Product Portfolio Selection as a Bi-level Optimization Problem

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Motivation

Product portfolio selection:

Investment in new products is a difficult decision

1) Markets are dynamic

- Stiff completion from other suppliers
- Uncertainty in future product demands

2) Impact on financial success

- Huge capital investments
- Long term benefits in the form of profits

Risks involved in decision making can be reduced by **Single & Bi-level** optimization models

Problem statement

Maximize Net resent value (NPV):

- Determine optimal portfolio of products to invest for a primary supplier to maximize NPV

Given:

- Demand from customers for products with certain specifications
- Suppliers supply multiple products with a profit margin to the customers at a predetermined price
- Production of a product require some resources (effort units)
- Each supplier is limited by resources they can spend on all the products combined (total effort units)
- Suppliers can acquire more resources (total effort units) at an additional cost

Single level formulation

Investment decisions on new products and resources

Ignores competition from other suppliers

Max $\sum_{t\in T}\frac{1}{(1+R)^t}\left(\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\pi_{t,i,q,c,j}\sum_{k\in I}P_{t,i,q,c,j,k}-\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\beta_{t,i,q,c,j}v_{t,i,q,c,j}-\gamma_{t,j}E_{t,j}\right)\ \forall j\in S_1$ Maximize NPV of Primary supplier

s.t $w_{t,i,q,c,j}=V_{0,i,q,c,j}+\sum_{t'\in T'_t}v_{t',i,q,c,j}\quad \forall\, t\in T, i\in P, q\in Q, c\in C, j\in S_1$ Investments in new products

$E_{t,j}=E_{0,j}+H\sum_{t'\in T'_t}x_{t',j}\quad \forall t\in T, j\in S_1$ Expansion of effort units

$x_{t,j}\le\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}w_{t,i,q,c,j}\quad \forall t\in T, j\in S_1$ Expansion only if product lines are open

$\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}(e_{t,i,q,c,j}\sum_{k\in I}P_{t,i,q,c,j,k})\le E_{t,j}\quad \forall t\in T, j\in S_1$ Restriction on total number of effort units

$\sum_{j\in S_1}P_{t,i,q,c,j,k}\le D_{t,i,q,c,k}\quad \forall t\in T, i\in P, q\in Q, c\in C, k\in I$ Production less than the demand

$\sum_{k\in I}P_{t,i,q,c,j,k}\le UB_{t,i,q,c,j}w_{t,i,q,c,j}\quad \forall t\in T, i\in P, q\in Q, c\in C, j\in S_1$ Production only from open plants

$P_{t,i,q,c,j,k}\in R^+\quad \forall t\in T, i\in P, q\in Q, c\in C, j\in S_1, k\in I$

$E_{t,j}\in R^+\quad \forall t\in T, j\in S_1$

$w_{t,i,q,c,j}, v_{t,i,q,c,j}, x_{t',i}\in\{0,1\}\quad \forall t\in T, i\in I, q\in Q, c\in C, j\in S_1$

Sets:

T- Time

S- Suppliers

I- Customers

P- Product type

Q- Quality specification

C- Capacity specification

Bi-level formulation

Upper level : Maximizes Net Present Value of the primary supplier by optimal investment strategy

Lower level : Minimizes the costs paid by market to the suppliers

$$\text{Max} \sum_{t\in T}\frac{1}{(1+R)^t}\left(\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\pi_{t,i,q,c,j}\sum_{k\in I}P_{t,i,q,c,j,k}-\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\beta_{t,i,q,c,j}v_{t,i,q,c,j}-\gamma_{t,j}\,E_{t,j}\right)\forall\, j\in S_1$$

Maximize NPV of Primary supplier

$$\text{s.t } w_{t,i,q,c,j}=V_{0,i,q,c,j}+\sum_{t'\in T_t'}v_{t',i,q,c,j} \qquad \forall\, t\in T, i\in P, q\in Q, c\in C, j\in S_1$$

Investments in new products

$$E_{t,j}=E_{0,j}+H\sum_{t'\in T_t'}x_{t',j} \qquad \forall t\in T, j\in S_1$$

Expansion of effort units

$$x_{t,j}\le\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}w_{t,i,q,c,j} \qquad \forall t\in T, j\in S_1$$

Expansion only if product lines are open

$$\text{Min}\sum_{t\in T}\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\sum_{j\in S}\left(\frac{\alpha_{t,i,q,c,j}}{(1+R)^t}\sum_{k\in I}P_{t,i,q,c,j,k}\right)$$

Minimize the costs paid by Market

$$\sum_{i\in P}\sum_{q\in Q}\sum_{c\in C}\left(e_{t,i,q,c,j}\sum_{k\in I}P_{t,i,q,c,j,k}\right)\le E_{t,j} \qquad \forall t\in T, j\in S$$

Restriction on total number of effort units

$$\sum_{j\in S}P_{t,i,q,c,j,k}=D_{t,i,q,c,k} \qquad \forall t\in T, i\in P, q\in Q, c\in C, k\in I$$

Production is equal to the demand

$$\sum_{k\in I}P_{t,i,q,c,j,k}\le UB_{t,i,q,c,j}w_{t,i,q,c,j} \qquad \forall t\in T, i\in P, q\in Q, c\in C, j\in S_1$$

Production only from open plants

$$P_{t,i,q,c,j,k}\in R^+ \qquad \forall t\in T, i\in P, q\in Q, c\in C, j\in S, k\in I$$

$$E_{t,j}\in R^+ \qquad \forall t\in T, j\in S_1$$

$$w_{t,i,q,c,j}, v_{t,i,q,c,j}\, x_{t,i}\in\{0,1\} \qquad \forall t\in T, i\in P, q\in Q, c\in C, j\in S$$

Bi-level duality based reformulation

- Bi-level is transformed to single level by dual reformulation
- Lower level LP of Bi-level formulation is equated to its corresponding dual using strong duality
- Primal and dual feasibility conditions are added
- Bilinear terms arising from multiplication of upper level variables are linearized using Glovers linearization

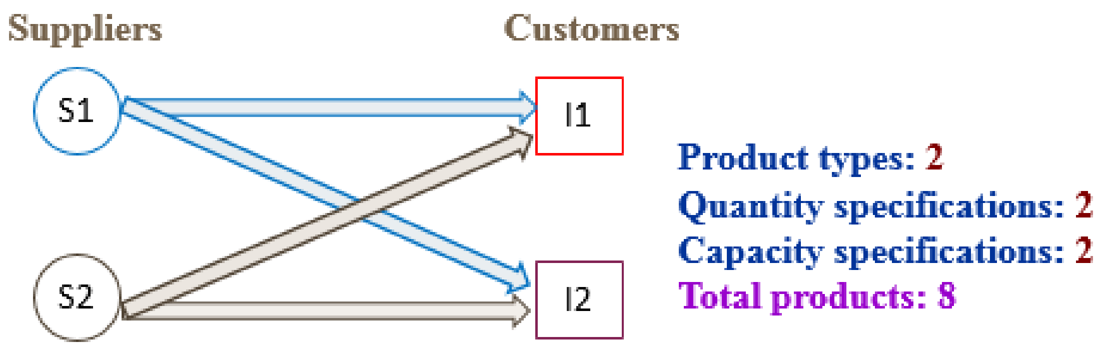
Dual reformulation: Upper level problem + strong duality condition + primal feasibility + dual feasibility + Glovers linearization

Example

- Two customers require two types of products
- Two suppliers supply these products
- Each product has quality and capacity specifications
- Time horizon - 5 years (5 Time Periods)
- Discount rate - 12%
- Effort units to produce a product with certain specification is given
- Each supplier is restricted by total number of effort units that can be spent
- Product prices, price margins and investment costs are given for each product

Example

Schematic representation :



Each arrow indicates possible flow of 8 products

Model Statistics:

	Single level problem	Bi-level with duality based reformulation
No of constraints:	179	512
No of continuous variables:	174	441
No of discrete variables:	85	85
Solution time	0.046 sec	0.234 sec
Optimality gap	0.00%	0.00%

Results

Model results	Single level problem	Bi-level with duality based reformulation
NPV of the primary supplier:	$ 8 MM	$ 4.1 MM
Profit from products:	$ 12.7 MM	$ 6.9 MM
Investment costs:	$ 3.3 MM	$ 1.7 MM
Expansion costs :	$ 1.4 MM	$ 1. 1 MM
Investment decisions:	**P1,Q1,C1 : time period 1** **P1,Q1,C2 : time period 1** **P2,Q1,C1 : time period 2** **P2,Q1,C2: time period 2** **P2,Q2,C1: time period 2**	**P1,Q1,C1 :time period 1** **P1,Q1,C2: time period 1** **P2,Q2,C1: time period 2**
Resource expansion decisions:	**Time periods 1 to 5**	**Time periods 1 and 2**

Aggressive investment strategy by Single level formulation due to the assumption that market will choose to buy from a supplier irrespective of price

Conservative investment strategy by Bi-level formulation due to consideration of rational market behavior (market buys from supplier with lowest price)

AIR PRODUCTS

CAPD CENTER

Conclusions

- Single and Bi-level formulations help in deciding product portfolio selection
- Absence of competition in single level formulation results in aggressive investment strategy due to overestimation of demands
- Bi-level structure allows to considers market preferences and overcomes the drawbacks of Single level formulation

Future work

- Consider demand uncertainties through stochastic programming