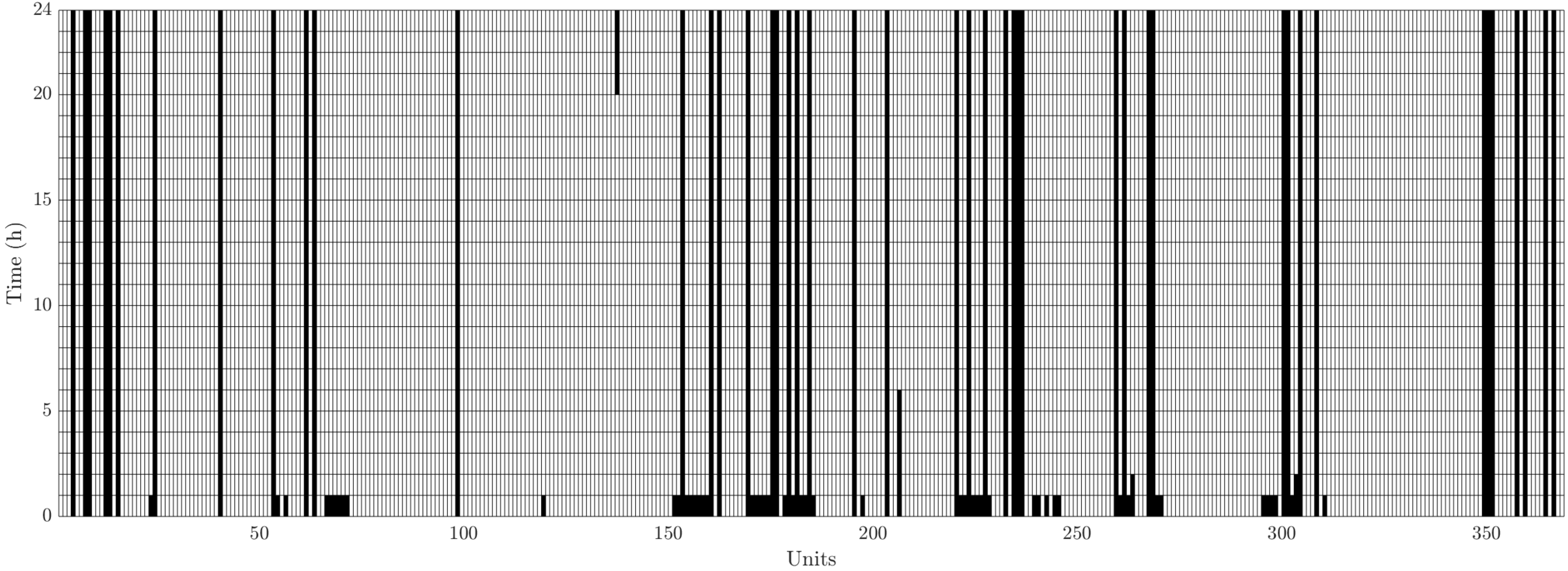
AC Network-Constrained Unit Commitment (AC-NCUC)

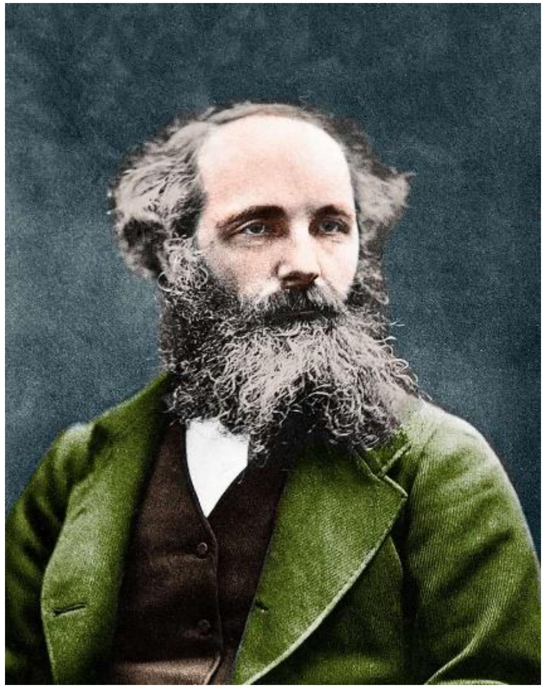
Antonio J. Conejo, Gonzalo Constante, Feng Qiu

April 20, 2021

Binary decisions!



Nonlinearities and non-convexities!



$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\Downarrow$$

$$p_{nm}(\cdot) = v_n^2 y_{\mathrm{L}nm} \cos(\theta_{\mathrm{L}nm}) - v_n v_m y_{\mathrm{L}nm} \cos(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) + \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \cos(\theta_{\mathrm{S}nm})$$
$$q_{nm}(\cdot) = -v_n^2 y_{\mathrm{L}nm} \sin(\theta_{\mathrm{L}nm}) - v_n v_m y_{\mathrm{L}nm} \sin(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) - \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \sin(\theta_{\mathrm{S}nm})$$

$$p_{nm}(\cdot) = \quad v_n^2 y_{\mathrm{L}nm} \cos\left(\theta_{\mathrm{L}nm}\right) - v_n v_m y_{\mathrm{L}nm} \cos(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) + \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \cos\left(\theta_{\mathrm{S}nm}\right)$$
$$q_{nm}(\cdot) = -v_n^2 y_{\mathrm{L}nm} \sin\left(\theta_{\mathrm{L}nm}\right) - v_n v_m y_{\mathrm{L}nm} \sin(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) - \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \sin\left(\theta_{\mathrm{S}nm}\right)$$

$$\Downarrow$$

$$p_{nm}(\cdot) = G_{nm}(e_m e_n + f_m f_n) - B_{nm}(e_m f_n - e_n f_m) - G_{nm}(e_n^2 + f_n^2)$$
$$q_{nm}(\cdot) = -G_{nm}(e_m f_n - e_n f_m) - B_{mn}(e_m e_n + f_m f_n) + (B_{nm} - b_{nm}^{\mathrm{shunt}})(e_n^2 + f_n^2)$$

The Unit Commitment (UC) problem consists of

- determining, for a planning horizon (typically 1 day), the hourly start-up and shut-down schedule of all production units
- so that the electric demand is supplied and total operating costs are minimized,
- while satisfying different technical and security constraints (network and production units).

Objective function (cost)

minimize

$$\sum_{t\in\mathcal{T}}\Bigg(\sum_{j\in\mathcal{G}^{\mathrm{c}}\cup\mathcal{G}^{\mathrm{g}}}\big(c_j(p_j(t)) + c_j^{\mathrm{U}} y_j(t) + c_j^{\mathrm{D}} z_j(t)\big) + \sum_{j\in\mathcal{G}^{\mathrm{w}}} c_j(p_j^{\mathrm{W}}(t)) + \sum_{j\in\mathcal{G}^{\mathrm{ng}}} c_j(p_j(t)) + \sum_{n\in\mathcal{N}}\big(c_n^{\mathrm{LOL}} D_n^{\mathrm{shed}}(t) + c_n^{\mathrm{shed}} Q_n^{\mathrm{shed}}(t)\big)\Bigg) \tag{1}$$

subject to:

Start-up and shut-down logic, ramping

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0; \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ t = 2, \dots, |\mathcal{T}| \tag{2}$$

$$p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}} v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t); \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ t = 2, \dots, |\mathcal{T}| \tag{3}$$

$$p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}} v_j(t) + S_j^{\mathrm{D}}\, z_j(t); \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ t = 2, \dots, |\mathcal{T}| \tag{4}$$

$$v_j^{\mathrm{ini}} - v_j(1) + y_j(1) - z_j(1) = 0; \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{5}$$

$$p_j(1) - p_j^{\mathrm{ini}} \leq R_j^{\mathrm{U}} v_j^{\mathrm{ini}} + S_j^{\mathrm{U}}\, y_j(1); \ \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{6}$$

$$p_j^{\mathrm{ini}} - p_j(1) \leq R_j^{\mathrm{D}} v_j(1) + S_j^{\mathrm{D}}\, z_j(1); \ \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{7}$$

Power buffer

$$\underline{P}_j v_j(t) \leq p_j(t) \leq \overline{p}_j(t) \leq \overline{P}_j v_j(t); \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ \forall t \in \mathcal{T} \tag{8}$$

$$\underline{P}_j \leq p_j(t) \leq \overline{p}_j(t) \leq \overline{P}_j; \quad \forall j \in \mathcal{G}^{\mathrm{ng}},\ \forall t \in \mathcal{T} \tag{9}$$

$$\overline{p}_j(t) \leq p_j(t-1) + R_j^{\mathrm{U}} v_j(t-1) + S_j^{\mathrm{U}} y_j(t); \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ t = 2, \ldots, |\mathcal{T}| \tag{10}$$

$$\overline{p}_j(t) \leq p_j(t-1) + R_j^{\mathrm{U}}; \quad \forall j \in \mathcal{G}^{\mathrm{ng}}, \qquad t = 2, \ldots, |\mathcal{T}| \tag{11}$$

$$\overline{p}_j(t) \leq \overline{P}_j \left[v_j(t) - z_j(t+1) \right] + z_j(t+1) S_j^{\mathrm{D}}; \qquad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}},\ \forall t \in \mathcal{T} \tag{12}$$

$$\overline{p}_j(1) \leq p_j^{\mathrm{ini}} + R_j^{\mathrm{U}} v_j^{\mathrm{ini}} + S_j^{\mathrm{U}} y_j(1); \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{13}$$

$$\overline{p}_j(1) \leq p_j^{\mathrm{ini}} + R_j^{\mathrm{U}}; \quad \forall j \in \mathcal{G}^{\mathrm{ng}} \tag{14}$$

Reactive power bounds

$$\underline{Q}_j v_j(t) \leq q_j(t) \leq \overline{Q}_j v_j(t);\ \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}}, \forall t \in \mathcal{T} \tag{15}$$

$$\underline{Q}_j \leq q_j(t) \leq \overline{Q}_j; \quad \forall j \in j \in \mathcal{G}^{\mathrm{ng}},\ \forall t \in \mathcal{T} \tag{16}$$

$$\underline{Q}_j^{\mathrm{N}} \leq q_j^{\mathrm{N}}(t) \leq \overline{Q}_j^{\mathrm{N}}; \quad \forall j \in j \in \mathcal{G}^{\mathrm{nuc}},\ \forall t \in \mathcal{T} \tag{17}$$

Minimum up time

$$\sum_{t=1}^{L_j} [1 - v_j(t)] = 0; \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{18}$$

$$\sum_{i=t}^{k+T_j^{\mathrm{U}}-1} v_j(t) \geq T_j^{\mathrm{U}} y_j(t); \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}}, \qquad \forall t \in [L_j + 1, \ \ldots, |\mathcal{T}| - T_j^{\mathrm{U}} + 1] \tag{19}$$

$$\sum_{i=t}^{|\mathcal{T}|} [v_j(t) - y_j(t)] \geq 0; \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}}, \qquad \forall t \in [|\mathcal{T}| - T_j^{\mathrm{U}} + 2, \ \ldots, |\mathcal{T}|] \tag{20}$$

Minimum down time

$$\sum_{t=1}^{F_j} v_j(t) = 0; \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}} \tag{21}$$

$$\sum_{i=t}^{k+T_j^{\mathrm{D}}-1} [1 - v_j(t)] \geq T_j^{\mathrm{D}} z_j(t); \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}}, \qquad \forall t \in [F_j + 1, \ \ldots, |\mathcal{T}| - T_j^{\mathrm{D}} + 1] \tag{22}$$

$$\sum_{i=t}^{|\mathcal{T}|} [1 - v_j(t) - z_j(t)] \geq 0; \quad \forall j \in \mathcal{G}^{\mathrm{c}} \cup \mathcal{G}^{\mathrm{g}}, \qquad \forall t \in [|\mathcal{T}| - T_j^{rmD} + 2, \ \ldots, |\mathcal{T}|] \tag{23}$$

Weather-dependent units

$$0 \leq p_j^{\mathrm{W}}(t) \leq \overline{P}_j^{\mathrm{W}}(t); \quad \forall j \in \mathcal{G}^{\mathrm{w}},\ \forall t \in \mathcal{T} \tag{24}$$

$$\underline{Q}_j^{\mathrm{W}}(t) \leq q_j^{\mathrm{W}}(t) \leq \overline{Q}_j^{\mathrm{W}}(t); \quad \forall j \in \mathcal{G}^{\mathrm{w}},\ \forall t \in \mathcal{T} \tag{25}$$

Reserve

$$\sum_{j \in \Omega_r^{\mathrm{G}}} \overline{p}_j(t) - p_j(t) \geq R_r(t); \quad \forall r \in \mathcal{R},\ \forall t \in \mathcal{T} \tag{26}$$

Power balance

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j(t) + \sum_{j\in\Lambda_n^{\mathrm{W}}} p_j^{\mathrm{W}}(t) + \sum_{j\in\Lambda_n^{\mathrm{N}}} p_j^{\mathrm{N}}(t) + D_n^{\mathrm{shed}}(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m\in\Lambda_n} \left[G_{nm}c_{nm}(t) - B_{nm}s_{nm}(t) - G_{nm}c_{nn}(t)\right]; \quad \forall n \in \mathcal{N},\ \forall t \in \mathcal{T} \tag{27}$$

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} q_j(t) + \sum_{j\in\Lambda_n^{\mathrm{W}}} q_j^{\mathrm{W}}(t) + \sum_{j\in\Lambda_n^{\mathrm{N}}} q_j^{\mathrm{N}}(t) + Q_n^{\mathrm{shed}}(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} Q_i(t) = \sum_{m\in\Lambda_n} \left[-G_{nm}s_{nm}(t) - B_{mn}c_{nm}(t) + (B_{nm} - b_{nm}^{\mathrm{shunt}})c_{nn}(t)\right]; \quad \forall n \in \mathcal{N},\ \forall t \in \mathcal{T} \tag{28}$$

Transmission and voltage bounds

$$
\begin{aligned}
&(G_{nm}c_{nm}(t) - B_{nm}s_{nm}(t) - G_{nm}c_{nn}(t))^2 + \Big(- G_{nm}s_{nm}(t) - B_{nm}c_{nm}(t) \\
&\quad + (B_{nm} - b_{nm}^{\text{shunt}})c_{nn}(t)\Big)^2 \leq \overline{S}_{nm}^2; \forall n \in \mathcal{N}, \forall m \in \Lambda_n, \ \forall t \in \mathcal{T} \qquad (29)
\end{aligned}
$$

$$
\underline{V}_n^2 \leq c_{nn}(t) \leq \overline{V}_n^2; \quad \forall n \in \mathcal{N}, \ \forall t \in \mathcal{T} \qquad (30)
$$

Non-convexities

Nonlinear and non-convex!

$$c_{nn}(t) = e_n^2(t) + f_n^2(t); \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \tag{31}$$

$$c_{nm}(t) = e_m(t)e_n(t) + f_m(t)f_n(t); \qquad \forall n \in \mathcal{N}, \forall m \in \Lambda_n, \ \forall t \in \mathcal{T} \tag{32}$$

$$s_{nm}(t) = e_m(t)f_n(t) - e_n(t)f_m(t); \qquad \forall n \in \mathcal{N}, \forall m \in \Lambda_n, \ \forall t \in \mathcal{T} \tag{33}$$

Unserved energy

$$0 \leq D_n^{\text{shed}}(t) \leq \sum_{i \in \Lambda_n^{\text{D}}} D_i(t); \quad \forall n \in \mathcal{N},\ \forall t \in \mathcal{T} \tag{34}$$

$$0 \leq Q_n^{\text{shed}}(t) \leq \sum_{i \in \Lambda_n^{\text{D}}} Q_i(t); \quad \forall n \in \mathcal{N},\ \forall t \in \mathcal{T} \tag{35}$$

1. Why?
2. A three-step solution approach
3. Benders (if needed)
4. Two case studies

Why?

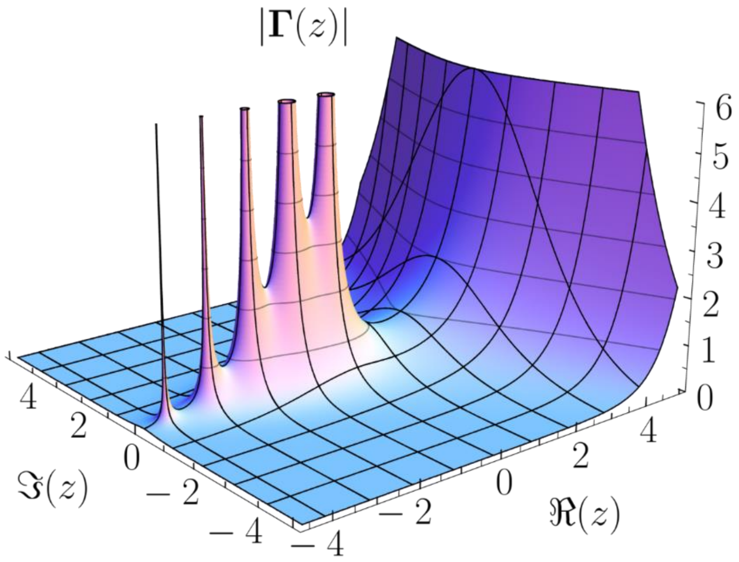
$\Longrightarrow$ Increasing integration of weather-dependent renewable power.

$\Longrightarrow$ Increasing demand (electric vehicles, data centers).

Reactive power / voltage constraints may alter commitment decisions.

$$p_{nm}(\cdot) = \quad v_n^2 y_{\mathrm{L}nm} \cos(\theta_{\mathrm{L}nm}) - v_n v_m y_{\mathrm{L}nm} \cos(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) + \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \cos(\theta_{\mathrm{S}nm})$$

$$q_{nm}(\cdot) = -v_n^2 y_{\mathrm{L}nm} \sin(\theta_{\mathrm{L}nm}) - v_n v_m y_{\mathrm{L}nm} \sin(\delta_n - \delta_m - \theta_{\mathrm{L}nm}) - \frac{1}{2} v_n^2 y_{\mathrm{S}nm} \sin(\theta_{\mathrm{S}nm})$$



$$\min_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{y}\in\mathbb{B}^m} \quad f^{\text{CONVEX}}(\boldsymbol{x},\boldsymbol{y})$$

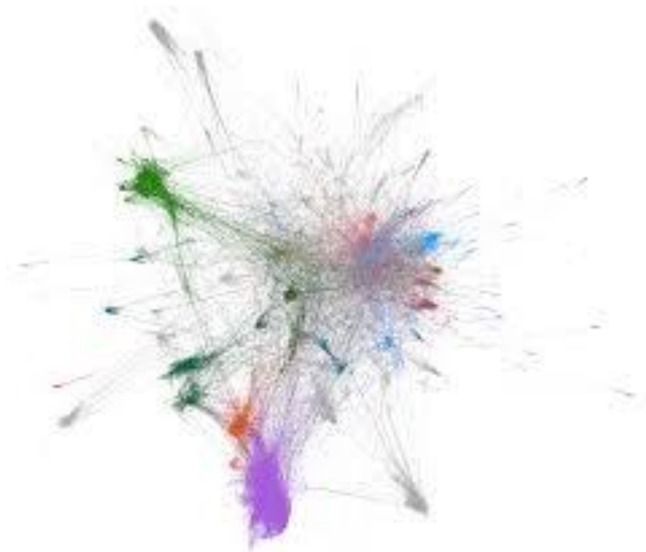
$$\text{s.t.} \quad \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0}$$

$$\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \leq \boldsymbol{0}$$

Large-scale

Nonlinear and nonconvex

Binary decisions



A three-step approach:

1. Linear NCUC (CPLEX, GUROBI)
2. Second order conic NCUC (CPLEX, GUROBI)
3. Convex OPF (MOSEK)

1. Solve a mixed-integer linear DC-NCUC and get (i) an initial solution and (ii) the set of potentially congested transmission lines.

2. Solve a mixed-integer second-order conic relaxation of the AC-NCUC using as initial solution the solution in 1 and an active set strategy on transmission line constraints (bounds).

3. Fixing the commitment decisions to those in 2, solve a sequence of continuous convex problems to ensure AC feasibility.

1

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{y}\in\mathbb{B}^m} \quad & f^{\text{CONVEX}}(\boldsymbol{x},\boldsymbol{y}) \\ \text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0} \\ & \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \le \boldsymbol{0} \end{aligned}$$

$\Longrightarrow$

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{y}\in\mathbb{B}^m} \quad & \boldsymbol{c}^\top [\boldsymbol{x} \;\; \boldsymbol{y}] \\ \text{s.t.} \quad & \boldsymbol{A}[\boldsymbol{x} \;\; \boldsymbol{y}] = \boldsymbol{b} \end{aligned}$$

$\Downarrow$

$[\boldsymbol{x}^0 \;\; \boldsymbol{y}^0]$

Potentially congested lines

$\Longleftrightarrow$

Large-scale

Nonlinear and nonconvex

Binary decisions

2

$$[\boldsymbol{x}^0 \;\; \boldsymbol{y}^0]$$

Potentially congested lines

$$\Downarrow$$

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^n, \boldsymbol{y}\in\mathbb{B}^m} \quad & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}) \\ \text{s.t.} \quad & \boldsymbol{h}^{\text{SOC}}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0} \\ & \boldsymbol{g}^{\text{SOC}}_{\text{REDUCED}}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0} \end{aligned}$$

$$\Downarrow$$

$$[\boldsymbol{x}^{\text{C}} \;\; \boldsymbol{y}^{\text{C}}]$$

Before 3

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^n} \quad & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) \\ \text{s.t.} \quad & \boxed{\begin{aligned} \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) &= \mathbf{0} \\ \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) &\leq \mathbf{0} \end{aligned}} \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \min_{\boldsymbol{x}\in\mathbb{R}^n} \quad & f^{\text{CONVEX}}(\boldsymbol{x}) \\ \text{s.t.} \quad & \boxed{\boldsymbol{u}^{\text{CONVEX}}(\boldsymbol{x}) - \boldsymbol{v}^{\text{CONVEX}}(\boldsymbol{x}) \leq \mathbf{0}} \end{aligned}$$

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \quad f^{\text{CONVEX}}(\boldsymbol{x})$$
$$\text{s.t.} \quad \boxed{\boldsymbol{u}^{\text{CONVEX}}(\boldsymbol{x}) - \boldsymbol{v}^{\text{CONVEX}}(\boldsymbol{x}) \leq \mathbf{0}}$$

3

$$\boldsymbol{y}^{\text{C}}$$

$$\Downarrow$$

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \quad f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}})$$
$$\text{s.t.} \quad \boldsymbol{u}^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) - \boldsymbol{v}^{\text{CONVEX}}_{\text{LINEARIZED}}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) \leq \mathbf{0}$$

$$\Downarrow$$

$$[\boldsymbol{x}^* \quad \boldsymbol{y}^*]$$

Checking AC feasibility

3

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\mathbb{R}^n} & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) \\ \text{s.t.} & \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) = \boldsymbol{0} \\ & \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}^{\text{C}}) \leq \boldsymbol{0} \end{array}$$

$\Downarrow$ Reformulate

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\mathbb{R}^n} & f^{\text{CONVEX}}(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{u}^{\text{CONVEX}}(\boldsymbol{x}) - \boldsymbol{v}^{\text{CONVEX}}(\boldsymbol{x}) \leq \boldsymbol{0} \end{array}$$

$\Downarrow$ Convexify

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\mathbb{R}^n} & f^{\text{CONVEX}}(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{u}^{\text{CONVEX}}(\boldsymbol{x}) - \boldsymbol{v}^{\text{CONVEX}}_{\text{LINEARIZED}}(\boldsymbol{x}) \leq \boldsymbol{0} \end{array}$$

Checking AC feasibility 3

Power balance

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i = \sum_{m\in\Lambda_n} \Big[G_{nm}(e_m e_n + f_m f_n) - B_{nm}(e_m f_n - e_n f_m) - G_{nm}(e_n^2 + f_n^2)\Big]$$

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} q_j - \sum_{i\in\Lambda_n^{\mathrm{D}}} Q_i = \sum_{m\in\Lambda_n} \Big[-G_{nm}(e_m f_n - e_n f_m) - B_{mn}(e_m e_n + f_m f_n) + (B_{nm} - b_{nm}^{\text{shunt}})(e_n^2 + f_n^2)\Big]$$

$\Downarrow$ Lift!

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i = \sum_{m\in\Lambda_n} \Big[G_{nm}c_{nm} - B_{nm}s_{nm} - G_{nm}c_{nn}\Big]$$

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} q_j - \sum_{i\in\Lambda_n^{\mathrm{D}}} Q_i = \sum_{m\in\Lambda_n} \Big[-G_{nm}s_{nm} - B_{mn}c_{nm} + (B_{nm} - b_{nm}^{\text{shunt}})c_{nn}\Big]$$

$$c_{nn} = e_n^2 + f_n^2$$

$$c_{nm} = e_m e_n + f_m f_n$$

$$s_{nm} = e_m f_n - e_n f_m$$

Checking AC feasibility 3

$$c_{nn} = e_n^2 + f_n^2$$
$$c_{nm} = e_m e_n + f_m f_n$$
$$s_{nm} = e_m f_n - e_n f_m$$

$$\Big\Downarrow \quad xy = \frac{1}{4}\left[(x+y)^2 - (x-y)^2\right]$$

$$e_n^2 + f_n^2 \le c_{nn}$$
$$c_{nn} \le e_n^2 + f_n^2$$
$$(e_n + e_m)^2 + (f_n + f_m)^2 - 4c_{nm} \le (e_n - e_m)^2 + (f_n - f_m)^2$$
$$(e_n - e_m)^2 + (f_n - f_m)^2 + 4c_{nm} \le (e_n + e_m)^2 + (f_n + f_m)^2$$
$$(e_n - f_m)^2 + (e_m + f_n)^2 + 4s_{nm} \le (e_n + f_m)^2 + (e_m - f_n)^2$$
$$(e_n + f_m)^2 + (e_m - f_n)^2 - 4s_{nm} \le (e_n - f_m)^2 + (e_m + f_n)^2$$

Linearize!

1. Linear NCUC
2. Second order conic NCUC
3. Convex OPF

$$
\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{y}\in\mathbb{B}^m} \quad & f^{\text{CONVEX}}(\boldsymbol{x},\boldsymbol{y}) \\
\text{s.t.} \quad & \boldsymbol{h}^{\text{SOC}}(\boldsymbol{x},\boldsymbol{y}) = \mathbf{0} \\
& \boldsymbol{g}^{\text{SOC}}_{\text{REDUCED}}(\boldsymbol{x},\boldsymbol{y}) \leq \mathbf{0}
\end{aligned}
$$

Too much?

2

Benders

$$\min_{\boldsymbol{x}_1 \in \mathbb{R}^{n_1}, \boldsymbol{y} \in \mathbb{B}^m, \eta \in \mathbb{R}} \quad \boldsymbol{c}^\top [\boldsymbol{x}_1 \ \ \boldsymbol{y} \ \ \eta]$$
$$\text{s.t.} \quad \boldsymbol{A}[\boldsymbol{x}_1 \ \ \boldsymbol{y} \ \ \eta] = \boldsymbol{b}$$

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{y} \in \mathbb{B}^m} \quad & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}) \\ \text{s.t.} \quad & \boldsymbol{h}^{\text{SOC}}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0} \\ & \boldsymbol{g}^{\text{SOC}}_{\text{REDUCED}}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0} \end{aligned} \quad \Leftrightarrow$$

Commitment decisions, $\boldsymbol{y}^k$ $\Downarrow$

$\Uparrow$ Benders' cuts
Network information

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^n} \quad & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^k) \\ \text{s.t.} \quad & \boldsymbol{h}^{\text{SOC}}(\boldsymbol{x}, \boldsymbol{y}^k) = \boldsymbol{0} \\ & \boldsymbol{g}^{\text{SOC}}_{\text{REDUCED}}(\boldsymbol{x}, \boldsymbol{y}^k) \leq \boldsymbol{0} \end{aligned}$$

Benders

Trick: Include "physical" cuts in the master problem!

$$\boxed{\begin{array}{cc} \min\limits_{\boldsymbol{x_1}\in\mathbb{B}^{n_1},\boldsymbol{y}\in\mathbb{B}^m,\eta\in\mathbb{R}} & \boldsymbol{c}^\top\left[\boldsymbol{x_1}\quad \boldsymbol{y}\quad \eta\right] \\ \text{s.t.} & \boldsymbol{A}[\boldsymbol{x_1}\quad \boldsymbol{y}\quad \eta] = \boldsymbol{b} \end{array}}$$

Commitment decisions, $\boldsymbol{y}^k$ $\Downarrow$

$\Uparrow$ Benders' cuts
Network information

Benders' approach

$$\begin{array}{ll} \min\limits_{\boldsymbol{x_1}\in\mathbb{R}^{n_1},\boldsymbol{y}\in\mathbb{B}^m,\eta\in\mathbb{R}} & \boldsymbol{c}^\top [\boldsymbol{x_1} \;\; \boldsymbol{y} \;\; \eta] \\ \text{s.t.} & \boldsymbol{A}[\boldsymbol{x_1} \;\; \boldsymbol{y} \;\; \eta] = \boldsymbol{b} \end{array}$$

$\longrightarrow$ Unit-related constraints

Commitment decisions, $\boldsymbol{y}^k$ $\Downarrow$

$\Uparrow$ Benders' cuts
Network information

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}\in\mathbb{R}^n} & f^{\text{CONVEX}}(\boldsymbol{x}, \boldsymbol{y}^k) \\ \text{s.t.} & \boldsymbol{h}^{\text{SOC}}(\boldsymbol{x}, \boldsymbol{y}^k) = \boldsymbol{0} \\ & \boldsymbol{g}^{\text{SOC}}_{\text{REDUCED}}(\boldsymbol{x}, \boldsymbol{y}^k) \leq \boldsymbol{0} \end{array}$$

$\longrightarrow$ Network-related constraints

Can we make the master network-aware?

Adding “physical” cuts

- More expensive master problem
- Scalable and linear
- Does not cut feasible solutions

- Less iterations
- Better convergence

For each node

$$\Downarrow$$

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m\in\Lambda_n} \Big[G_{nm}c_{nm}(t) - B_{nm}s_{nm}(t) - G_{nm}c_{nn}(t)\Big]$$
$$\sum_{j\in\Lambda_n^{\mathrm{G}}} q_j(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} Q_i(t) = \sum_{m\in\Lambda_n} \Big[-G_{nm}s_{nm}(t) - B_{mn}c_{nm}(t) + (B_{nm} - b_{nm}^{\mathrm{shunt}})c_{nn}(t)\Big]$$
$$\underline{V}_n^2 \leq c_{nn}(t) \leq \overline{V}_n^2$$

$2000 \times 4 \times 24 = 192,000$ constraints!

For each line

$$\Downarrow$$

$$c_{nm}(t) = c_{mn}(t)$$
$$s_{nm}(t) = -s_{mn}(t)$$
$$c_{nm}^2(t) + s_{nm}^2(t) \leq c_{nn}(t)c_{mm}(t)$$
$$p_{ij}^2(t) + q_{ij}^2(t) \leq \overline{S}_{ij}(t)$$

$3206 \times 4 \times 24 = 307,776$ constraints!

Copper plate relaxation

$4 \times 24 = 96$ linear constraints! $\Longrightarrow$

$$\sum_{j \in \mathcal{G}} p_j(t) - \sum_{i \in \mathcal{D}} D_i(t) \geq \sum_{n \in \mathcal{N}} G_{nn} c_{nn}(t)$$

$$\sum_{j \in \mathcal{G}} q_j(t) - \sum_{i \in \mathcal{D}} Q_i(t) \geq \sum_{n \in \mathcal{N}} B_{nn} c_{nn}(t) - \sum_{(nm) \in \mathcal{L}} b_{nm}^{\text{shunt}} (c_{nn}(t) + c_{mm}(t))$$

$$\underline{V}_n^2 \leq c_{nn}(t) \leq \overline{V}_n^2$$

$$\boxed{\begin{aligned} \min_{\boldsymbol{x_2} \in \mathbb{R}^{m_2}, \boldsymbol{y} \in \mathbb{B}, \eta \in \mathbb{R}} \quad & \boldsymbol{c}^\top [\boldsymbol{x_2} \;\; \boldsymbol{y} \;\; \eta] \\ \text{s.t.} \quad & \boldsymbol{A}[\boldsymbol{x_2} \;\; \boldsymbol{y} \;\; \eta] = \boldsymbol{b} \\ & \boldsymbol{G}\boldsymbol{x_2} \leq \boldsymbol{0} \end{aligned}}$$

$\longrightarrow$ Unit-related constraints

$\longrightarrow$ Network-aware constraints

Commitment decisions, $\boldsymbol{y}^k$ $\Downarrow$

$\Uparrow$ Benders' cuts
Network information

C. Coffrin, H. Hijazi, and P. van Hentenryck, "Network Flow and Copper Plate Relaxations for AC Transmission Systems," https://arxiv.org/abs/1506.05202

Summary

1. Good initial solution: Mixed-integer linear NCUC
2. Commitment solution: Mixed-integer second order conic NCUC
3. AC feasibility: Continuous convex OPF

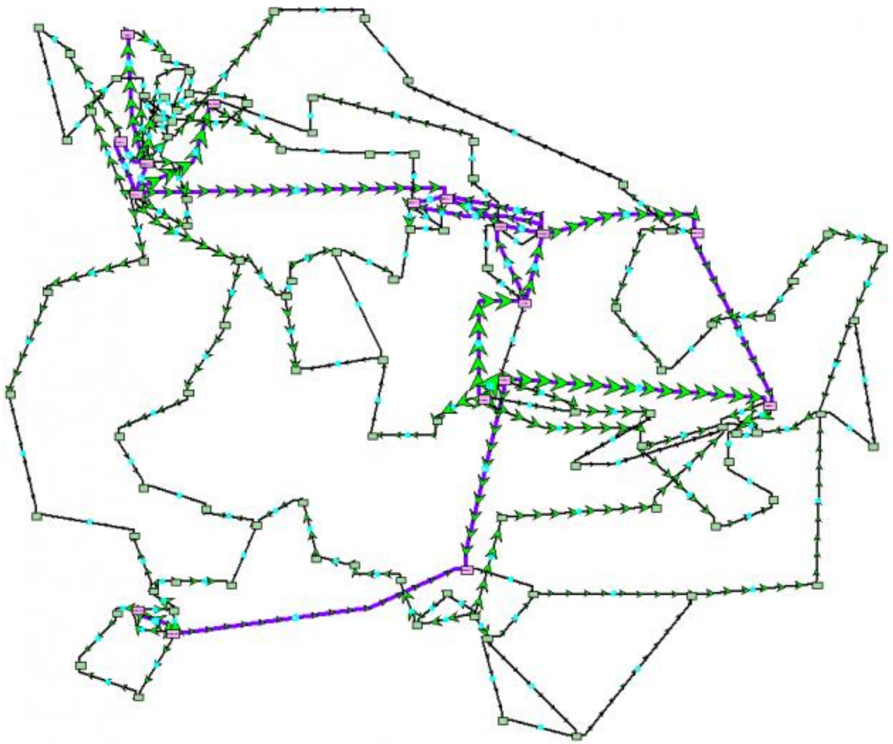
Case Studies

Illinois 200-bus 49-generator 245-line system

https://electricgrids.engr.tamu.edu/electric-grid-test-cases/activsg200/

Data

- Entirely synthetic
  - Based on real data
  - Preserves statistical properties
- 49 generating units
  - 1 nuclear (15% installed capacity)
  - 6 wind (15% installed capacity)
  - 42 thermal (70% installed capacity)
  - 33 units with commitment (binary) variables
- 245 transmission lines

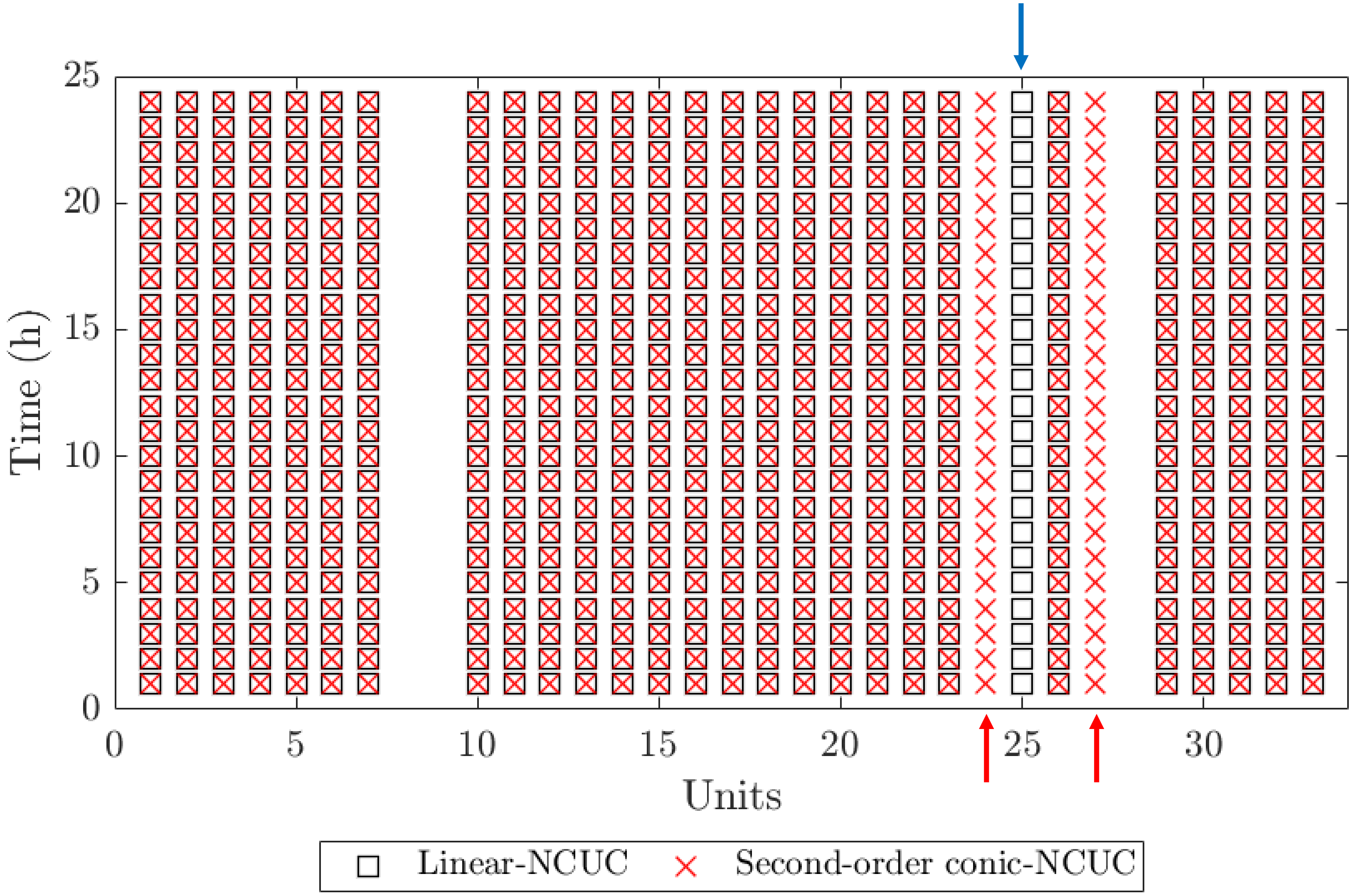


Results

Model	Solver	Loading level	# Start ups[1]	Generation (MW)	Objective ($)	Time* (s)
Mixed-integer linear	CPLEX	Low	0	29344.51	482444.82	15.38
		Medium	0	39126.02	484188.40	25.39
		High	6	48907.52	509953.48	23.65
Mixed-integer second-order conic	CPLEX	Low	0	29889.96	482472.70	236.14
		Medium	0	39428.27	484247.05	313.66
		High	7	49396.99	512111.75	977.09
Continuous second-order conic	Mosek	Low	-	29528.43	482473.35	407.83
		Medium	-	39432.99	484248.75	434.29
		High	-	49360.09	512142.89	750.66

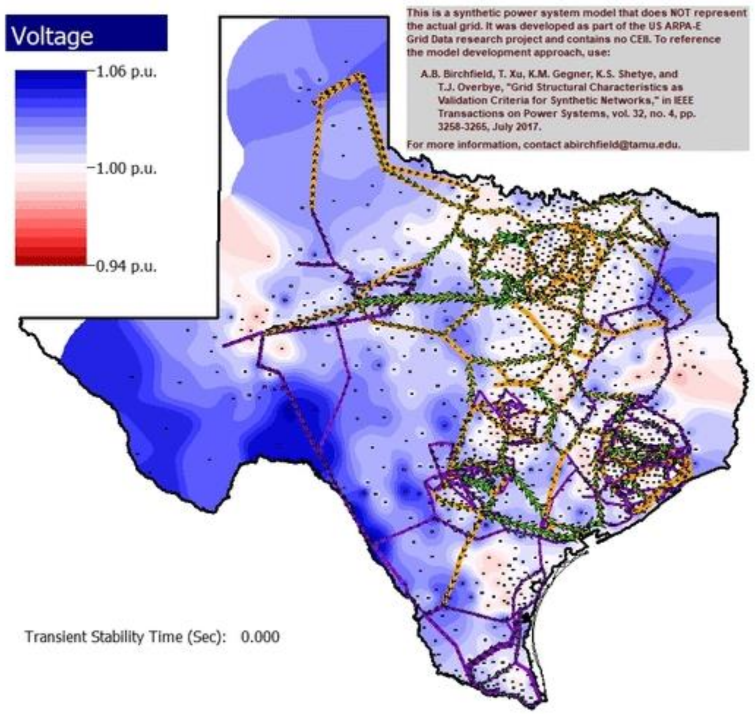
[1]Number of units that start up/shutdown in the planning horizon.
*Windows-based laptop with a processor Intel Core i7 2.60 GHz and 16 GB of RAM.

Results



25
20
15
10
5
0
Time (h)
0
5
10
15
20
25
30
Units
Linear-NCUC
Second-order conic-NCUC

Texas 2000-bus 544-unit 3206-line system



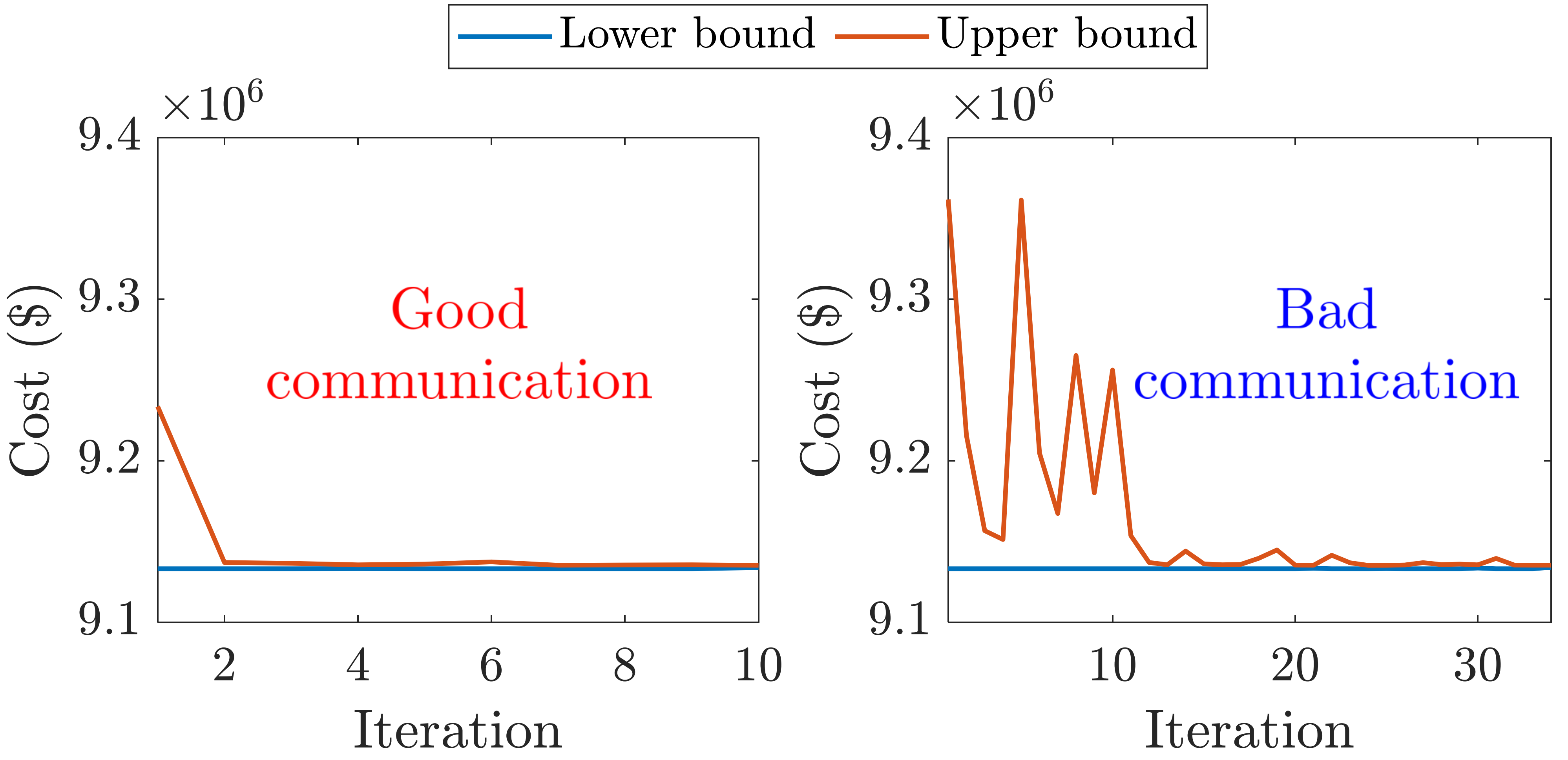
https://my.syncplicity.com/share/wubzq34byp7h2g4/ACTIVSg2000

Data

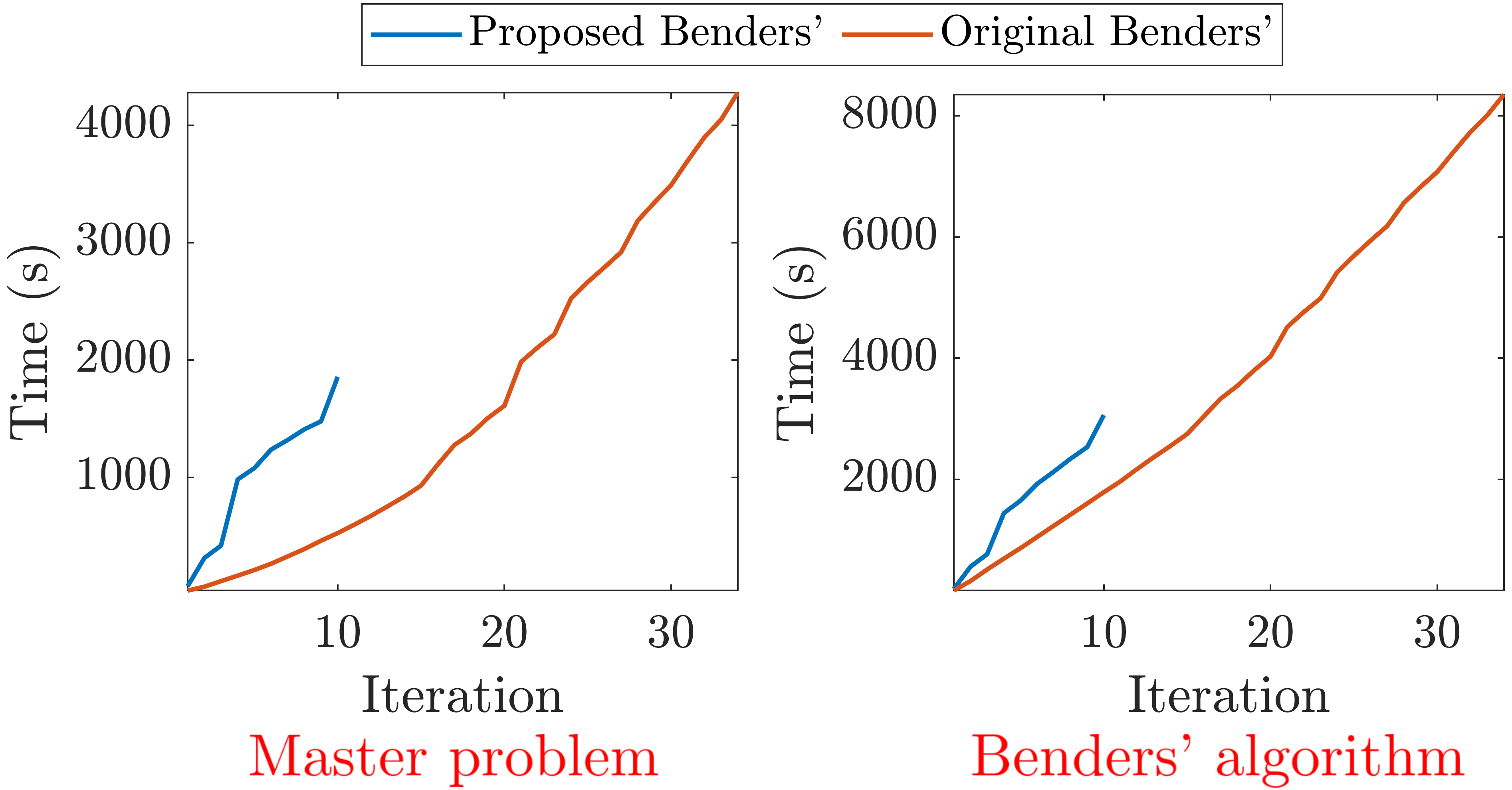
- Entirely synthetic
- 544 generating units
  - 39 coal units - 15.06% of total installed capacity
  - 367 gas units - 66.27% of total installed capacity
  - 25 run-of-the-river hydro units - 2.70% of total installed capacity
  - 87 wind units - 9.96% of total installed capacity
  - 22 solar units - 0.68% of total installed capacity
  - 4 nuclear units - 5.34% of total installed capacity
- 3206 transmission lines

$\Rightarrow$ 369 units with commitment variables!

Benders' bounds

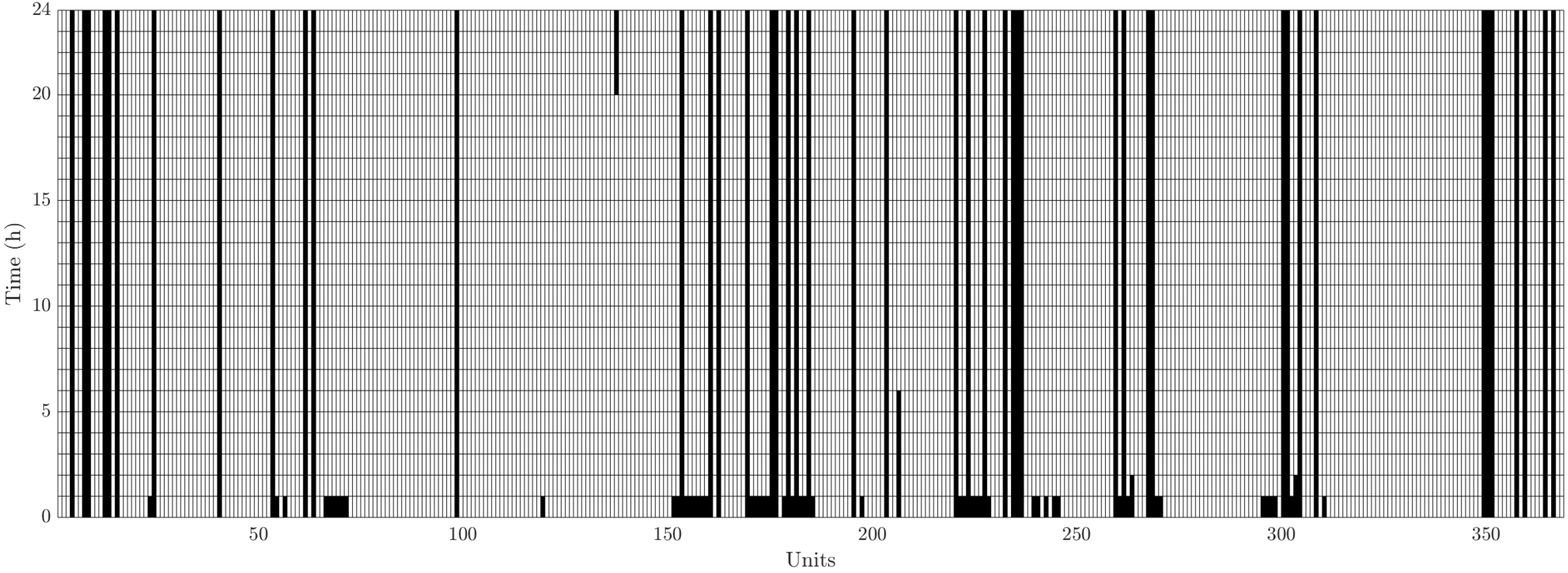


Computing time

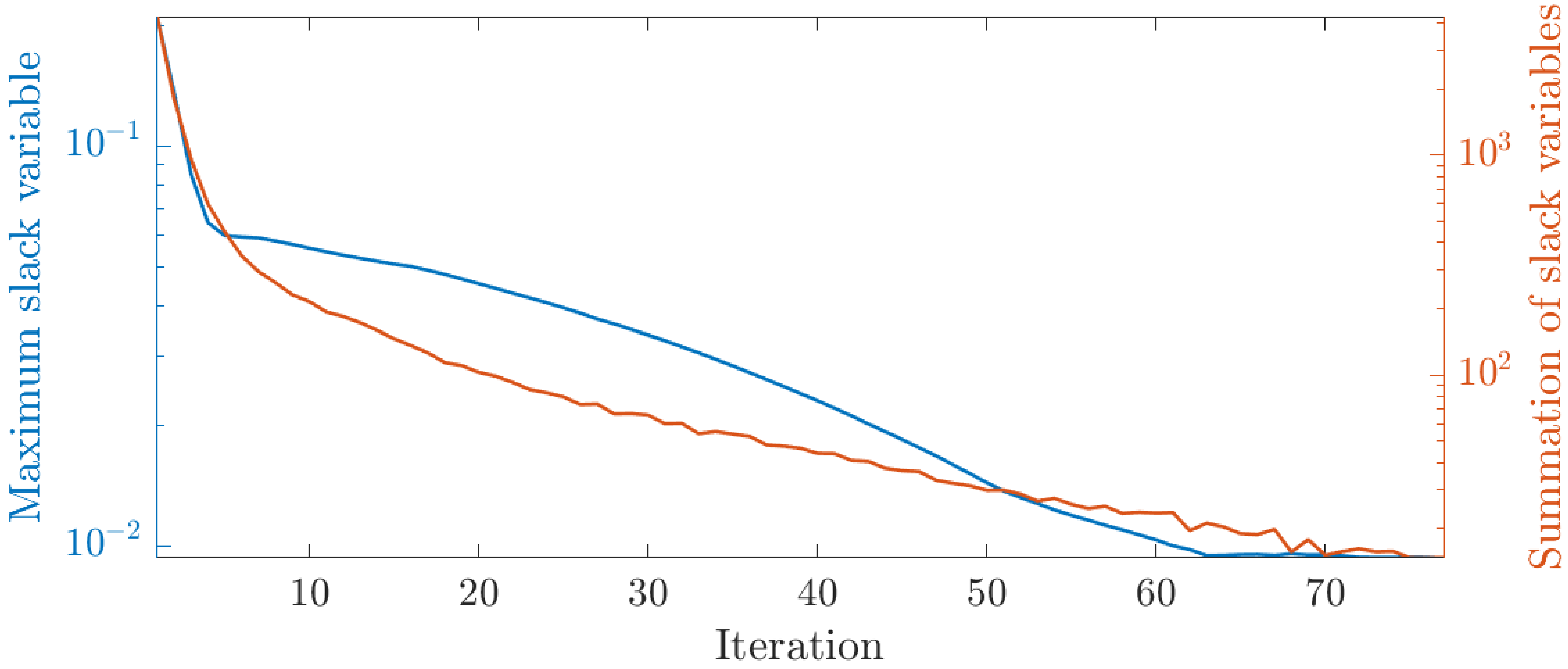


> 60% improvement!

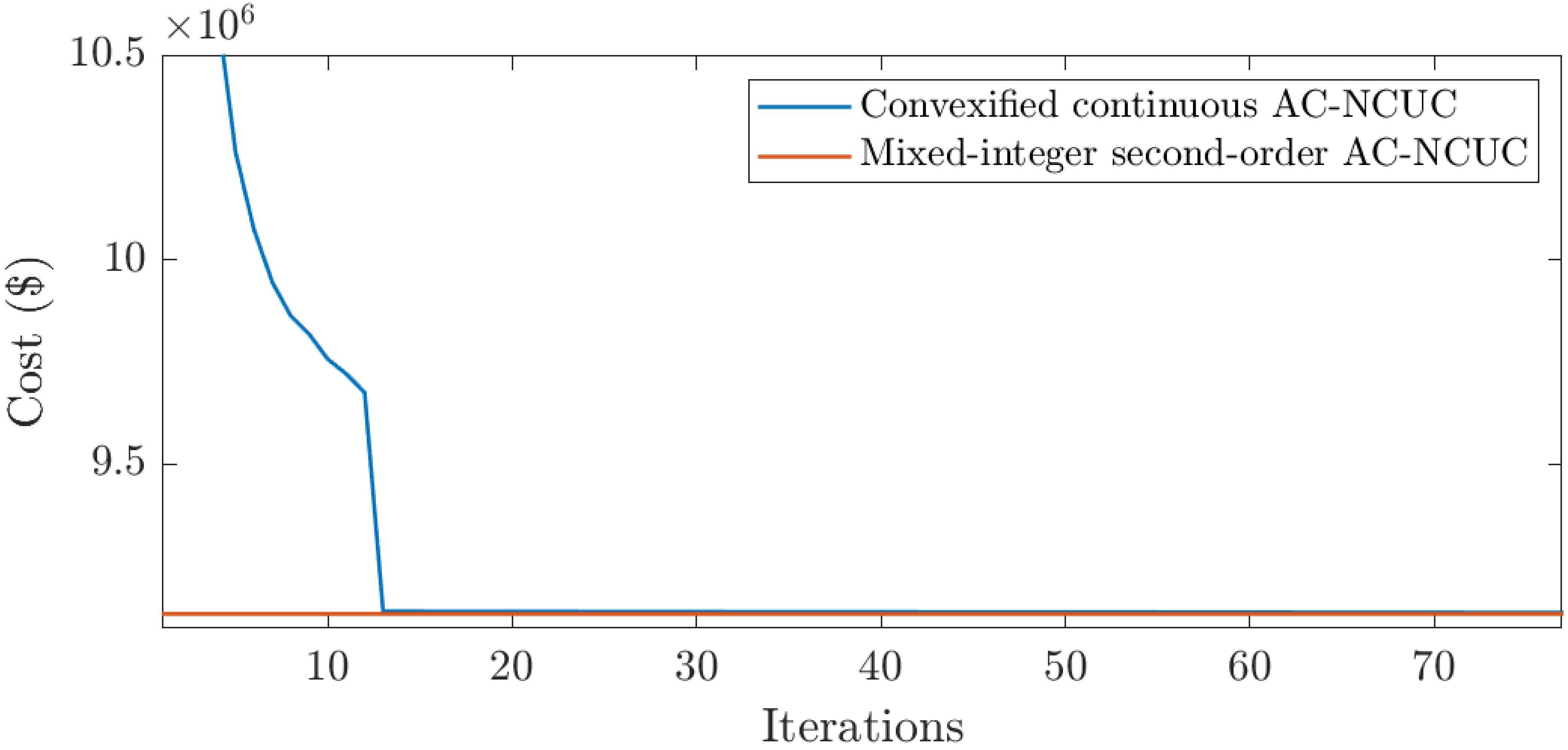
Commitment



Feasibility step



Feasibility step



Thank you!