heat carrie rconsumption, USD/kg;

T–stream temperature, K;

U–heat transfer coefficient, kW·h/m^2·K;

ΔQ–heat quantity, kW;

$\Delta T_{\min}$–minimum allowed temperature difference, K;

r–specific heat of evaporation, kW·h/kg·K;

γ–correlation coefficient;

Φ–total sum of reduced capital and operating costs for hea texchanger network, USD/ year;

$Z^{(k)}$ –total sum of coupled elementary streams with their HEN elementary units belonging to the HEN structure at current iteration

INDICES

c–cold stream;

h–hot stream;

hu–heating steam;

cu–cooling water;

aut – autonomous heat exchange;

he–recuperative heat exchanger;

col–cooler / condenser;

reb–heater/ boiler;

i–hot stream number;

j–cold stream number;

l –elementary stream;

k–iteration number.

REFERENCES

1. Klemeš J.J., Kravanja, Z. Forty years of heat integration: pinch analysis (PA) and mathematical programming (MP) // Current Opinion in Chemical Engineering. 2013. V. 2. № 4. P. 461.

2. Smith, R., Klemesh J.J.., TovazhnyanskyL., KapustenkoP., UlyevM. (2000) [The Basics of Heat Process Integration] In: *Library of ITE Journal* [Biblioteka Zhurnala ITE], Kharkov: NTU "KhPI" (In Russian)

3. TsirlinA., Akhremenkov A., GrigorevskyI. (2008) [Minimum Irreversibility, Optimal Distribution of Surface and Thermal Capacity for HENs] *Theoretical Foundations of Chemical Engineering*, Vol. 42, No. 2, pp. 214-221. (In Russian)

4. Linnhoff, B. Pinch analysis — A state of the art overview // Transactions of the Institution of Chemical Engineers. 1993. № 38. P. 745.

5. Biegler,L.T., I.E. Grossmann, A.W. Westerberg. Systematic methods of chemical process design, New Jersey: Prentice Hall PTR; 1997.

6. Ostrovskii G.M., Ziyatdinov N.N., Lapteva T.V. (2012) [Process Systems Optimization], Moscow: KNORUS (In Russian).

7. Zamora, J. M., and Grossmann, I. E. A global MINLP optimization algorithm for the synthesis of heat exchanger networks with no stream splits // Comput. Chem. Eng. 1998. V.22. № 3. P. 367.

8. Furman, K. C. and Sahinidis, N.V. Computational complexity of heat exchanger network synthesis // Comput. Chem. Eng. 2001. № 25. P. 1371.

9. Yee, T.F. Simultaneous optimization models for heat integration - II. Synthesis of heat exchanger networks / T.F. Yee, I.E. Grossmann // Comput. Chem. Eng. 1990. V. 14. № 10. P. 1165.

10. Papoulias, S. A., and Grossmann, I. E. A structural optimization approach in process synthesis. II. Heat recovery networks // Comput. Chem. Eng., 1983. V.7. № 6. P. 707.

11. Chen, Y., Grossmann, I. E., and Miller, D.C., Computational Strategies for Large-Scale MILP Transshipment Models for Heat Exchanger Network Synthesis // Comput. Chem. Eng. 2015. V. 82. P. 68.
12. Cerda J, Westerberg AW. Synthesizing heat exchanger networks having restricted stream/stream matches using transportation problem formulation // Chem. Eng. Sci., 1983b; 38(10):1723-1740.
13. Athier, G., Floquet, P., Pibouleau, L., and Domenech, S. Synthesis of heat exchanger network by simulated annealing and NLP procedures // American Institute of Chemical Engineering Journal. 1997. V.43. № 11. P. 3007.
14. Ghiasvand A., Fazlali A.R., Ghiasi T.S., Shoorehdeli M.A., Mohammadi A.H. Optimization of Heat Exchanger Networks Using an Evolutionary Method. In: Advances in Energy Research, V. 18, Nova Science Publishers, Inc., USA, 2014.
15. M.A.S.S. Ravagnani, A.P. Silva, P.A. Arroyo, A.A. Constantino Heat exchanger network synthesis and optimisation using genetic algorithm // Applied Thermal Engineering. 2005. V. 25 № 7. P. 1003.
16. Ziyatdinov N.N., Ostrovskii G.M., Emel'yanov I.I. Designing a Heat–Exchange System upon the Reconstruction and Synthesis of Optimal Systems of Distillation Columns // Theoretical Foundations of Chemical Engineering. 2016. V. 50. № 2. P. 178.
17. Ziyatdinov N.N., Emel'yanov I.I and Le Quang Tuen Method for the Synthesis of Optimum Multistage Heat Exchange Networks // Theoretical Foundations of Chemical Engineering. 2018. V. 52. № 6. P. 614
18. Ostrovskii G.M., Berezhinskii T.A. (1984) [Optimization of Chemical Engineering Processes: Theory and Practice], Moscow: Chemistry (In Russian).
19. Ostrovskii G.M., Volin Yu.M. (1975) [Simulation of Complex Process Systems], Moscow; Chemistry (in Russian).
20. Ostrovskii G.M., Ziyatdinov N.N., and Emel'yanov I.I. Synthesis of Optimal Systems of Simple Distillation Columns with Heat Recovery // Doklady Chemistry. 2015. V. 461, № 2, P. 89.

21. Yee, T.F. Simultaneous optimization models for heat integration — III. Process and heat exchanger network optimization / T.F. Yee, I.E. Grossmann, Z. Kravanja // Comput. Chem. Eng. 1990. V. 14. № 11. P. 1185.
22. J. Ponce-Ortega, A. Jimenez-Gutierrez, I. Grossmann, Optimal synthesis of heat exchanger networks involving isothermal process streams // Comput. Chem. Eng. 2008. № 32. P. 1918.