Production Planning with Uncertain Power Supply

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Setting

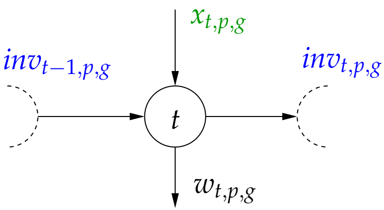
- Two plants, two products
- Each has an inventory with given maximum capacity
- Customer demand known for every *time bucket*

⇒ Plan production at each time bucket

... while minimizing total prod./inv. cost

Model Notation

- Variables: production $x_{t,p,g}$, inventory $inv_{t,p,g}$.
- Constraints: for all $t \in T, p \in \{P_1, P_2\}, g \in \{LIN, LOX\}$:
  - Production capacity: $x_{t,p,g} \leq C_{pro}$
  - Conservation: $x_{t,p,g} + inv_{t-1,p,g} = w_{t,p,g} + inv_{t,p,g}$



- Obj. function: $\sum_{t,p,g} (c_{\text{prod}} \cdot x_{t,p,g})$

Disruptions

Both plants are subject to power interruptions.

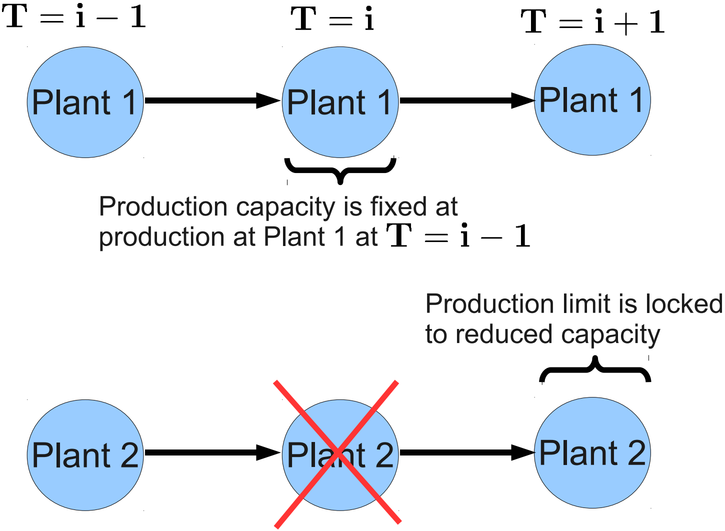
Part of a “flexible supply” contract with the power company. Clauses might specify

- How many interruptions there might be
- Duration (# time buckets)
- What plant(s) are affected

However,

- Short notice ($\approx$ hours, even minutes)
- Unknown duration (apart from per-contract limits)

Interruption Model



Uncertainty in power interruptions

Optimization problems under uncertainty may be dealt with

- Robust Optimization[1]: ideal when uncertainty is **limited** but the scenarios seem all equally likely.
- Stochastic Programming[2]: more suited when a probability distribution is known.

Our case is prototypical for Robust Optimization: there are a few interruptions which, however, are all equally likely.

[1] A. Ben-Tal, L. El Ghaoui, A. Nemirovski, Robust Optimization. Princeton Univ. Press, 2009. http://sites.google.com/site/robustoptimization

[2] J.R. Birge, F. Louveaux, Introduction to Stochastic Programming, Springer, 1997.

Robust Model

Intuition

Production $x_{t,p,g}$ is influenced by an opponent that can shut off a plant under certain conditions.

$$\begin{aligned} \min \quad & \sum_{t,p,g} (c_{\text{prod}} \cdot x_{t,p,g}) \\ & \text{[production constraints]} \\ & \min_{U} \{\text{inv. of gas } G \text{ at time } T\} \geq 0 \end{aligned}$$

Opponent's problem

The **inner** problem is a worst-case estimate of the inventory at every time bucket. Its objective function value

$$z(inv) = \min_{U}\{\text{inv. of gas } G \text{ at time } T\}$$

depends on the production levels, but has to be computed by *implicitly* solving the optimization problem.

Uncertainty - I

Define

$$\xi_{t,p} = \begin{cases} 1 & \text{if an interruption occurs at time } t \text{ plant } p \\ 0 & \text{o.w.} \end{cases}$$

Then,

$$U = \left\{ \xi_{t,p} \in \{0,1\} \ \forall (t,p) \in (T,P) \ \middle| \ \sum_{p \in P} \xi_{t,p} \leq 1 \ \forall t, \ \sum_{t \in T, p \in P} \xi_{t,p} \leq K \right\}$$

Scenarios

Each $\boldsymbol{\xi} \in U$ corresponds to an interruption scenario.

Uncertainty - II

Approach based on Soyster ('73): "solve" the inner problem by taking its dual.

- Construct inner optimization problems (uncertainty embedded)
- Take the duals and embed dual feasibility constraints to outer model
- Enforce non-negativity on dual objectives
- Dual feasibility, dual optimality and primal feasibility will be satisfied when outer problem solution is optimal.

Outer Model

$$
\begin{aligned}
&\text{minimize} && \sum_{t\in T, p\in P, g\in G} c_{t,p,g} x_{t,p,g} \\
&\text{subject to} && \\
&\forall(t) && \sum_{g\in G} x_{t,p,g} \leq \; cap_p \\
&\forall(t,p) && x_{t,p,1} \leq \; (ratio + flex)\, x_{t,p,2} \\
&\forall(t,p) && x_{t,p,1} \geq \; (ratio - flex)\, x_{t,p,2} \\
&\forall(t,g) && \sum_{p\in P} w_{t,p,g} \geq \; d_{t,g} \\
&\forall(t,p,g) && \min_{U} \{inv_{t,p,g}\} \geq 0 \\
&\forall(t,p,g) && x_{t,p,g}, w_{t,p,g} \geq 0
\end{aligned}
$$

Duality trick

$$
\begin{aligned}
\min \quad & \sum_{t,p,g} (c_{\text{prod}} \cdot x_{t,p,g}) \\
& \text{[prod. constraints]}, \\
& \min\{c^\top x : Ax \geq b, x \in \mathbb{R}^n_+\} \geq 0
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\min \quad & \sum_{t,p,g} (c_{\text{prod}} \cdot x_{t,p,g}) \\
& \text{[prod. constraints]}, \\
& \max\{u^\top b : u^\top A \leq c, u \in \mathbb{R}^m_+\} \geq 0
\end{aligned}
$$

Uncertainty - III

Need to use strong LP duality, therefore relax integrality

$$U^R = \left\{ \xi_{t,p} \in [0,1] \;\; \forall (t,p) \in (T,P) \;\middle|\; \sum_{p \in P} \xi_{t,p} \leq 1 \;\; \forall t, \quad \sum_{t \in T, p \in P} \xi_{t,p} \leq K \right\}$$

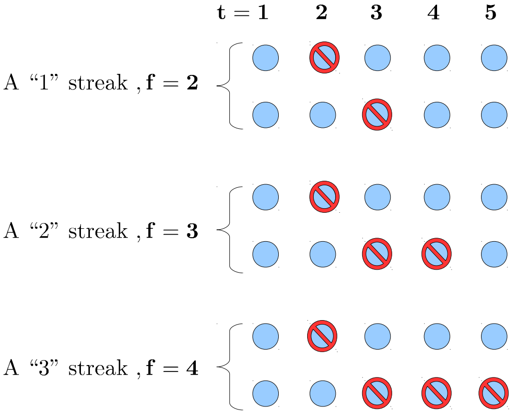
However in this case following holds since $U \subset U^R$

$$\min_{U^R} \left\{ inv_{t,p,g} \right\} \leq \min_{U} \left\{ inv_{t,p,g} \right\}$$

Implies over-conservatism in the inner problems, i.e. perceived severance of uncertainty is amplified!

Better Description of Uncertainty - I

A streak of interruptions at any plant locks the production at the other plant to reduced mode.
This is not included in the base model, in which only streaks of length 1 are included.



Changing the Robustness Target

Replace

$$inv_{t,p,g} = \sum_{i=1}^{t} \bar{x}_{i,p,g} - \sum_{i=1}^{t-1} w_{i,p,g} + inv_{0,p,g} \tag{1}$$

with

$$\sum_{p \in P} inv_{t,p,g} = \sum_{p \in P} \left\{ \sum_{i=1}^{t} \bar{x}_{i,p,g} - \sum_{i=1}^{t-1} w_{i,p,g} + inv_{0,p,g} \right\} \tag{2}$$

- Total production in model 1 is larger than the one in model 2
- Model 2 is immunized against interruptions while model 1 requires more refinement in uncertainty to reach same level of immunization.

Solving the robust model

- LP Duality allows to implicitly solve the above problem
- We know what the opponent wants, and we can model it
- The inner problem is a Mixed Integer Linear Program

$\Rightarrow$ It needs to be strengthened

This can be done by

- cut separation in the inner problem's primal, or
- column generation in its dual

Can significantly improve the total cost

Numerical Study

- Current results suggest that
  - The new uncertainty description is more realistic
  - Cuts further enhance accuracy, reduce the increase in conservatism caused by linear relaxation
  - Aggregation of demand over plants reduces conservatism and provides a better description
- Simulation
  - A scenario generator creates all possible scenarios
  - The production levels obtained from the optimization problem are tested for feasibility for all scenarios.

Test #1: production levels

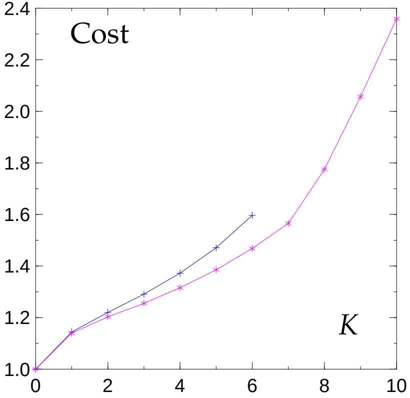
- Two plants, two products, 6 time buckets.
- K: max. number of interruptions ($\approx$ uncertainty level).
- More interruptions $\rightarrow$ production "spreads" over time

t	$K = 0$	1	2	3	4	5	6
1	100	100	100	100	100	100	100
2	106	106	106	106	106	106	106
3	130	130	130	130	130	130	130
4	99	142	138	141	131	118	124
5	210	168	172	168	178	192	185
6	149	149	149	149	149	149	185

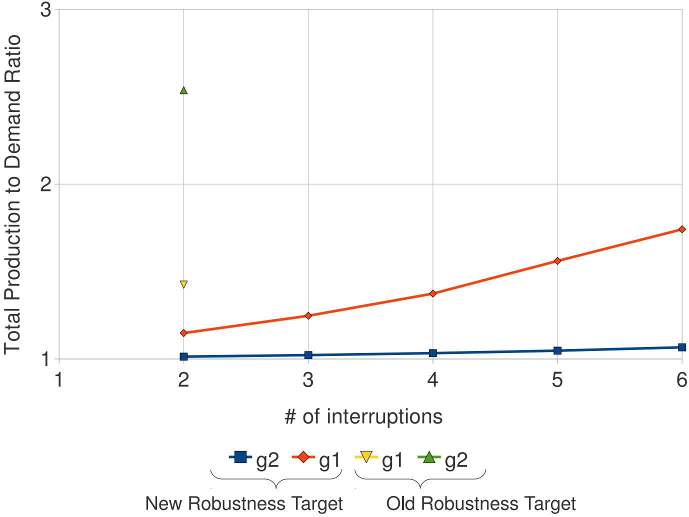
Test #2: policy vs. production cost

Two plants, two products, 14 time buckets. Cost is normalized to 1 (for $K = 0$, i.e., no interruptions).

- K: max. number of interruptions ($\approx$ uncertainty level).
- Blue (upper curve): cost with less accurate model, purple (lower): more accurate
- Real cost: between purple line and Cost=1



Test #3: Alternative Robust Formulation



Current

- Try several uncertainty sets (contract policies).
- Further tightening of current uncertainty sets.
- Incorporate even more realistic operational constraints.