Dynamic RTO with Energy Pricing with application to a Gas Pipeline Network

ESI MEETING, Spring 2011

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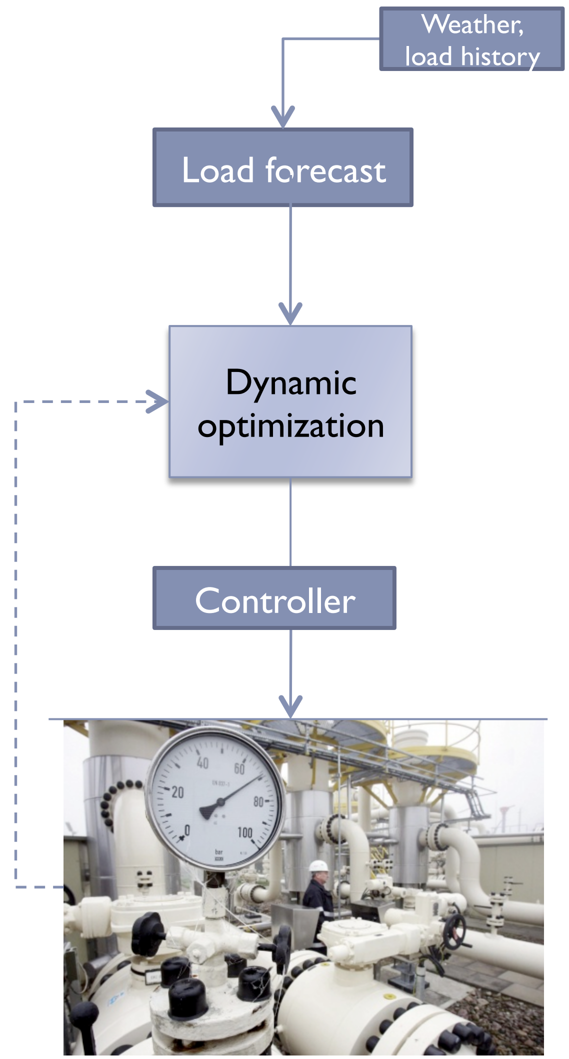
Background: Gas pipeline optimization

- Gas pipeline networks – Branched network consisting of compressor stations, suppliers and consumers (chemical industries, power plants, residential/commercial heating)
- Modeling & control challenge – Highly coupled nonlinear model describing flow of gas through a pipe.
- Optimization scope – Compression energy for the supplier to be minimized, while satisfying gas demands, contract pressures and physical constraints.
  - Moving Horizon Optimization – Repeated solution of dynamic optimization over a finite horizon.



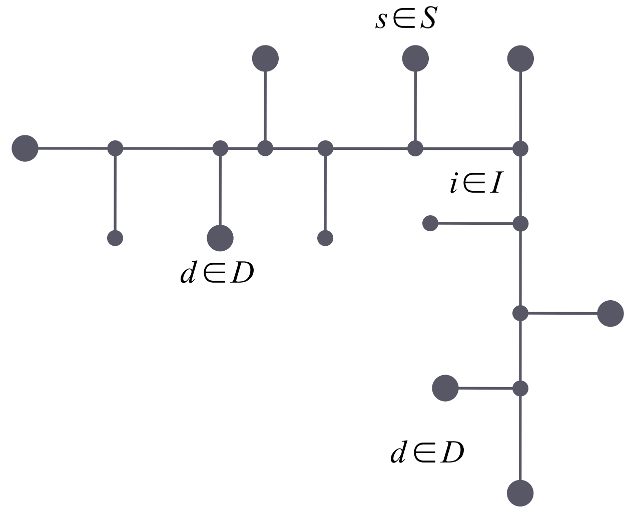
Pipeline modeling [Baumrucker & Biegler, 09]

- Pipe segment equations

  Material balance $\frac{M_W A_i L_i}{RT_{ref}}(\bar{P}_{i,t+1} - \bar{P}_{i,t}) = \int_t^{t+1}(q_i^{in} - q_i^{out})dt \quad \forall i \in I, t \in T$

  Momentum balance $\left.\frac{dP}{dz}\right|_{i,k,t} = \frac{-f_{i,k,t} RT_{ref} q_{i,k,t}|q_{i,k,t}|}{2D_i A_i^2 M_W P_{i,k,t}} \quad \forall i \in I, t \in T, k \in K$

  Network inventory $mass_{i,t} = \frac{\bar{P}_{i,t} M_W A_i L_i}{RT_{ref}} \quad \forall i \in I, t \in T$



- Node equations
  - Flow balance: no accumulation at nodes.
  - Pressure balance: pressure at a node is equal to the pressure either into or out of the connected arc.
- Compressor equation: $Power_{s,t} = q_{s,t}^{Supply} \frac{C_p T_{ref}}{\eta}\left[\left(\frac{P_{s,t}}{P_{inlet}}\right)^{(\gamma-1)/\gamma} - 1\right] \quad \forall s \in S, t \in T$
- Laminar friction factor/Colebrook-White turbulent friction factor
- Switches for permitting flow reversals, and non-smooth flow transitions (laminar to turbulent)

SET DEFINITIONS

I	Arcs (pipe segments)
J	Nodes (intersection)
$K = \{in, out\}$	End-points of the pipe
S	Suppliers in network
D	Demands in network
$T = \{0,\ldots,t_f\}$	Time points

B.T. Baumrucker and L.T. Biegler. MPEC strategies for cost optimization of pipeline operations. doi:10.1016/j.compchemeng.2009.07.012

Dynamic Optimization Formulation

- Formulation (NLP):

$$\min \; F(x, u, p) + Regularization$$

$$s.t.\;$$ Discretized dynamic pipeline model

Physical Constraints

Terminal Constraints ($x_N \in \varepsilon$)

- Objective function (Energy minimization):

$$F = \underbrace{\sum_{s\in S} \int_{t=t_0}^{t=t_0+T_P} Power_{s,t}\,dt}_{\text{Total Energy}}$$

$$\text{Regularization} = \underbrace{\rho \sum_{s\in S} \sum_{t\in T\setminus\{t_0\}} (Power_{s,t} - Power_{s,t-1})^2}_{\text{Smoothing term}}$$

Term	Meaning
x	Pressures & flow rates at various points
u	Supplier discharge pressures and supplier flow rates for $t \in T$
p	Forecast of future demand loads for $t \in T$

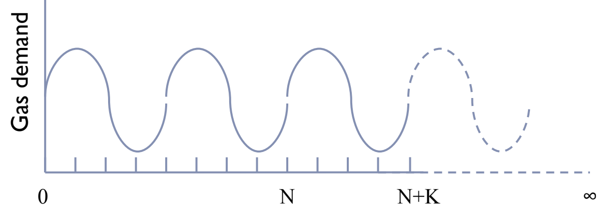
Constraints:

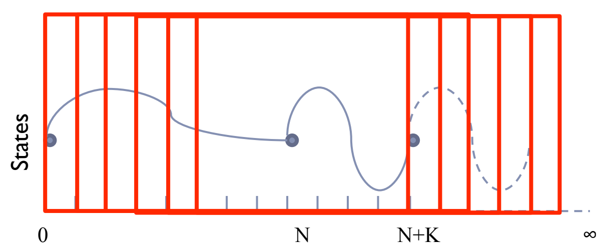
- **Delivery points:** Contract pressures should be satisfied.
- **Supplier points**: Compressor limits on minimum & maximum discharge, pressures and work.
- **Linepack/Inventory targets:** Restoring sustainable gas pipeline inventory and pressure.

$$\sum_i mass_{i,t_f} \geq \sum_i mass_{i,0}$$

Nonlinear MPC Formulation

- Gas demand is diurnal and periodic.





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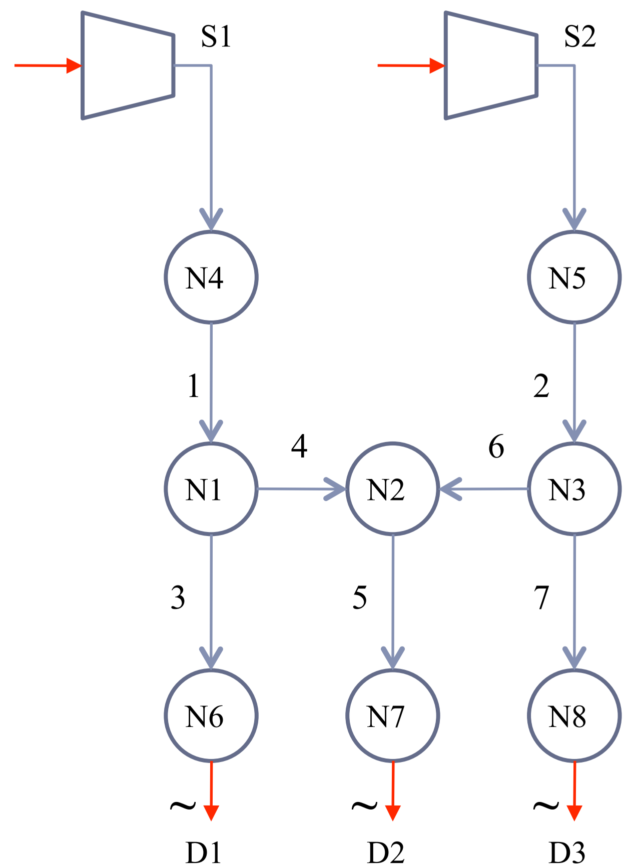
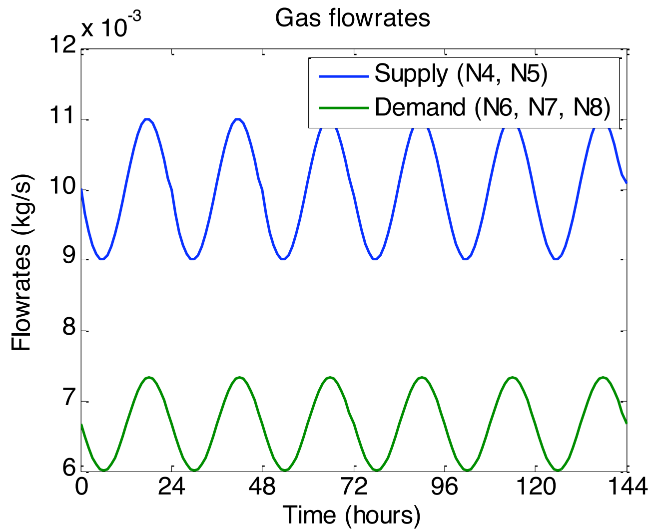
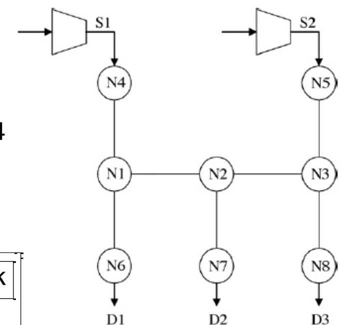
Scenarios & Simulation details

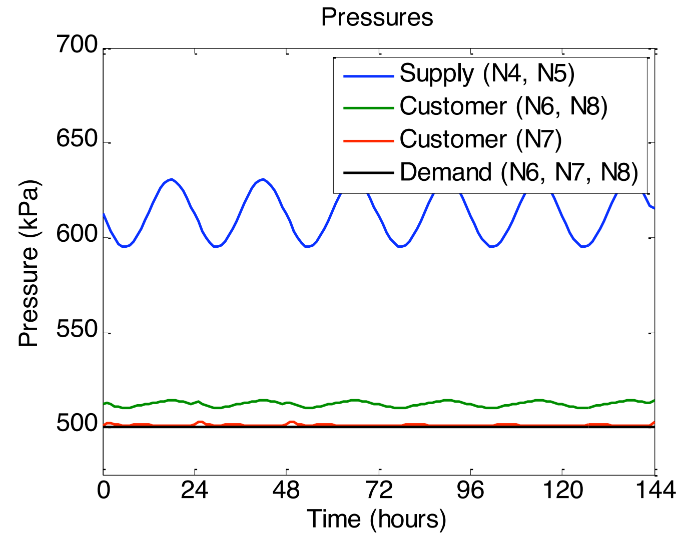
- Gas demand: Average of 24 kg/h. Sinusoidal with a period of 24-h and 5% amplitude of oscillations.
- Contract pressure: 500 kPa.
- Length of horizon (T_P) : 2 days.
- Time discretization: 1-h.
- NLP size:
  - Equations: 12,066
  - Variables: 12,302
- Solver used: IPOPT (Interior point Optimizer for large scale NLPs).

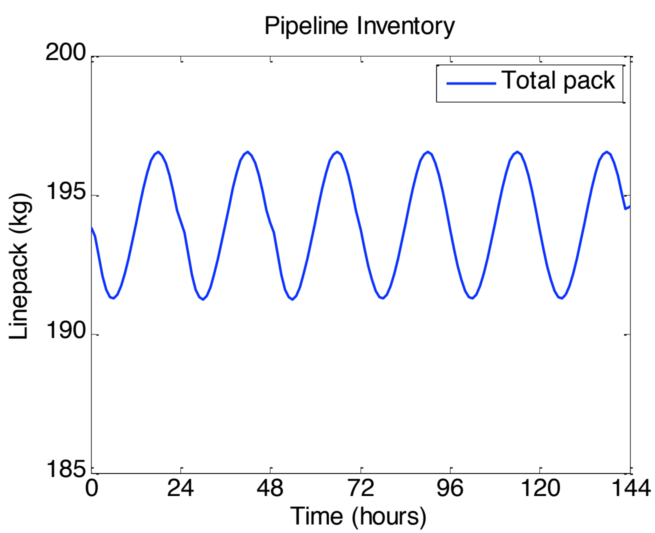


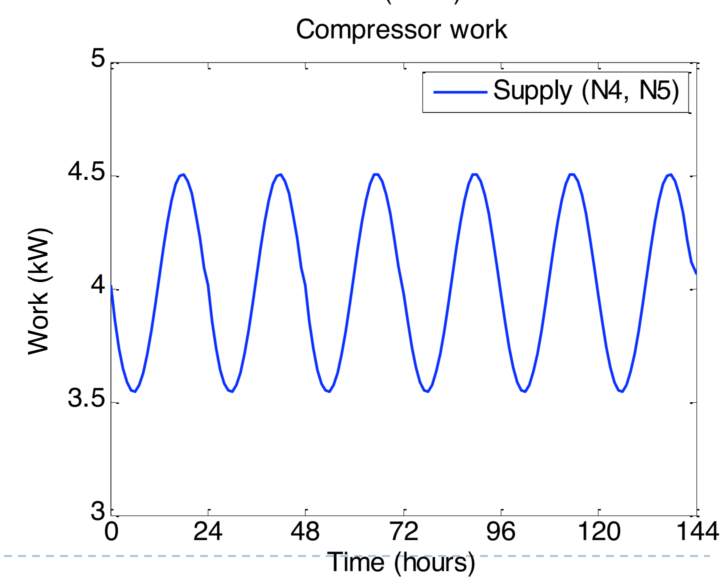
Case study 1 – Energy minimization



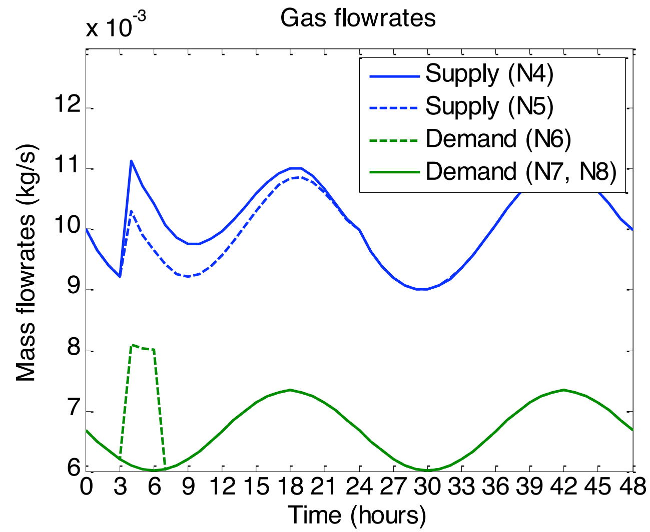


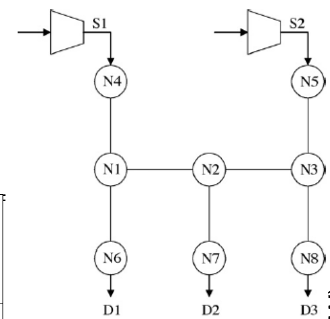


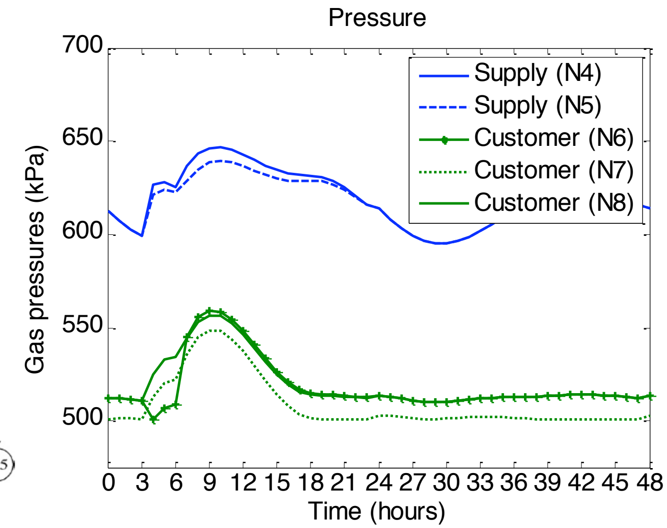


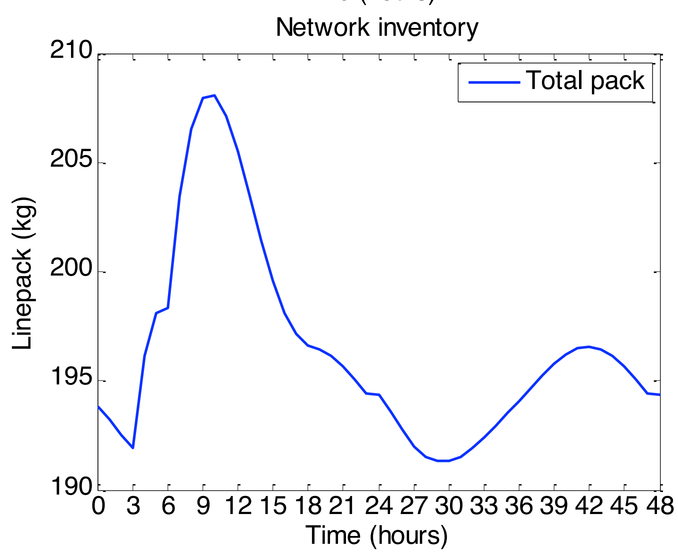


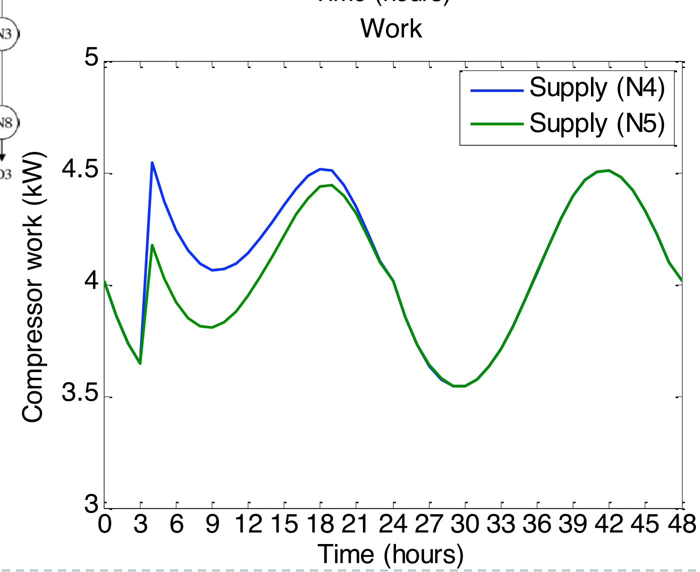
Case study 2 – Energy minimization w/ Disturbance









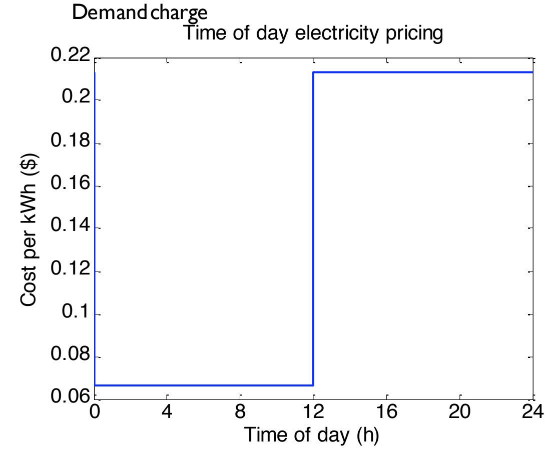


Electricity pricing

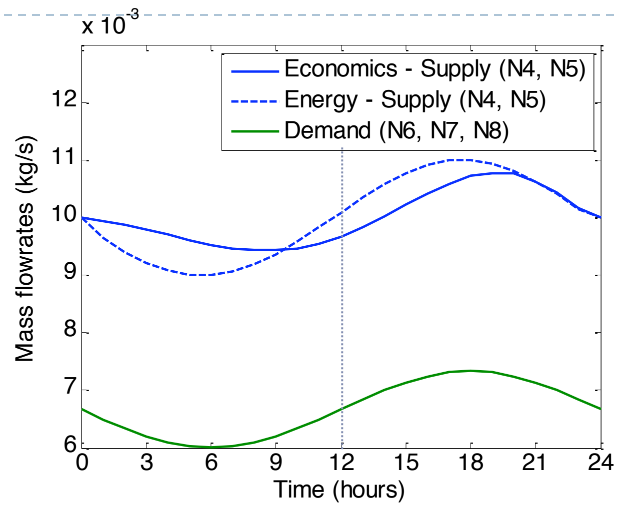
- Complex energy pricing schemes: Electricity prices vary through the day and consumers are encouraged to use more when power is cheaper.
  - *Time of day pricing*: Two 12-h time periods. Cheaper electricity during off-peak periods. Variations are seasonal.
  - *Day ahead pricing*: Hourly electricity prices decided at the beginning of the day.
  - *Real-time pricing*: Prices vary real-time (hourly) based on spot-market.
- Objective function modified to include economics:

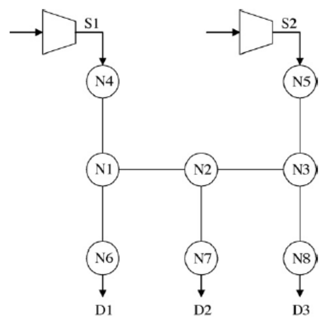
$$J(t_0) = \underbrace{\sum_{s\in S} \int_{t=t_0}^{t=t_0+T_P} Power_{s,t}\,\boldsymbol{Cost_{s,t}}\,dt}_{\text{Energy charge}} + \underbrace{\rho \sum_{s\in S} \sum_{t\in T\setminus\{t_0\}} (Power_{s,t} - Power_{s,t-1})^2}_{\text{Smoothing term}} + \underbrace{c_d \max_t (\sum_{s\in S} Power_{s,t})}_{\text{Demand charge}}$$

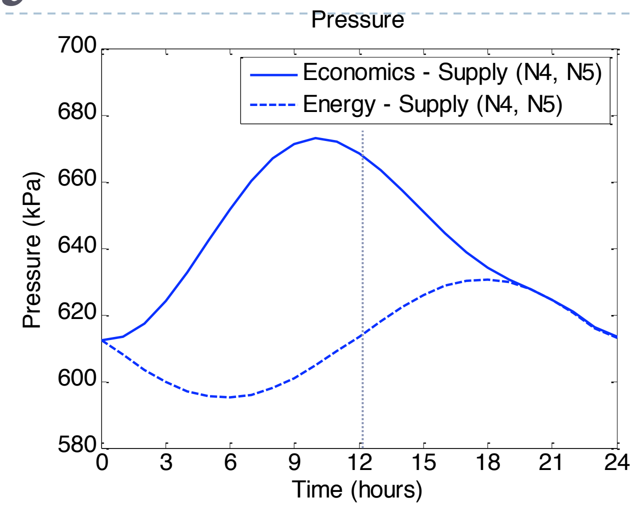
- Time of day pricing data (SRP Utility):
  - Off-peak: $Cost_t = \$0.0662$ per kWh
  - On-peak: $Cost_t = \$0.2130$ per kWh
  - Demand charge coefficient $(c_d) = 0.1435$

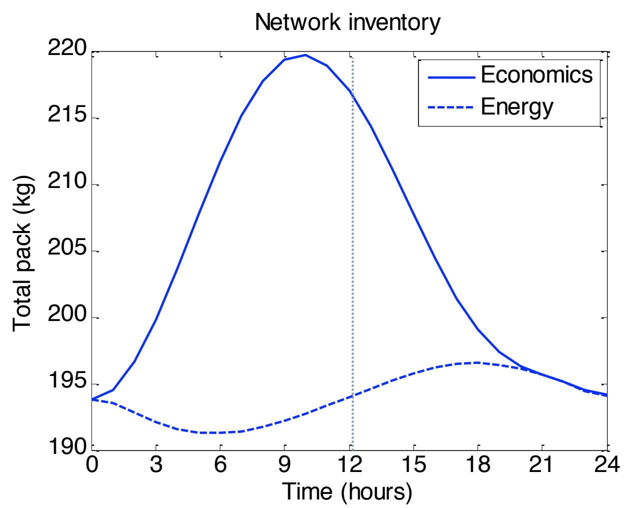


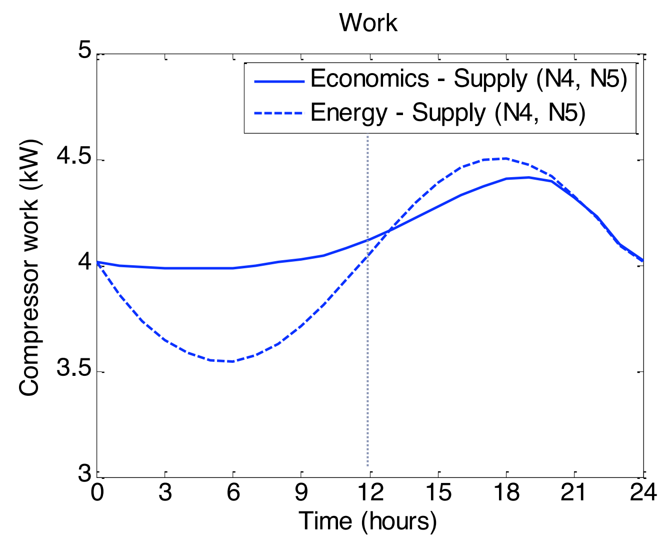
Case study 3 – Economics











- Cost savings of 1.80% over a month due to flattening of work profiles & reduction of demand charge though 2.91% more work is done.

Conclusions

Pipeline optimization & Control:

- Moving horizon control scheme developed for optimal operation of pipelines – handles varying demand forecasts, inventory depletion, satisfies all constraints.
- State-of-art NLP solvers : ~10 secs of CPU time per moving horizon iteration.

Control theory:

- Stability of Economically-oriented NMPC with Periodic Constraints [Huang & Biegler, 2010]
  - With long enough prediction horizon stability can be proved for an equality terminal constraint or discount factor MPC formulation.

Current:

- Working on simulations for a larger pipeline network (3 supplier and 22 demand nodes) from literature.

Thank you

Questions & Comments?