Rolling Horizon Approach for Optimal Production-Distribution Coordination of Industrial Gases Supply-chains

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Background and Motivation

Deterministic models

- Consider fixed data (based on future projections, forecast data, etc.)
  - Demand, market prices, process reliability, etc.
  - Exact data is unavailable or very expensive
  - Easier to solve

Uncertainty Management

- Realistic decision making (much more expensive to solve)
- Require efficient solution method for deterministic problem in order to tackle the problem under uncertainty

Main Goal

Develop a Robust decision making model to improve the optimal production, inventory and distribution coordination under uncertainty

Main Goal

Deterministic Production Distribution of Industrial Gases Supply Chains Management

Reduce the computational effort (Analyzing **Large-scale optimization techniques**)

Production Distribution of Industrial Gases Supply Chains Management **under uncertainty**

Robust Optimization (worst case)

- No recourse actions (short term problems)
- All decisions are here and now
- Some parameters are affected by an uncertainty set (simplest case upper and lower bounds)

Two Stage Stochastic programming

- Special attention to the size of the problem
- Discrete scenarios (based on probability distribution)

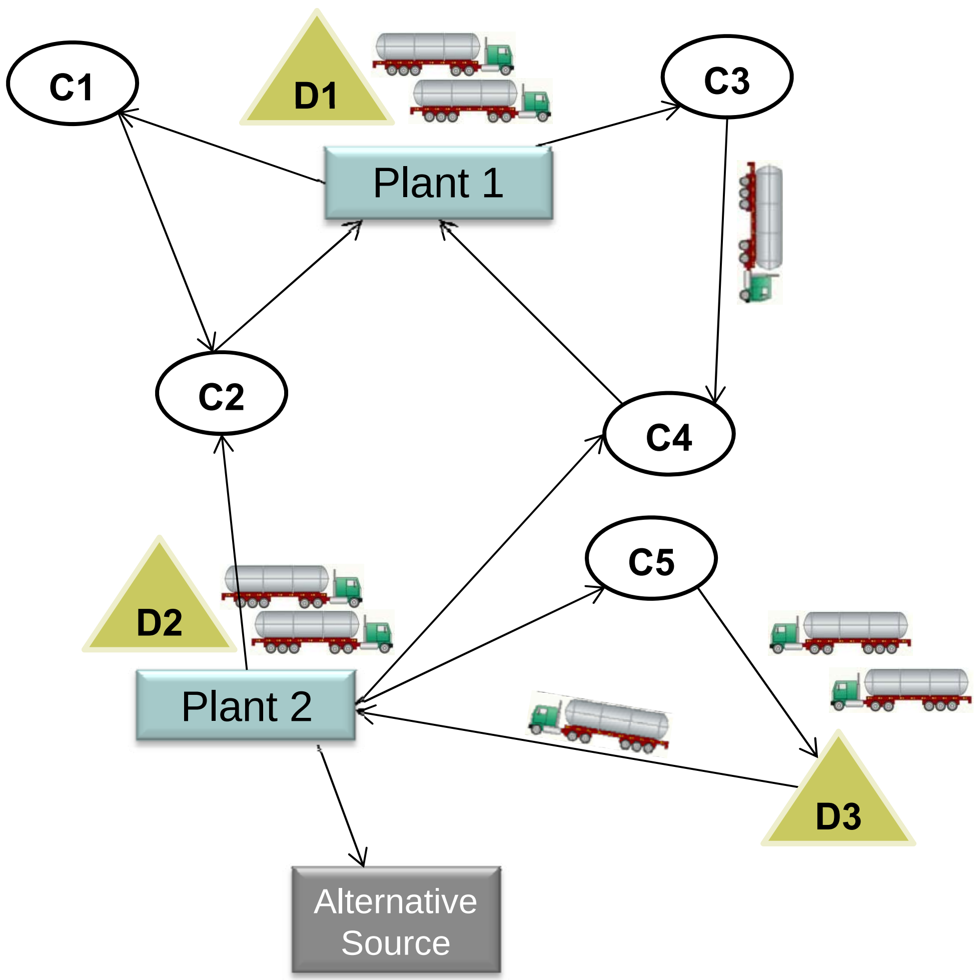
Deterministic Production Distribution of Industrial Gases Supply Chains Management

Reduce the computational effort (Analyzing **Large-scale optimization techniques**)

Rolling Horizon

- Solves the problem in a sequence of iterations, in each iteration part of the planning horizon will be modeled in detail (“detailed time block”) and the rest of the time horizon is represented in an aggregated manner (“the aggregated time block”).

Problem Statement and Main Assumptions



Given

- Plants, Products, Operating Modes and Production Limits
- Daily Electricity Prices (off-peak and peak)
- Customers and their demand/consumption profiles
- Max/Min inventory at production sites and customer locations
- Alternative sources and product availabilities
- Depots, Truck availabilities and capacities, Distances
- Fixed Planning Horizon (usually 1-2 weeks)

Decisions in each time period t

- Modes and production rates at each plant
- Inventory level at customer location and plants
- How much product to be delivered to each customer through which route

Objective Function

- Minimize total production and distribution cost over planning horizon

Main Assumptions – Distribution Side

- Two time periods per day (peak and off-peak) are considered
- Trucks do not visit more than 4 customers in a single delivery

Rolling Horizon approach

- The planning horizon in detail ("the detailed time block"),
- Rest of the horizon is represented in an aggregate manner ("the aggregate time block").

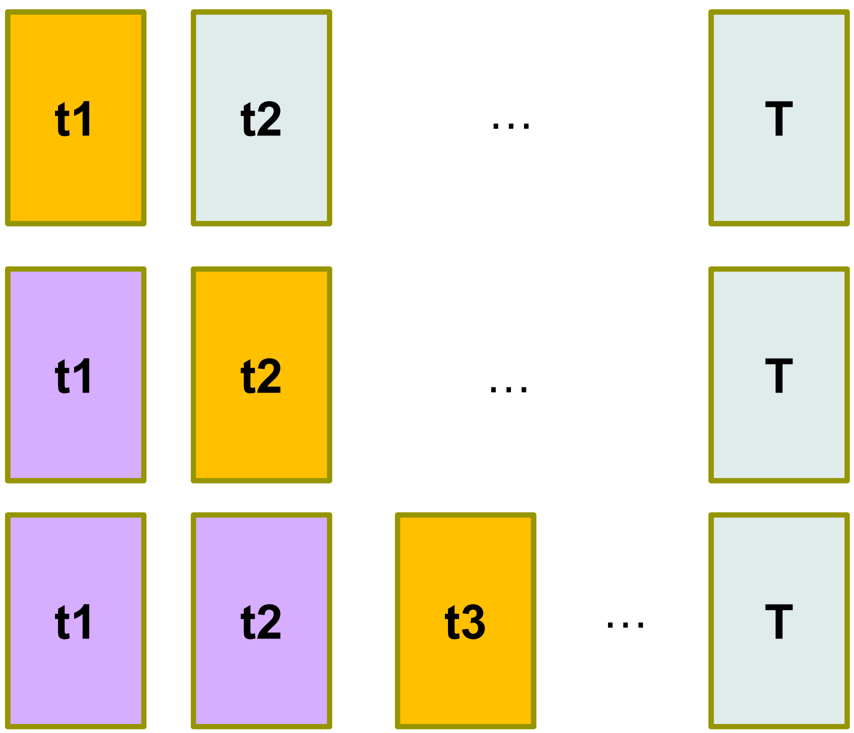
$$\min TotalCost(x, y)$$

Full space

$$s.t.$$

$$Ax_s^t = Bx_s^{t-1} \qquad \forall s \in S, t \in T$$

$$Cx_s^t = Dy_s^t \qquad \forall s \in S, m \in M, t \in T$$



Detailed block
Aggregate block
Fixed variables

$\forall t \in T$ → **Sub-problems**

RH Relaxed model

- The planning horizon in detail (“the detailed time block”),
- Rest of the horizon is represented in an aggregate manner (“the aggregate time block”).
  - A) Relaxed model (binary variables are replaced by continuous variables [0,1]
  - B) Aggregate model (Distribution side constraints are replaced by a tailored distribution model)

DETAILED BLOCK
PDC model

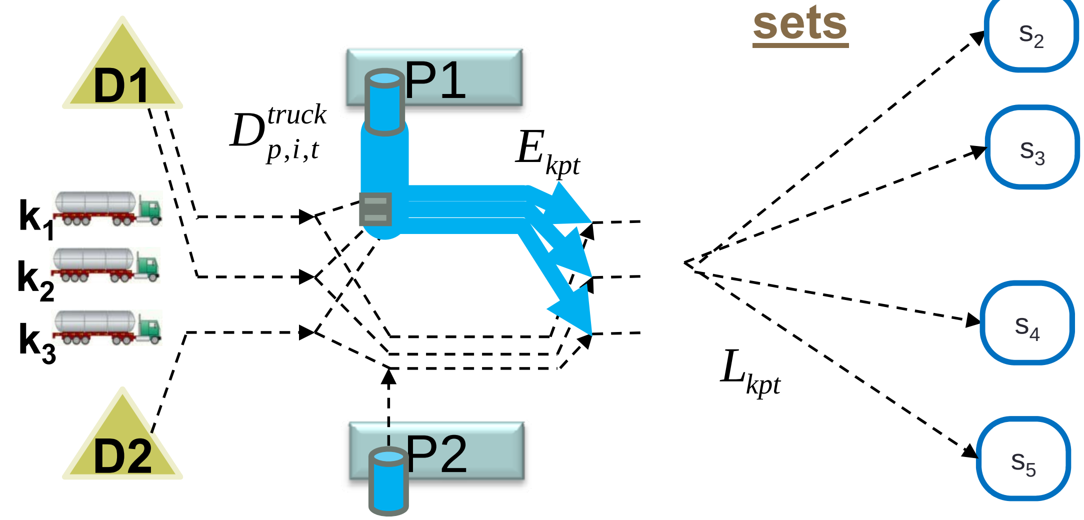
Detailed Production

Main binary variables
mode selection
B_{pmt}

Production mode
Same production mode
Material Balances

Detailed Distribution

Depots **Plants** **Customer sets**



Relaxed variables (option a)

- Y_RH(k,source,time) whether or not truck is loaded at source at time period t
- y_route_RH(k,r,time) whether or not truck k delivers to route r (a customer set) in time period t

Rolling Horizon approach

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Aggregate BLOCK

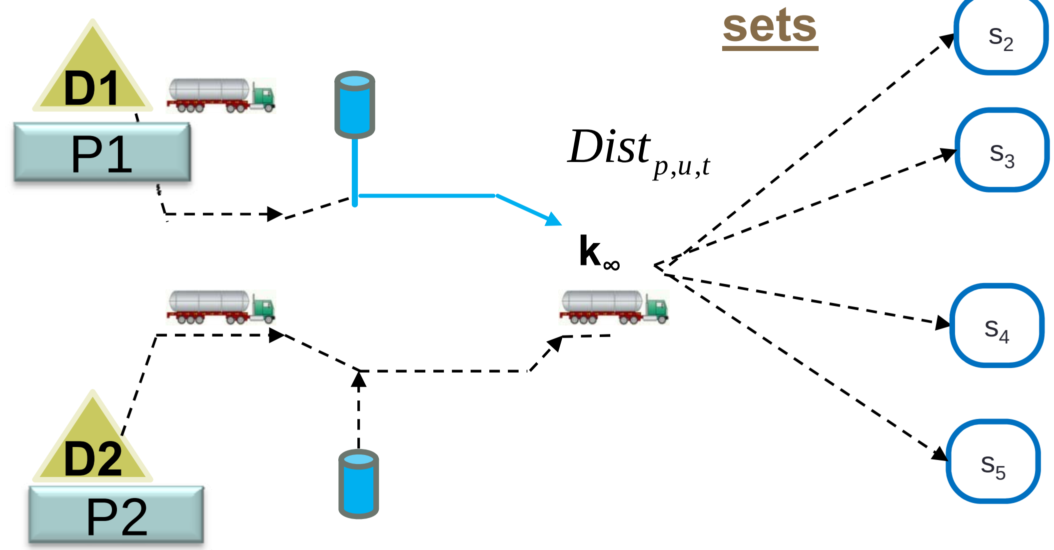
Aggregate Distribution model (option b)

Detailed Production

Main binary variables
mode selection
B_{pmt}

Production mode
Same production mode
Material Balances

Depots **Plants** **Customer sets**



Products are shipped directly from plant to customer (disregarding the route generation)

Rolling Horizon (illustrative example iteration 2)

Fixed:
$Y_{k,source,time}$, $B\_on_{p,m,time}$, $Y\_route_{k,r,time}$

Detailed time block

Aggregate time block

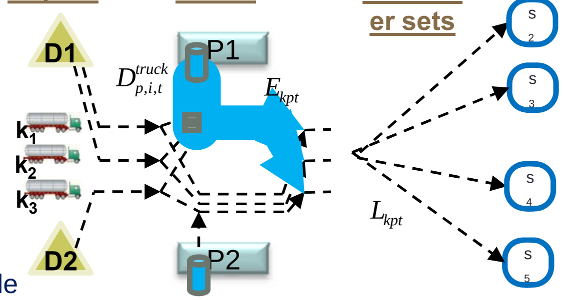
Relaxed approach

Detailed Distribution constraints

Detailed Production

Main binary variables
mode selection
B_{pmt}

Production mode
Same production mode
Material Balances



Relaxed variables (continuous 0—1)

- Y_RH(k,source,time) whether or not truck is loaded at source at time period t
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Aggregate approach

Aggregate Distribution model

Detailed Production

Main binary variables
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Production mode
Same production mode
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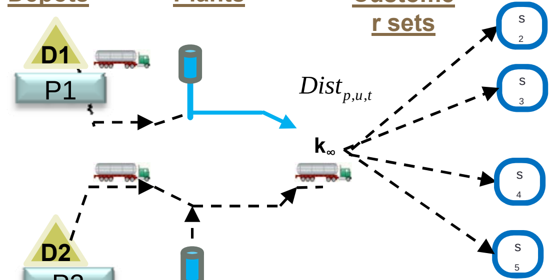
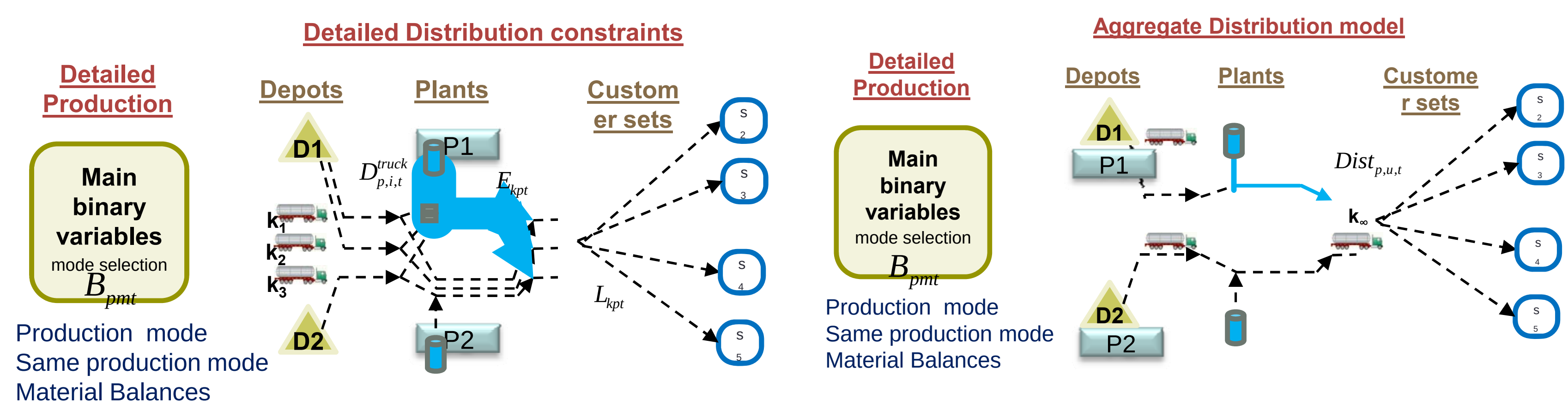


Rolling Horizon (illustrative example iteration 2)

Detailed time block

Aggregate time block

Aggregate approach



Aggregate model (original):

- Infinite number of trucks
- Time window for demand satisfaction
- Travel distance calculations
- Distribution cost

Aggregate model (v2):

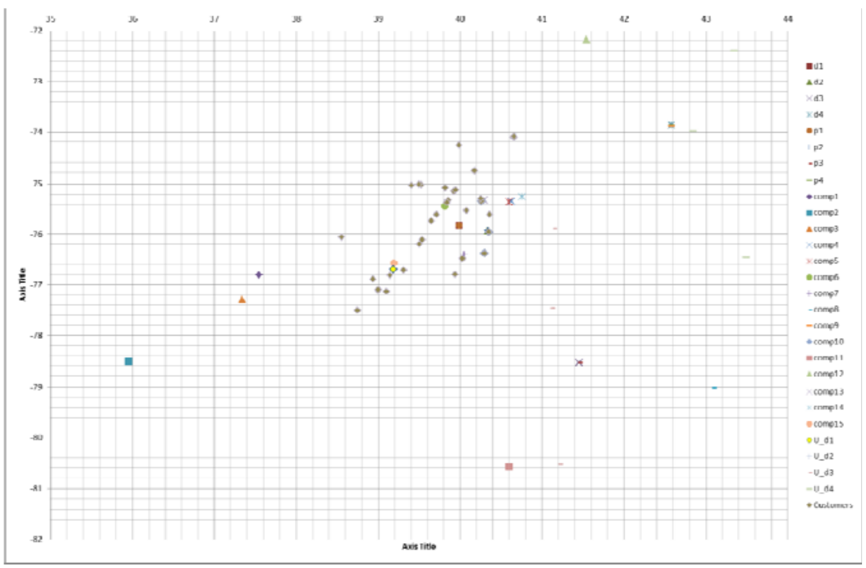
- Infinite number of trucks
- Demand (fixed one time period)
- Travel distance calculations
- Distribution cost
- Minimum truck load

Industrial Size Test Case – Results

- 4 Plants / Depots
- 2 products (LIN, LOX)
- 2-3 production modes for each plant
- 15 alternative sources
- 229 customers

19 sources

10,032 possible routes reduced to **1,900** by the route generation algorithm



- 14 time periods (peak and off-peak)
- 46 trucks (25 for LIN, 12 for LOX)
- Min/max inventory, distances, electricity prices, truck deliveries, etc.

Industrial Size Test Case – Results

- 4 Plants / Depots
- 2 products (LIN, LOX)
- 2-3 production modes for each plant
- 15 alternative sources
- 229 customers
- 14 time periods (peak and off-peak)
- 46 trucks (25 for LIN, 12 for LOX)
- ntory, distances, electr eries, etc.

19 sources

10,032 possible routes reduced to 1,900 by the route generation algorithm

RH M2: 7 times faster!

3.22 % worse

RH Aggregate: 21 times faster!

4.73 % worse

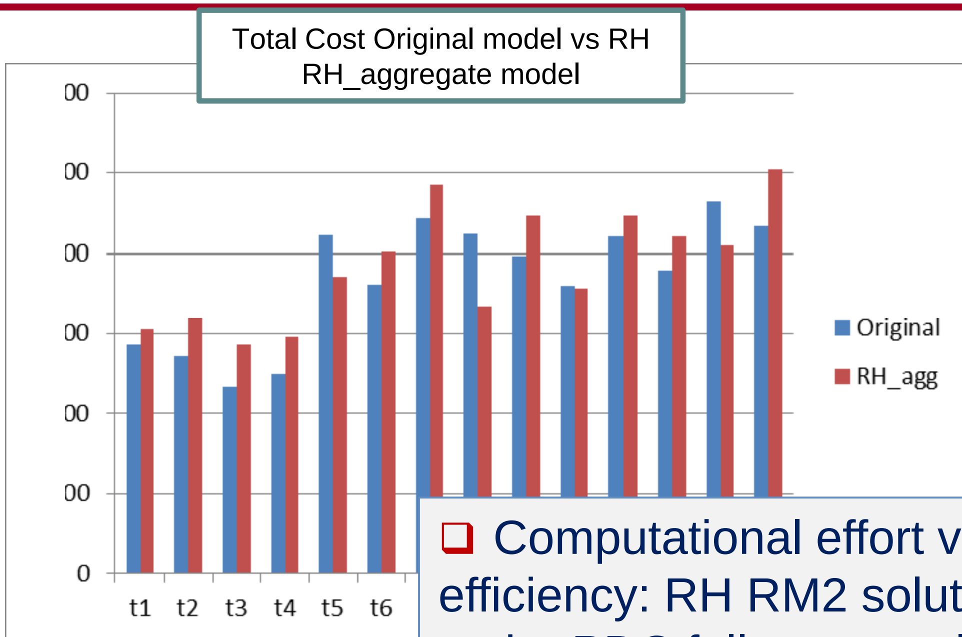
RH Agg_v2:

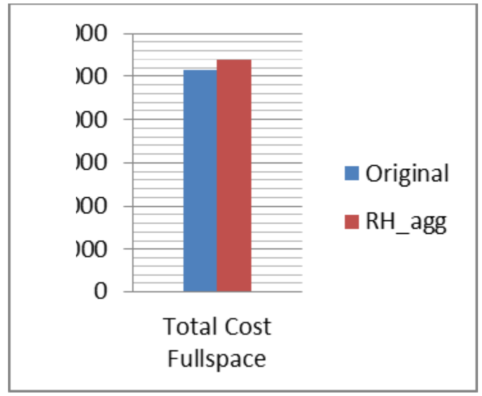
200 times faster!

3.3 % worse

		Original	RH RM2	Aggregate	Agg v2
Model Size	Binary variables	26,486	-	-	-
	Continuous variables	69,902	-	-	-
	Constraints	35,233	-	-	-
	Total cost (normalized)	100	103.22	104.73	103.35
CPU results	Time	**41,511s**	**1,297**	**368s**	**204s**
	Nodes	**104,880**	**1,817**	**936**	**900**
	Relative gap	**4.7%**	**1.9%**	**1%**	**1%**

12 hrs PDC vs RH RM2





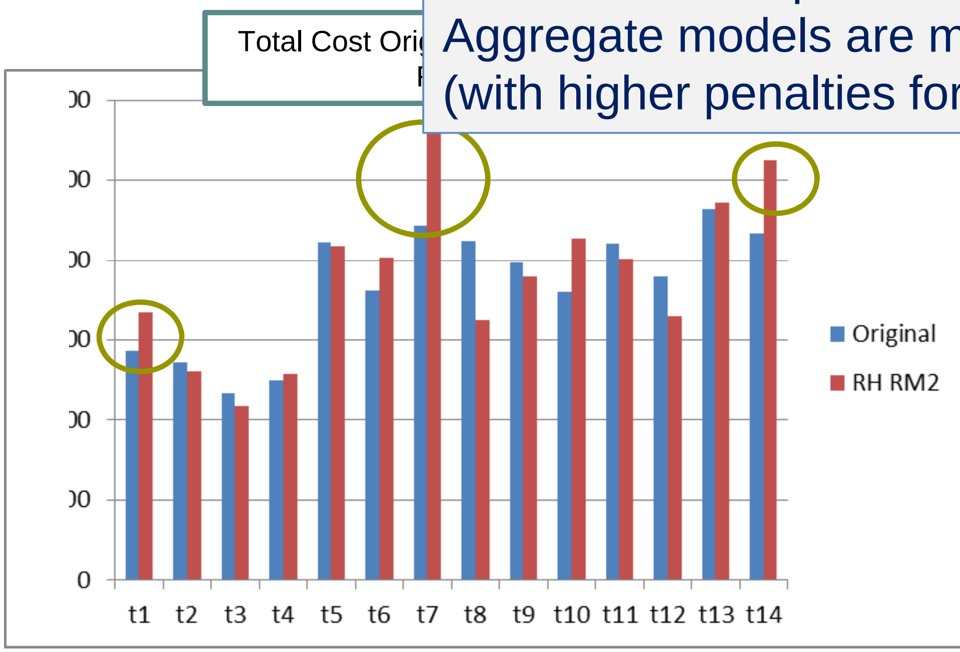
- ❑ Computational effort vs solution efficiency: RH RM2 solution is the **closest** to the PDC full-space solution, while Aggregate models are much more **faster** (with higher penalties for sub optimality).

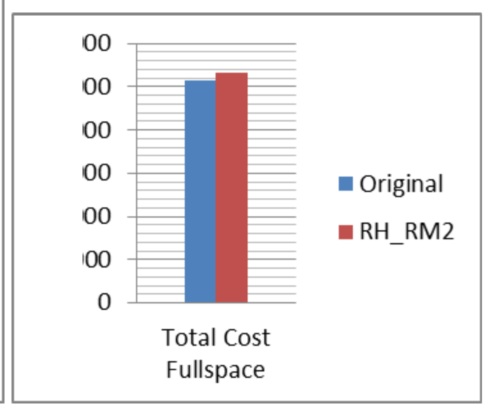
Total Cost

iginal Model (12 hrs) 100

iginal Model (3hr) 102.21

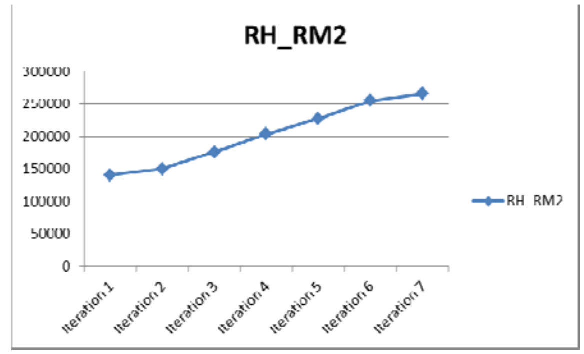
- ❑ RH agg 104.73
- ❑ **RH RM2 103.22**
- ❑ **RH agg_v2 103.35**

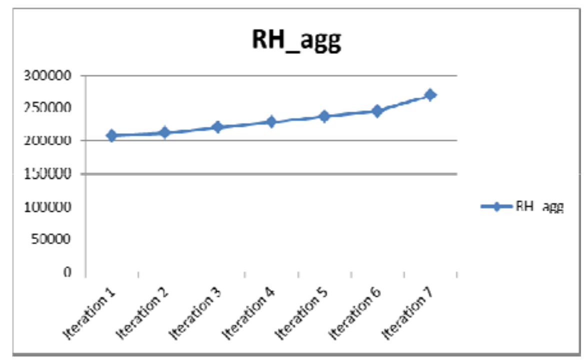


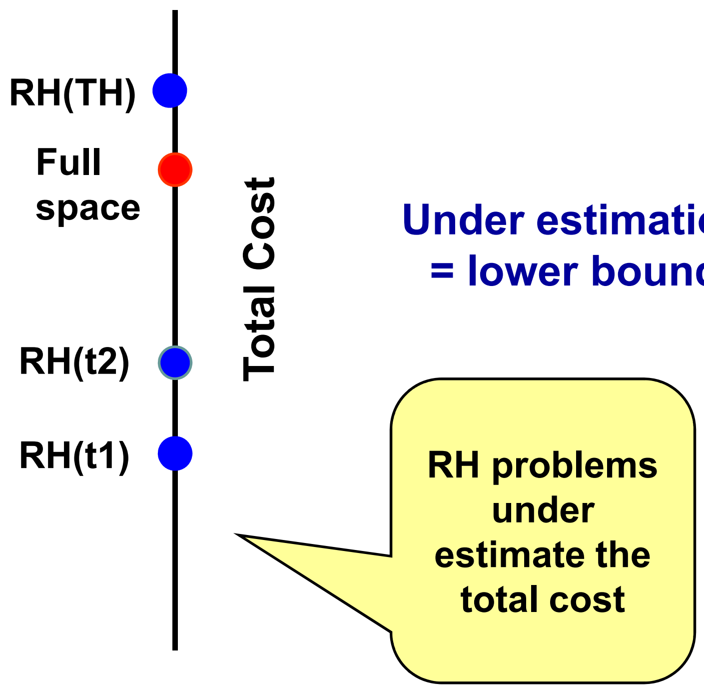


Rolling horizon approach

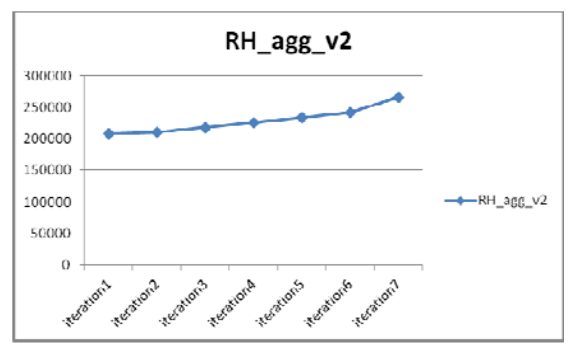
- ❑ Decompose the full-space problem in a series of iterations.
- ❑ Detailed time block
- ❑ Aggregate time block







Under estimation = lower bound



Remarks

Proposed framework provides optimal production and distribution coordination reducing the computational effort

- Proposed rolling horizon approach (both relaxed and aggregate models) are **computationally faster** than the simultaneous approach with a **small penalty** for sub-optimality.
- Aggregate model changes the search in the Branch and Bound method, while in the original model **symmetric results** impact the solution.
- Alternative sources have been widely used by the Rolling horizon approach (comp1, comp6 and comp10; in time periods t1-t4, t7, t9-t13). Otherwise, in the original model were narrowly used (comp7, comp10, and comp 13 used in time periods t7, t11, and t13).

Work under development:

- Receding horizon: we consider that the information after iteration 1 could be used as decision making, and then explore the next time period as a new RH problem.
- Hybrid decomposition approach.