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OF NEW JERSEY

Optimizing Work and Heat Flows in Sustainable Chemical Processes Using Attainable Regions

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Introduction

- There is a growing pressure to reduce global CO_2 emissions
- Chemical conversion of fossil fuels for the production of energy and chemicals are the major source of anthropogenic CO_2 emissions
- This means that there is a need to:
  - Develop processes with reduced CO_2 emissions
  - Incorporate renewable energy into processes
  - Utilize new feeds such as CO_2
- How do we design and optimize these processes?
  - Can we develop simple methods to analyze these processes to understand the relationship between material, energy and work flows?

- Image from: https://cdn.pixabay.com/photo/2017/06/04/04/16/warming-2370285_1280.jpg

Examples of Opportunities: Mature Technologies Conversion of Methane to Methanol

Significant potential exists for improving process performance even for this mature technology.

	Tons of CH_4 Consumed/ Ton of Methanol produced	Tons of CO_2 Emitted/ Ton of Methanol produced
Current Processes	0.53	0.09
Target (without incorporating renewable energy)	0.43	-0.19

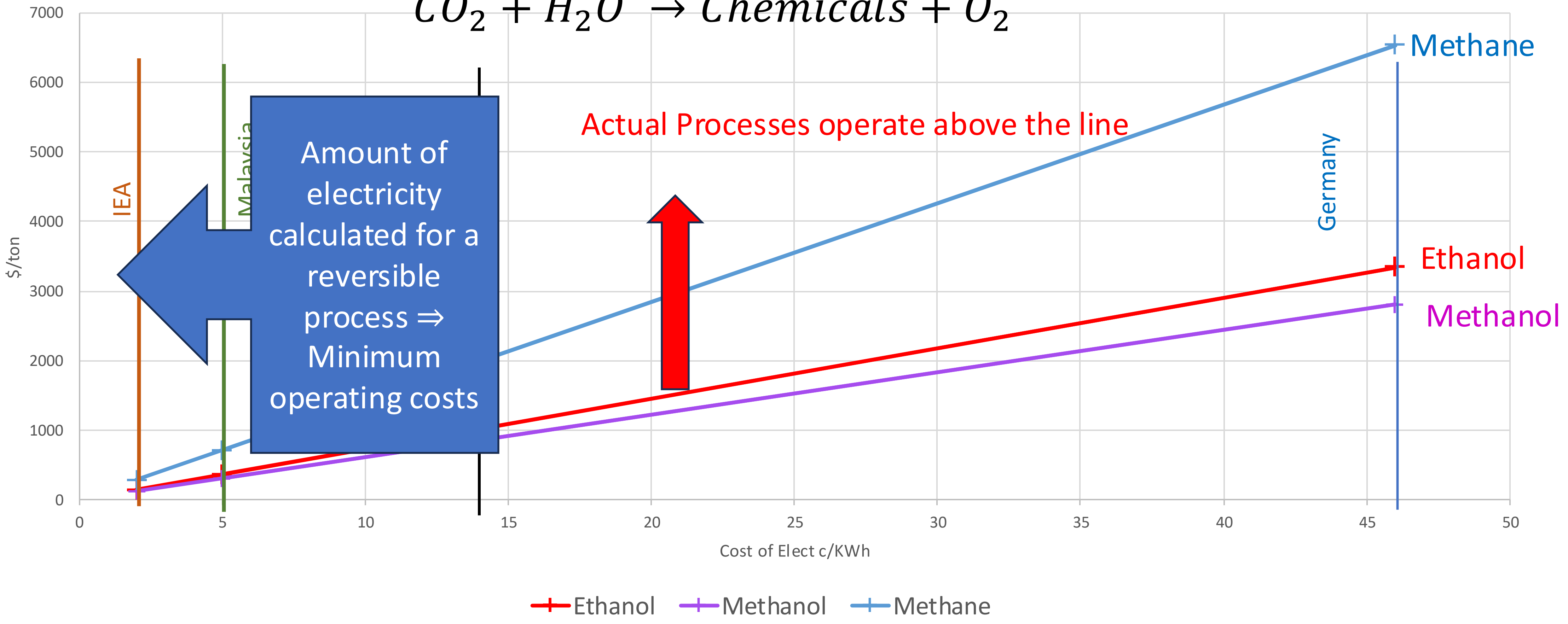
Examples of Opportunities: Emerging Technologies. Conversion of Cellulosic Biomass to Ethanol

- For every mole of C in the feed:
  - 0.35 is converted to ethanol
  - O.65 is emitted as CO_2

- Can we do better than a 35 % carbon conversion efficiency?

Examples of Opportunities: New Technologies Conversion of CO_2 to chemicals

$$CO_2 + H_2O \rightarrow Chemicals + O_2$$

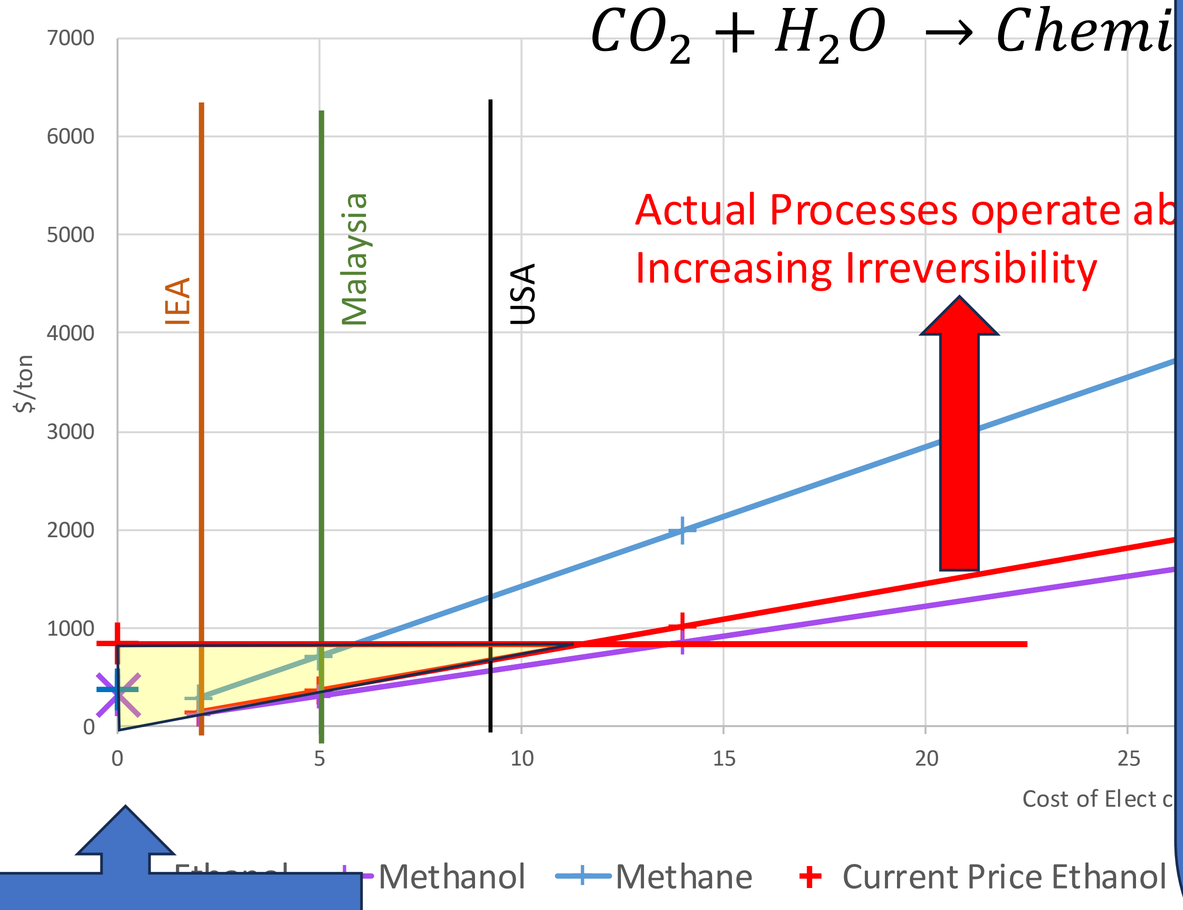


Amount of electricity calculated for a reversible process ⇒ Minimum operating costs

Actual Processes operate above the line

Examples of Opportunities: Conversion of CO_2 to che

$$CO_2 + H_2O \rightarrow Chemi$$

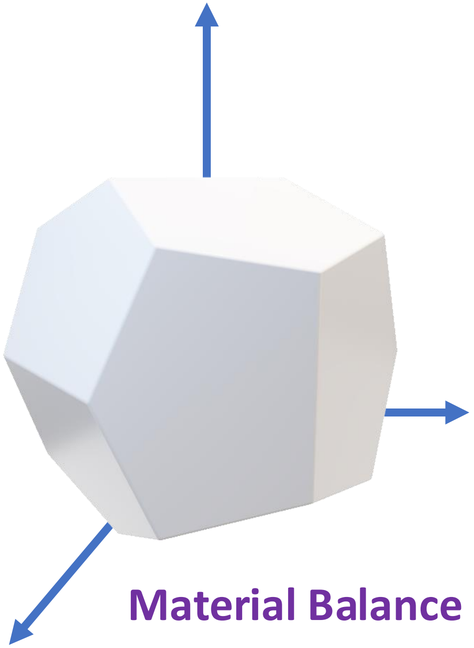


Actual Processes operate ab
Increasing Irreversibility

Current Selling prices

- Any inefficiency in process design or operation further increases electricity consumption.
- We may need to reconsider our design paradigms and methods to enhance process efficiency and thereby more closely approach process reversibility (minimum electricity consumption)
- This talk is based on the premiss that the choice of reaction pathways and operating conditions contributes to major process irreversibility

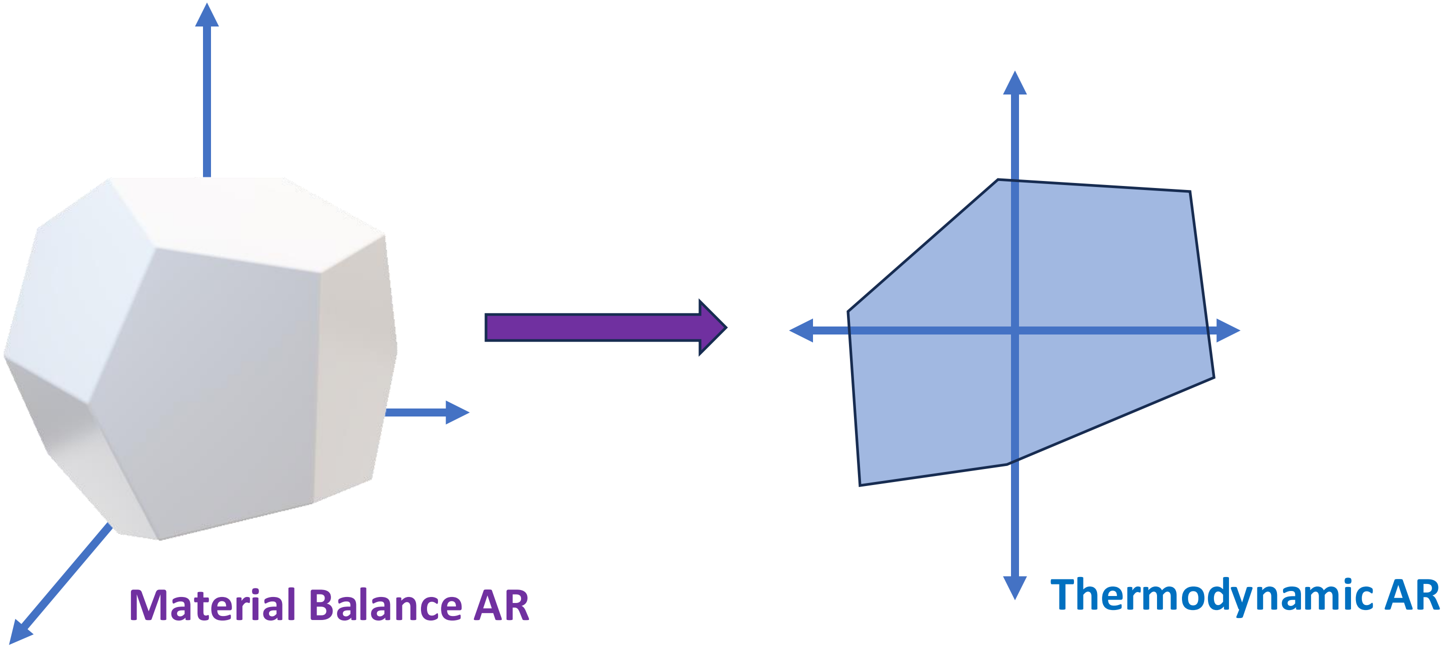
Approach Used in this Research



Material Balance Limited Attainable Region

- Start by finding the ***Material Balance Limited Attainable Region***
  - Region of all concentrations stoichiometrically compatible with a given feed
  - We will discuss some of the important properties of this region

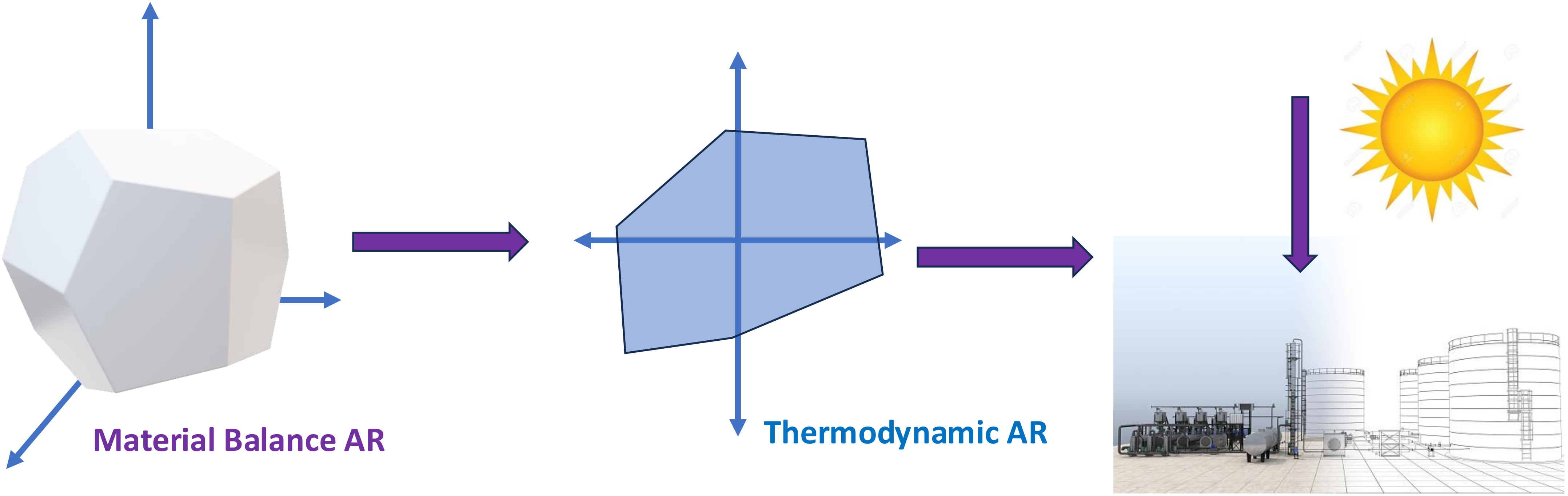
Approach Used in this Research



We will then use the ***Material Balance Limited Attainable Region*** to calculate the ***Thermodynamic Attainable Region***

- A point in the **Thermodynamic Attainable Region** represents the quantity of heat and work flow between the process and the environment
  - We will discuss some of the important properties of this region

Approach Used in this Research



Material Balance AR

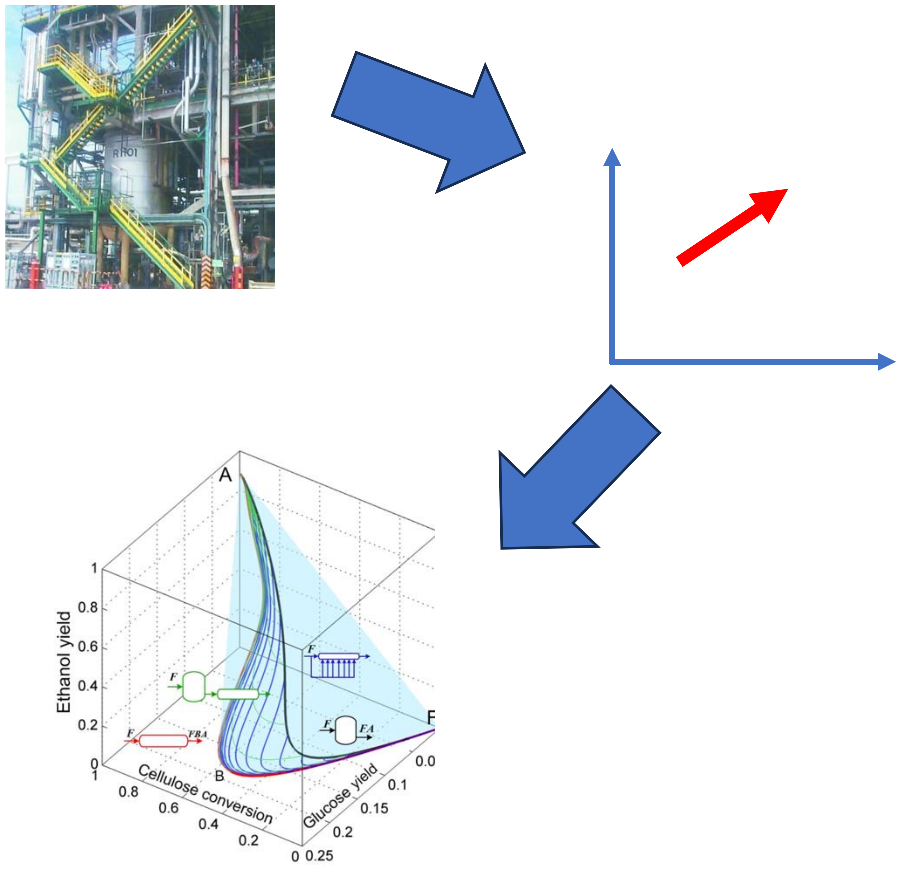
Thermodynamic AR

- We will then discuss the implication of the heat and workflows on process
- Will finally consider the addition of work and heat using renewable energy
- We will use a geometric approach based on vectors

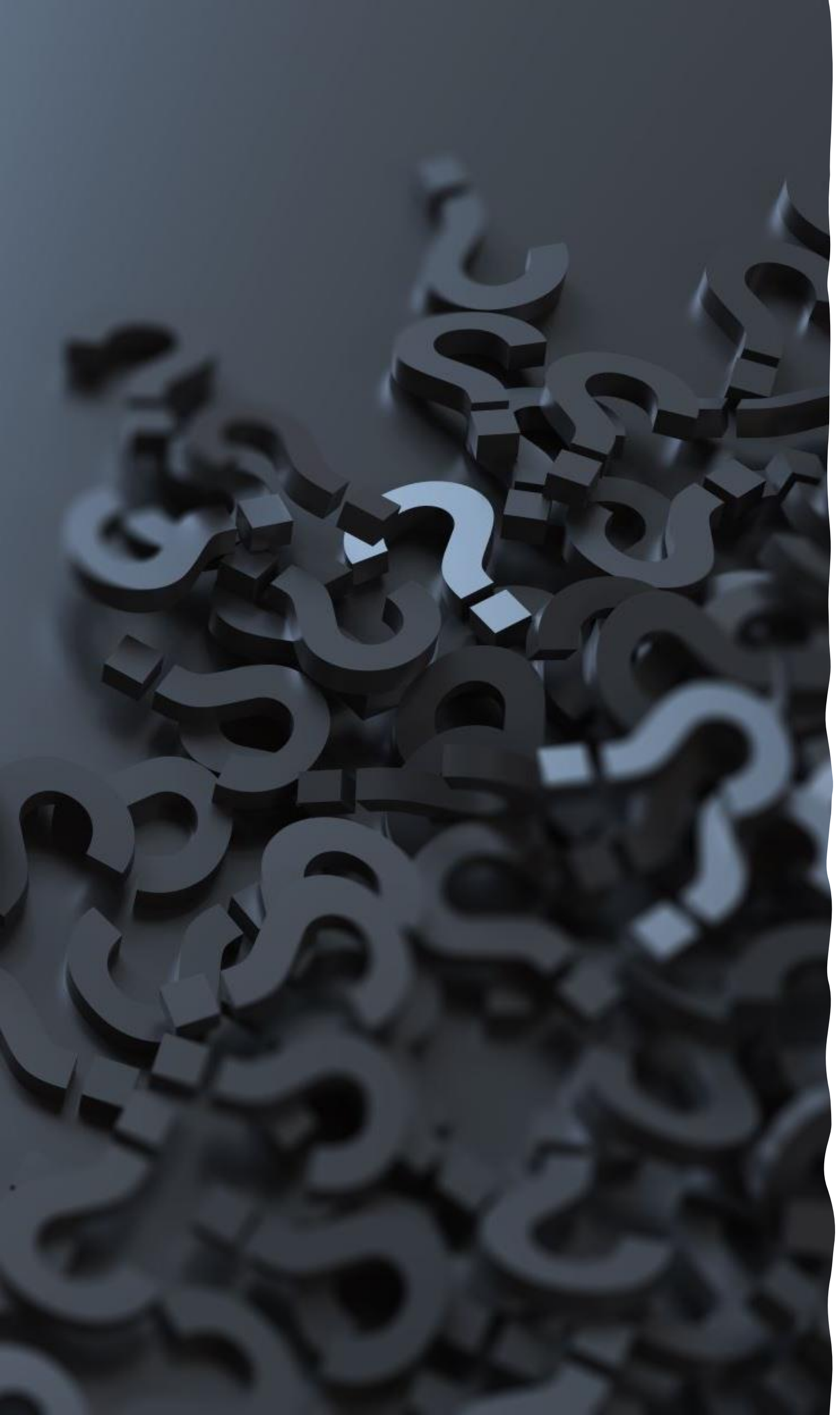
What is an Attainable Region (AR)?

Fritz Horn (1964) addressed the problem of finding an optimal reactor

- He proposed finding the set of all possible outputs for all possible reactors, for a given feed and reaction kinetics.
- He called this set of outputs the **Attainable Region (AR)**
  - If one could find the AR, one could search the region for a given objective function and find the point corresponding to the optimal value of the objective function;
  - This point would correspond to the output of the optimal reactor.
- He was unable to develop a method to find the AR
- Glasser and Hildebrandt (1987) developed a geometrical-based approach to determine the AR for reaction and mixing
  - Other researchers have since used this approach and developed it further



Scott, F., Conejeros, R. & Aroca, G. Attainable region analysis for continuous production of second generation bioethanol. *Biotechnol Biofuels* **6**, 171 (2013). https://doi.org/10.1186/1754-6834-6-171

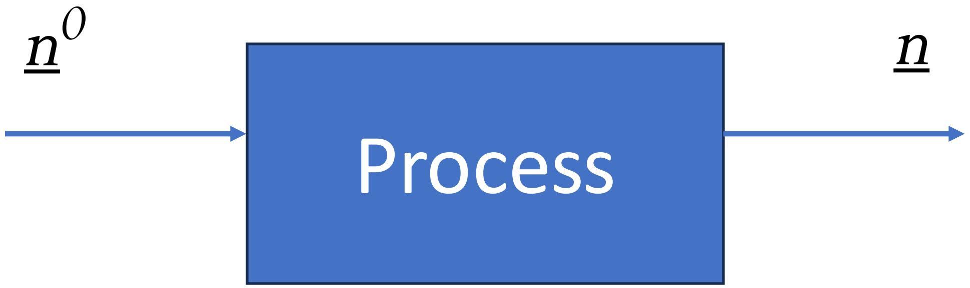


Extension of the AR to other situations

- We can extend the concept of the Attainable Region (AR) to other situations
- In general, we can consider an **AR** to be the set of all possible outputs (sets of values of the choice variables) of a problem that satisfy the problem's constraints.
- The dimensionality of the region is $\mathcal{R}^{m,}$ where m is related to the number of variables.

Definition: Material Balance Limited Attainable Region AR^{MB}

- The Material Balance Limited Attainable Region (AR^{MB}) is the set of all possible outputs $\underline{\boldsymbol{n}}$ for a given feed $\underline{\boldsymbol{n}}^{\boldsymbol{o}}$ that is consistent with the material balance constraints, where $\underline{\boldsymbol{n}}$ is the molar flowrate
  - The AR^{MB} does not take other constraints such as kinetics, equilibrium, energy, etc. into account



$\underline{n}^{0}$

Process

$\underline{n}$

Previous work on the AR^{MB}

- The AR^{MB} has previously been referred to as the stoichiometric compatibility class or reaction hyperplane.
- Methods exist for finding the AR^{MB}
- In this talk, we will discuss some of the properties of the AR^{MB} that are useful for process synthesis
  - Use an example to demonstrate this

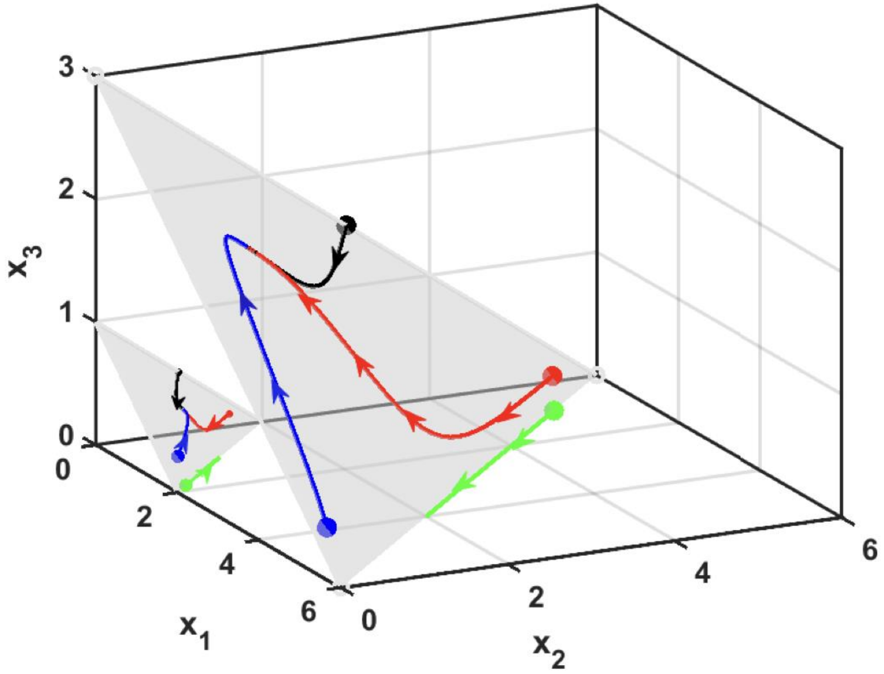
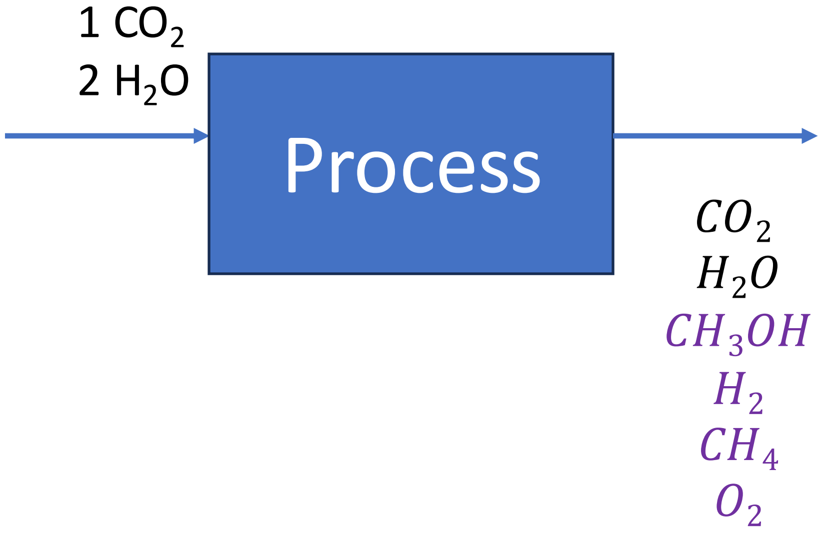


Fig. 1. Chemical stoichiometric compatibility classes and trajectories of the system in Example 3.

Taken from: Zhang, X., Fang, Z., Gao, C., & Dochain, D. (2023). On the relation between ω-limit set and boundaries of mass-action chemical reaction networks. *Automatica*, *149*, 110828

Example: Production of Chemicals from CO_2 and H_2O



1 CO_2
2 H_2O

Process

CO_2
H_2O
CH_3OH
H_2
CH_4
O_2

- Consider:
  - a feed of CO_2 and H_2O, in the ratio 1:2
  - possible products: CH_3OH, H_2 and CH_4 and O_2

- There are 3 Independent Material Balances:
  - $H_2O \rightarrow H_2 + \frac{1}{2} O_2$ $\qquad \varepsilon_1$
  - $CO_2 + 2H_2O \rightarrow CH_3OH + {}^3/_2\, O_2$ $\qquad \varepsilon_2$
  - $CO_2 + 2H_2O \rightarrow CH_4 + 2O_2$ $\qquad \varepsilon_3$

Use linear programming to find the AR^{MB}

$$N_{CO2} = 1 - \varepsilon_2 - \varepsilon_3 \geq 0$$

$$N_{H2O} = 2 - \varepsilon_1 - 2\varepsilon_2 - 2\varepsilon_3 \geq 0$$

$$N_{CH3OH} = \varepsilon_2 \geq 0$$

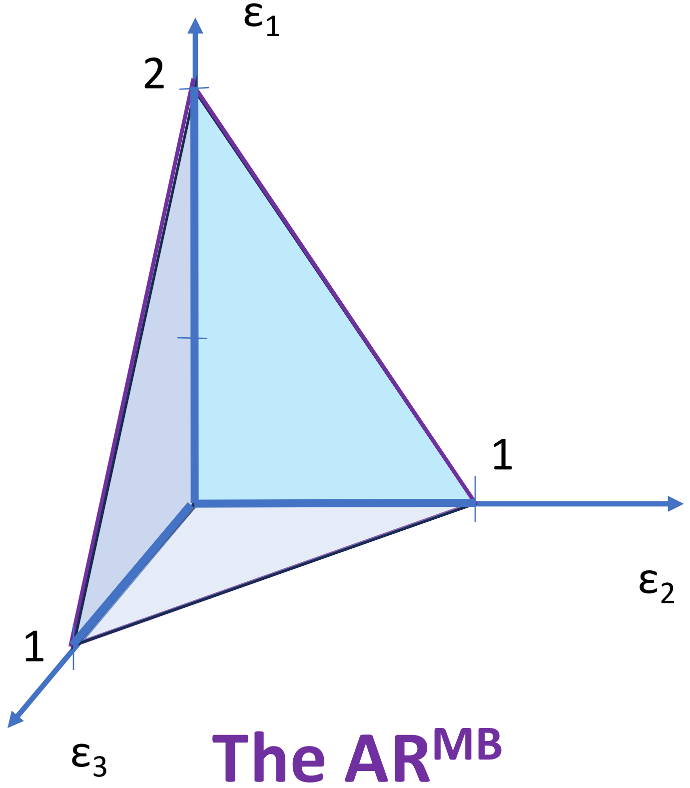
$$N_{H2} = \varepsilon_1 \geq 0$$

$$N_{CH4} = \varepsilon_3 \geq 0$$

$$N_{O2} = \frac{1}{2}\varepsilon_1 + \frac{3}{2}\varepsilon_2 + 2\varepsilon_3 \geq 0$$

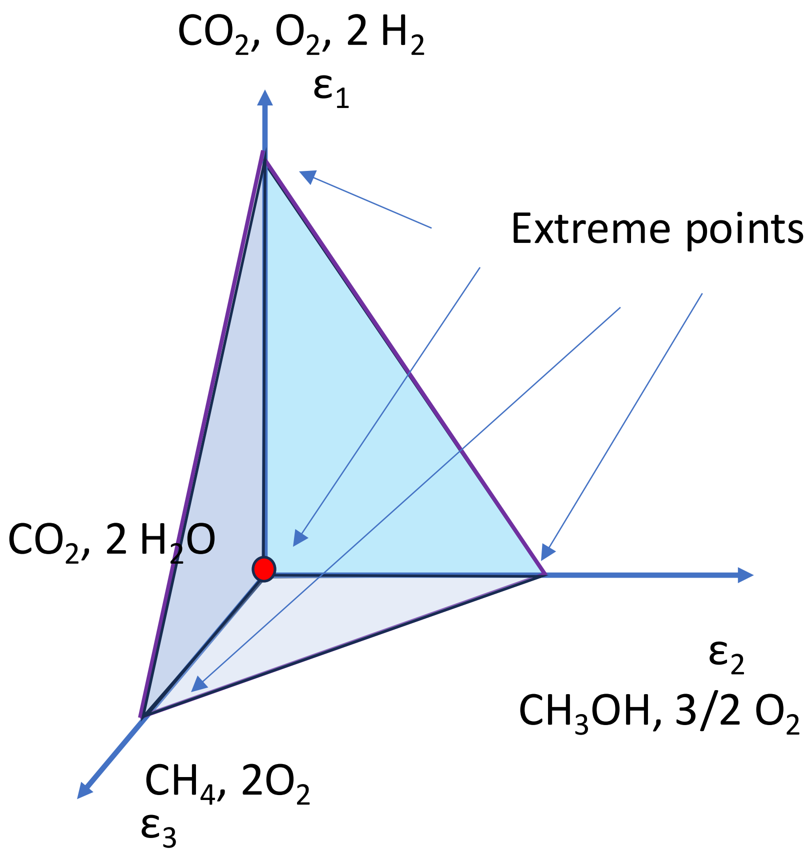
The AR^{MB}: Production of Chemicals from CO_2 and H_2O

- $H_2O \rightarrow H_2 + \frac{1}{2}O_2$ ε$_1$
- $CO_2 + 2H_2O \rightarrow CH_3OH + {}^3/_2\,O_2$ ε$_2$
- $CO_2 + 2H_2O \rightarrow CH_4 + 2O_2$ ε$_3$



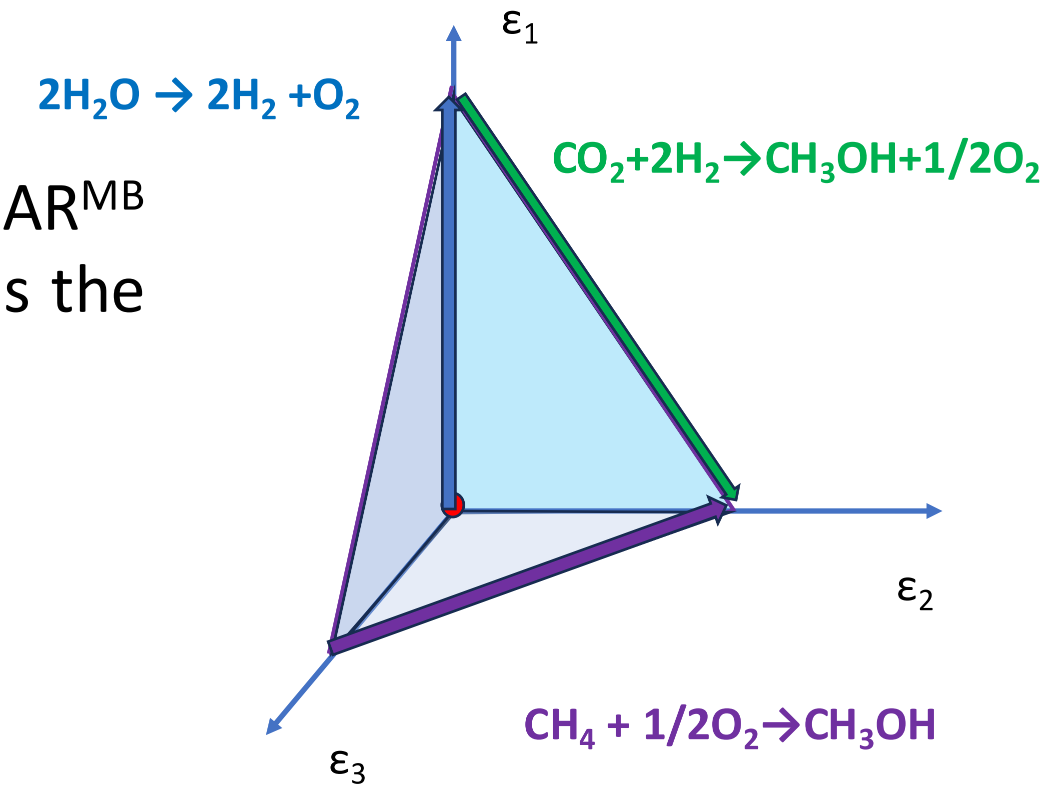
Properties of the AR^{MB}

- At extreme points, the moles of one or more species is zero.
- The AR^{MB} is the convex hull of the extreme points of the region
  - it follows that AR^{MB} is a convex, connected set.
  - the AR^{MB} contains the feed point



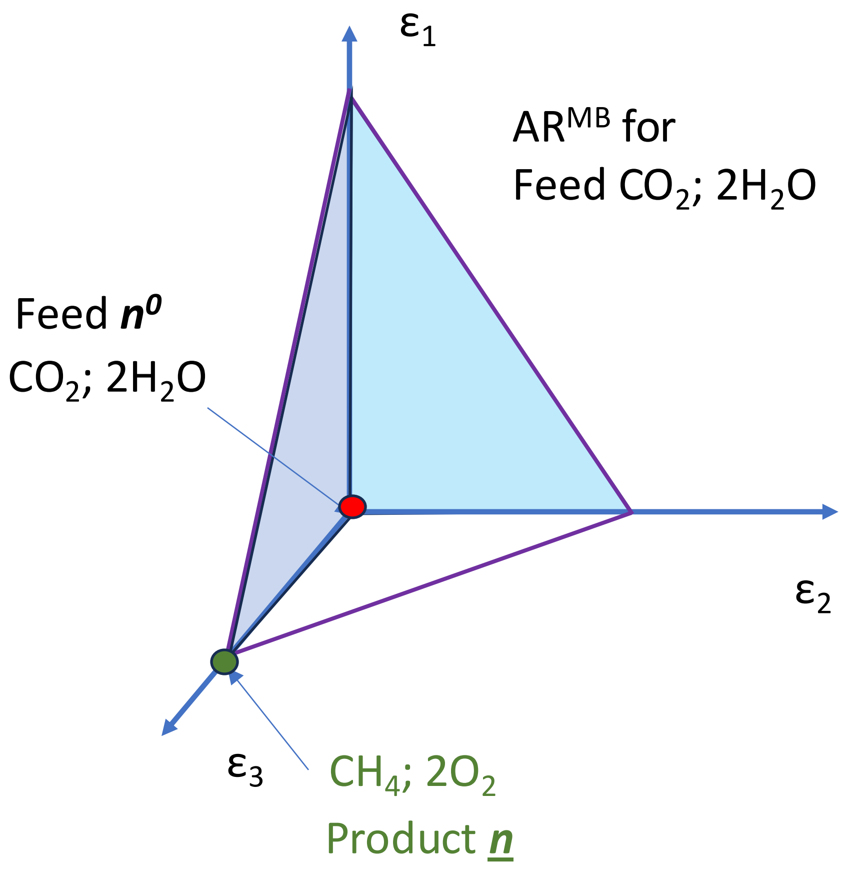
Properties of the AR^{MB}

- The vector between 2 points in the AR^{MB} corresponds to a material balance across the process



Properties of the AR^{MB}

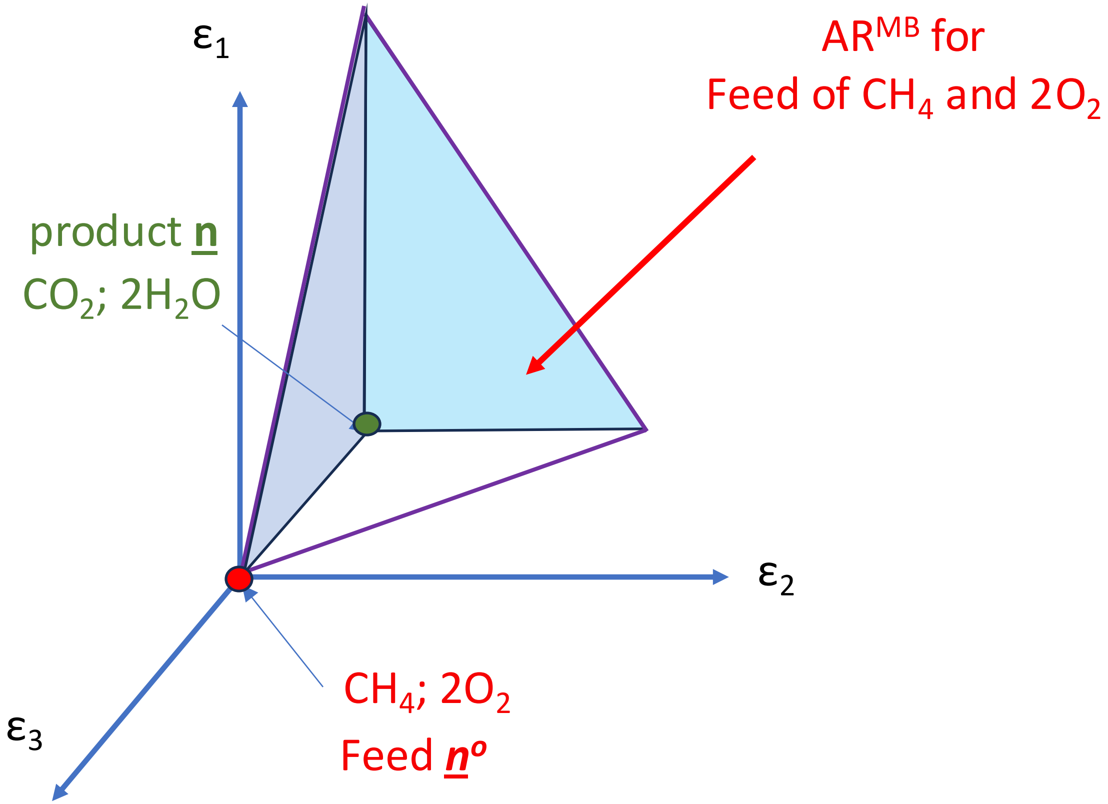
- Consider a product represented by a point $\underline{n}$ that lies in the AR^{MB} for feed $\underline{n}^o$.



Properties of the AR^{MB}

It follows that:

- If we find the AR^{MB} for feed $\underline{\boldsymbol{n}}$, then $\underline{\boldsymbol{n}}^{o}$ will lie in this AR^{MB}.

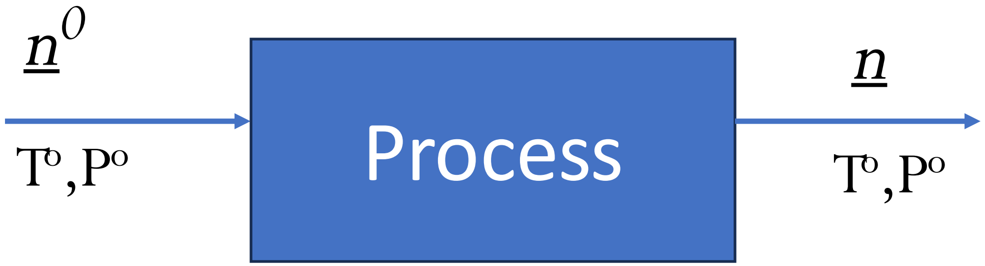


Definition: Enthalpy and Gibbs Free Energy Vectors

- We define
- $\underline{\hat{\boldsymbol{H}}}_f(T^o, P^o) = \left(\hat{H}_{f1}, \hat{H}_{f2}, \dots, \hat{H}_{fn}\right)^T$
  - where $\hat{H}_{fi}$ = enthalpy of formation of component i at temperature T^0 and pressure P^o

and

- $\underline{\hat{\boldsymbol{G}}}_f(T^o, P^o) = \left(\hat{G}_{f1}, \hat{G}_{f2}, \dots, \hat{G}_{fn}\right)^T$
  - where $\hat{G}_{fi}$ = Gibbs Free Energy of formation of component i at temperature T^0 and pressure P^o



Definition: Change in Enthalpy and Gibbs Free Energy across a process

Consider a process where the feed $\underline{\boldsymbol{n}}^{\boldsymbol{o}}$ and the product $\underline{\boldsymbol{n}}$ enter and leave at temperature T^o and pressure P^o

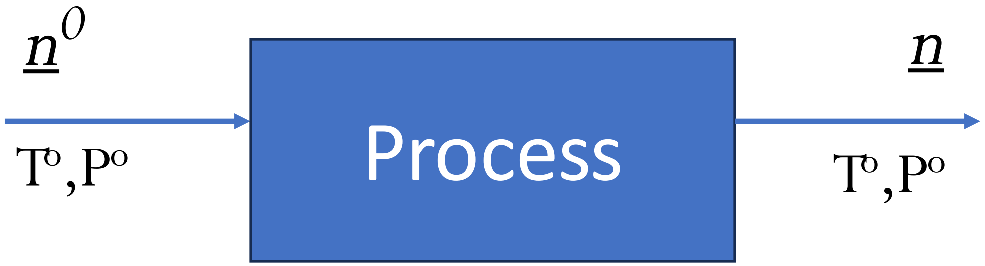
- We define

  Change in enthalpy across the process =

  $$\Delta H(T^o, P^o) = \left(\underline{\boldsymbol{n}} - \underline{\boldsymbol{n}}^{\boldsymbol{o}}\right)^T \cdot \underline{\hat{H}}_f(T^o, P^o)$$

  Change in Gibbs Free Energy across the process =

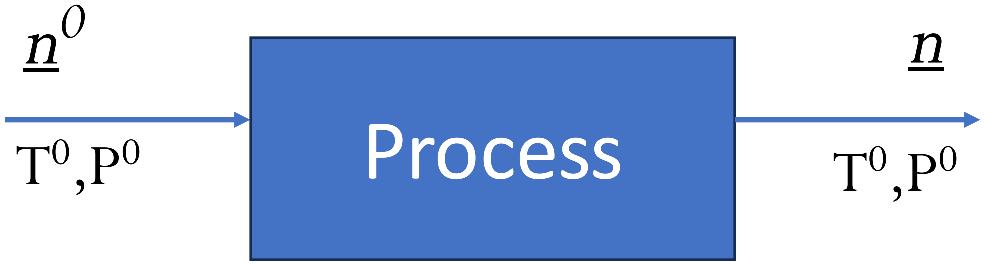
  $$\Delta G(T^o, P^o) = \left(\underline{\boldsymbol{n}} - \underline{\boldsymbol{n}}^{\boldsymbol{o}}\right)^T \cdot \underline{\hat{G}}_f(T^o, P^o)$$



Definition: Thermodynamic Attainable Region AR^T

We define:

The **Thermodynamic Attainable Region AR^T** as the set of all $\{\Delta H(T^o, P^o), \Delta G(T^o, P^o)\}$ for all products ***n*** that lie in the AR^{MB} corresponding to a feed ***n***^{o}



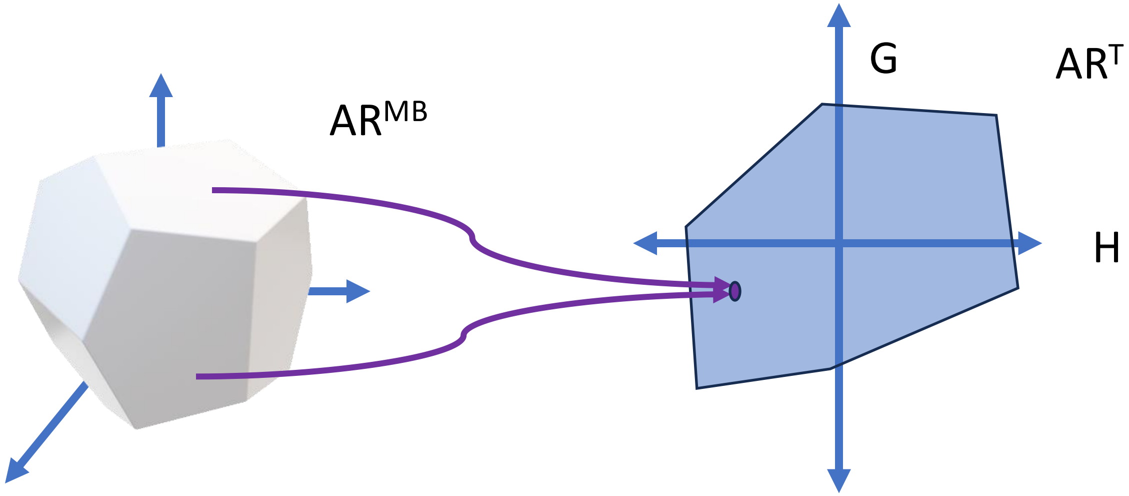
Relationship between the:

Material Balance
AR^{MB}

and the

Thermodynamic Attainable Region
AR^{T}

- Consider the transformation $\mathrm{T} : \mathcal{R}^{\mathrm{N}} \rightarrow \mathcal{R}^{2}$ where: $T(\underline{\boldsymbol{n}}) = \{\{\Delta H(T^o, P^o), \Delta G(T^o, P^o)\} = (\underline{\boldsymbol{n}} - \underline{\boldsymbol{n}}^{\boldsymbol{o}})^{\boldsymbol{T}} \cdot [\underline{\widehat{H}}_f(T^o, P^o), \underline{\hat{G}}_f(T^o, P^o)]$
- AR^{T} is thus a linear transformation of the elements of AR^{MB}.
- It follows that AR^{T} is a surjective mapping of the elements of AR^{MB},
  - equivalently there is a one-to-many relationship between elements of AR^{MB} and elements of AR^{T}.



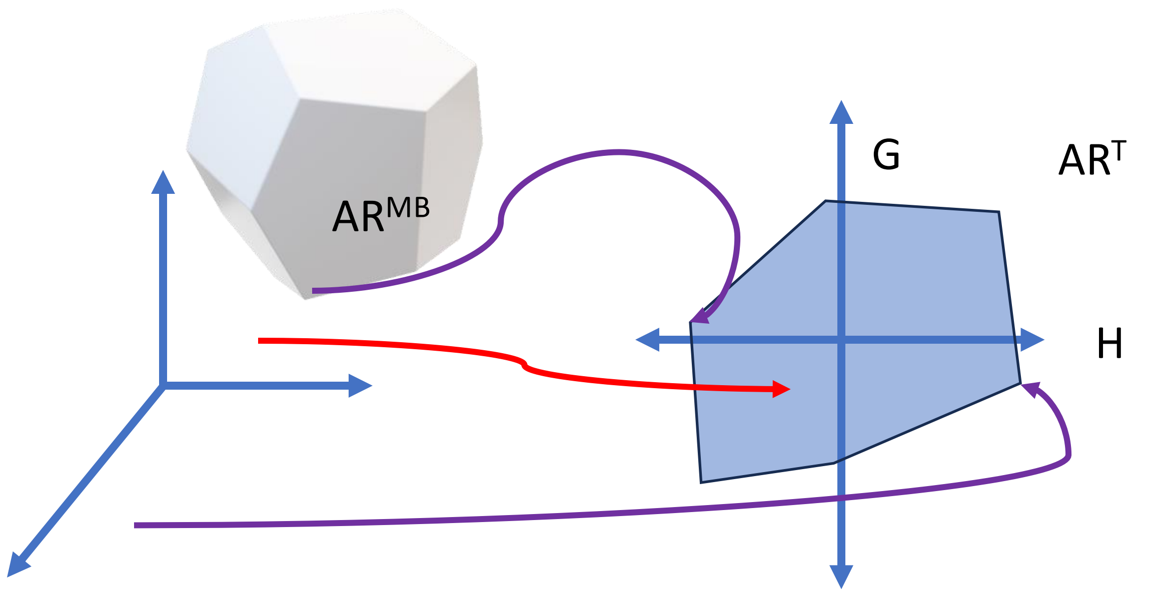
Relationship between the:

Material Balance AR^{MB}

and the

Thermodynamic Attainable Region AR^{T}

- Consequently, extreme points of the AR^{MB} are linearly transformed and are elements of AR^{T} but may not be extreme points of AR^{T}.



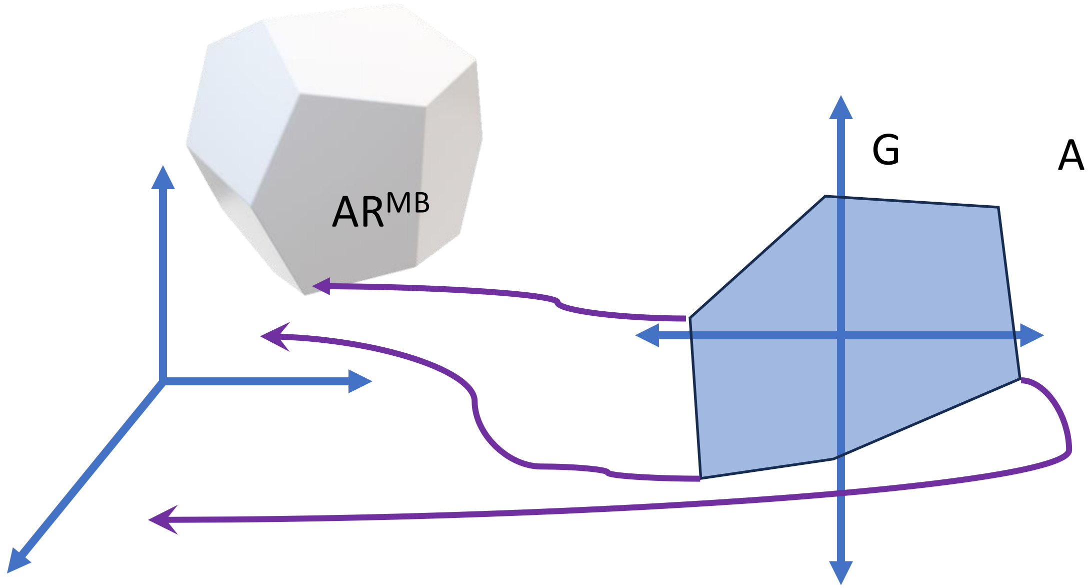
Relationship between the:

Material Balance AR^{MB}

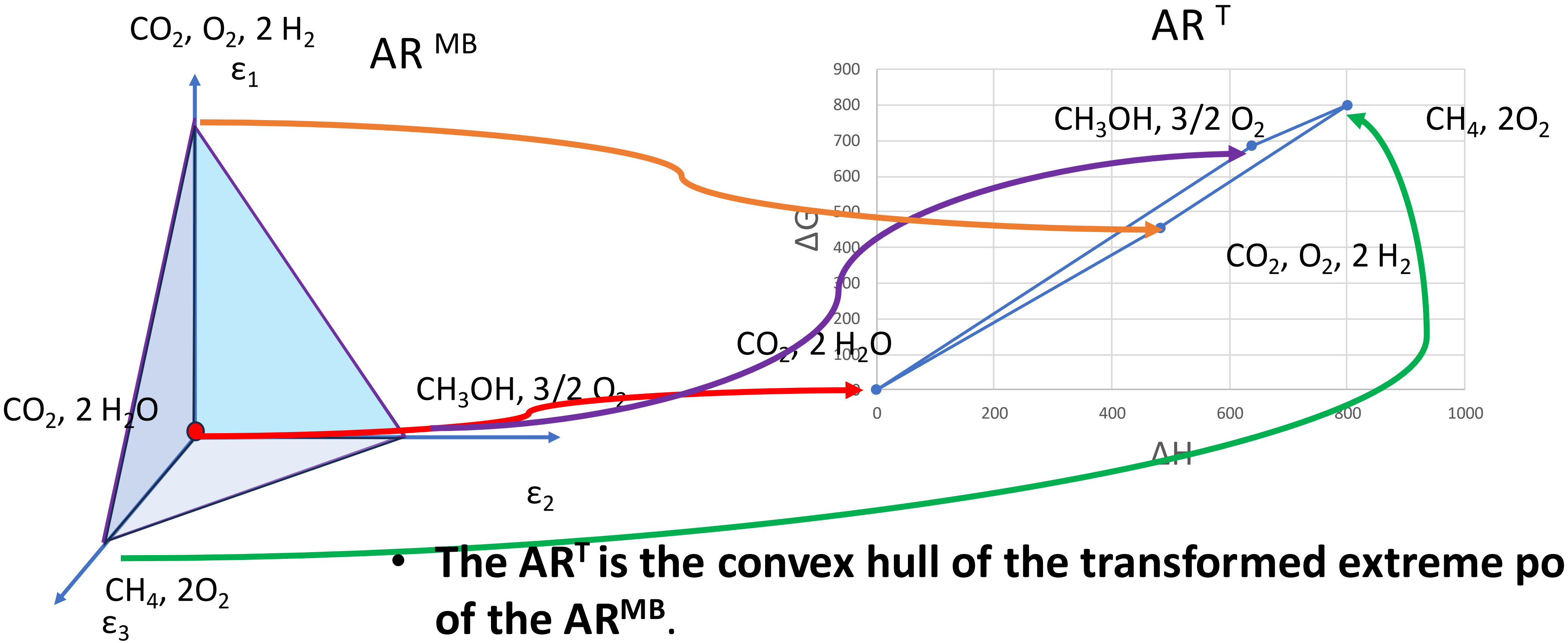
and the

Thermodynamic Attainable Region AR^{T}

- Consequently, extreme points of the AR^{MB} are linearly transformed and are elements of AR^{T} but may not be extreme points of AR^{T}.
- **However all extreme points of AR^{T} correspond extreme points of AR^{MB}.**



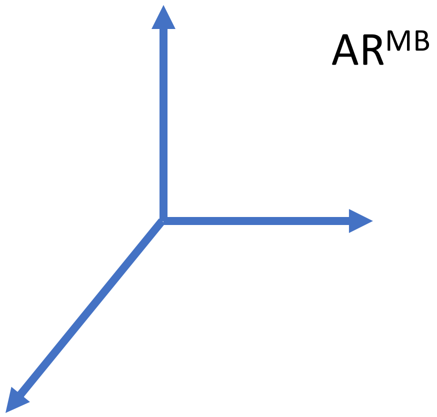
Example of AR^T: Finding the AR^T

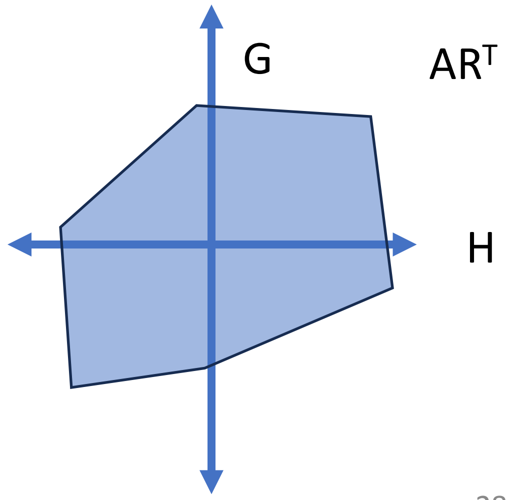


- **The AR^T is the convex hull of the transformed extreme points of the AR^{MB}.**
- It follows that AR^T is a convex, connected set that contains the feed point.

What does this all imply for calculating the AR^{T}

- Find the $\mathrm{AR}^{\mathrm{MB}}$ by solving the set of linear equations:
  - $n_i \leq 0$ for all components i in the systems
  - find all extreme points of the $\mathrm{AR}^{\mathrm{MB}}$
- Transform the extreme points of the $\mathrm{AR}^{\mathrm{MB}}$.
  - $T(\underline{\boldsymbol{n}}) = \{\{\Delta H(T^o, P^o), \Delta G(T^o, P^o)\} = (\underline{\boldsymbol{n}} - \underline{\boldsymbol{n}}^{\boldsymbol{o}})^{\boldsymbol{T}}.[\Delta H(T^o, P^o), \Delta G(T^o, P^o)]$
  - Find the convex hull of the transformed extreme points
- This convex hull corresponds to the AR^{T}
  - **No matter the dimension of the $\mathrm{AR}^{\mathrm{MB}}$ (ie no matter how many components in the process), the AR^{T} is always 2D**





Why is finding the AR^{MB} and the AR^{T} useful?

Before we answer this, we first need to introduce some more ideas:

- Representing Heat and Work Flows in the G-H Space
- Representing Processes in the G-H Space

Relationship between process heat and workflows and ΔH and ΔG

- The energy balance across a process gives:

$$\boldsymbol{\Delta H = Q(T) + W}$$

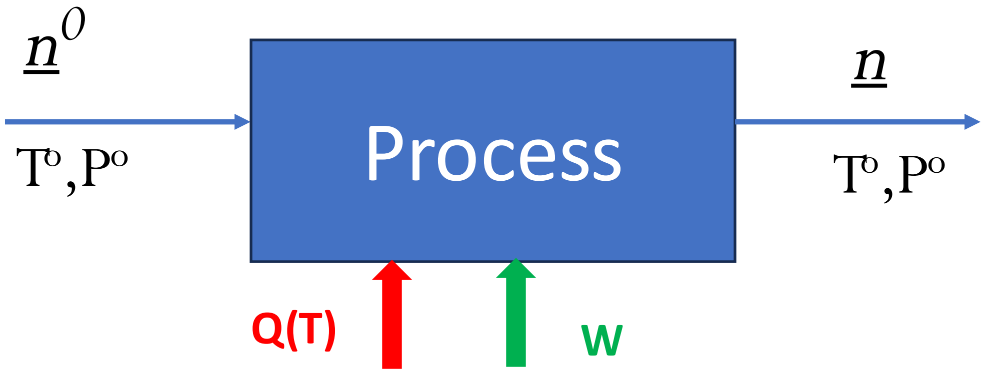
- The entropy balance across a reversible process gives:

$$\Delta S = Q(T)/T$$

- Combining these, gives for a reversible process, that:

$$\Delta G(T^o, P^0) = \Delta H\ (T^o, P^0) - T^o \Delta S\ (T^o, P^0)$$

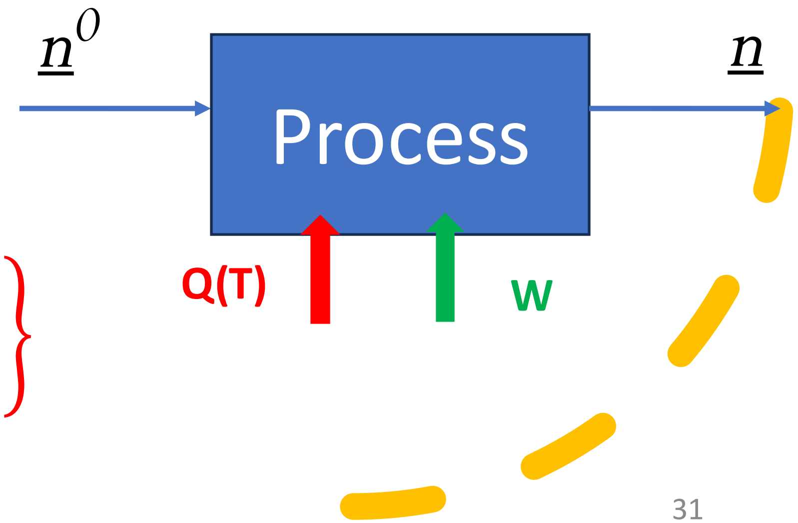
or $\boldsymbol{\Delta G = W + Q(T)\left(1 - {}^{T^o}\!/_{T}\right)}$



Representation of Heat and Work flows in the AR^T

<u>Result</u>: Heat and work flows can be represented as vectors in the GH plot

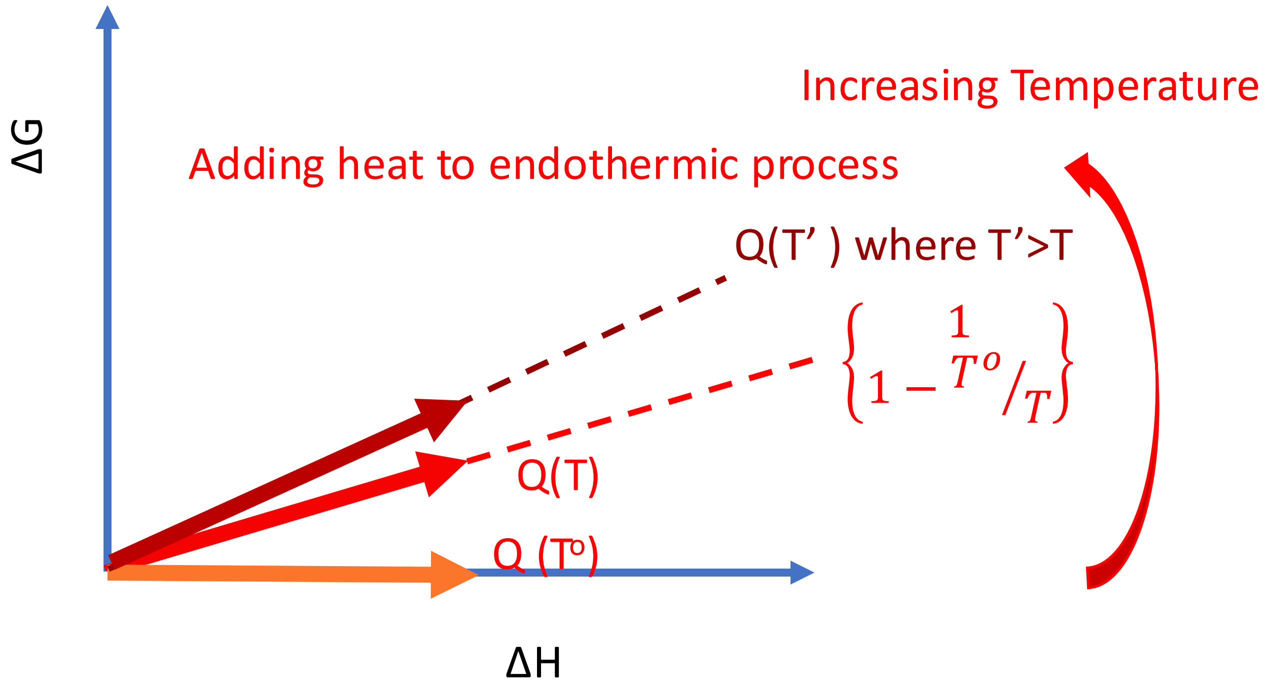
The ***reversible*** addition of work W and heat Q(T) at temperature T is described by:



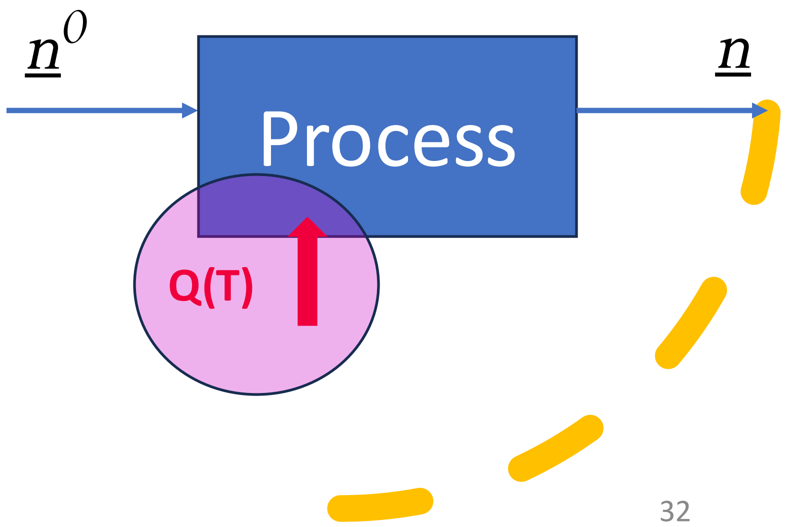
$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{T^o}/{T}\end{Bmatrix}$$

Interpretation of the AR^T

Vector representation of Heat addition



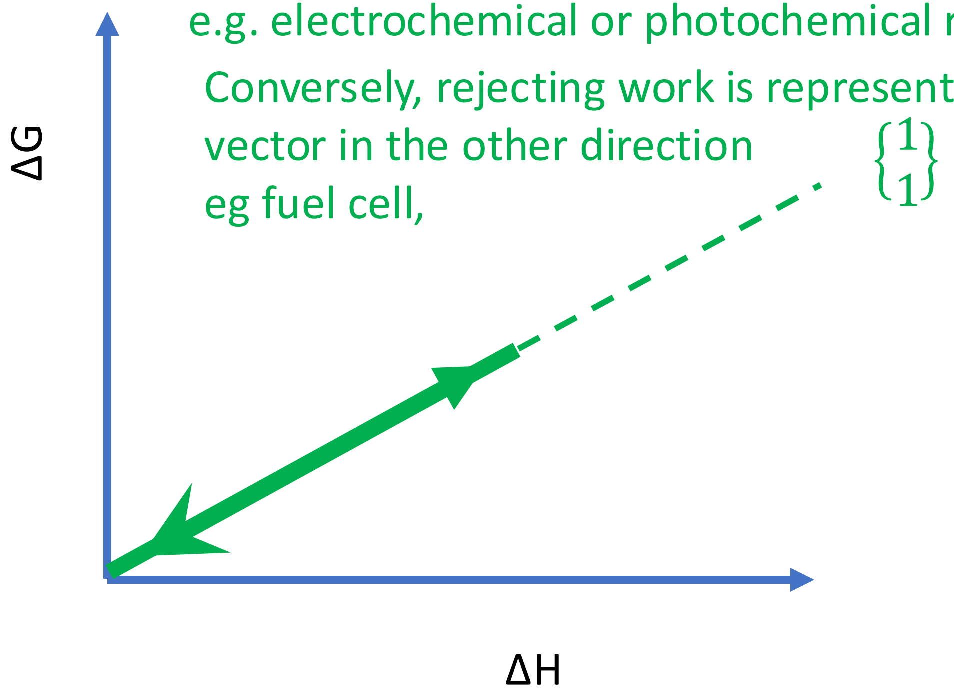
Consider a process where only heat Q(T) at temperature T is added



$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = Q\begin{Bmatrix}1\\ 1-{T^o}/{T}\end{Bmatrix}$$

Interpretation of the AR^T

Vector representation of reversible Work flow

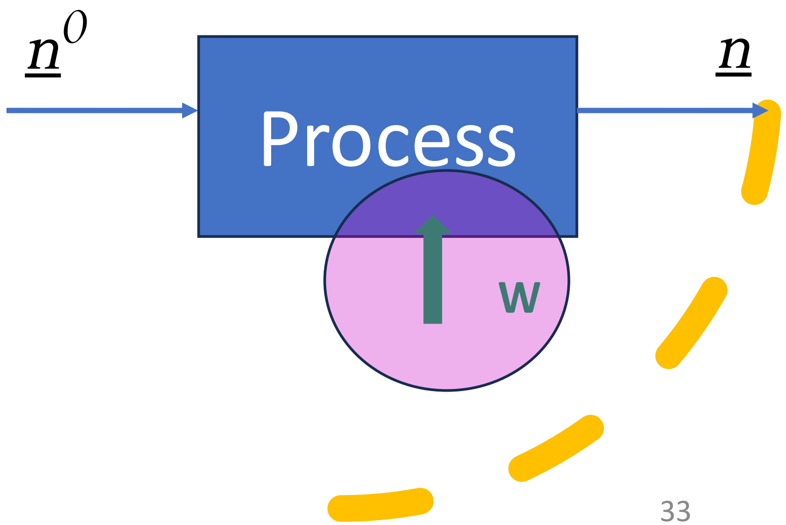


This direction corresponds to adding work;
e.g. electrochemical or photochemical reaction

Conversely, rejecting work is represented as a vector in the other direction
eg fuel cell,

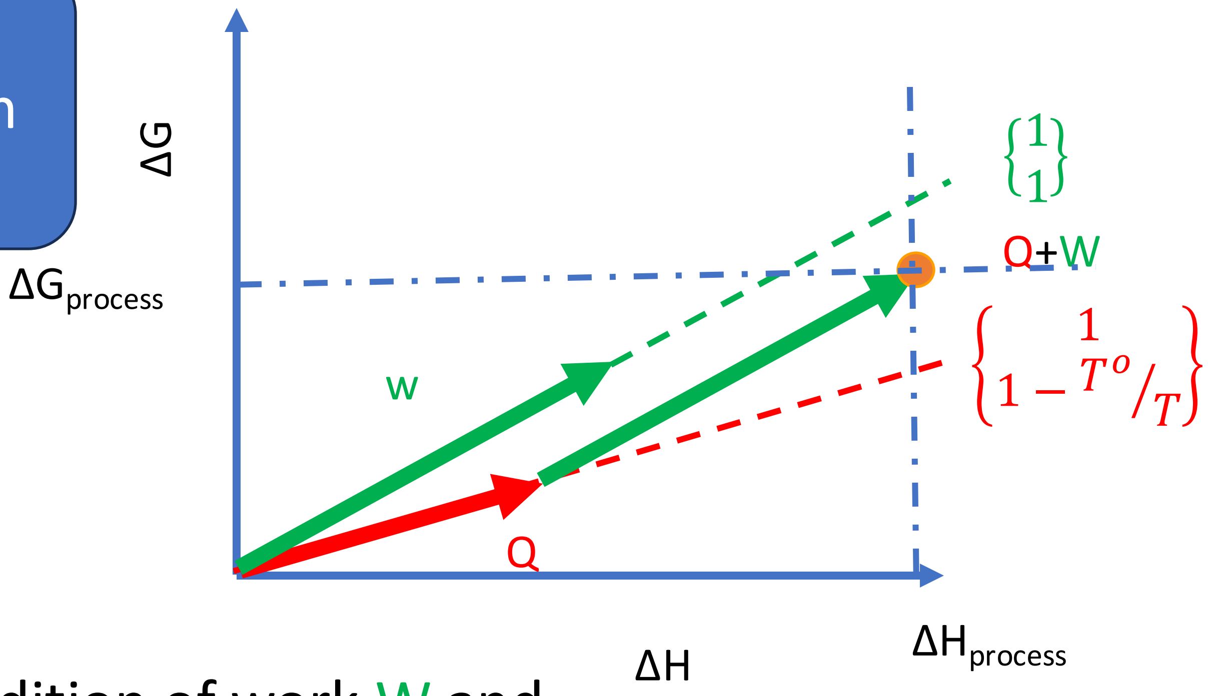
The reversible addition of work W is described by:

$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W \begin{Bmatrix}1\\1\end{Bmatrix}$$



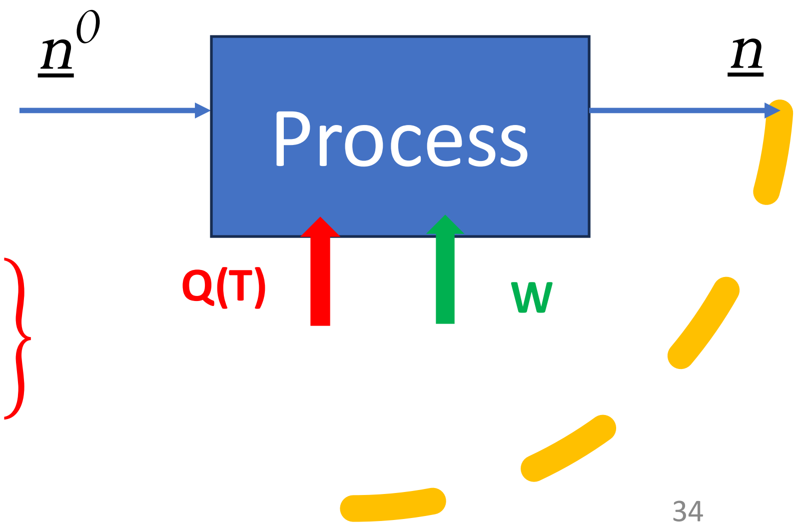
Vector Representation of process heat and work flows

Heat and work inputs must match process ΔH and ΔG in a reversible process



The addition of work W and heat Q(T) can be represented as a vector sum:

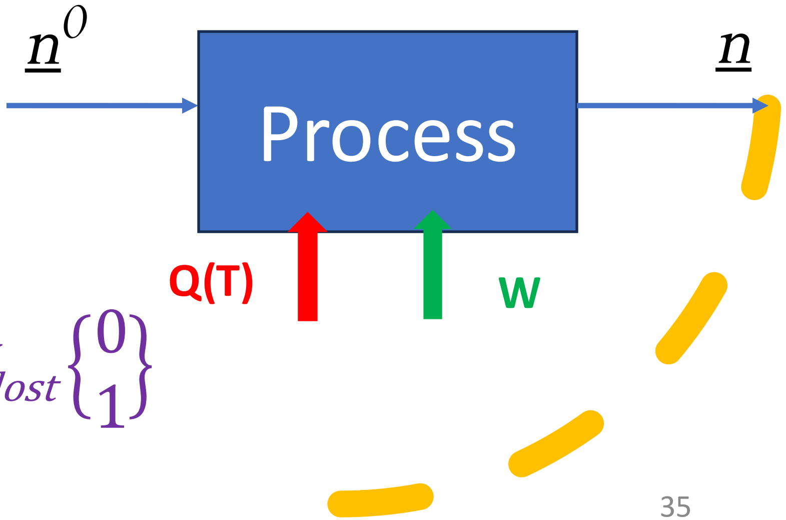
$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{T^o}/{T}\end{Bmatrix}$$



Reversible vs Irreversibility of processes

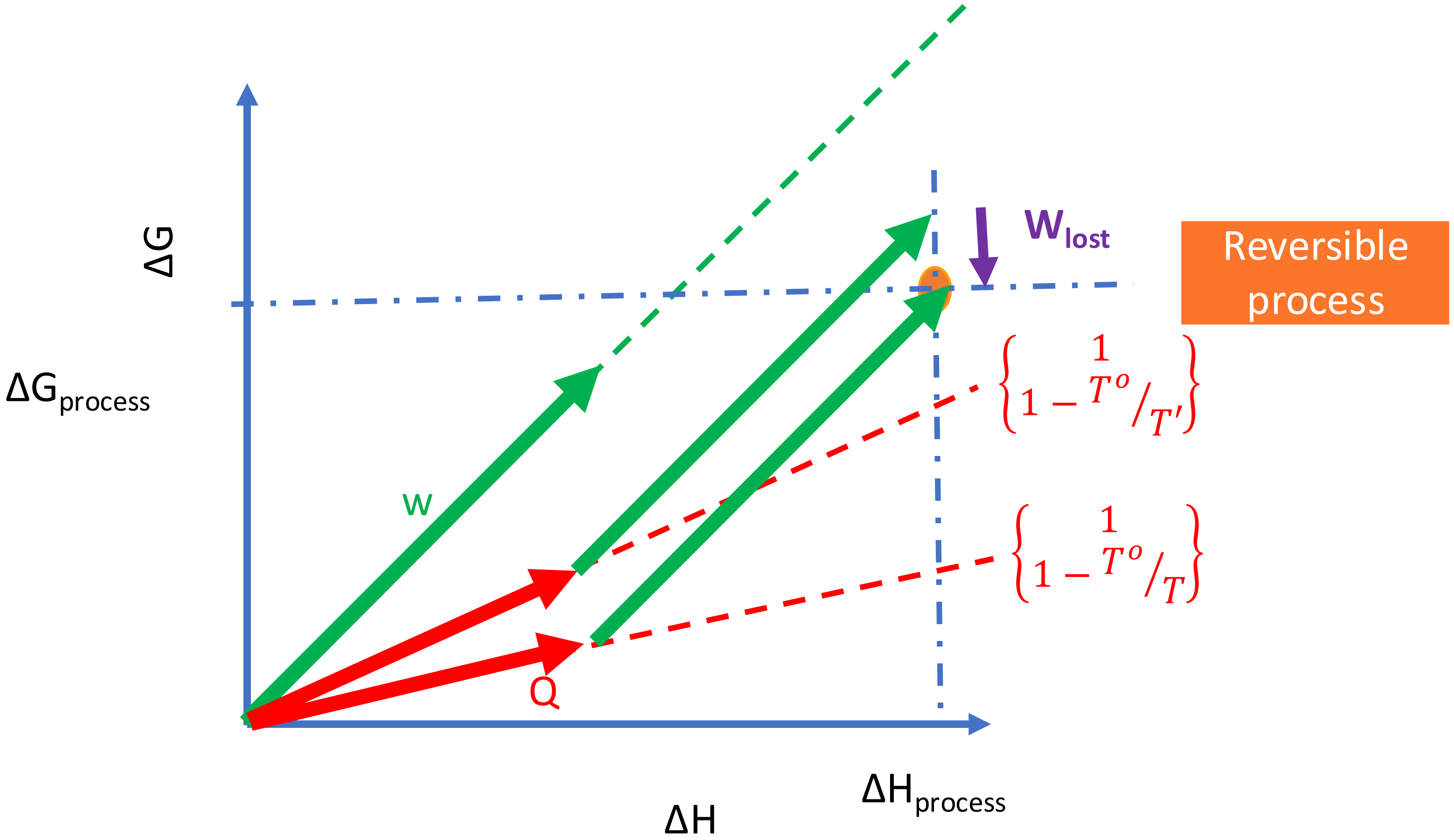
Comparing reversible and irreversible processes:

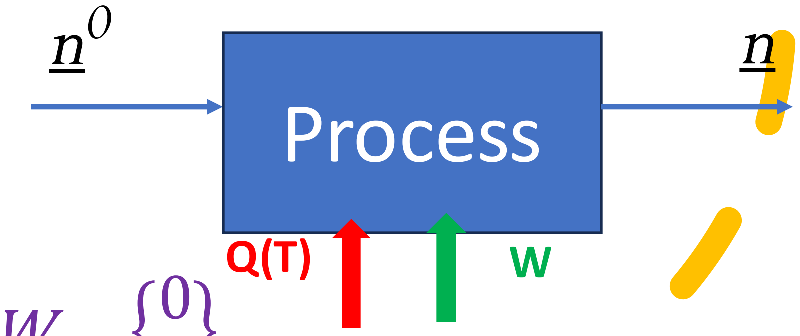
- The energy balance always balances
    - $\Delta H_{process} = W+Q$, but
- More work is added through either W or Q(T) than ΔG (the reversible limit)
    - $\Delta G_{process} = W + Q\,(1-T^0/T) - W_{lost}$,
- This extra work adds operating costs to the process, usually resulting in increased CO_2 emissions, extra electricity consumption etc.



$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{}^{T^o}/_{T}\end{Bmatrix} - W_{lost}\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

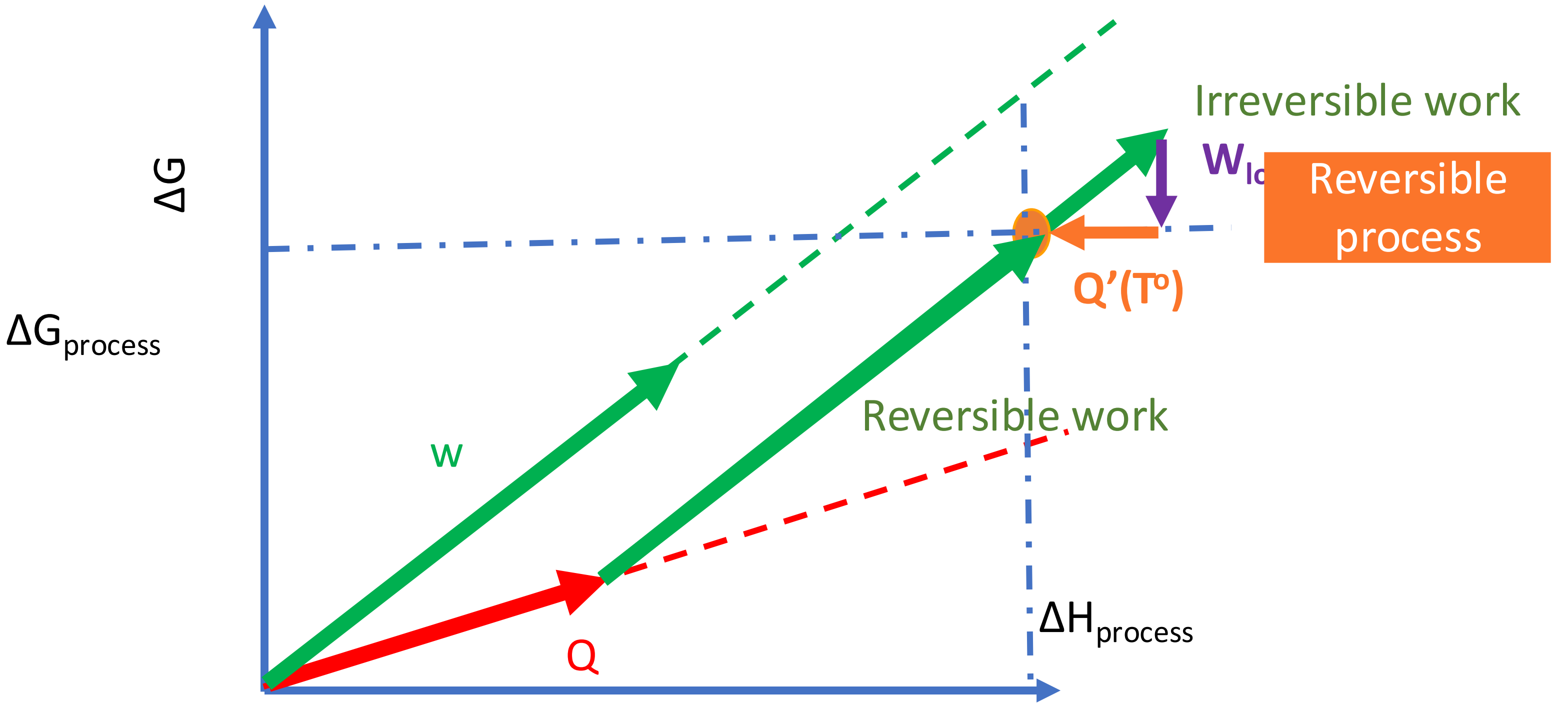
Effect of increasing the temperature of the heat added to process

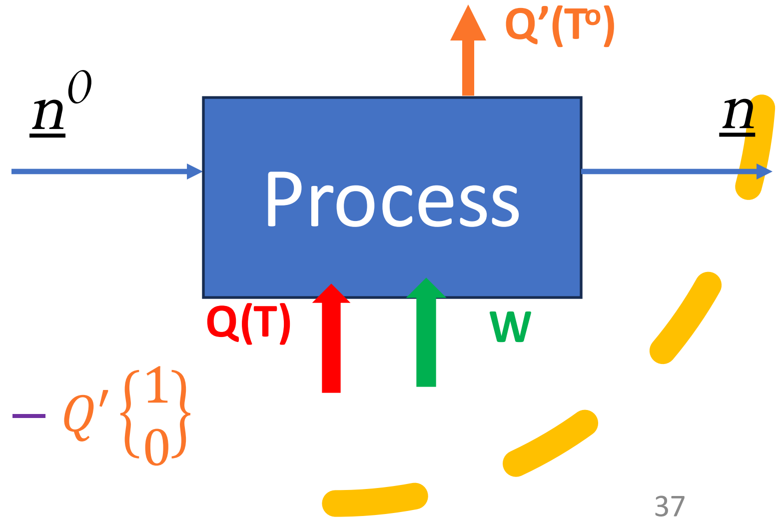




$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W_{irr}\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{}^{T^o}/_{T'}\end{Bmatrix} - W_{lost}\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

Adding work irreversibly

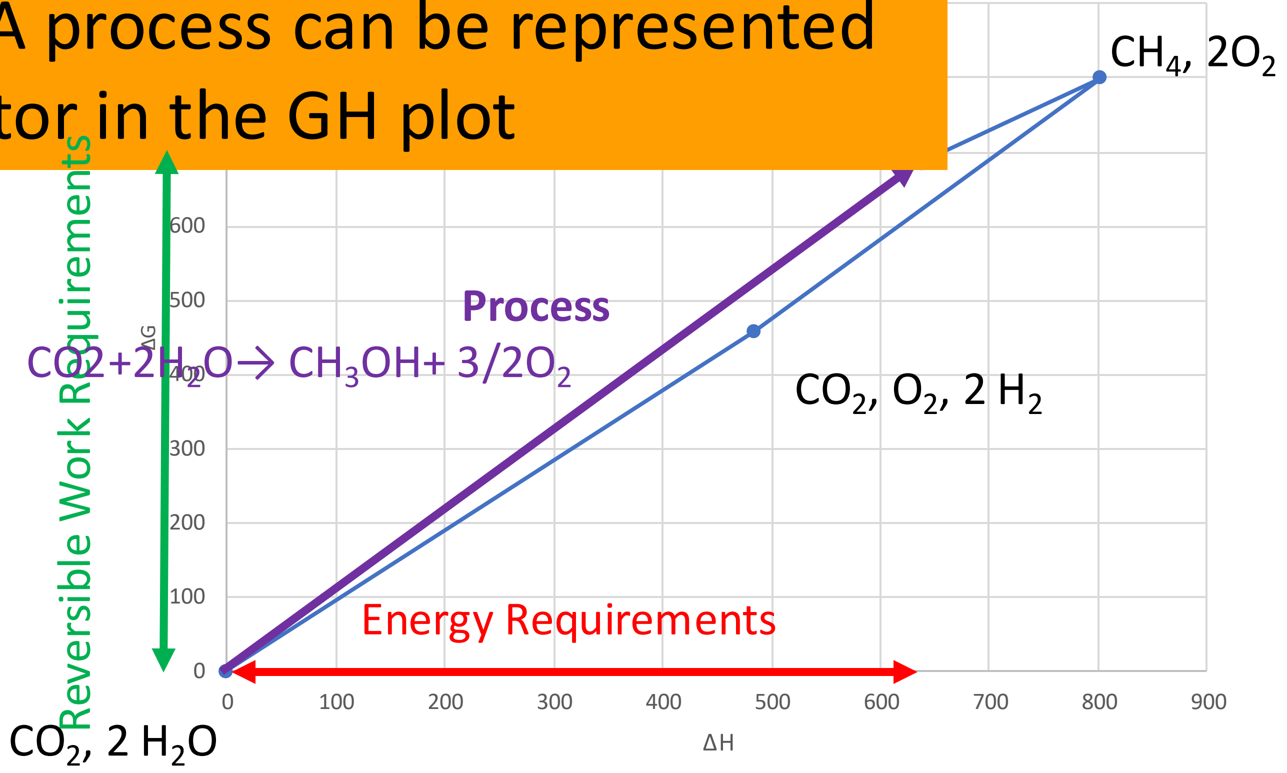


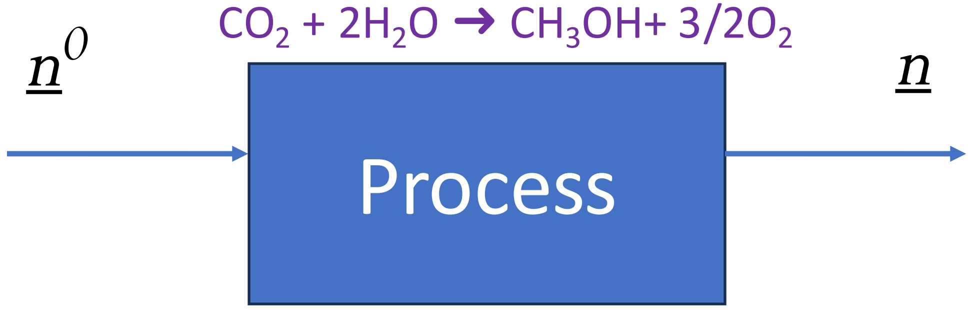


$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W_{irr}\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{}^{T^o}/_{T}\end{Bmatrix} - W_{lost}\begin{Bmatrix}0\\ 1\end{Bmatrix} - Q'\begin{Bmatrix}1\\ 0\end{Bmatrix}$$

Representing a Process as a vector

Result: A process can be represented as a vector in the GH plot

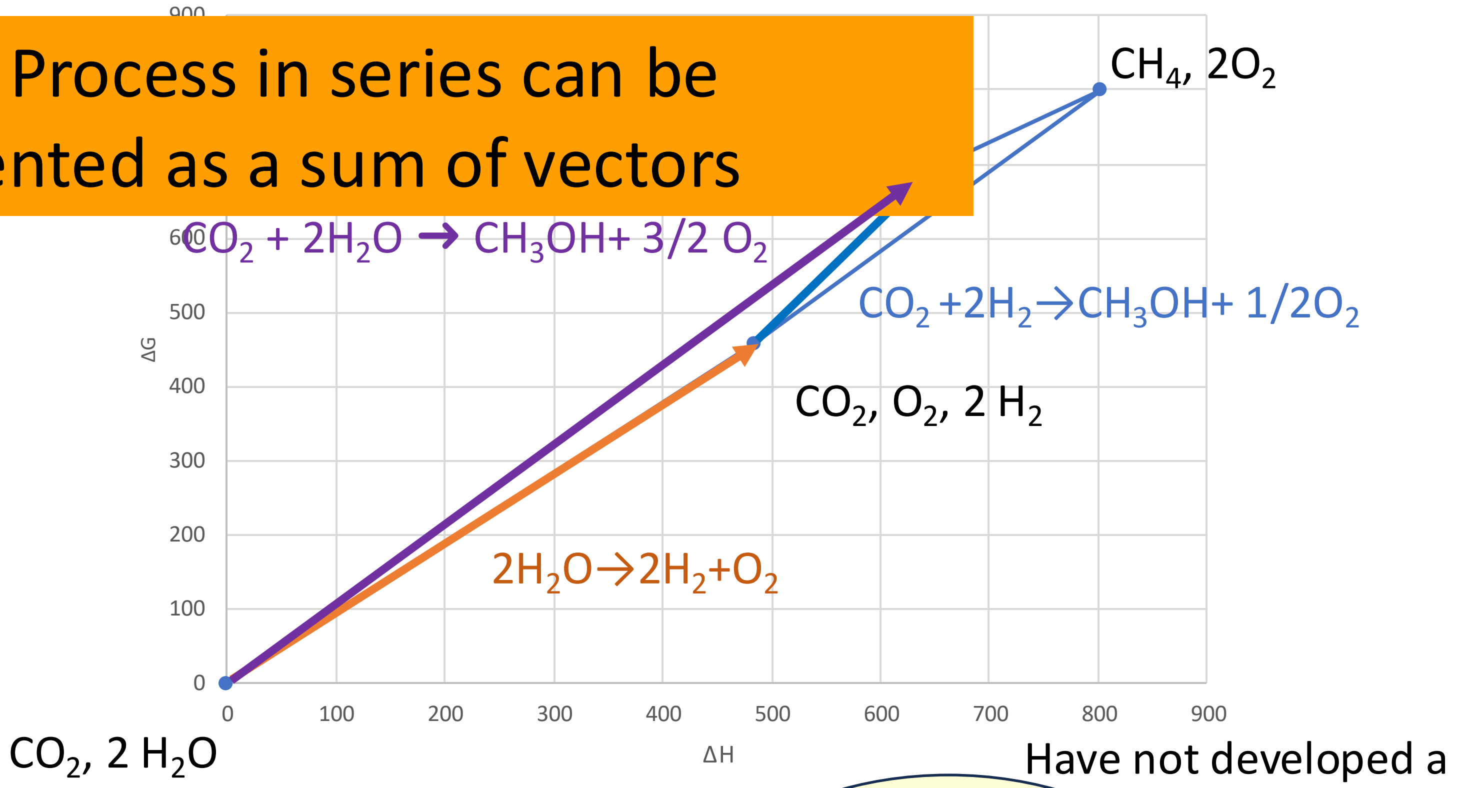


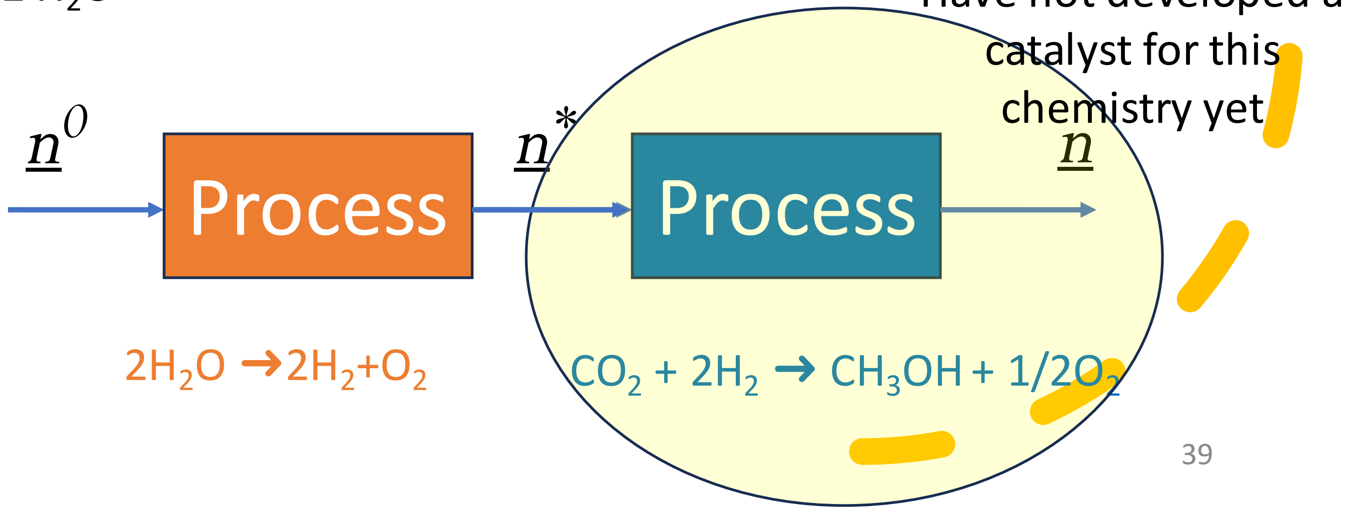

$CO_2 + 2H_2O \rightarrow CH_3OH + 3/2O_2$

Interpretation of the AR^T

Processes in Series

Result: Process in series can be represented as a sum of vectors





$2H_2O \rightarrow 2H_2 + O_2$

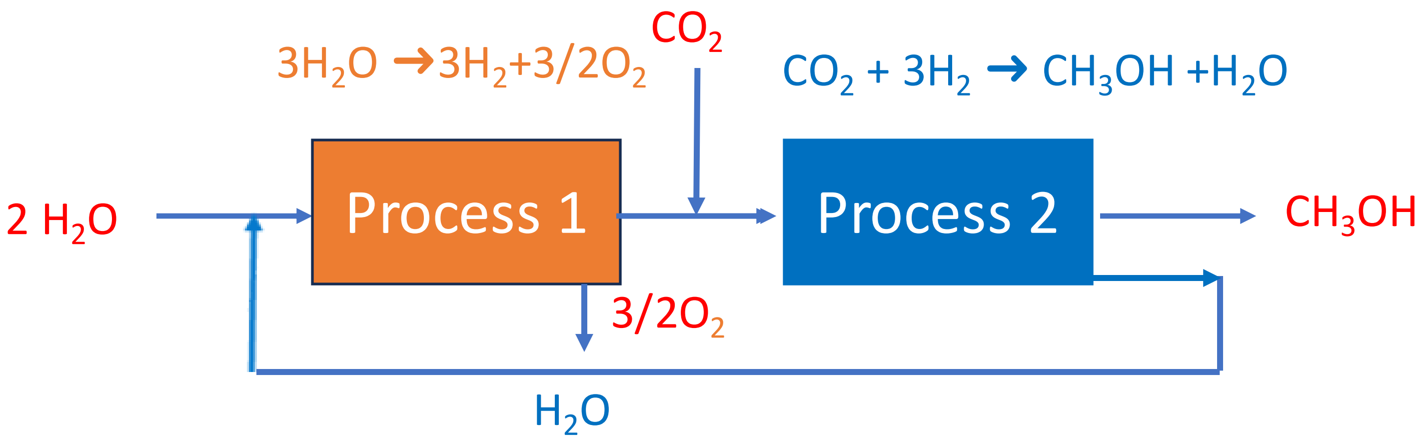
$CO_2 + 2H_2 \rightarrow CH_3OH + 1/2O_2$

Have not developed a catalyst for this chemistry yet

Putting this all together

- We have introduced the following concepts
  - The Mass Balance Attainable Region AR^{MB}
  - The Thermodynamic Attainable Region AR^{T}
  - Representing process heat and work flows as vectors
  - Representing processes as vectors
- How do we put this all together?

Example: Production of green methanol using existing catalytic chemistry



- Currently, we have catalysts for the reaction: $CO_2 + 3H_2 \rightarrow CH_3OH + H_2O$

- To use this chemistry, we need to electrolyze 3 moles H_2O to produce 3 moles of H_2 and 3/2 moles of O_2:
- We then react the 3 moles of H_2 with 1 mole CO_2 to produce 1 mole CH_3OH and 1 mole H_20
- The water produced in methanol synthesis can be recycled to give the overall material balance:

$$CO_2 + 2\ H_2O \rightarrow CH_3OH + 3/2\ O_2$$

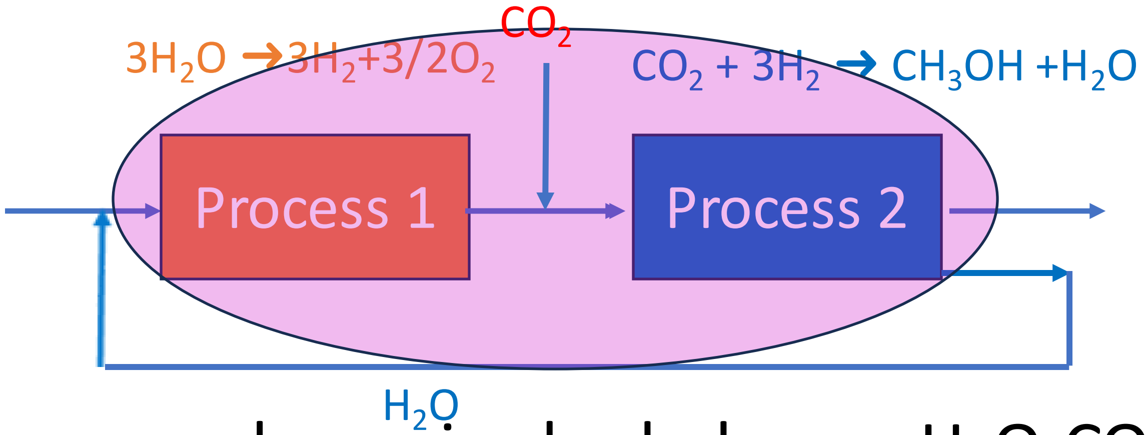
Finding the AR^{MB} and AR^{T}

Independent Material Balances:

$$H_2O \rightarrow H_2 + \frac{1}{2}O_2 \qquad \varepsilon_1$$

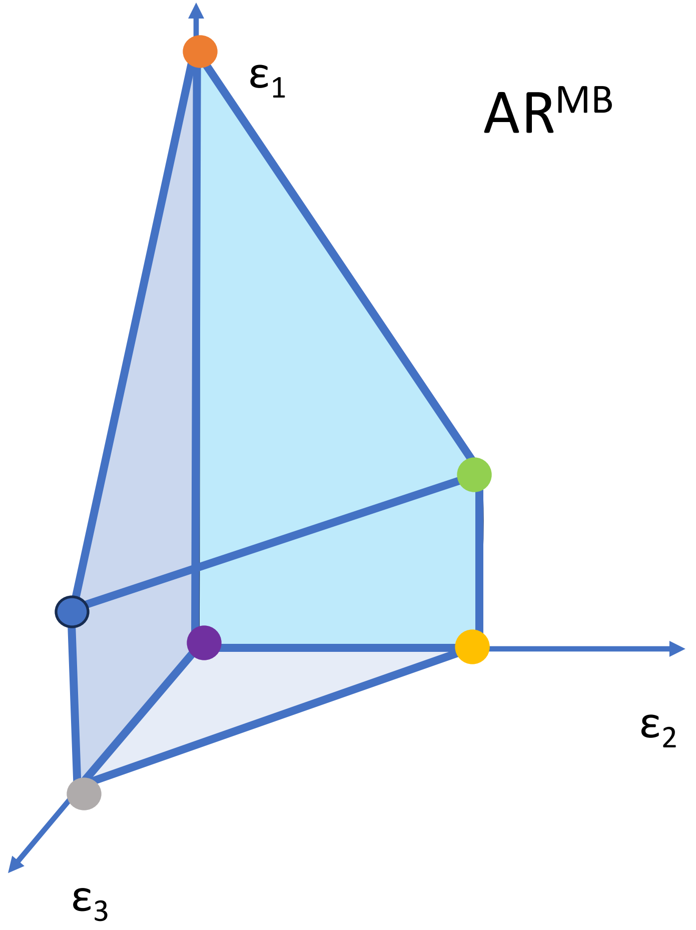
$$CO_2 + 2H_2O \rightarrow CH_3OH + {}^3/_2\, O_2 \qquad \varepsilon_2$$

$$CO_2 + 2H_2O \rightarrow CH_4 + 2O_2 \qquad \varepsilon_3$$

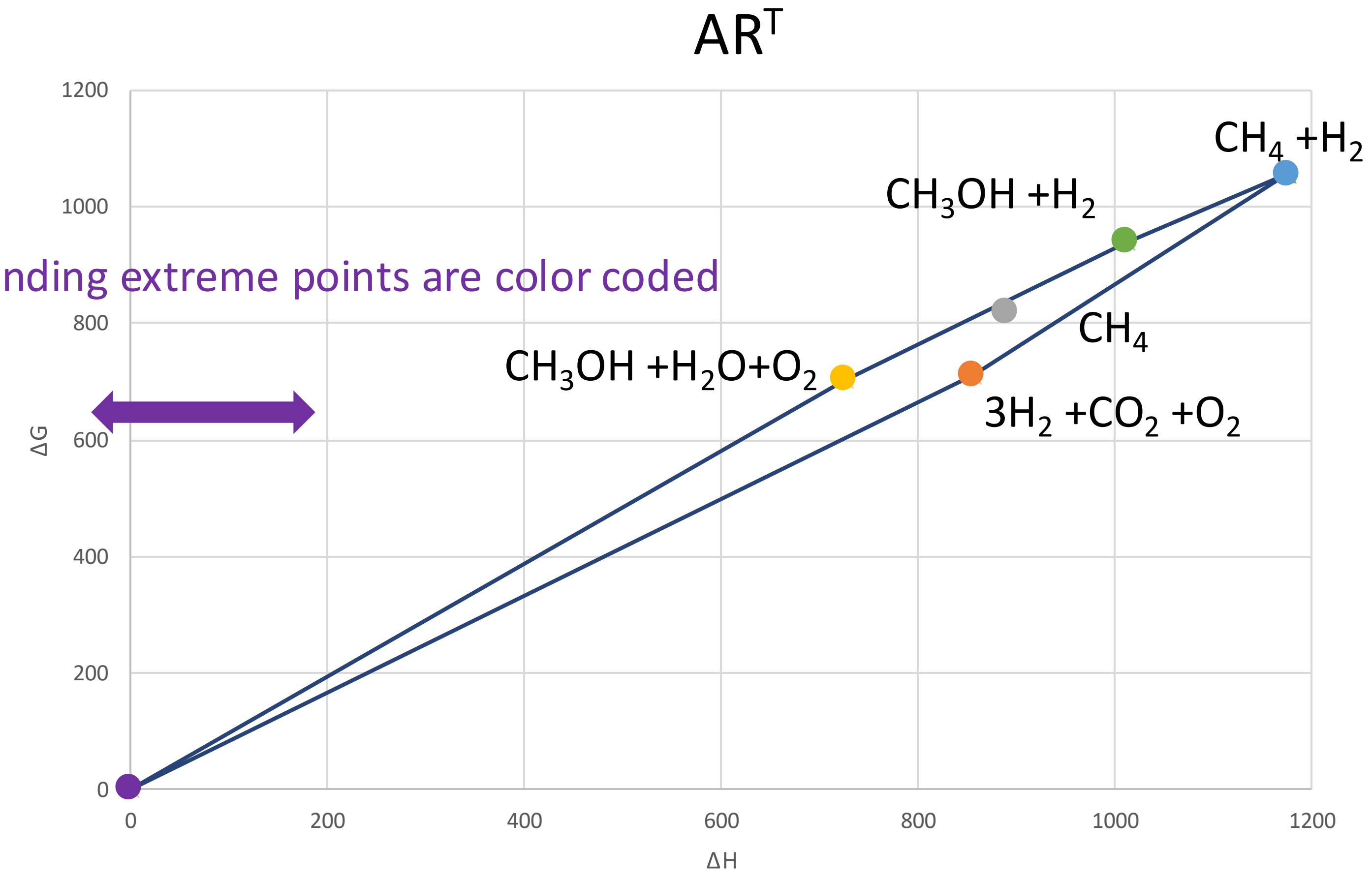


- Feed to process shown in shaded area: $H_2O:CO_2 = 3:1$
- Process 1
  - Require $\varepsilon_1 = 3$ giving $3H_2O \rightarrow 3H_2 + 3/2O_2$
- Process 2:
  - Require $\varepsilon_1 = -3$ $\varepsilon_2 = 1$ giving $CO_2 + 3H_2 \rightarrow CH_3OH + H_2O$
- Overall Process
  - Described by $\varepsilon_1 = 0$; $\varepsilon_2 = 1$; $\varepsilon_3 = 0$ giving $CO_2 + 2H_2O \rightarrow CH_3OH + 3/2O_2$

AR^{MB} and AR^{T} for a feed of: 1 mole CO_2 and 3 moles H_2O

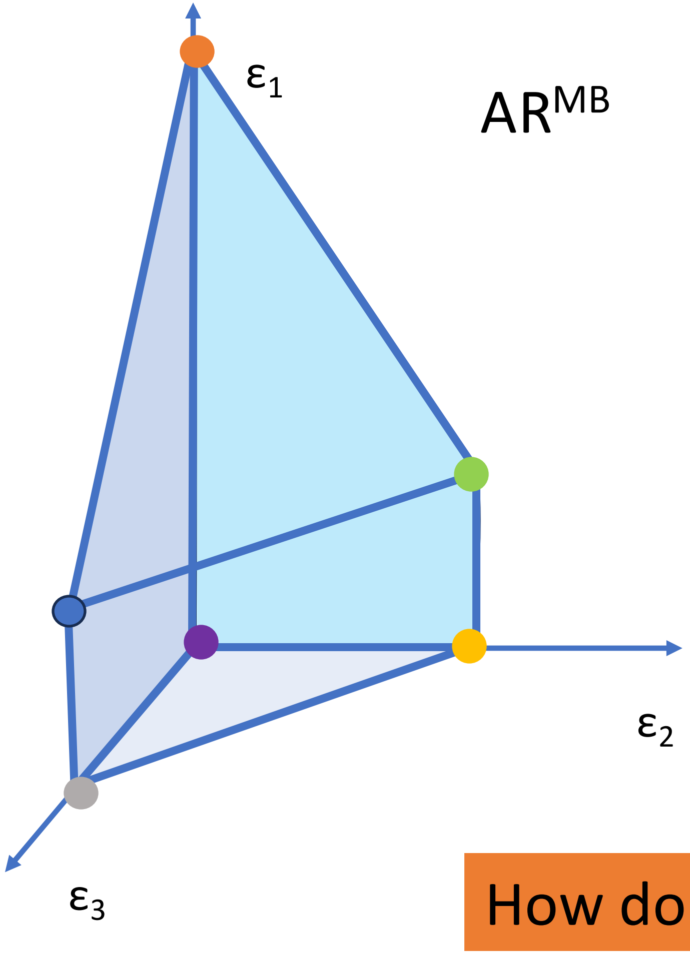


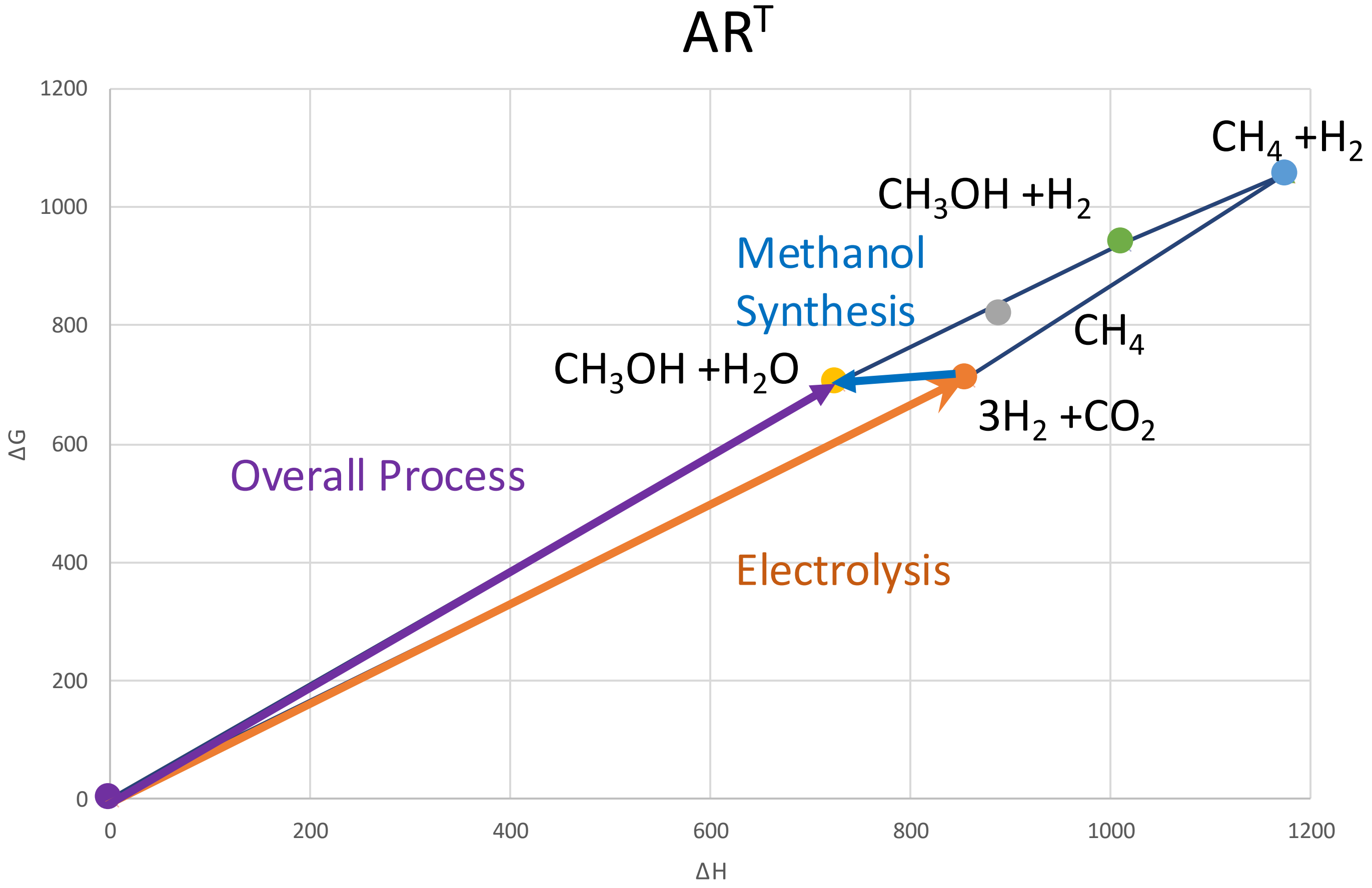
Corresponding extreme points are color coded



Please note: I will leave O_2 out of labels going forward

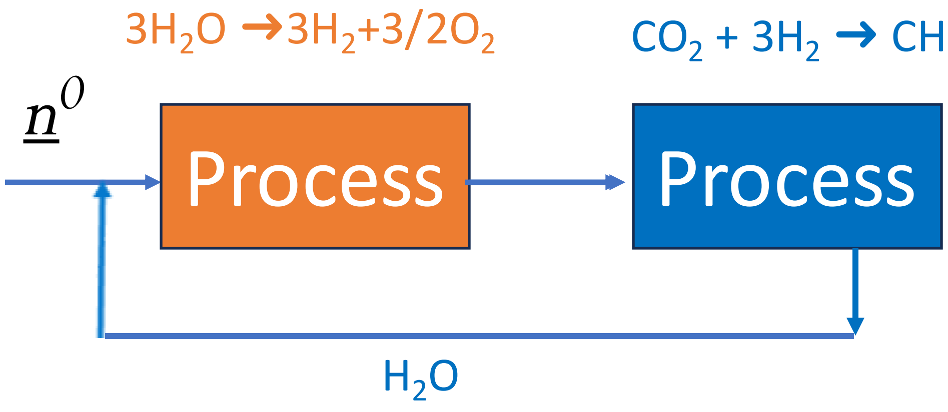
Representing the processes as vectors in the GH space

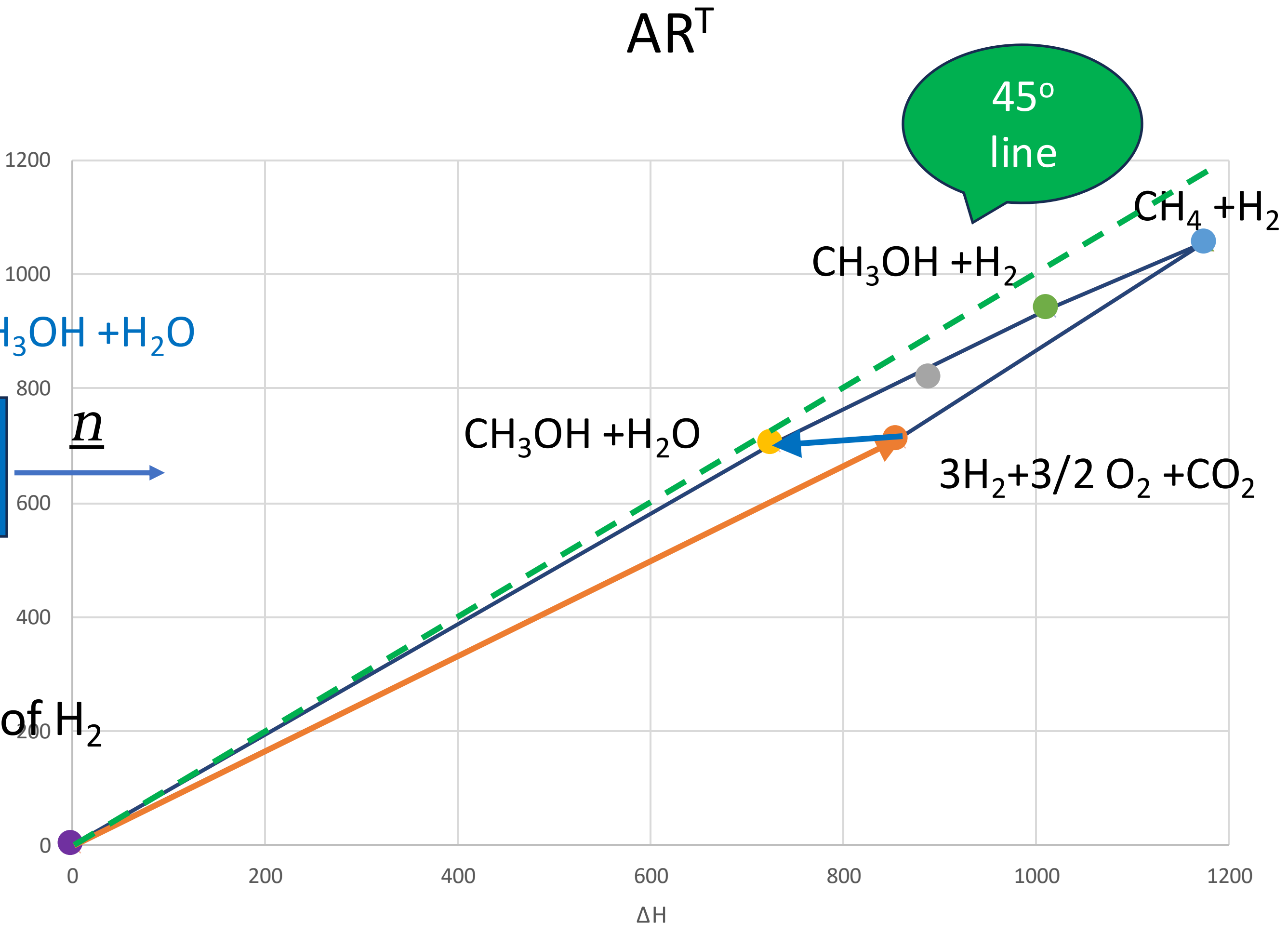




How do we add or reject heat and work in this process?

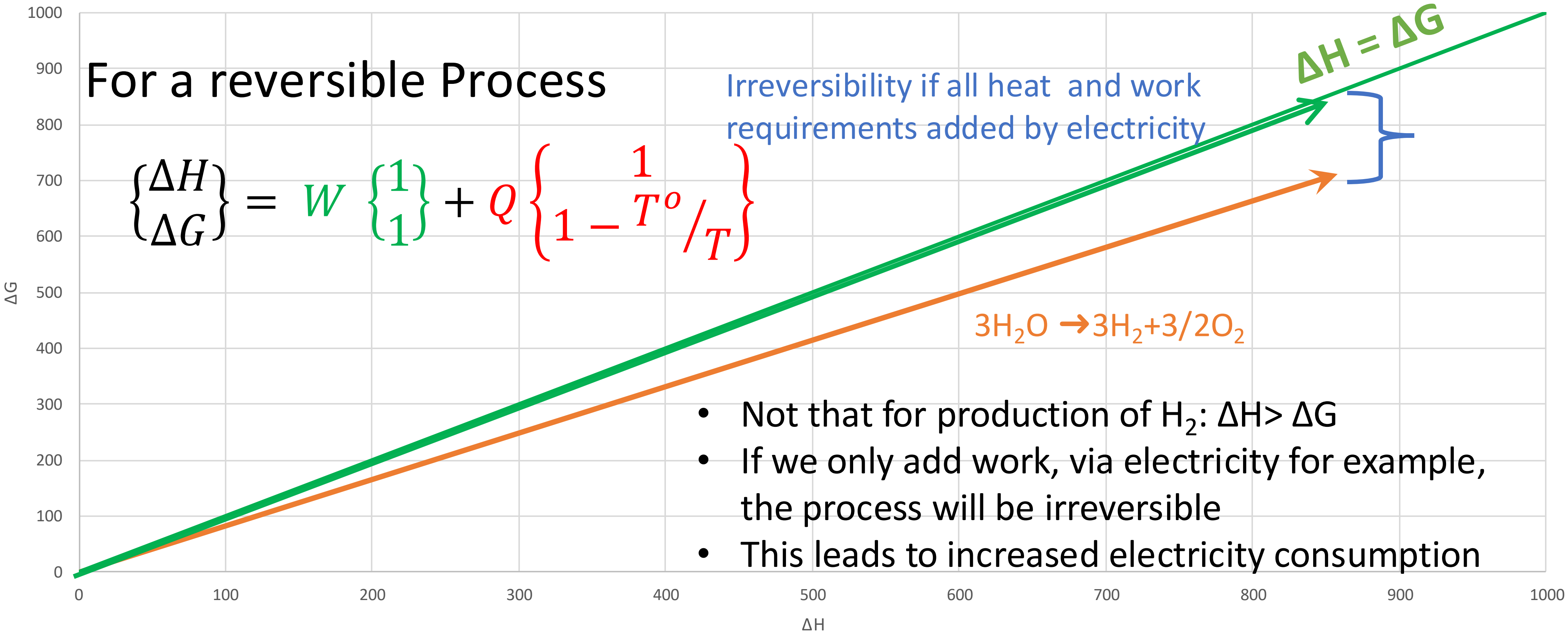
Electrolysis: matching heat and work vectors to the process vector





- The orange vector represents the production of H_2
- Both ΔG and ΔH >0
  - Requires work and heat addition,
  - But ΔG< ΔH
- Can we add heat and work reversibly?

Consider Adding Work only for H_2 Production

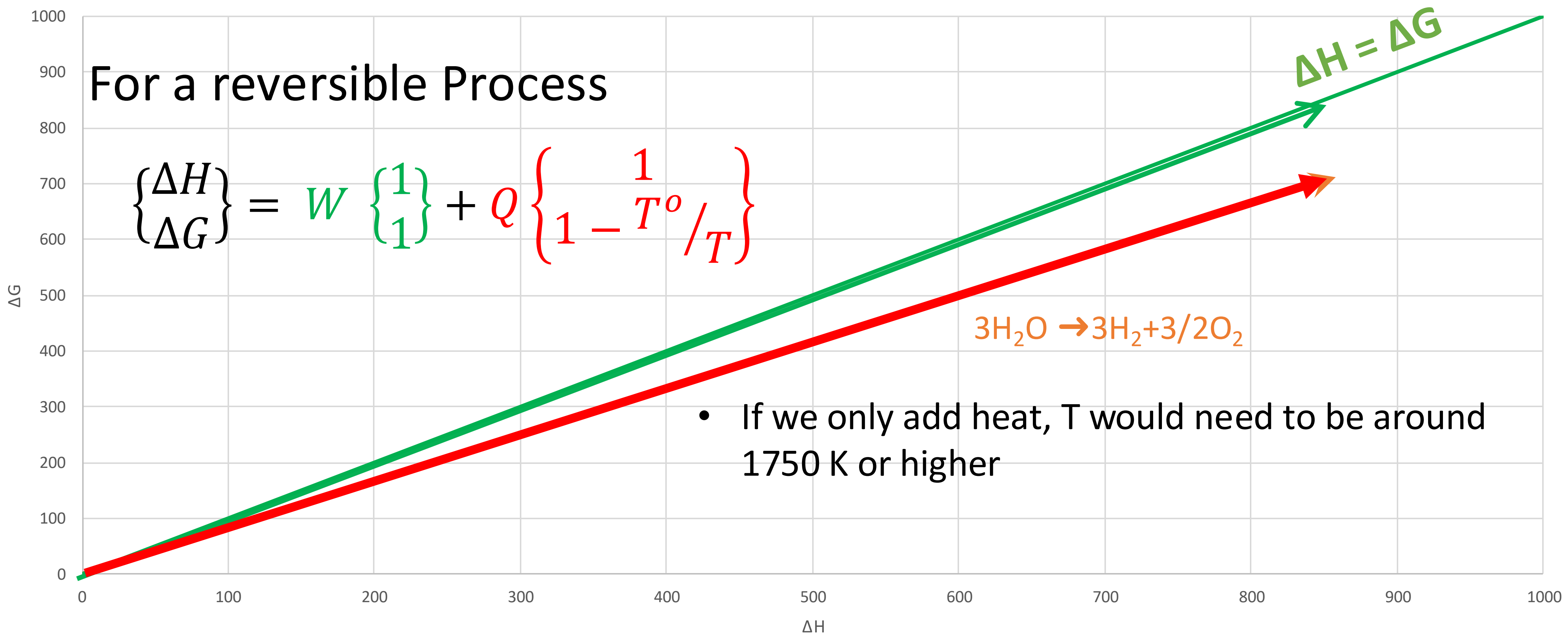


For a reversible Process

$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{T^o}/{T}\end{Bmatrix}$$

- Not that for production of H_2: ΔH> ΔG
- If we only add work, via electricity for example, the process will be irreversible
- This leads to increased electricity consumption

Consider Adding Heat Only for H_2 Production

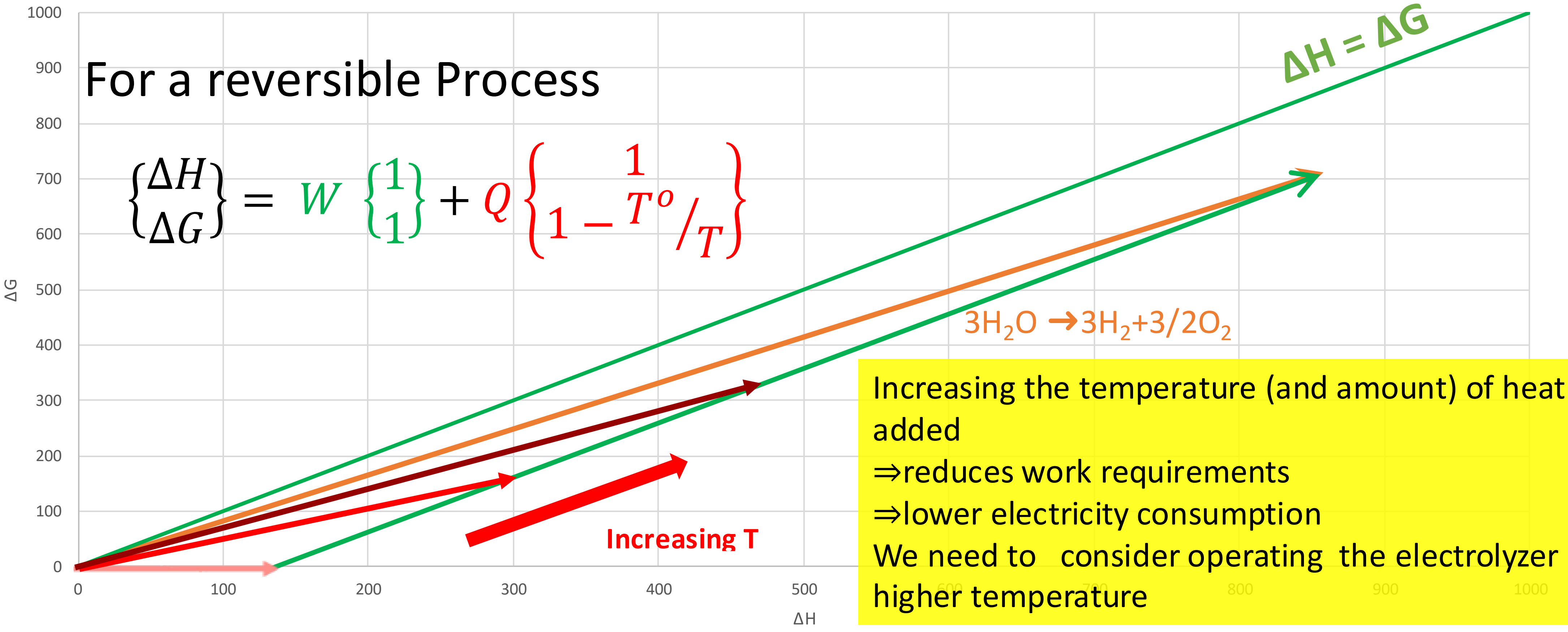


For a reversible Process

$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-{T^o}/{T}\end{Bmatrix}$$

- If we only add heat, T would need to be around 1750 K or higher

Adding Heat and Work to H_2 Production: Improves Process Reversibility



For a reversible Process

$$\begin{Bmatrix}\Delta H\\ \Delta G\end{Bmatrix} = W\begin{Bmatrix}1\\ 1\end{Bmatrix} + Q\begin{Bmatrix}1\\ 1-T^o/T\end{Bmatrix}$$

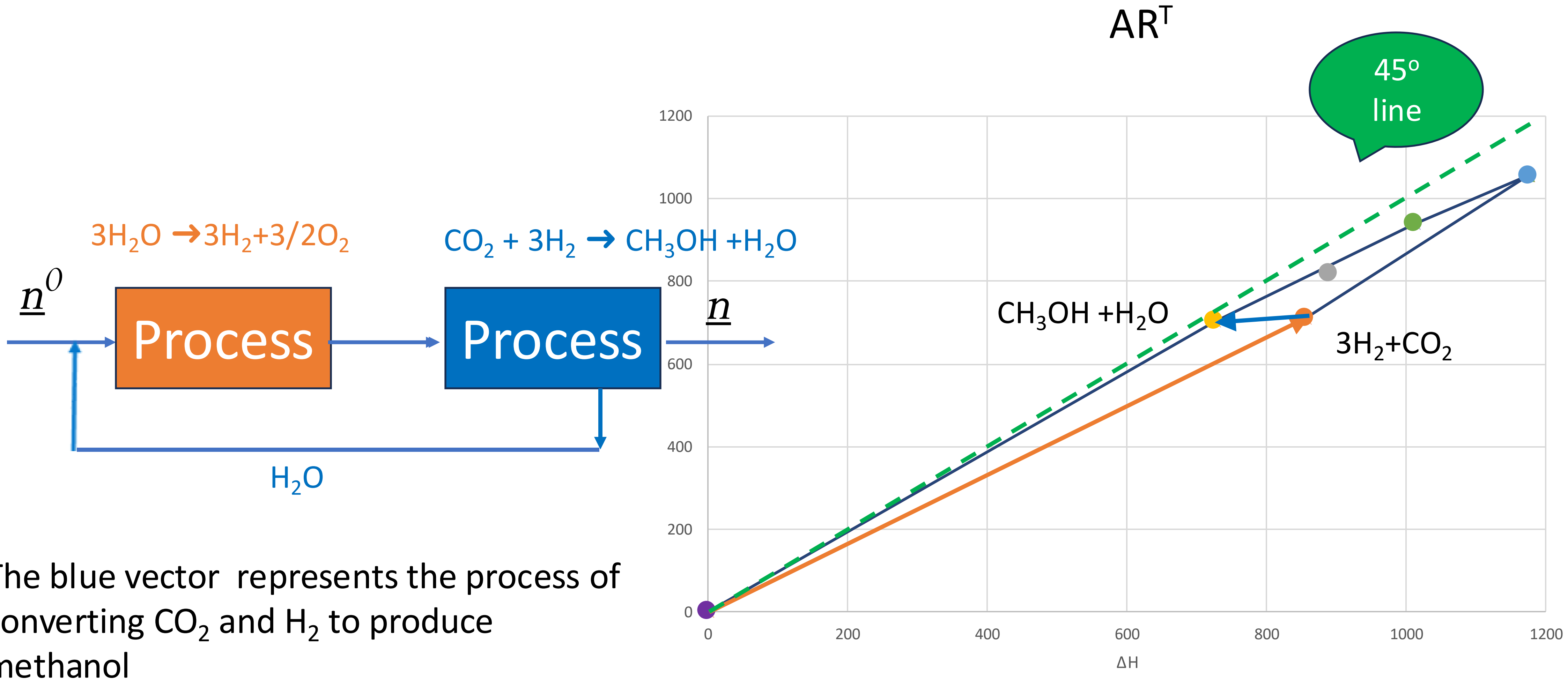
Increasing the temperature (and amount) of heat added

⇒reduces work requirements

⇒lower electricity consumption

We need to consider operating the electrolyzer at a higher temperature

Synthesis: matching heat and work vectors to the process vector

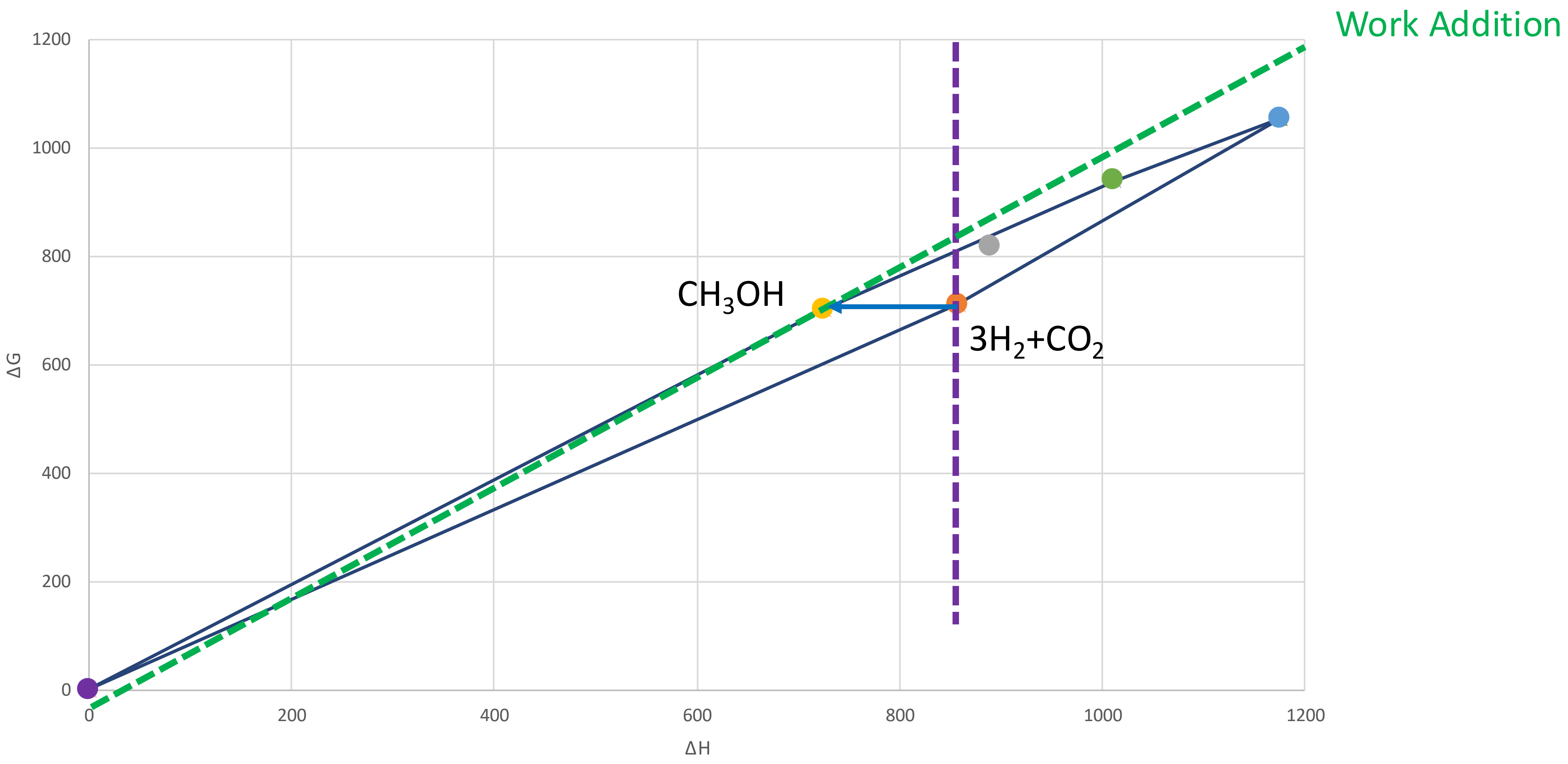


$3H_2O \rightarrow 3H_2 + 3/2O_2$

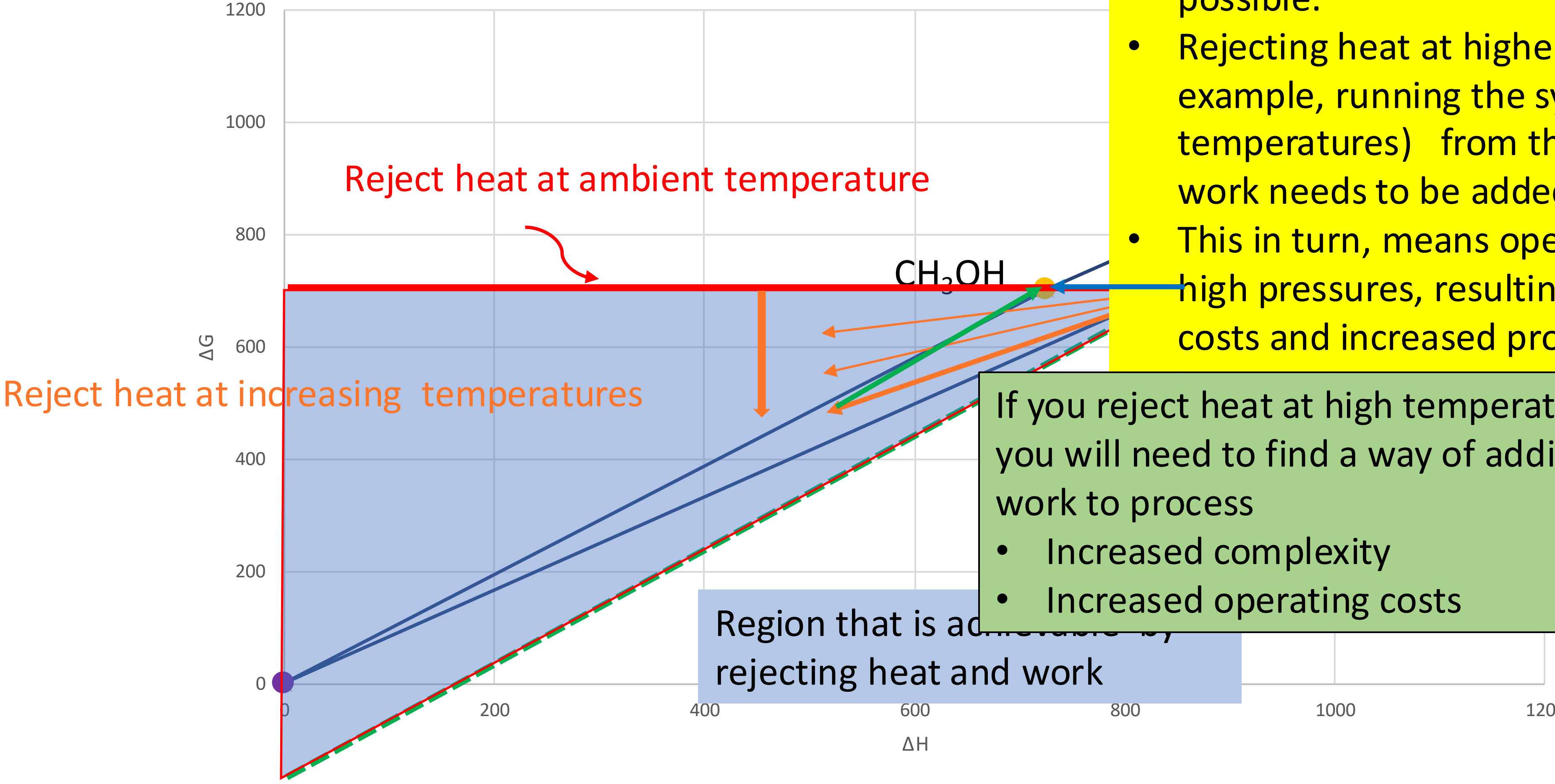
$CO_2 + 3H_2 \rightarrow CH_3OH + H_2O$

- The blue vector represents the process of converting CO_2 and H_2 to produce methanol
- Note that ΔG is slightly negative
- Process rejects heat and a bit of work

Matching Process Vector with Heat and Work Flows



Impact of rejecting heat at high tem



- Ideally, heat from the process should be rejected at as close to ambient temperature as possible.
- Rejecting heat at higher temperatures (for example, running the synthesis reactor at high temperatures) from the process means that work needs to be added to the process,
- This in turn, means operating the reactors at high pressures, resulting in increased operating costs and increased process complexity

If you reject heat at high temperature, you will need to find a way of adding work to process

- Increased complexity
- Increased operating costs

Applications of this approach

- By applying these techniques, we can improve the methane-to-methanol process

	Tons of CH_4 Consumed/ Ton of Methanol produced	Tons of CO_2 Emitted/ Ton of Methanol produced
Current Processes	0.53	0.09
New Process*	0.47	-0.11
Target (without incorporating renewable energy)	0.43	-0.19

Presented at FOCAPD 24; under review for Computers and Chemical Engineering

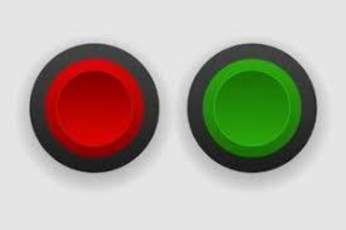
Putting this into practice

- In 2006 we were asked if we could develop a CTL technology for China
    - China at that time was short of fuel and electricity (similar to SA)
- We started by designing a pilot plant,
    - Tight schedule (18 months), tight budget
- We were responsible:
    - Process Synthesis
    - Project Management
    - Catalyst selection
    - Overseeing commissioning and operation
- Pilot plant using conventional semi-batch coal gasification
    - Located in Shaanxi, China
    - Commissioned in 2008, 5 bpd
- Demonstrated that we could reduce CO_2 emissions compared to conventional technologies and reduce operating and capital costs.

And more recently

How small can we make an XTL process?

- Worked together with ThyssenKrupp SA
  - A feasibility study was done (late 2013) on a 500 bpd GTL plant in TX, US
  - CAPEX in $/bpd is close to a recent (2012) ~100,000 bpd GTL case in US
- Can we design a 1 ton per day waste to electricity or fuel process?
  - Robust, simple process



Conclusions

- We have introduced a simple graphical representation of processes
- We can use vectors in a simple G-H plot to represent
    - Heat and workflows
    - Process
    - Chemical transformations
- You would like to match the heat and workflow vectors to the process vectors.
    - This implies that the heat and work added to the process is incorporated into the chemical transformations
    - Helps with screen chemistries, catalysts, and equipment choices

Conclusions Continued

- Reversibility sets the target for process performance
    - It allows one to determine the room for improvement in existing technologies
    - It allows one to determine sources of process inefficiencies
    - It allows one to set performance targets for new processes
- As energy costs increase, the balance in design will probably move towards reducing "energy" consumption, perhaps at the expense of increased capital costs.
    - It is, therefore, important during the early stages of process development to try to minimize inefficiencies resulting from choices of process operating conditions.

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Acknowledgements

The ideas presented have been developed over many years,with contributions from many different people.

Thanks, in particular, to Celestin Sempuga, James Fox, Ephraim Llane, Ralph Muvhiiwa, David Glasser, Brendon Hausberger, Bilal Patel.

We would like to thank the funders, which include:

Rutgers University
University of the Witwatersrand
University of South Africa
National Research Foundation of SA

Questions?