Lagrangean Decomposition Algorithm for Supply Chain Redesign in Electric Motors Industry

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➢ Introduction of the Supply Chain Model

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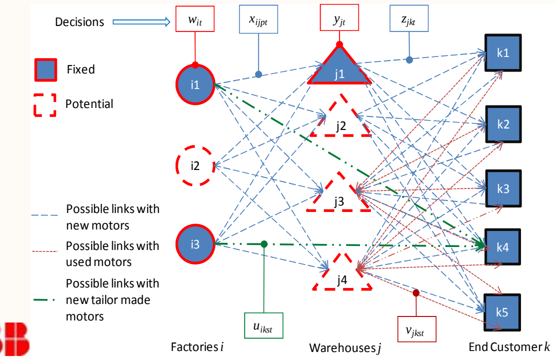
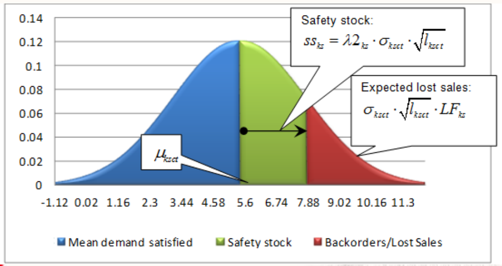


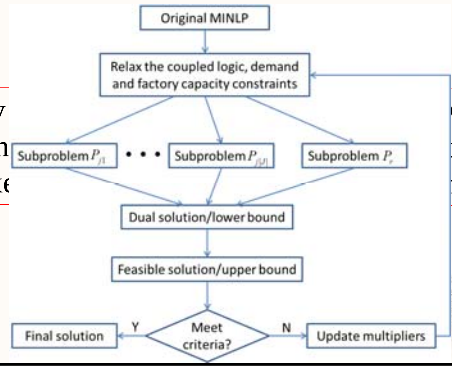
Demand, Safety stock & Back-orders

- *Uncertainty* **modeled with Poisson distribution**
  - *Mean demand* *is given by the average failure rate*
  - *Standard deviation* *indicates demand (failure) variability*



➢ Lagrangean Decomposition Algorithm

Capacity constrain are relaxe



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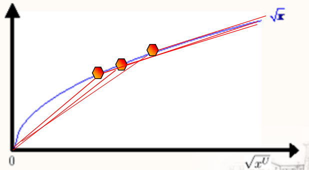
Lagrangean Decomposition Algorithm

- Handling of nonlinear terms of subproblems:
  - Adaptive linear approximation



Factories*i* Warehouses*j* 8/1/2013 5

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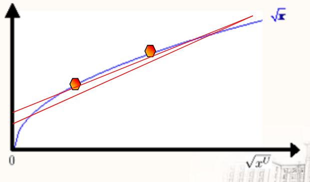
Lagrangean Decomposition Algorithm

- Feasibility scheme:
  - Adaptive linear approximation



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Lagrangean Decomposition Algorithm

- Multipliers initialization:

$$\lambda_{muctkpt0} = \left(\begin{array}{c} \sum_i \sum_j \sum_p g_j \cdot \mu^{new}_{ijkpt} \cdot \chi + \sum_i \sum_j \sum_p gp_i \cdot \mu^{new}_{ijkpt} \cdot \chi \\ + \sum_{j \in SC} \sum_p gr_{jp} \cdot \mu^{used}_{jkpt} \cdot \chi + \sum_i \sum_{j \in SC} \sum_p c1_{ij} \cdot \mu^{new}_{ijkpt} \cdot \chi \\ + \sum_i \sum_j \sum_p c2_{jk} \cdot \mu^{new}_{ijkpt} \cdot \chi + \sum_{j \in SC} 2 \cdot c2_{jk} \cdot \chi \cdot \sum_p \mu^{used}_{jkpt} \\ + \sum_i \sum_j \sum_p \theta1_{jp} \cdot \mu^{new}_{ijkpt} \cdot t1_{ijp} + \sum_i \sum_j \sum_p \theta2_{kp} \cdot \mu^{new}_{ijkpt} \cdot t2_{jkp} \\ + \sum_j \sum_p h1_{jp} \cdot ss_{jpt} + \sum_{s \in KT_{ks}} \sum_{c \in KSC_{ksc}} h2_k \cdot \lambda 2_{ks} \cdot \sigma_{ksct} \cdot \sqrt{l_{ksct}} \end{array} \right) \Bigg/ \left(\begin{array}{c} \sum_i \sum_j \sum_p \mu^{new}_{ijkpt} \\ + \sum_{j \in SC} \sum_p \mu^{used}_{jkpt} \end{array} \right)$$

$$\lambda_{tolkst0} = \left(\begin{array}{c} \sum_{j \in SC} \sum_{s \in KT_{ks}} gr'_{js} \cdot \tau^{used}_{jkst} \cdot \chi + \sum_i \sum_k \sum_{s \in KT_{ks}} c3_{ik} \cdot \tau^{new}_{ikst} \cdot \chi \\ + \sum_{j \in SC} \sum_k 2 \cdot c2_{jk} \cdot \chi \cdot \sum_{s \in KT_{ks}} \tau^{used}_{jkst} \\ + \sum_k \sum_{s \in KT_{ks}} \sum_{c \in KSC_{ksc}} h2_k \cdot \lambda 2_{ks} \cdot \sigma_{ksct} \cdot \sqrt{m_{ksct}} \end{array} \right) \Bigg/ \left(\begin{array}{c} \sum_i \tau^{new}_{ikst} \\ + \sum_{j \in SC} \tau^{used}_{jkst} \end{array} \right)$$

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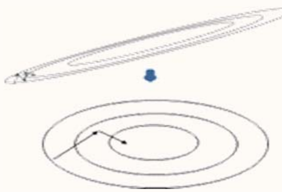
Lagrangean Decomposition Algorithm

- Multipliers Scaling: avoid ill-conditioning



$$\lambda_{zkt} = \alpha_z \lambda'_{zkt}$$
$$\lambda_{vskt} = \alpha_v \lambda'_{vskt}$$
$$\lambda_{muctkpt} = \alpha_{mu} \lambda'_{muctkpt}$$
$$\lambda_{tolks} = \alpha_t \lambda'_{tolks}$$
$$\lambda_c = \alpha_c \lambda'_c$$

$$g' = \left(\frac{1}{\alpha_z} g^T_{zkt}, \frac{1}{\alpha_v} g^T_{vskt}, \frac{1}{\alpha_{mu}} g^T_{muctkpt}, \frac{1}{\alpha_t} g^T_{tolks}, \frac{1}{\alpha_c} g^T_c \right)^T$$

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Results Analysis

Element	Case 1	Case 2	Case 3
Factory	3	7	7
Warehouse	7	5	5
Customer	27	27	27
Standard motor	32	32	99
Special motor	49	49	396
Service level	4	4	4
Time horizon	5	5	5

Item	Case 1	Case 2	Case 3
Number of constraints	326,151	430,707	1,471,287
Number of variables	190,414	236,853	807,713
Number of binary variables	14,444	16,339	47,654

ABB

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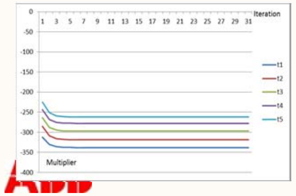
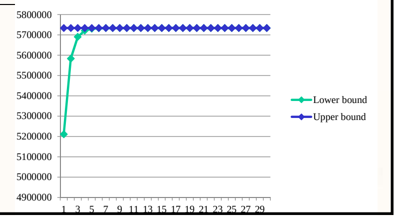
Results Analysis-Case 1

Name	Optimal objective/$	Error/%	Time consumed
MINLP	No feasible solution	—	3:23:05
MILP-2	5747911.872	0.24	0:11:35
MILP-5	5748118.909	0.24	2:49:01
LD (30 iterations)	5733962.14	0	0:20:51
LD (7 iterations)	5733962.14	0.005*	0:05:48*





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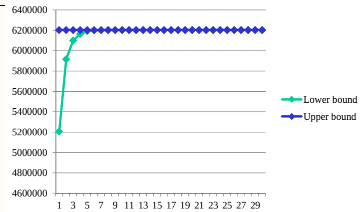
Results Analysis-Case 2

Name	Optimal objective/$	Error/%	Time consumed
MINLP	6537842.475	5.40	8:58:56
MILP-2	6354304.153	2.44	0:13:32
MILP-5	6354304.153	2.44	23:56:11
LD (30 iterations)	6202732.33	0	0:21:45
LD (8 iterations)	6202732.33	0.004*	0:06:24*



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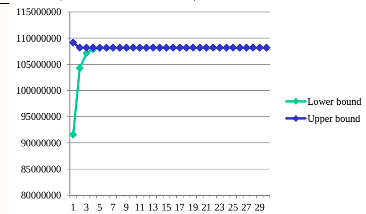
Results Analysis-Case 3

Name	Optimal objective/$	Error/%	Time consumed
MINLP	No feasible solution	—	100:02:28
MILP-2	1.205469E+8	11.43	3:08:8
MILP-5	No feasible solution	—	23:28:51
LD (30 iterations)	108178792.52	0	6:08:15
LD (7 iterations)	108178792.52	0.003*	1:35:00*



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➢ Conclusion

- Supply chain for electric motors is complex due to reverse flows
- MILP approximation only computationally feasible with few points => less accuracy
- LD algorithm can obtain optimal solution efficiently, especially if convergence is within ~0.005%

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Thanks!