**EWO Meeting March 10, 2010**

# Optimal Scheduling under Variable Electricity Pricing and Availability

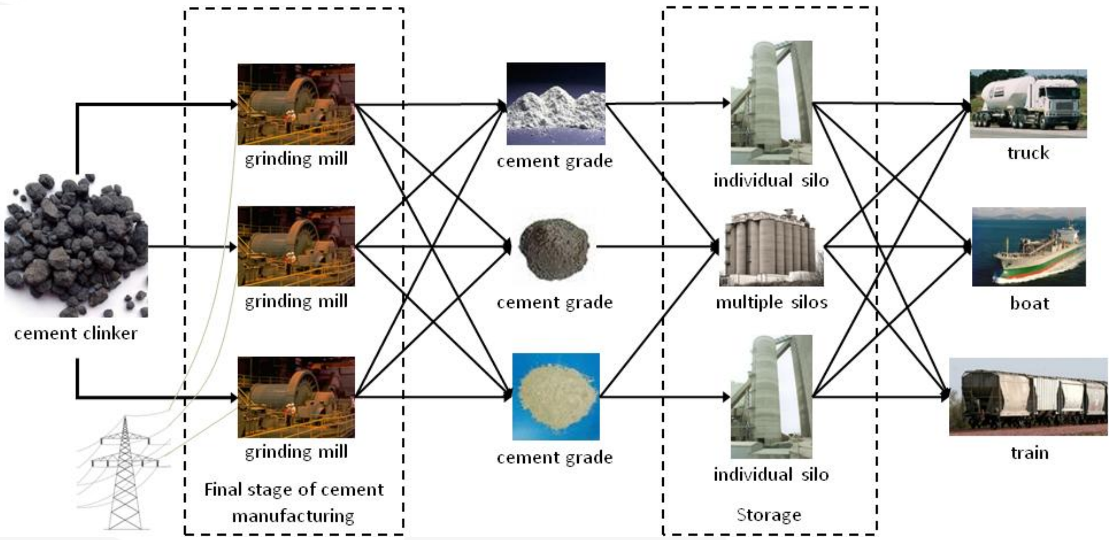
Pedro M. Castro
Iiro Harjunkoski
Ignacio E. Grossmann

LNEG ABB Carnegie Mellon CAPD CENTER

Lisbon, Portugal; Ladenburg, Germany; Pittsburgh, USA

# Motivating problem

- Multiproduct, single stage plant
  - Intensive use of electricity

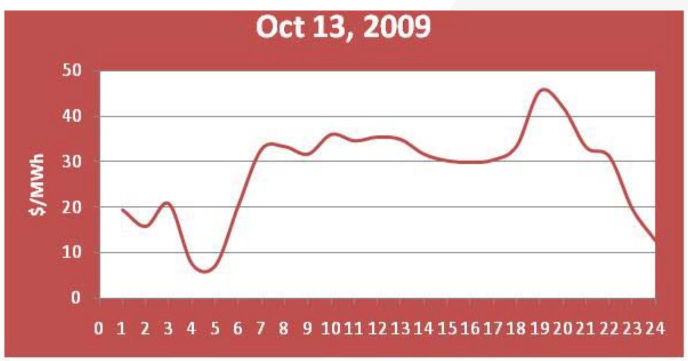


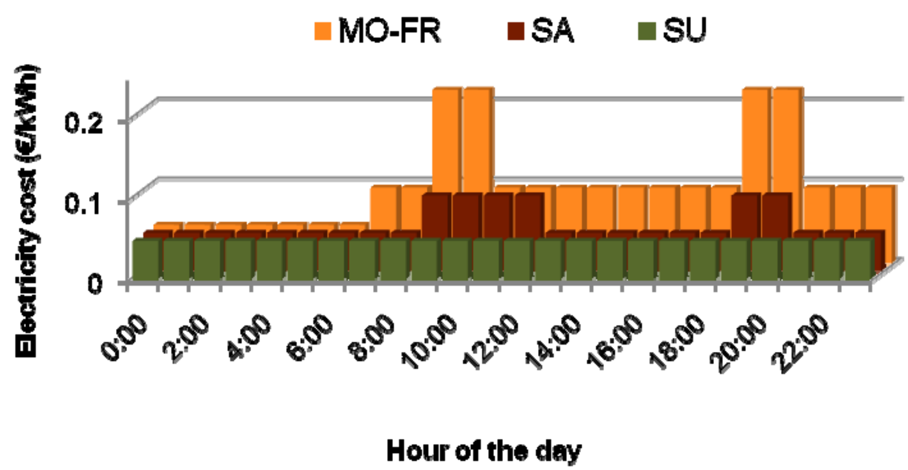

- When and where to produce a certain grade?
- How much to keep in storage?
  - Meet product demands (multiple due dates for each product)
- Minimize total energy cost
  - Meet power availability constraints

# Electricity usage

- Contracts between electricity supplier and plants
  - Energy cost [€/kWh]
    - Varies up to factor of 5 during the day
  - Maximum power consumption [MW]
    - Harsh cost penalties if levels are exceeded

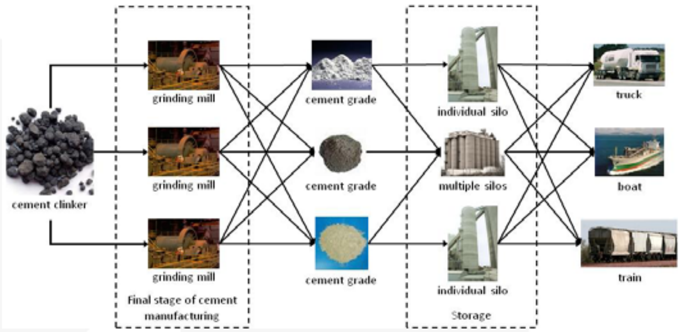
- Optimal scheduling with large impact on electricity bill
  - Produce in low-cost periods



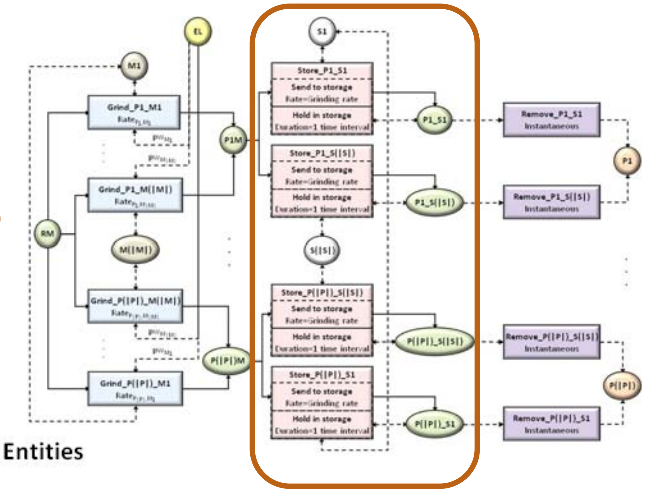



# Process modeling

- From flowsheet to Resource-Task Network







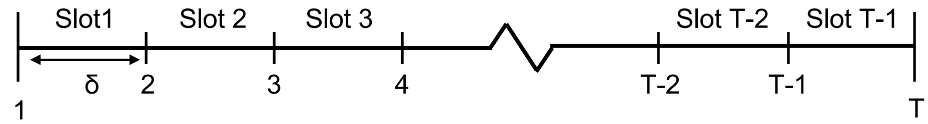
Shared storage units

- Convert problem data

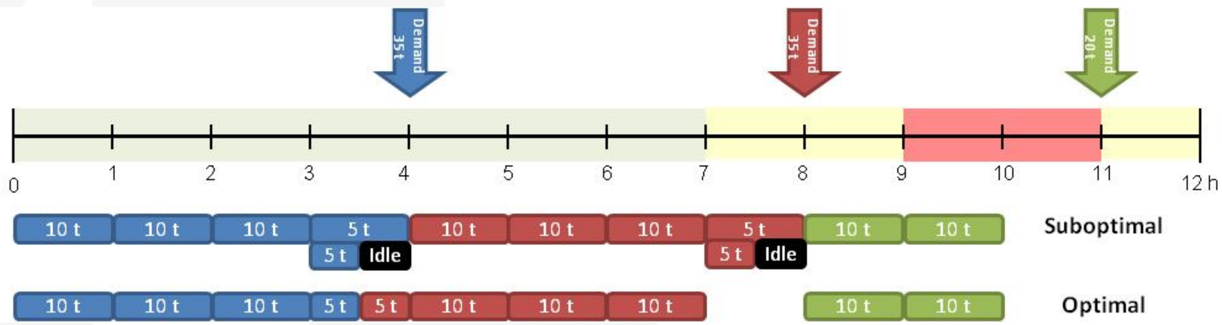
**Real Entities** | **RTN Virtual Entities**

| Real Entities | Data | | Description | | RTN data | RTN Virtual Entities |
|---|---|---|---|---|---|---|
| m- machine | $p_{p,m}$ | ⇨ | processing rate | ⇨ | $\rho_i^{max}$ | i- task |
| p- product | $d_{p,dy,hr}$ | ⇨ | product demands | ⇨ | $\Pi^{out}_{r,t}$, $\Pi^{out}_{r,td}$ | r- resource |
| hr- hour | $ec_{hr,dy}$ | ⇨ | electricity cost | ⇨ | $ce_t$ | t-time point |
| dy-day | | ⇨ | constant cost/power periods | ⇨ | $lb_{e,tp}$, $ub_{e,tp}$ | td- demand point |
| s- storage unit | $pwx_{hr,dy}$ | ⇨ | available power | ⇨ | $\Pi^{in}_{r,t}$, $\Pi^{in}_{r,tp,e}$ | tp-time period |
| | $cap_s$ | ⇨ | storage capacity | ⇨ | $R_r^{end\ max}$ | e- energy cost level |
| | $im_{s,p}$ | ⇨ | initial amount in storage | ⇨ | $R^0_r$ | |
| | $pw_{p,m}$ | ⇨ | structural parameters | ⇨ | $\mu_{r,i}$, $\bar{\mu}_{r,i}$, $\nu_{r,i}$, $\bar{\nu}_{r,i}$, $\lambda_{r,i}$ | |

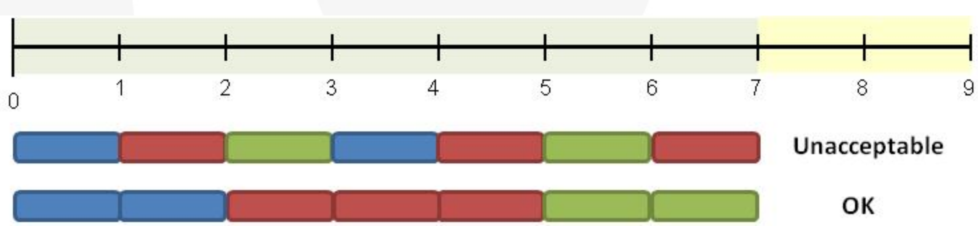
# 1.Discrete-time



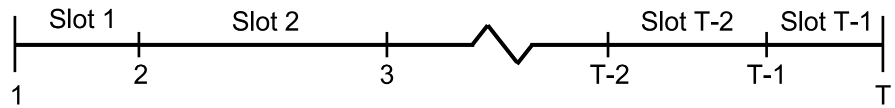

- Elegant and compact formulation
- Discrete-events handled naturally
  - Time intervals of 1 hour (δ) for 1 week horizon
- Minor limitations
  - Can lead to slightly suboptimal solutions



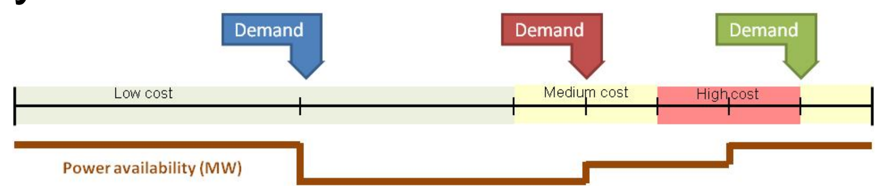

  - With too many changeovers



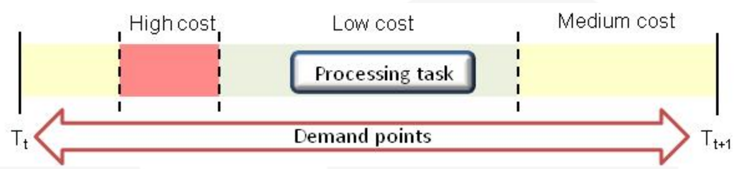
# 2.Continuous-time



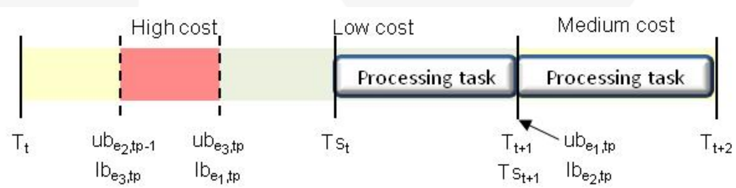

- General and accurate formulation
- Difficult to account for discrete events
  - Location of event points unknown a priori
    - Electricity pricing & availability
    - Product due dates




- Location of event points
  - At demand points

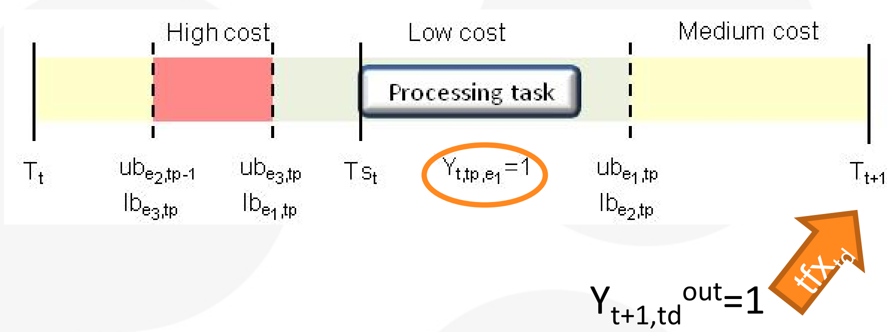



  - At some energy pricing/availability levels



# Mathematical formulation (MILP)

- Castro et al. (2009)
  - Binary variables
    - $N_{i,t}$ task *i* executed at interval *t*
    - $Y_{t,tp,e}$ time period *tp* of level *e* active during interval *t*
    - $Y_{t,td}^{out}$ event point *t* is demand point *td*




- New constraints
  - Starting time greater than energy level lower bound

$$Ts_t \geq \sum_{e\in E} \sum_{tp\in TP_e} lb_{e,tp} Y_{t,tp,e} \quad \forall t \in T, t \neq |T|$$

  - Tasks must end before level upper bound

$$Ts_t + \sum_{i\in I_e^c} \frac{\bar{\mu}_{r,i}\xi_{i,t}}{\rho_i^{max}} \leq \sum_{tp\in TP_e} ub_{e,tp} Y_{t,tp,e} + H\left(1 - \sum_{i\in I_e^c} \bar{\mu}_{r,i} N_{i,t}\right)$$

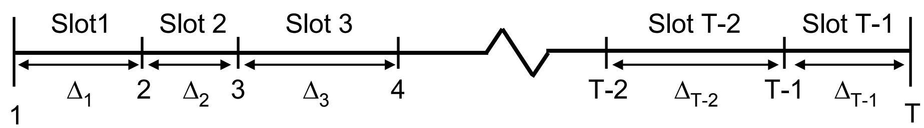
  - And before end of slot

$$T_{t+1} - Ts_t \geq \sum_{i\in I^c} \frac{\bar{\mu}_{r,i}\xi_{i,t}}{\rho_i^{max}} \quad \forall r \in R^{TC}, t \in T, t \neq |T|$$

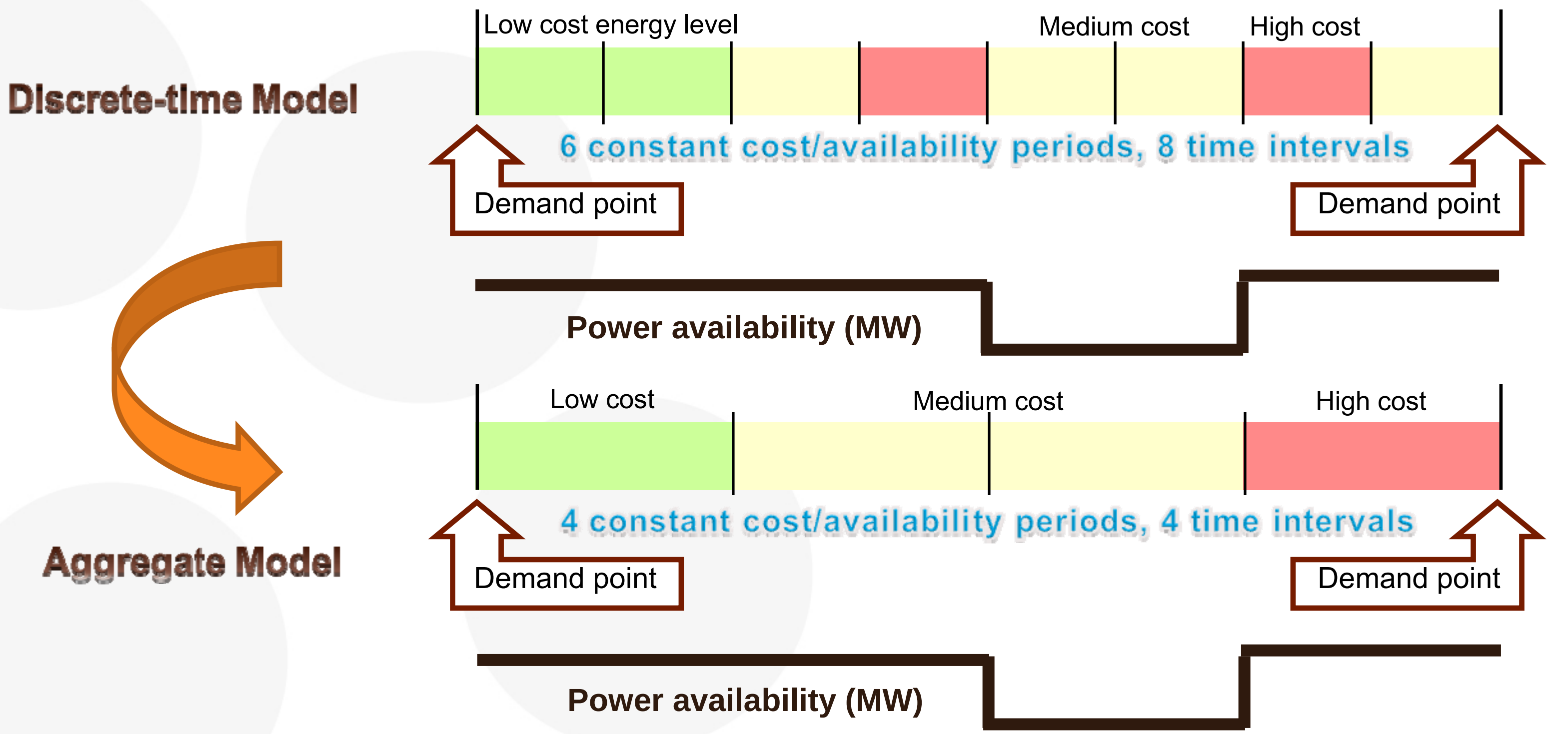
  - Absolute time of a demand point equal to a due date

$$T_t \leq \sum_{td\in TD} tfx_{td} Y_{t,td}^{out} + H\left(1 - \sum_{td\in TD} Y_{t,td}^{out}\right)$$

# Aggregate model




- Looks in between consecutive demand points
- Merges periods with same energy pricing/power level




- Only valid for single stage plants

# Important properties aggregate model

- Can be viewed as a planning approach
  - Not concerned with actual timing of events
- Continuous-time within a time interval without event points
  - Resource balances for equipments and utilities replaced by two sets of constraints
    - Time interval duration greater than sum of processing times

$$\Delta_t \geq \sum_{i \in I^e} \frac{\overline{\mu}_{r,i} \xi_{i,t}}{\rho_i^{\max}} \quad \forall r \in R^{TC}, t \in T, t \neq |T|$$

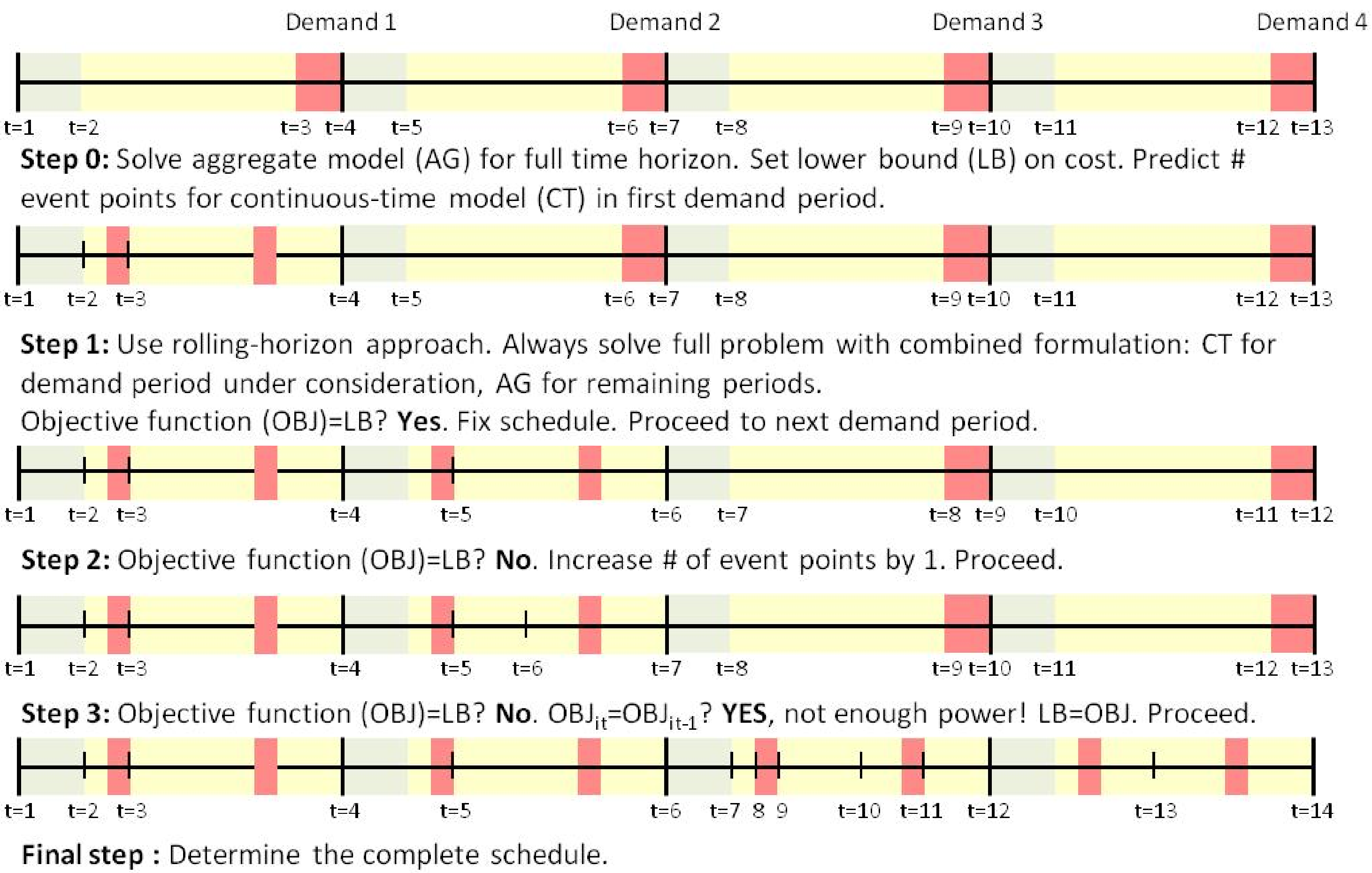
    - Energy balances (soft constraints due to slacks $S_{r,t}$)

$$\Delta_t \cdot (\Pi_{r,t}^{in} + S_{r,t}) \geq \sum_{i \in I^e} \frac{-\mu_{r,i} \xi_{i,t}}{\rho_i^{\max}} \quad \forall r \in R^{UT}, t \in T, t \neq |T|$$

- Predicts # slots to generate detailed schedule

# New rolling-horizon algorithm

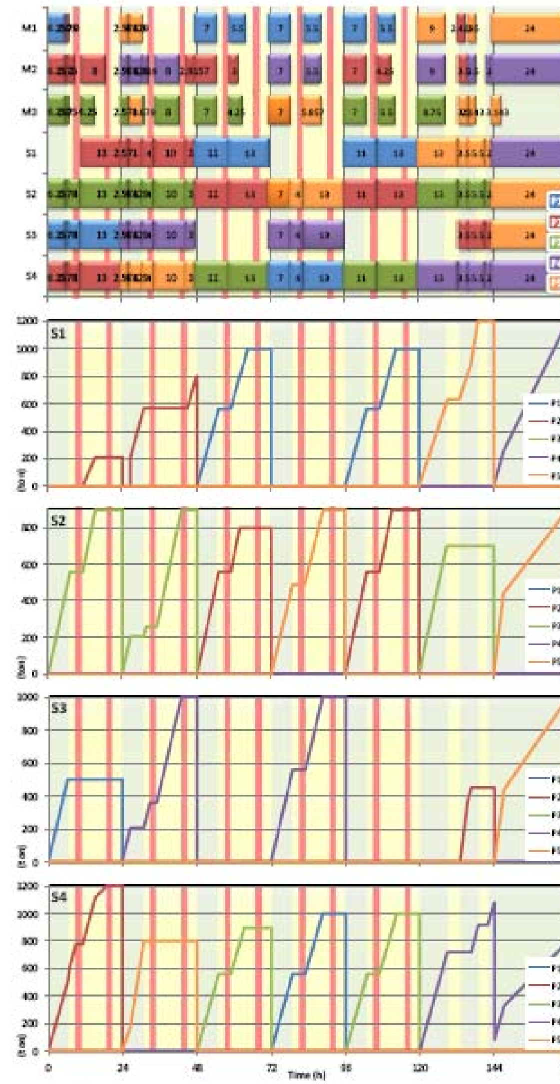
- Calls combined aggregate/continuous-time model
  - Time grid is part continuous and part discrete



# Computational statistics

| Case | (P,M,S) | Power | Model | \|T\| | RMIP | MIP [€] | CPUs | Gap (%) |
|---|---|---|---|---|---|---|---|---|
| EX5a | (3,2,2) | R | DT | 169 | 31351 | 31798 | 7200 | 0.02 |
| | | | AG | 20 | 29657 | 29657 | 0.24 | 0 |
| | | | RH | 17 | 41124 | 41124 | 7.06 | 0 |
| | | | CT | 10 | 25625 | 94901 | 9829 | 0 |
| EX6 | (3,2,3) | U | DT | 169 | 43250 | 43259 | 7200 | 0.02 |
| | | | AG | 19 | 43250 | 43250 | 0.37 | 0 |
| | | | RH | 21 | 43250 | 43250 | 5.57 | 0 |
| | | | CT | 9 | 35517 | Inf. | 2811 | - |
| EX7 | (3,3,4) | U | DT | 169 | 68282 | 68282 | 19.9 | 0 |
| | | | AG | 18 | 68282 | 68282 | 0.7 | 0 |
| | | | RH | 12 | 68282 | 68282 | 3.12 | 0 |
| | | | CT | 12 | 48852 | no sol. | 7200 | - |
| EX8 | (3,3,5) | R | DT | 169 | 101139 | 104622 | 7200 | 0.22 |
| | | | AG | 19 | 104375 | 104375 | 2.05 | 0 |
| | | | RH | 31 | - | 151257 | 17330 | 0.16 |
| EX9 | (4,3,4) | U | DT | 169 | 87817 | 87868 | 7200 | 0.06 |
| | | | AG | 19 | 87817 | 87817 | 0.71 | 0 |
| | | | RH | 25 | 87817 | 87817 | 917 | 0 |
| EX10 | (5,3,4) | U | DT | 169 | 86505 | 86582 | 7200 | 0.09 |
| | | | AG | 19 | 86505 | 86550 | 3.57 | 0 |
| | | | RH | 23 | 86550 | 86550 | 1508 | 0 |

- DT difficult to close optimality
- CT limited to small problems
- RH generates full schedule relatively fast



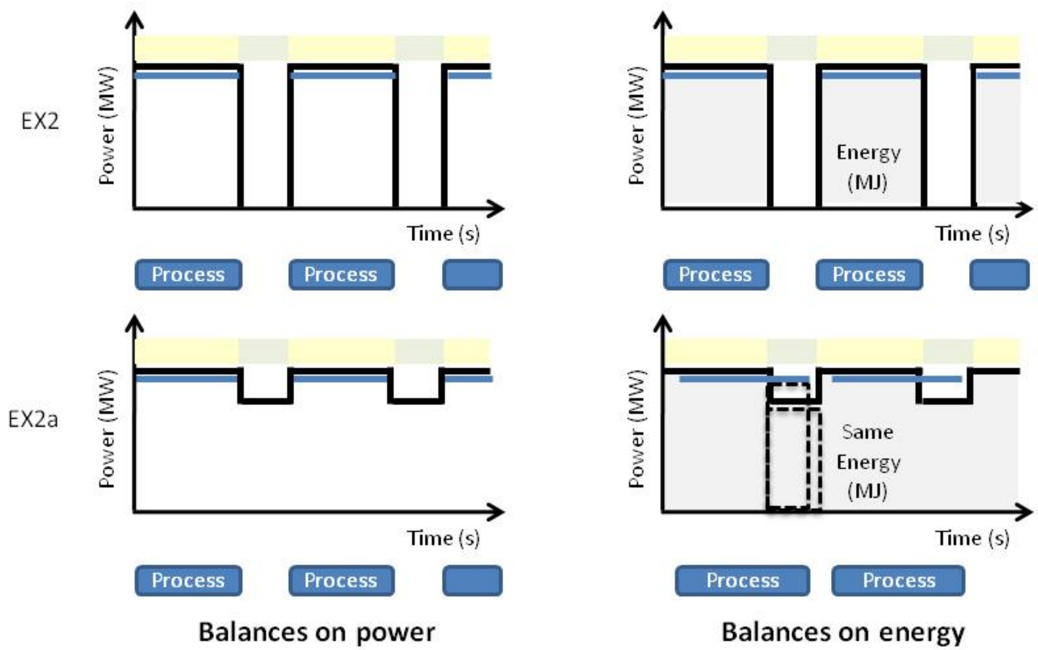
# Conclusions

- Problems with unlimited power availability
  - Aggregate model is rigorous and very powerful
    - Lower degree of degeneracy than DT
    - 1/10 problem size
    - 4 orders magnitude reduction CPUs
  - Rolling-horizon algorithm finds practical, global optimal solutions efficiently
    - Considers the whole remaining problem simultaneously

| EX10 | | | |
|---|---|---|---|
| Day | \|T\| | LB (k€) | OBJ (k€) |
| MO | 4 | 86.55 | 86.674 |
| | 5 | 86.55 | 86.55 |
| TU | 9 | 86.55 | 86.573 |
| | 10 | 86.55 | 86.55 |
| WE | 12 | 86.55 | 86.55 |
| TH | 16 | 86.55 | 86.55 |
| FR | 18 | 86.55 | 86.55 |
| SA | 21 | 86.55 | 86.601 |
| | 22 | 86.55 | 86.55 |
| SU | 23 | 86.55 | 86.55 |

# Energy vs. power balances

- Problems with restricted power
  - Discrete-time formulation is the best
    - Finds very good solutions (<0.8 %) rapidly (5 min)
  - Aggregate model is a relaxation (underestimates cost)
    - 5 h@4 MW ≠ 4 h@5 MW but have same energy




€21575

€18977

  - Rolling horizon may generate suboptimal solutions