Permeability Estimation in Heterogeneous Formations with Derivative-Free Optimization Based On Observed Data

Yan Zhang[1,2] and Nick Sahinidis[1,2]
[1]National Energy Technology Laboratory, Pittsburgh, PA
[2]Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA
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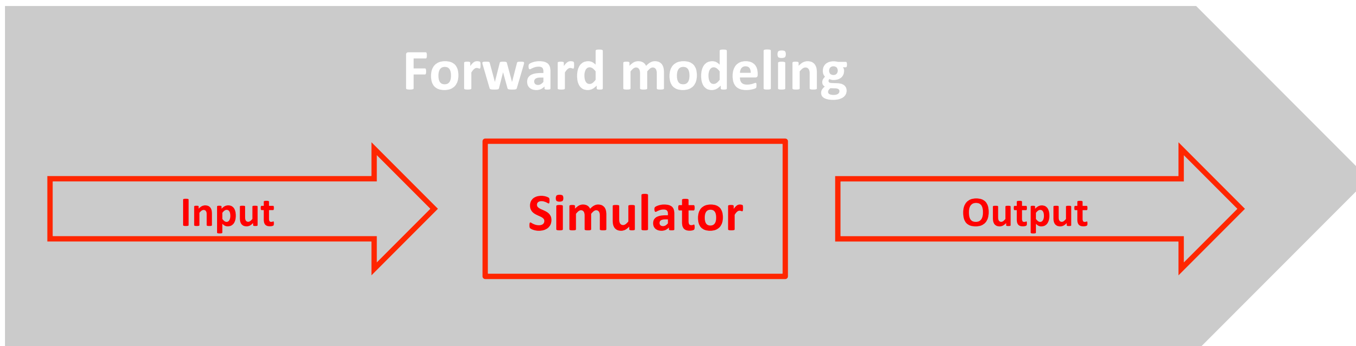
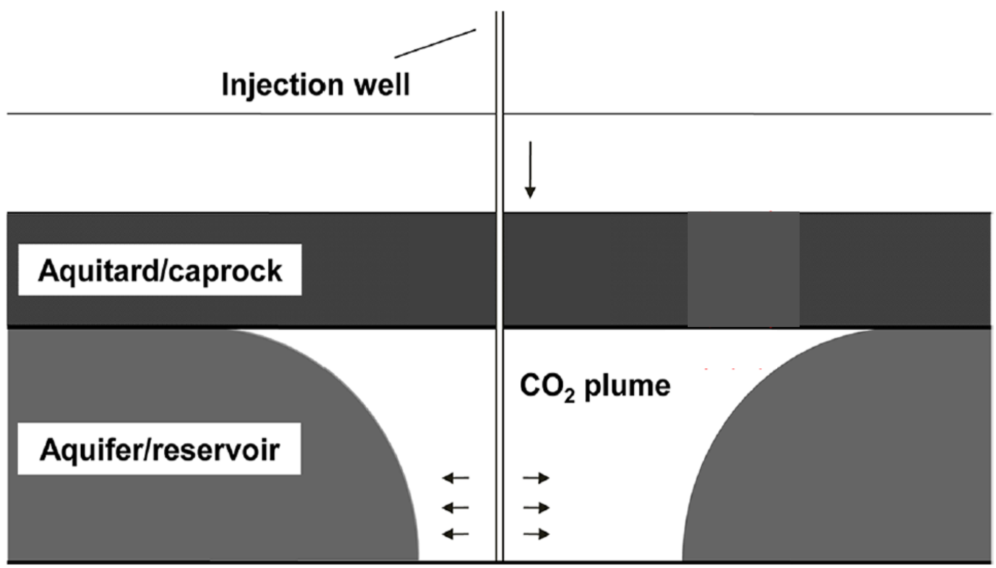
Outline

- Three key components of parameter estimation
- Work flow of parameter estimation
- Introduction to derivative-free optimization (DFO)
- Two case studies
- Conclusions

Three key components: System model

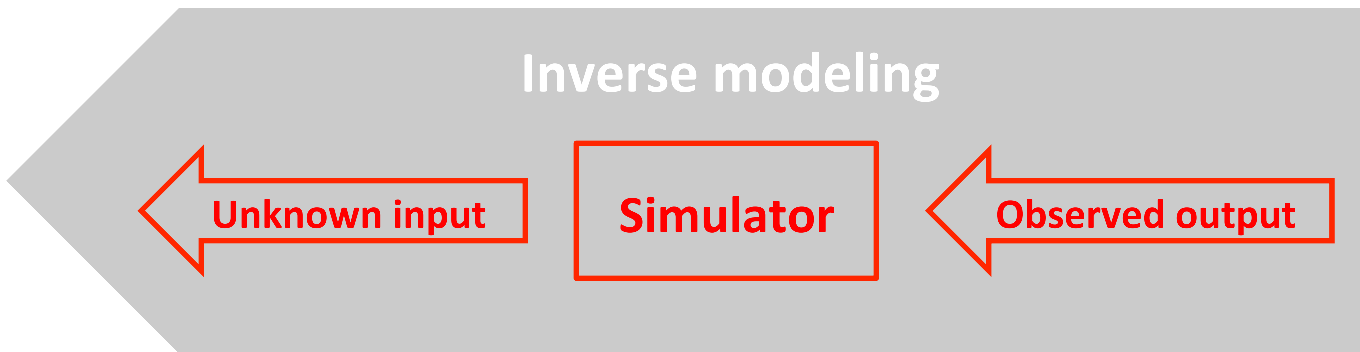


Porosity, permeability, etc

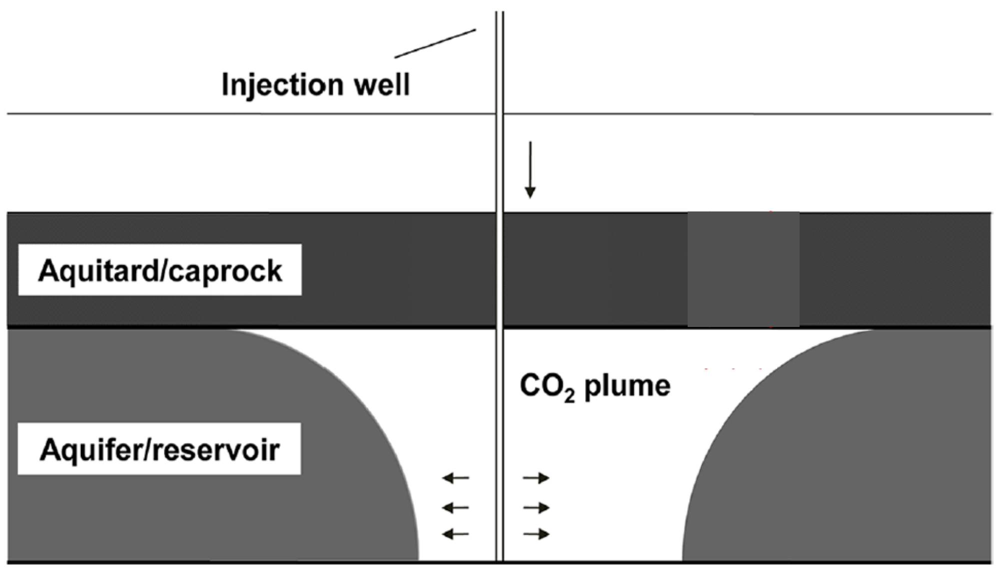


Pressure, gas saturation, etc

Three key components: System model



Unknown porosity, permeability, etc



Observed pressure, gas saturation, etc

Three key components: Fitness measure

- **Least squared error:**
  - **Mismatch between simulation and observation:** $\sum_i w_i \left(\frac{\boxed{y_i^{\mathrm{sim}}(p)} - y_i^{\mathrm{obs}}}{\sigma_i} \right)^2$

Simulation/black-box
No algebraic form for gradient

- **Maximum Likelihood**
  - **Likelihood of observation being present under parameter *p*:**

$$f\left(y^{\mathrm{obs}} | p\right) = \prod_i f_i\left(y_i^{\mathrm{obs}} | p\right) \propto \exp\left\{ -\frac{1}{2} \sum_i \left(\frac{y_i^{\mathrm{sim}}(p) - y_i^{\mathrm{obs}}}{\sigma_i} \right)^2 \right\}$$

- **Bayesian inference – posterior distribution**
  - **Prior probability distribution for *p*:** $f(p)$
  - **Posterior probability distribution for *p*:** $f\left(p | y^{\mathrm{obs}}\right) = f\left(y^{\mathrm{obs}} \middle| p\right) f(p)$

Three key components: Optimization algorithm

Derivative-based or algebraic solvers

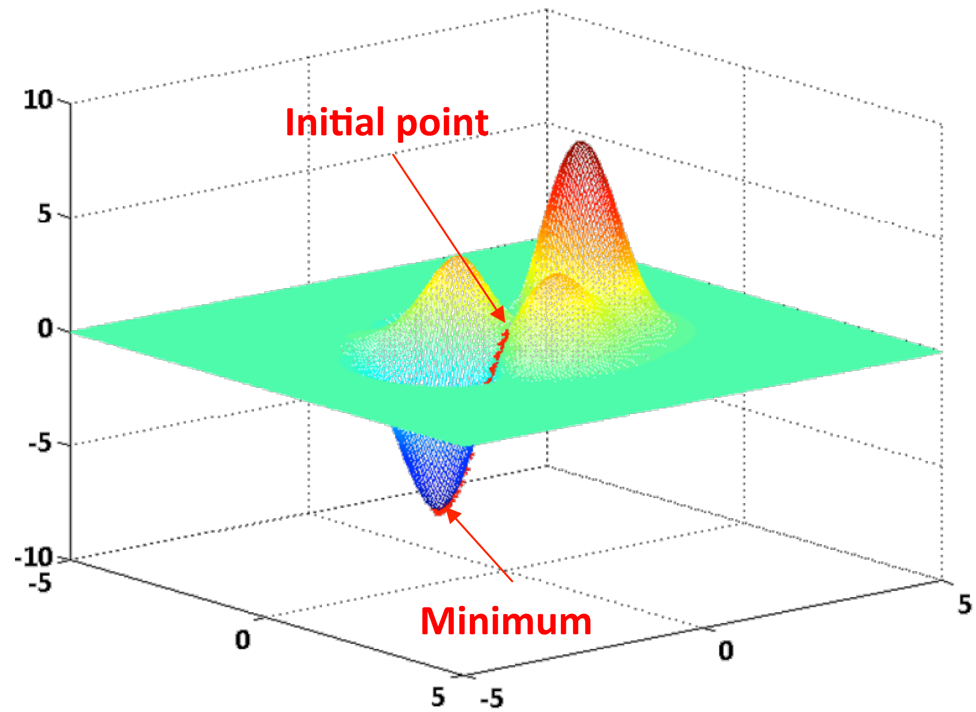
(BARON, CPLEX, IPOPT, CONOPT, etc.)

Pros:

Fast, efficient solutions with provably optimal solutions

Cons:

Need an algebraic model (access to derivative information)



Estimate gradient using central differencing

Three key components: Optimization algorithm

Derivative-free or black-box solvers

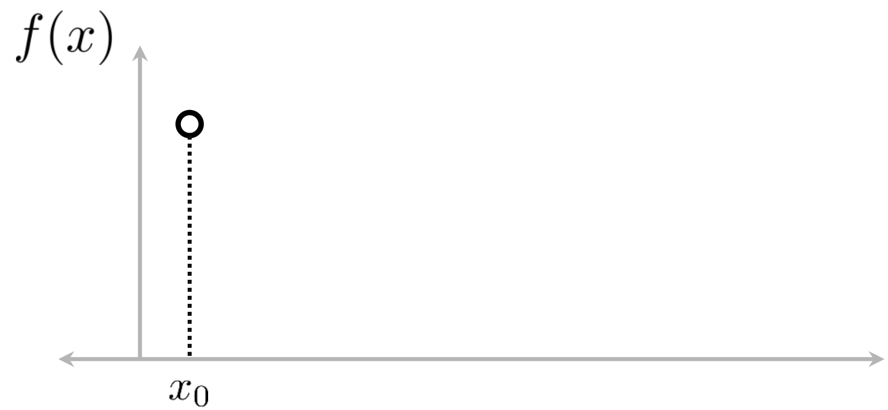
(Genetic algorithms, Matlab's fminsearch(), etc.)

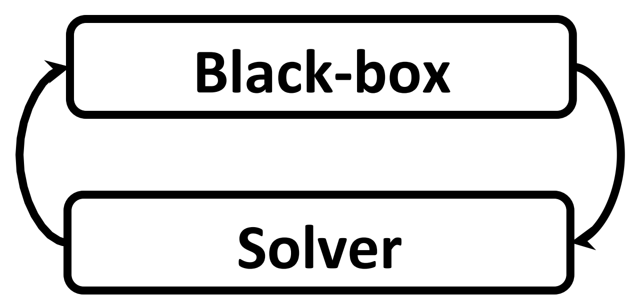
Pros:

Can solve problems where derivative information is unavailable, unreliable, or prohibitively expensive

Cons:

Slow and little to no guarantees that you have an optimal solution in the end





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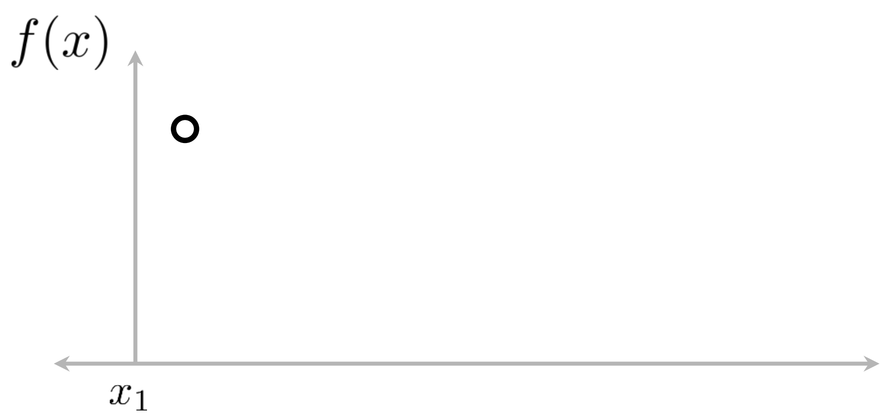

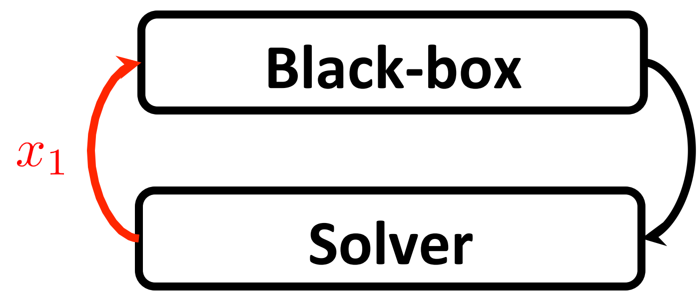

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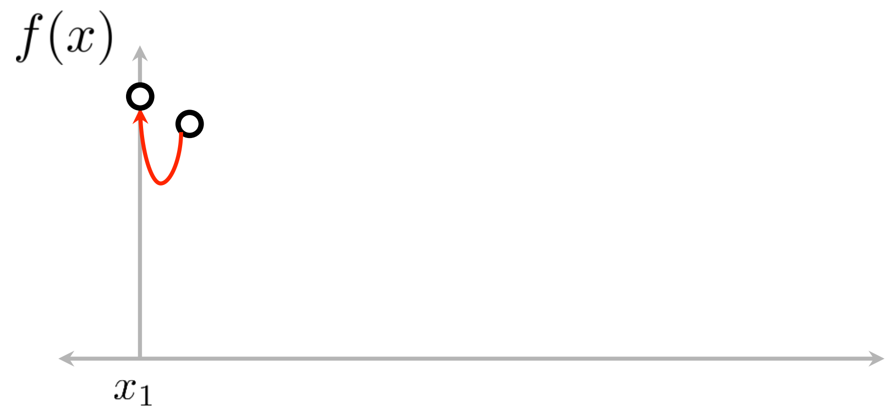

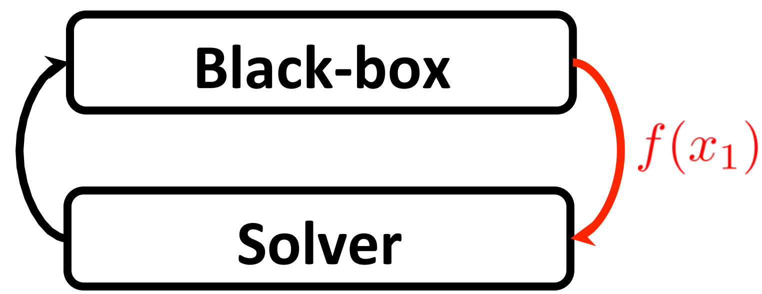

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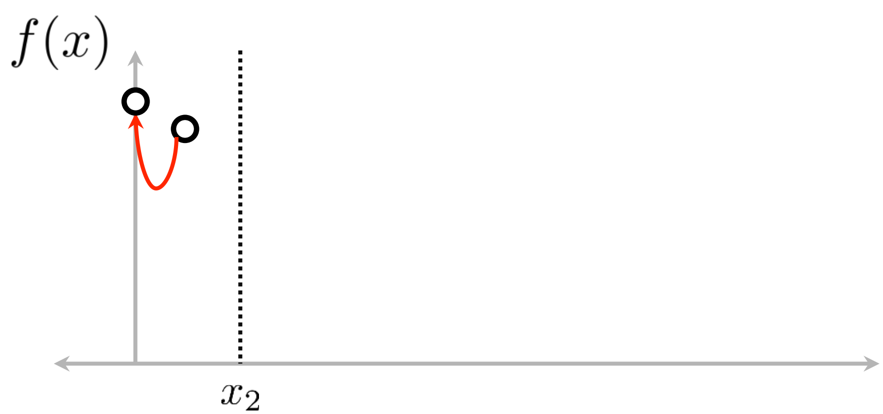

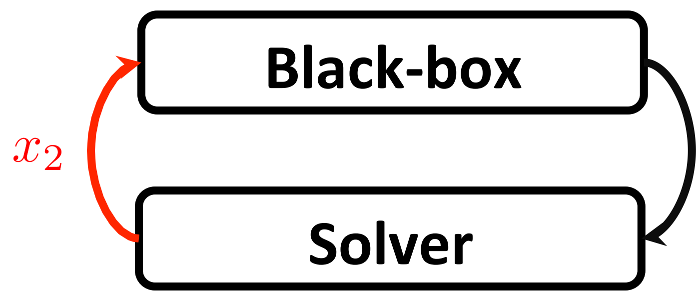

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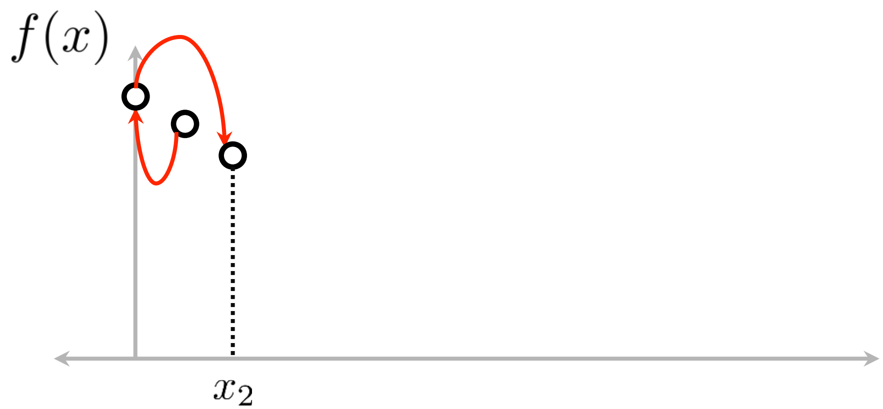

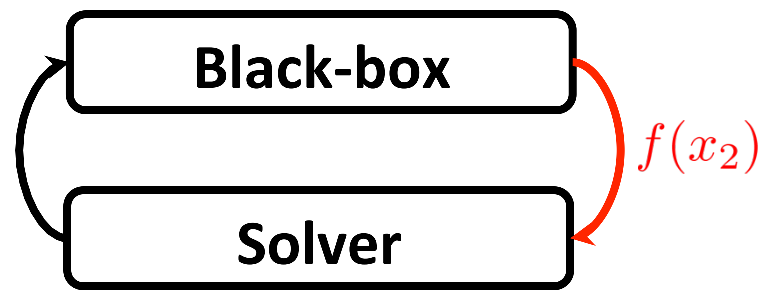

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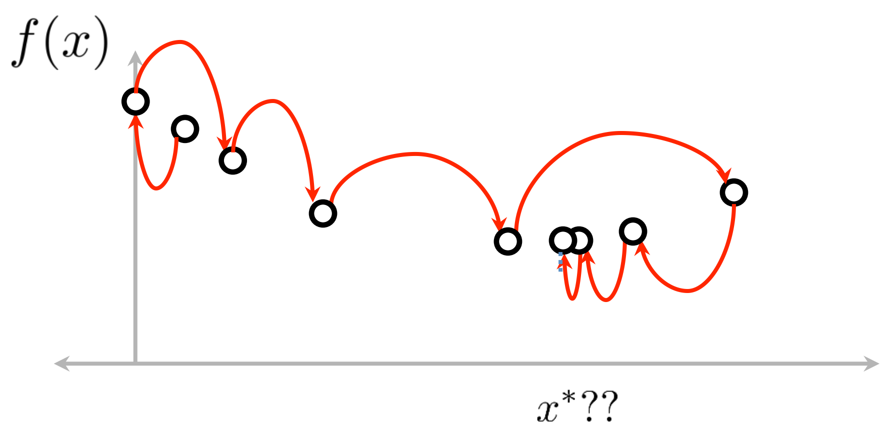
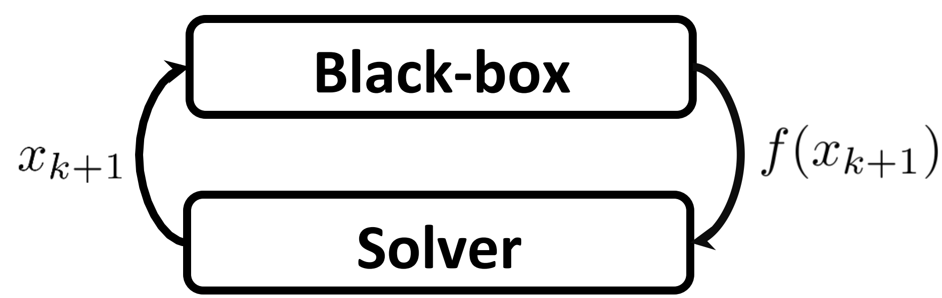
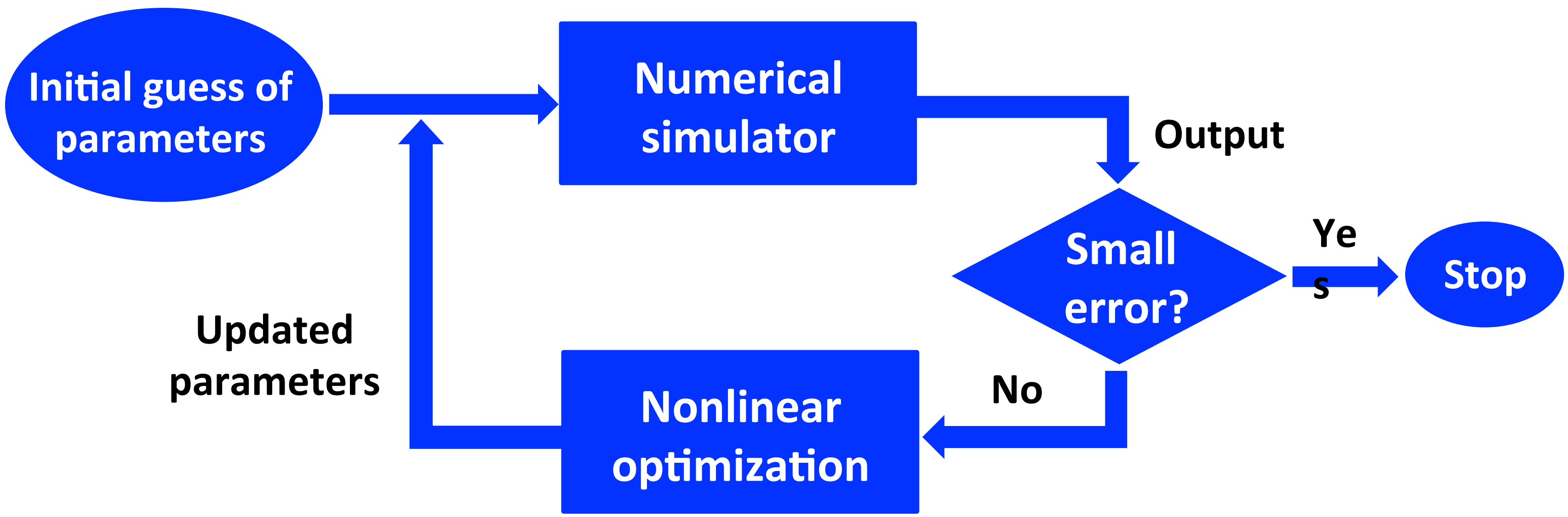




Procedure for estimating parameters



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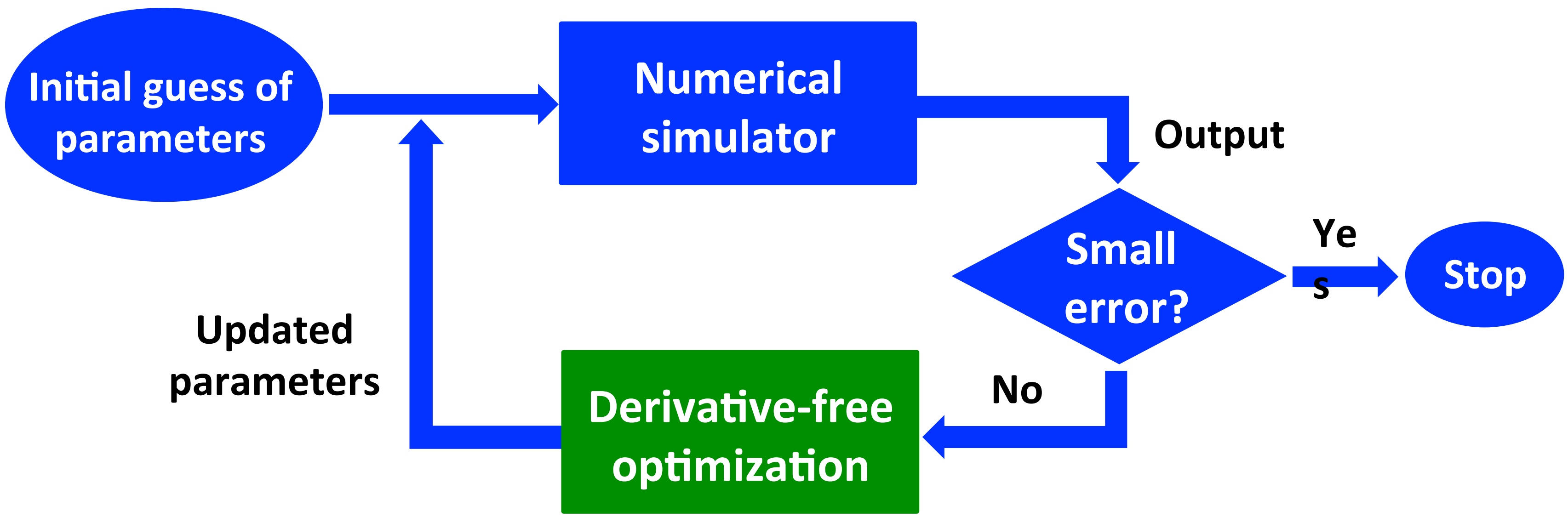
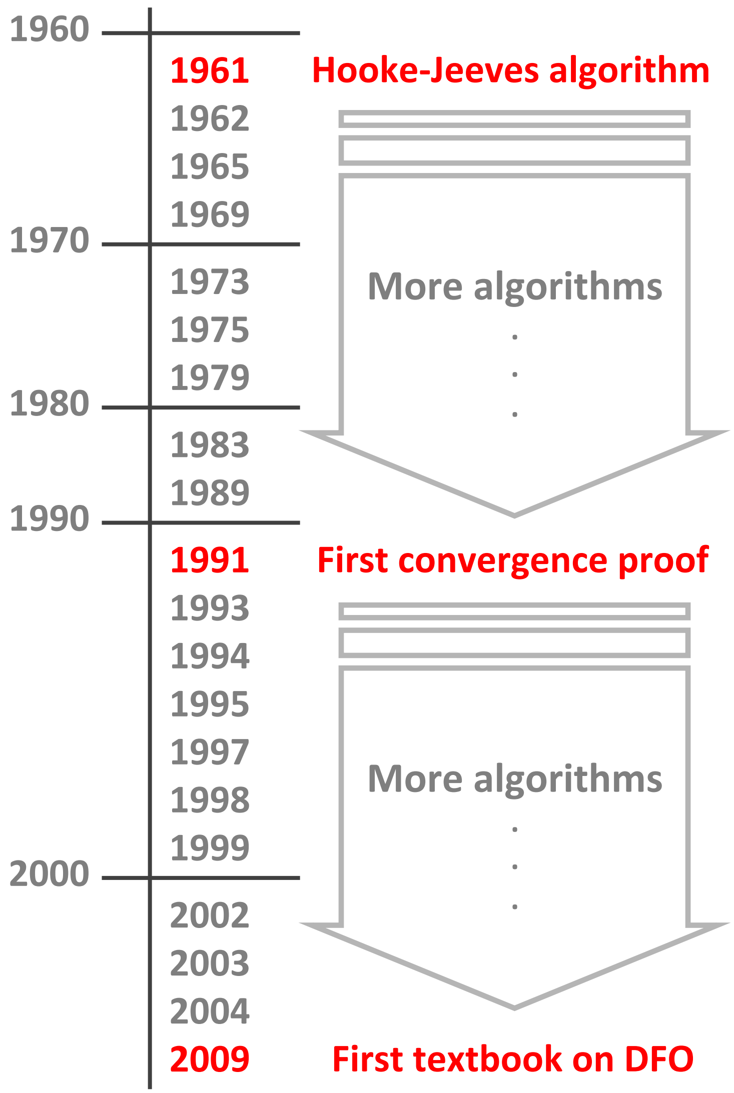
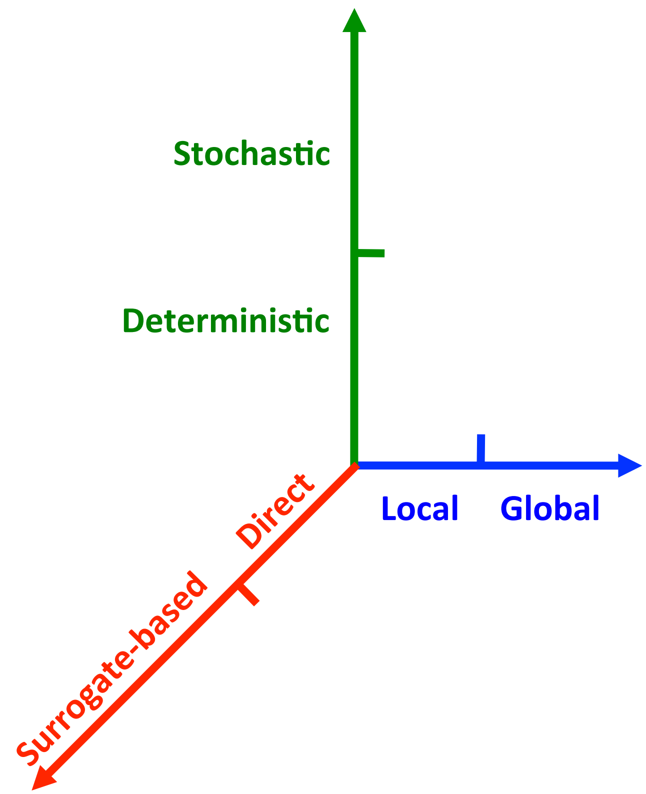


- **Performance of DFO solvers**
- **No systematic comparison of DFO solvers for parameter estimation**

Overview of Derivative-free optimization





Local DFO algorithms and software

Direct	• Nelder-Mead simplex algorithm	FMINSEARCH
	• Generalized pattern search and generating search set	DAKOTA PATTERN HOPSPACK SID-PSM NOMAD
Surrogate-based	• Implicit filtering	IMFIL
	• Trust-region methods	DFO BOBYQA NEWUOA

Global DFO algorithms and software

Direct

- Deterministic
    - Lipschitzian-based partitioning
    - Multilevel coordinate search

  `DAKOTA SOLIS-WETS`
  `DAKOTA DIRECT`
  `TOMLAB GLBSOLVE`
  `TOMLAB GLCSOLVE`
  `MCS`

- Stochastic
    - Hit-and-run
    - Simulated annealing
    - Genetic algorithms
    - Particle swarm

  `ASA`
  `CMA-ES`
  `DAKOTA`
  `EA`
  `GLOBAL`
  `PSWARM`

Surrogate-based

- Response surface methods

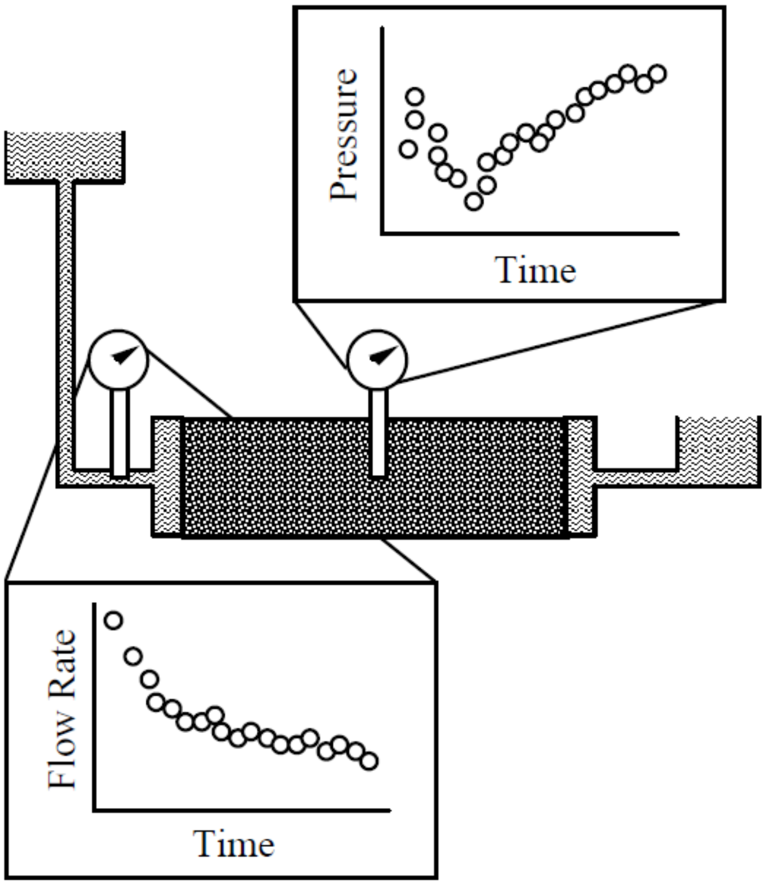
  `TOMLAB EGO`
  `TOMLAB RBF`

- Branch-and-fit

  `SNOBFIT`
  `TOMLAB LGO`

- Surrogate management framework

Case study I: Three parameters to be estimated



Water is injected at constant pressure into a 1D column filled with partially saturated sand

Known:

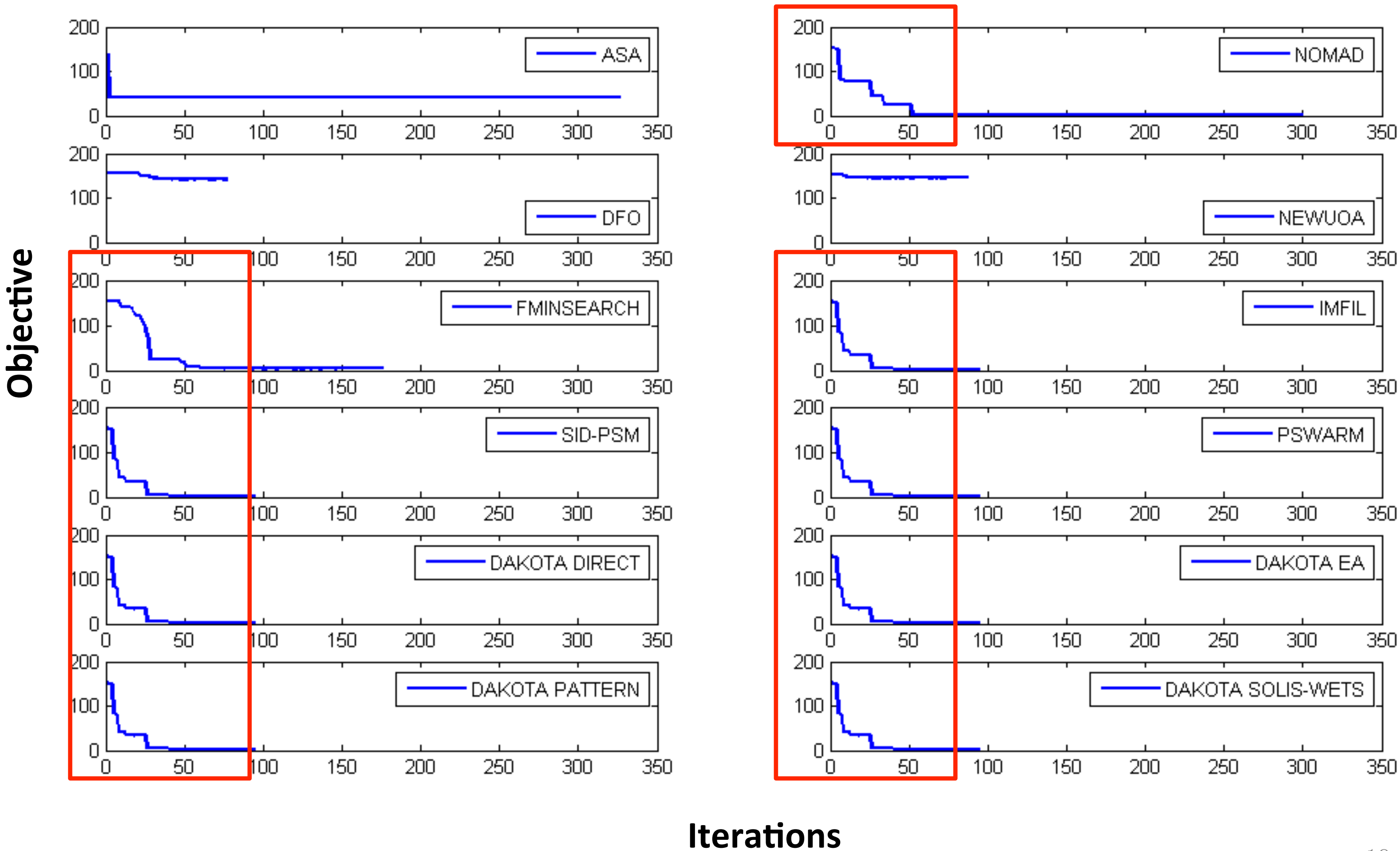
- **Pressure profile**
- **Flow rate profile**

Unknown:

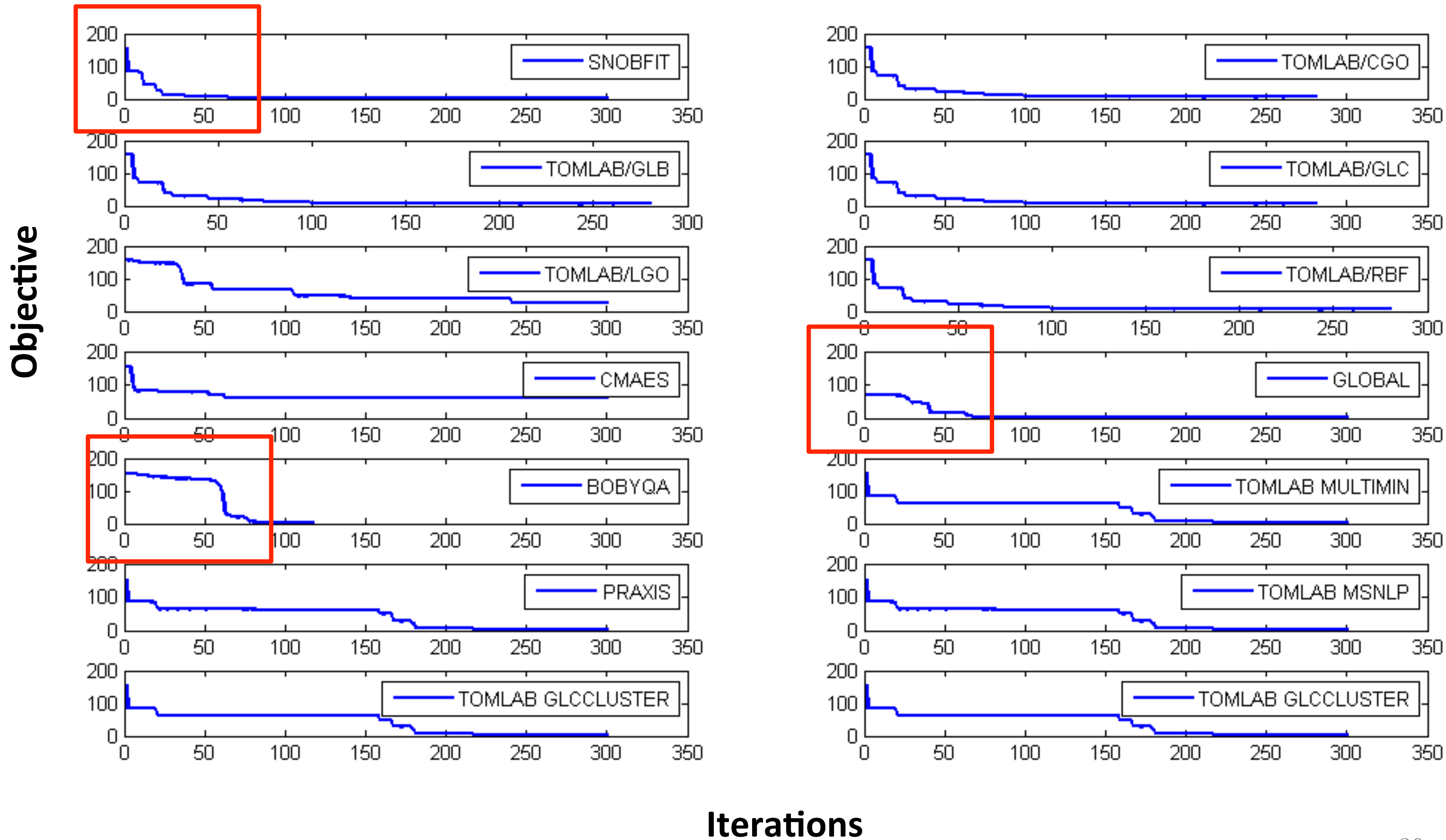
- **Porosity**
- **Permeability**
- **Initial gas saturation**

S. Finsterle, iTOUGH2 sample problems, LBNL-40042, 2007

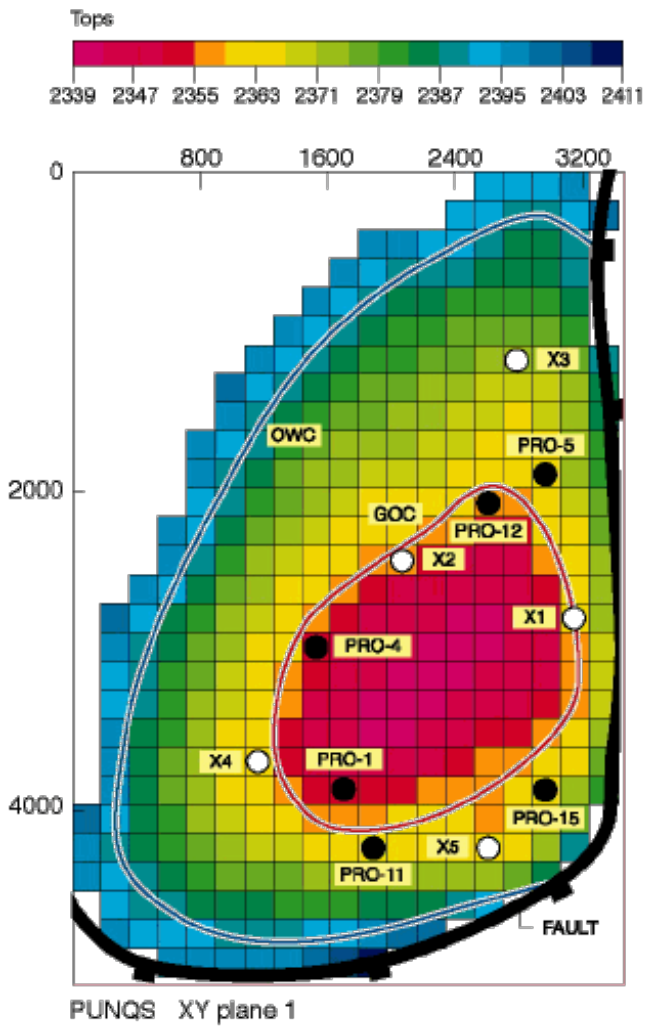
Performance of DFO solvers

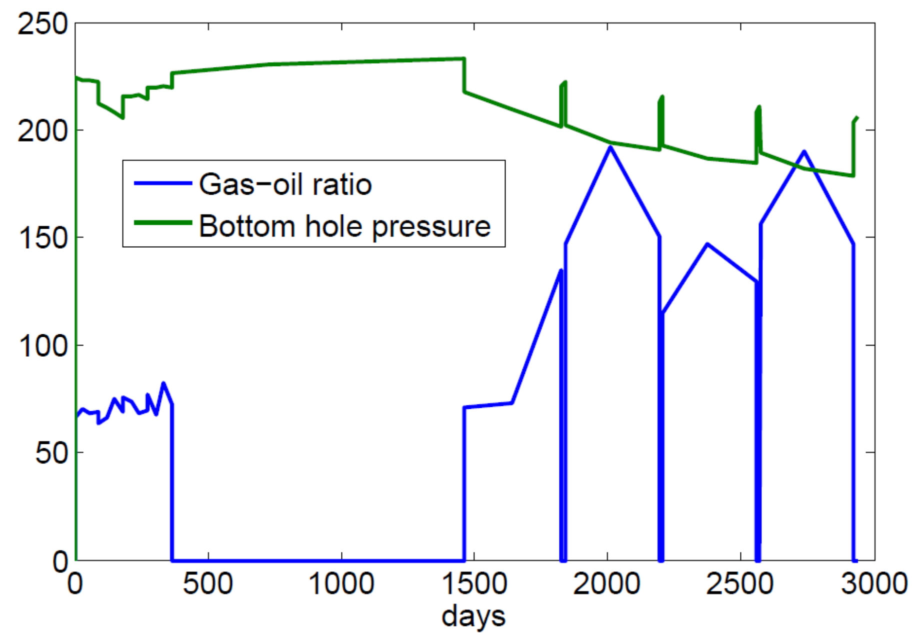


Performance of DFO solvers-cont'd



Case study II: 2660 parameters to be estimated



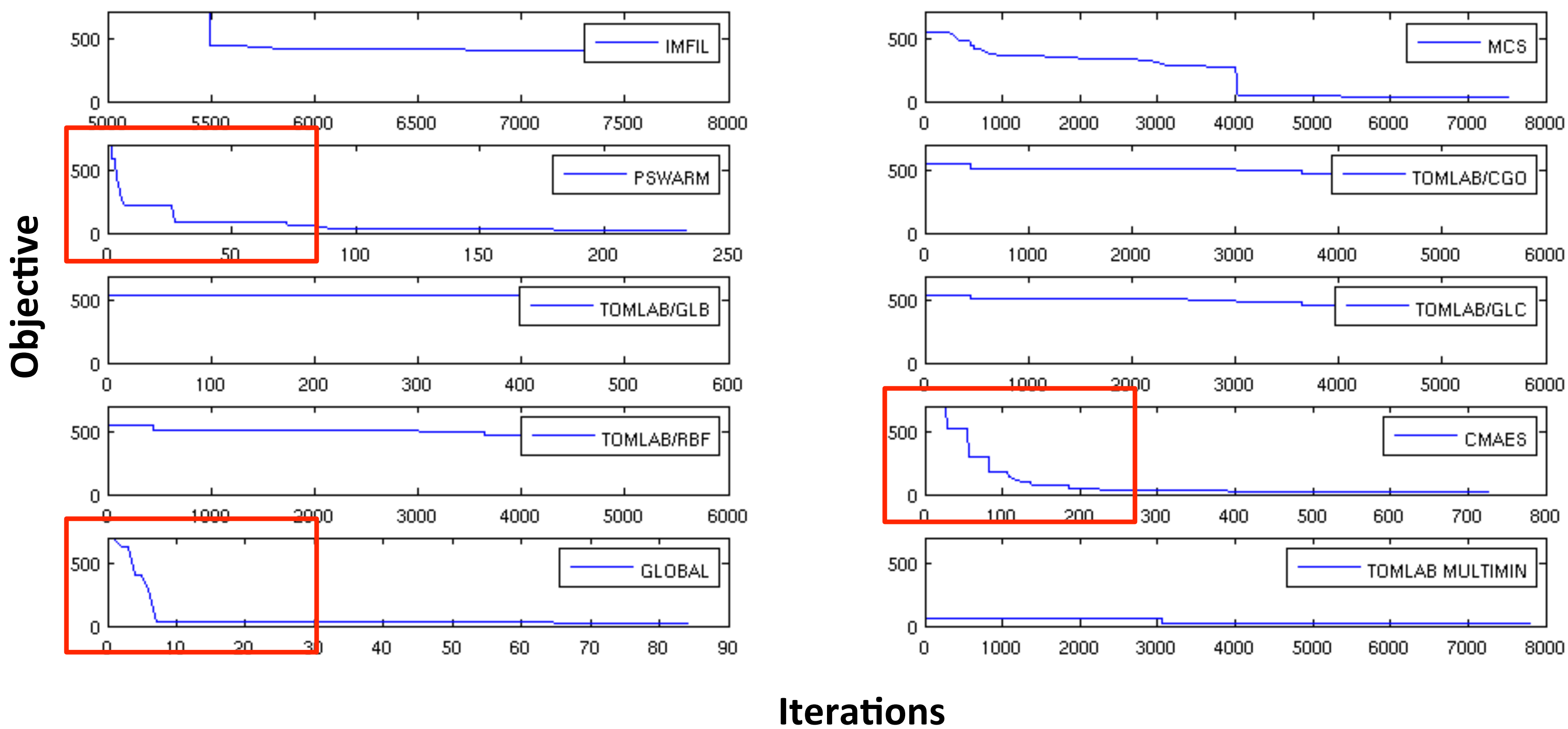


Production data for a well

- **Gridblocks: 19x28x5 = 2660**
- **2660 unknown porosities**
- **Permeability = function (porosity)**

http://www3.imperial.ac.uk/earthscienceandengineering/research/perm/punq-s3model

Performance of DFO solvers



Conclusions and future work

- **Conclusions**
  - **Systematic comparison of current DFO solvers in inverse modeling**
  - **DFO solver collection provides a robust package for inverse modeling**
    - **Small dimension problem:** `FMINSEARCH, SID-PSM, NOMAD, DAKOTA IMFIL, DFO, BOBYQA, NEWUOA`
    - **Large dimension problem:** `PSWARM, CMAES, GLOBAL`
  - **Performance depends on limits of computation resources such as CPU time and number of iterations**
- **Future work**
  - **Test with different measure of fitness**
  - **Apply DFO algorithms to permeability estimation of CO_2 sequestration in aquifer**