Hybrid Decomposition Approach for Optimal Production – Distribution Coordination of Industrial Gases Supply Chains

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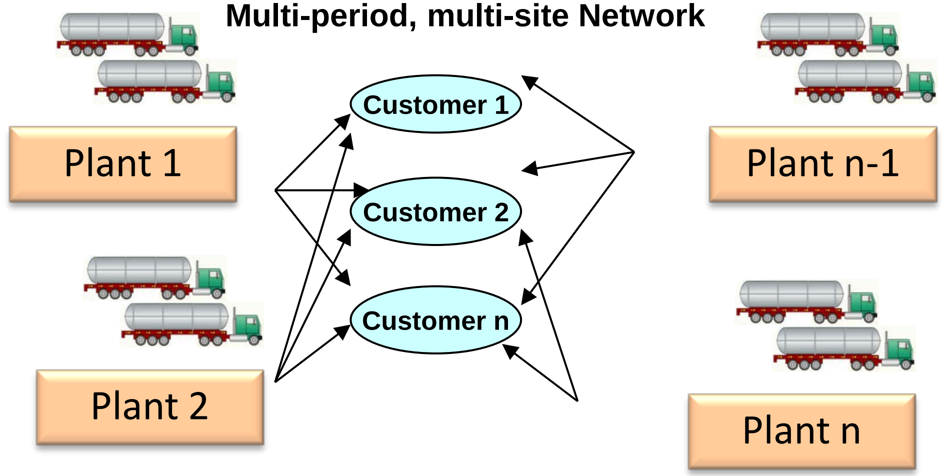
Motivation

Large Scale Optimization

- ❑ Multi-period, Multi-echelon supply chain problems with complex constraints
  - ❑ Production and inventory management
  - ❑ Transport assignments

Multi-period, multi-site Network



Challenges

- ❑ Combinatorial problem
- ❑ Complicating constraints
- ❑ Computational effort

Objective

❑ Propose a **hybrid approach** capable to take advantage of both LD and RH techniques and provide **tighter bounds** on the optimal solution and **better convergence** properties to the LD algorithm

Lagrangean Decomposition and Rolling Horizon Drawbacks

- ❑ Strong linking among variables
- ❑ Algorithm complexities (infeasible solutions)

} (LD)

- ❑ Penalties due to suboptimality ← (RH)

Introduction

Lagrangean Relaxation

- We assume that (P) is of the form:

$$\min_{x}\{fx \mid Ax \leq b, Cx \leq d, x \in X\}$$

  - x contains sign and integrality restrictions, i.e. $X = \mathbb{R}_{+}^{n-p} x \{0,1\}^{p}$
  - Ax≤b are considered to be complicating. Let λ to be a vector of nonnegative Lagrange multipliers.

- The Lagrangean relaxation of (P) is given by:

$$\min_{x}\{fx + \lambda(Ax \leq b) \mid Cx \leq d, x \in X\}$$

- Denote the Feasible set F(·) and the optimal value as v(·), then:

$$F(P) \subseteq F(LR_{\lambda})$$

$$v(P) \geq v(LR_{\lambda})$$

Guinard (2003); Jackson and Grossmann (2003); Terrazas-Moreno et al. (2011)

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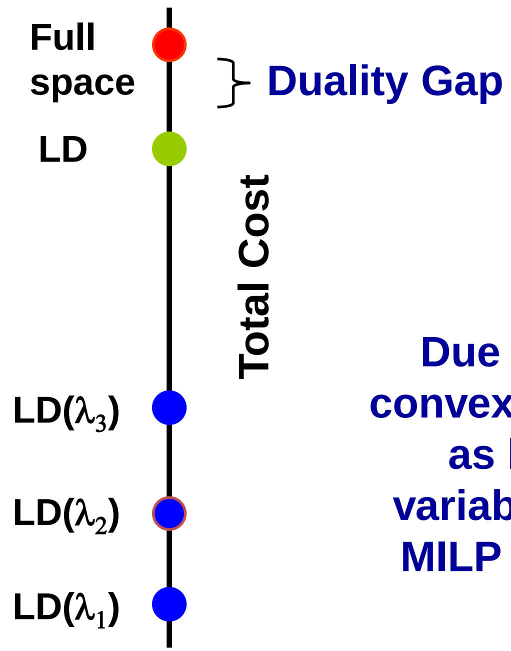
The Best bound

- The best bound we can obtain of LR_λ is solving the Lagrangean Dual.

$$\max_\lambda \left(\min_x \{fx + \lambda(Ax \le b) \mid Cx \le d, x \in X\} \right)$$



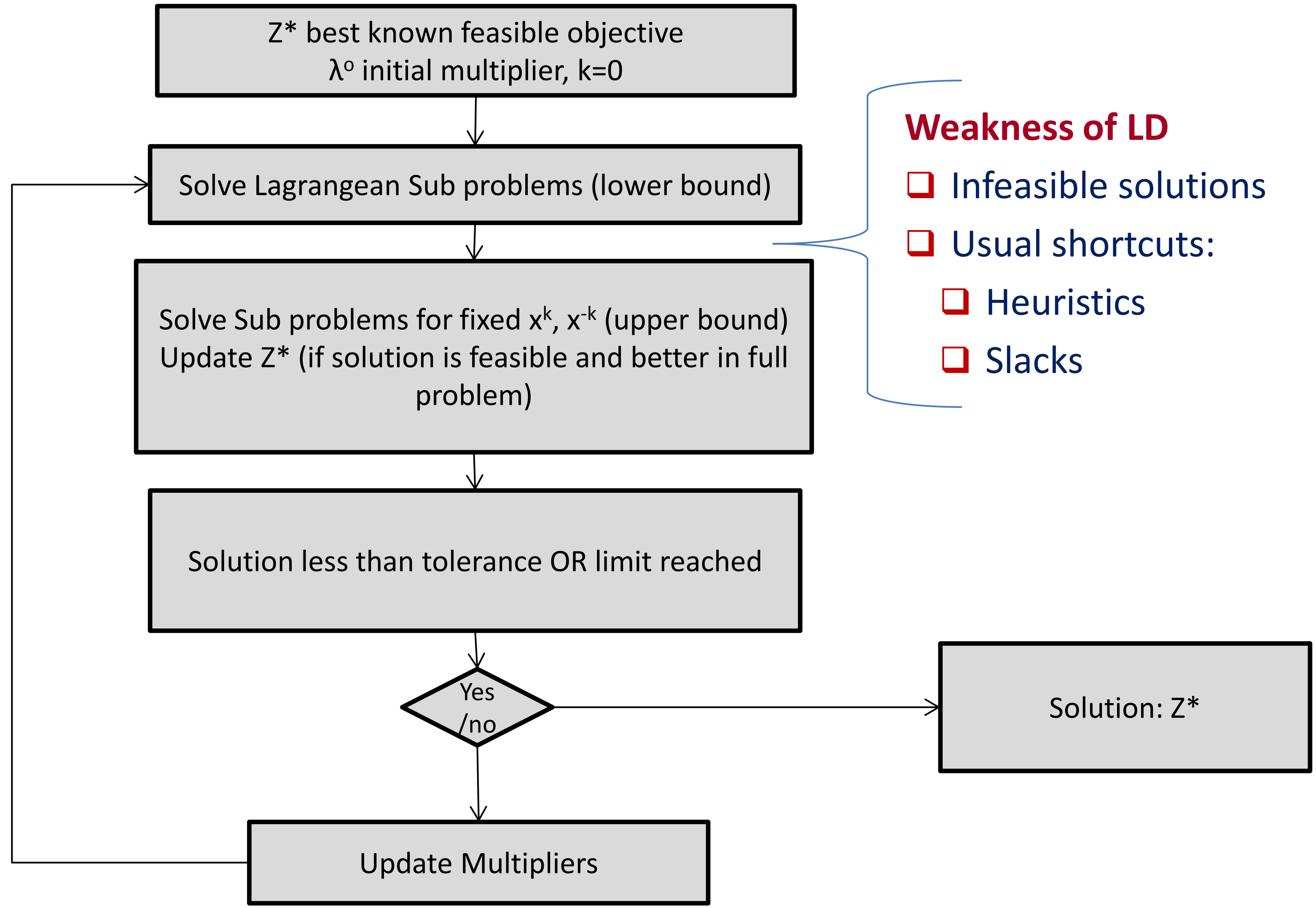
Due to Non-convexities such as binary variables in an MILP problem

Under estimation = lower bound

Guinard (2003); Jackson and Grossmann (2003); Terrazas-Moreno et al. (2011)

Solution approach

Lagrangean decomposition algorithm



Weakness of LD

- ❑ Infeasible solutions
- ❑ Usual shortcuts:
  - ❑ Heuristics
  - ❑ Slacks

Proposed Framework

- We assume that (P) is of the form:

$$\min_x \sum c^T x_t \qquad \textbf{Full space}$$

$$\sum A x_t \le b \qquad (P)$$

$$\mathrm{D} x_t \le d \quad \forall \mathrm{t}$$

$$x_t \in Xt, X = \mathbb{R}_+^{n-p} \times \{0,1\}^p$$

- The Lagrangean Relaxation

$$\min_x \sum c^T x_t + \sum \lambda_t \, (A x_t \le b)$$

$$\mathrm{D} x_t \le d \;\; \forall \mathrm{t} \qquad (LR)$$

$$x_t \in X, X = \mathbb{R}_+^{n-p} \times \{0,1\}^p$$

Full space (relaxing complicating constraints)

- The Lagrangean temporal decomposition (with fixed λ_t)

$$\min_{x_t} c^T x_t + \lambda_t \, (A x_t \le b)$$

$$\mathrm{D} x_t \le d \qquad (LR_t)\, \forall \mathrm{t}$$

$$x_t \in X_t, X = \mathbb{R}_+^{n-p} \times \{0,1\}^p$$

LR_λ Solving independent sub-problems

- Hybrid approach (with fixed λ_t)

$$\min_{x_t, \forall t \in T} c^T x_t + \lambda_t \, (A x_t \le b)$$

$$\mathrm{D} x_t \le \mathrm{d} \qquad (HLR_t)$$

Aggregate based constraints → $\mathrm{E} x_{t'} \le g \quad \forall t' \neq t$

$$x_{\hat{t}} \in X_t, X = \mathbb{R}_+^{n-p} \times \{0,1\}^p$$

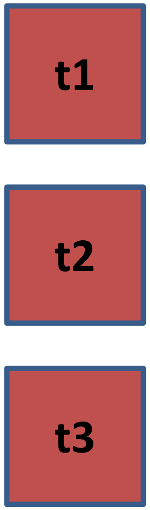
$$x_{\bar{t}} \in X, X = \mathbb{R}_+^{n-p} \times [0,1]^p \;\; \forall \bar{t} \neq \hat{t}$$

LP relaxation

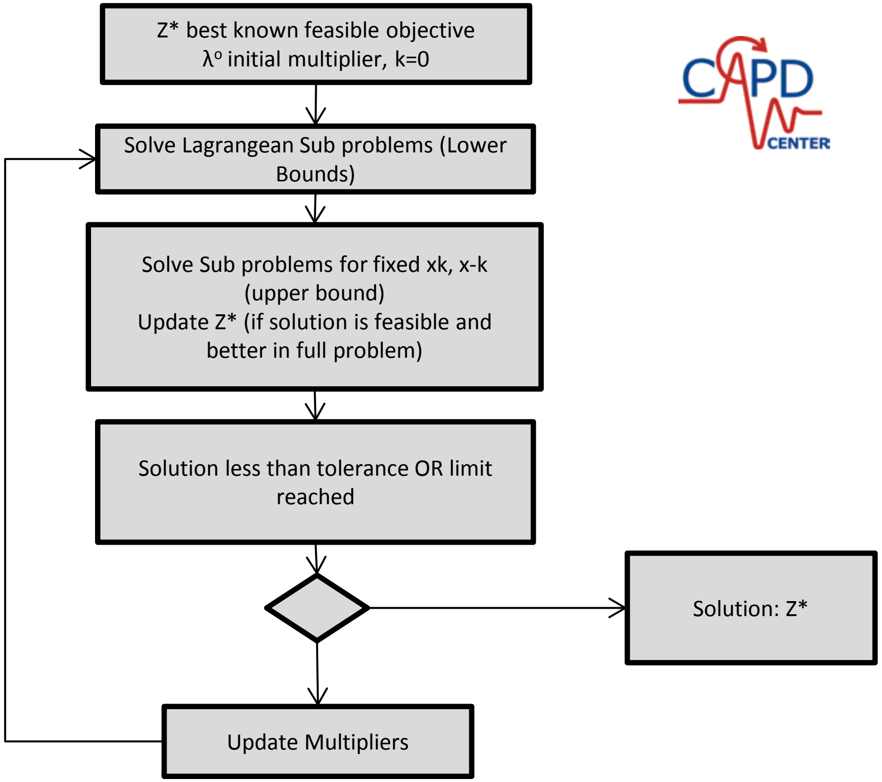
3 time period illustration

Temporal Lagrangean Decomposition (conventional approach)

Fix Multipliers and Solve each sub problem

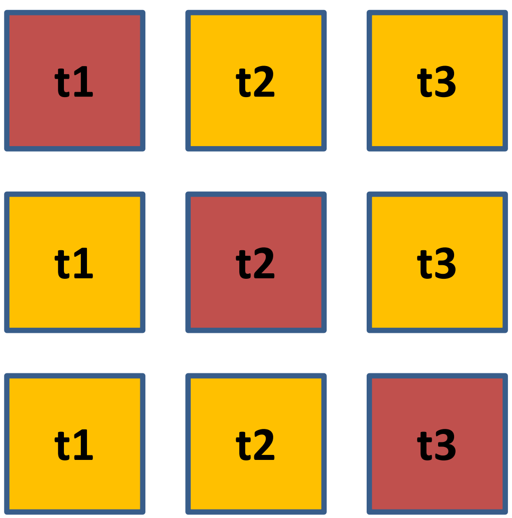


Fix variables and solve the Full space problem



Hybrid approach

Fix Multipliers and Solve each sub problem



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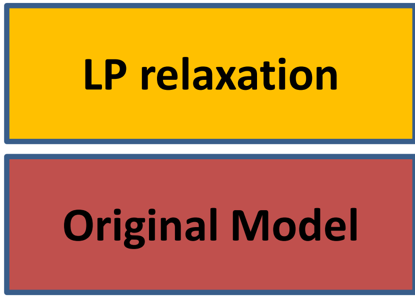
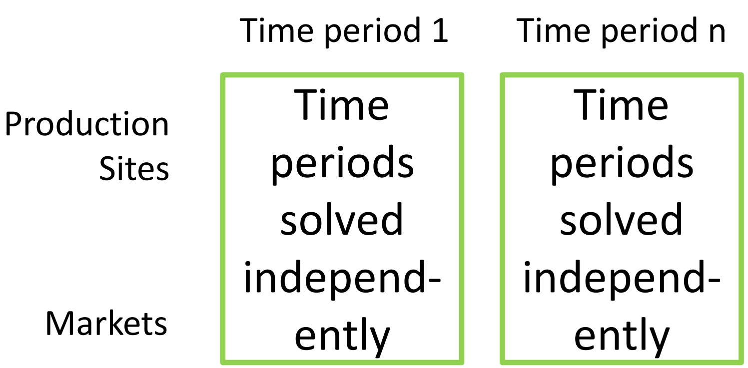


Lagrangean Decomposition

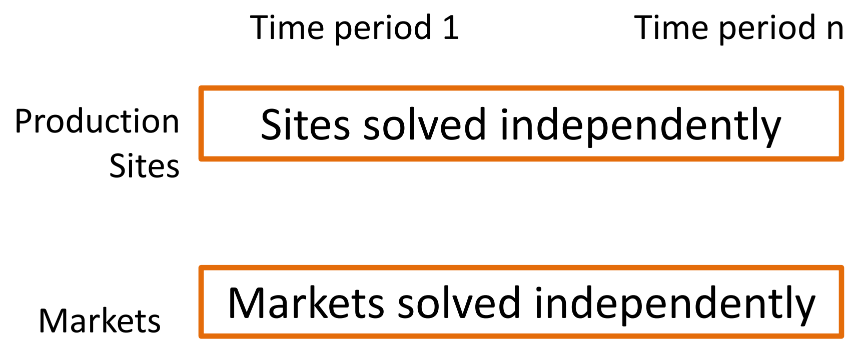
Temporal Decomposition

❑ A multi-period problem is separated by time periods, linked by the presence of inventory xs,t

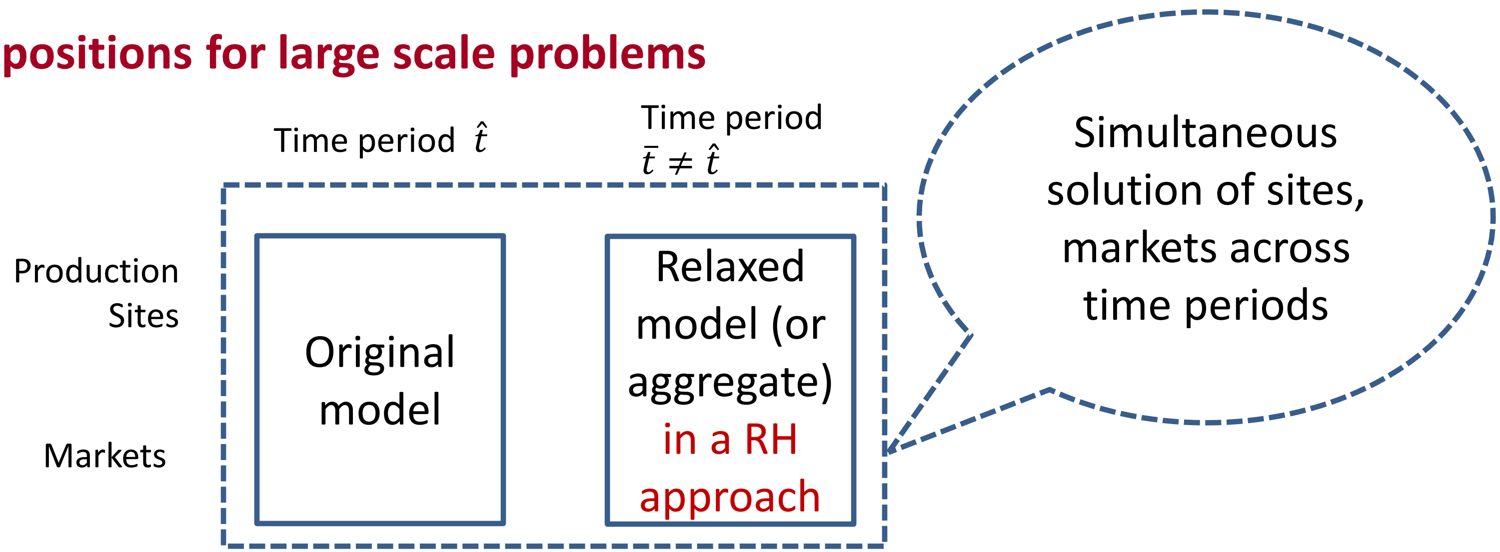


Spatial Decomposition

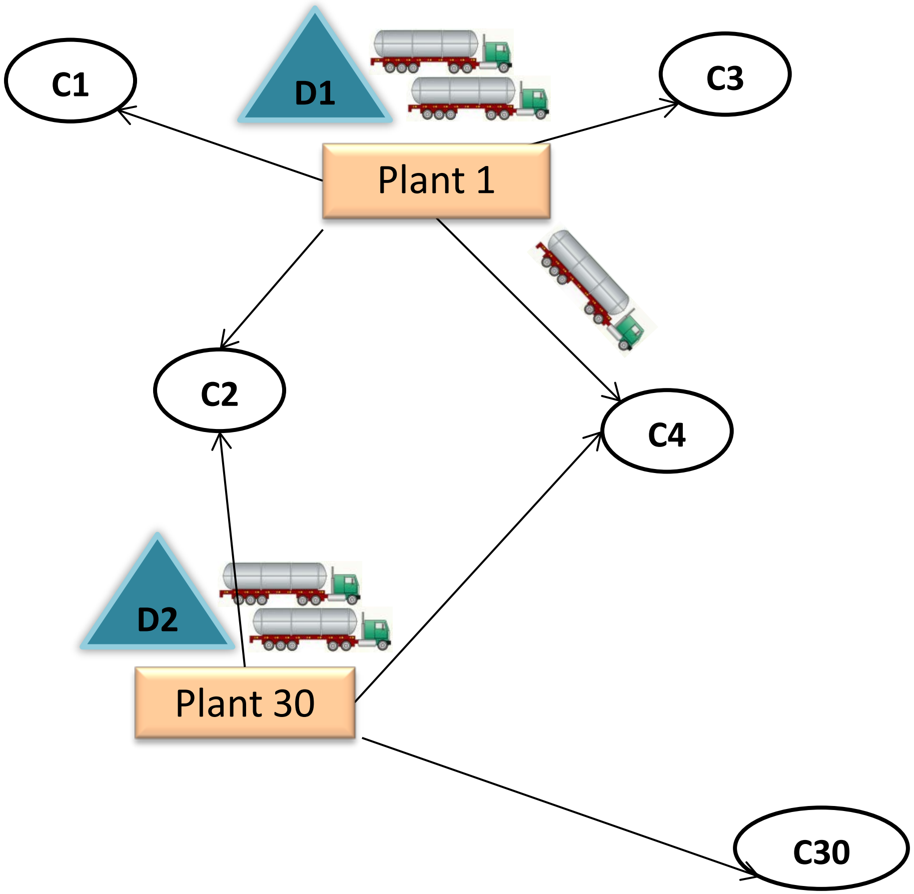
❑ In this case the multi-period, multi-echelon supply chain problem can be separate by sites and markets



Hybrid decompositions for large scale problems



Small example: Problem statement



3 products, 5 plants, 5 markets, 3 time periods

Given

- Plants, Products, and Production Limits
- Product prices and operation cost
- Customers and their demand
- Max/Min inventory
- Distribution costs
- Fixed Planning Horizon (3 days)

Decisions in each time period t

- Production rates at each plant
- Inventory level
- How much product to be delivered to each market
- Sales
- **Start up (binary)**

Objective Function

- Maximize Profit

Extended Case study: Jackson and Grossmann 2003

Small example: Mathematical model (MILP)

Multi product, multi site production planning

- Variables
  - Production
  - Distribution
  - Inventory
  - Plant start up
- Assumptions
  - Demand satisfaction
  - Safety stock
  - Production cost
  - Penalization cost
  - **Fixed cost**

$$\max \text{PROFIT} = \sum_t \sum_i [\sum_m \alpha_t^{i,m} \text{SL}_t^{i,m} - \sum_s \beta_t^{i,s}(P_t^{i,s} + \delta \text{PEN}_t^{i,s}) - \sum_s \sum_m \gamma_t^{i,s,m} F_t^{i,s,m}] \quad (8)$$

$$- \sum_t^T \sum_i^I \sum_s^S a_{i,s} Y_{i,s,t}$$

$$\text{PEN}_t^{i,s} \geq \text{QUOTA}^{i,s} - \text{INV}_t^{i,s} \quad \forall\, i \in \text{PR},\, s \in \text{SITES},\, t \in T \quad (4)$$

$$\text{PEN}_t^{i,s} \geq \text{INV}_t^{i,s} - \text{QUOTA}^{i,s} \quad \forall\, i \in \text{PR},\, s \in \text{SITES},\, t \in T \quad (5)$$

$$\text{SL}_t^{i,m} = \sum_{s \in \text{SITES}} F_t^{i,s,m} \quad \forall\, i \in \text{PR},\, m \in \text{MAR},\, t \in T \quad (6)$$

$$\text{SL}_t^{i,m} \leq \text{FCAST}_t^{i,m} \quad \forall\, i \in \text{PR},\, m \in \text{MAR},\, t \in T \quad (7)$$

$$P_t^{i,s} \leq \text{CAP}^{i,s} \text{tp}_t^{i,s} \quad \forall\, i \in \text{PR},\, s \in \text{SITES},\, t \in T \quad (18)$$

$$\sum_i \text{tp}_t^{i,s} = \tau_t \quad \forall\, s \in \text{SITES},\, t \in T \quad (19)$$

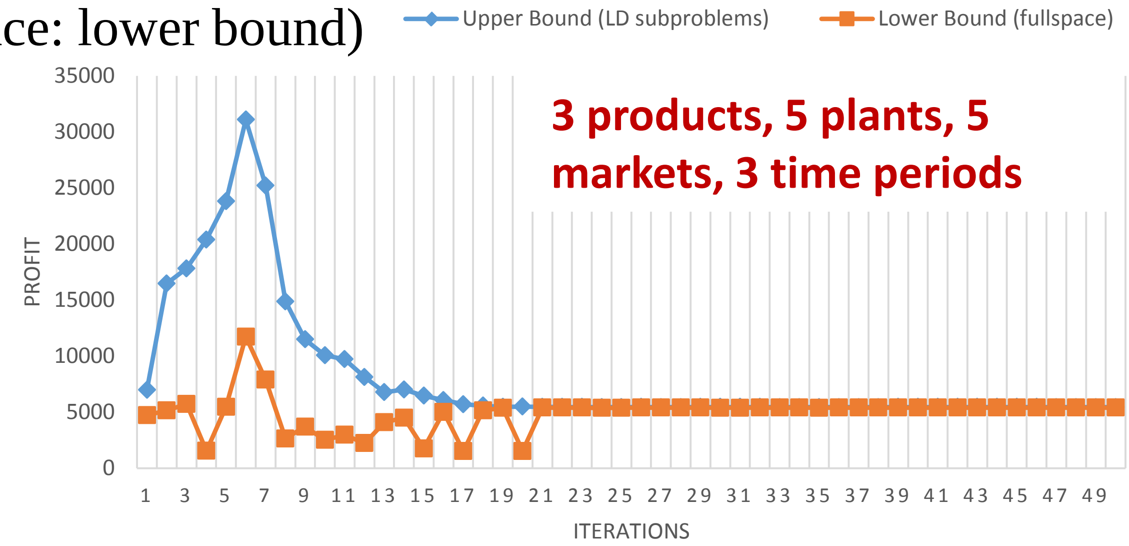
$$P_{i,s,t} \leq CAP_{i,s} Y_{i,s,t} \quad \forall i \in PR, s \in SITES, t \in T$$

Extended from Jackson and Grossmann 2003

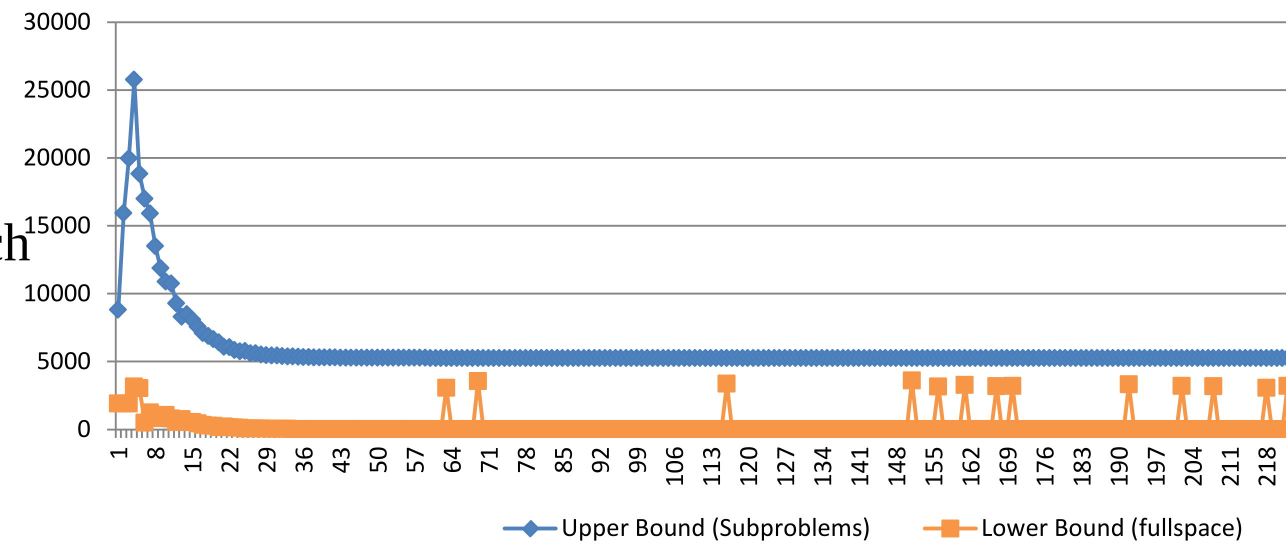
Small example: Results (MILP)

Profit (LD: Upper bound; Full space: lower bound)

- Temporal LD
  - 5402.895 $
  - 16.629 cpu seconds
  - Gap < 1 $ (0.0015%)



- Spatial LD
  - 3596.366 $
  - 115.448 cpu seconds
  - Gap > 1000 $ (33%)



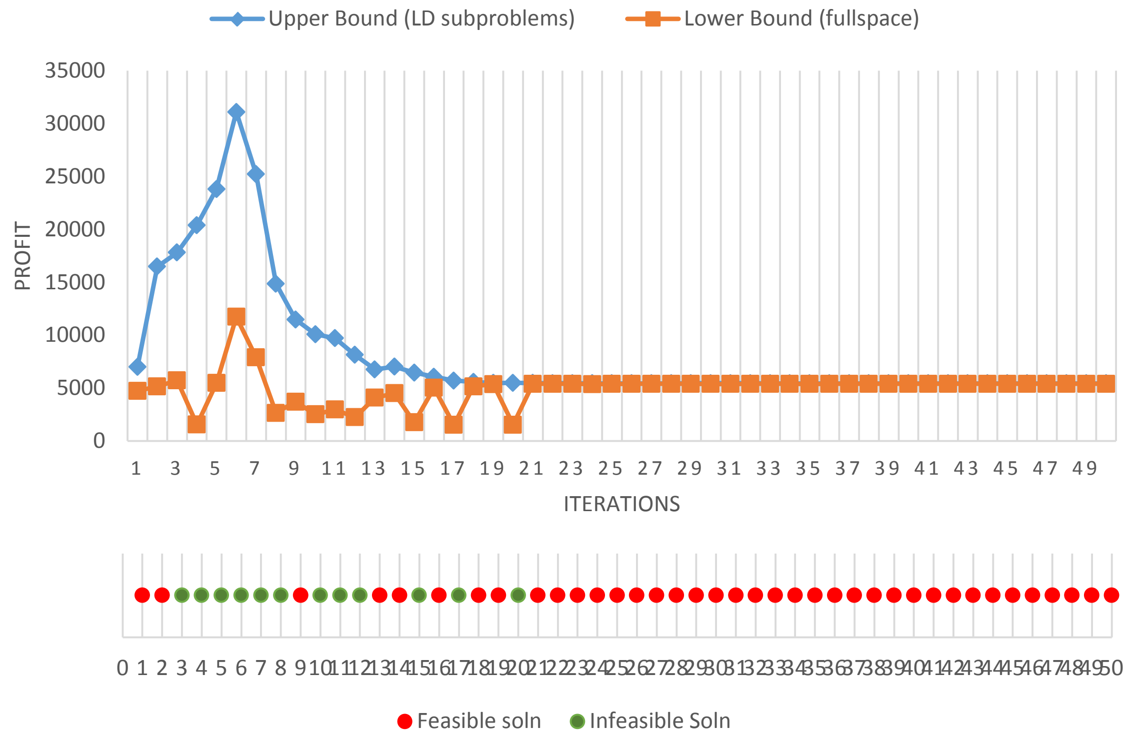
- Rolling Horizon approach
  - 5402.9730 $
  - 0.235 cpu seconds
- Full space problem
  - 5402.973
  - 0.219 cpu s

Case study extended from Jackson and Grossmann 2003

Small example: Hybrid vs Lagrangean Decomp

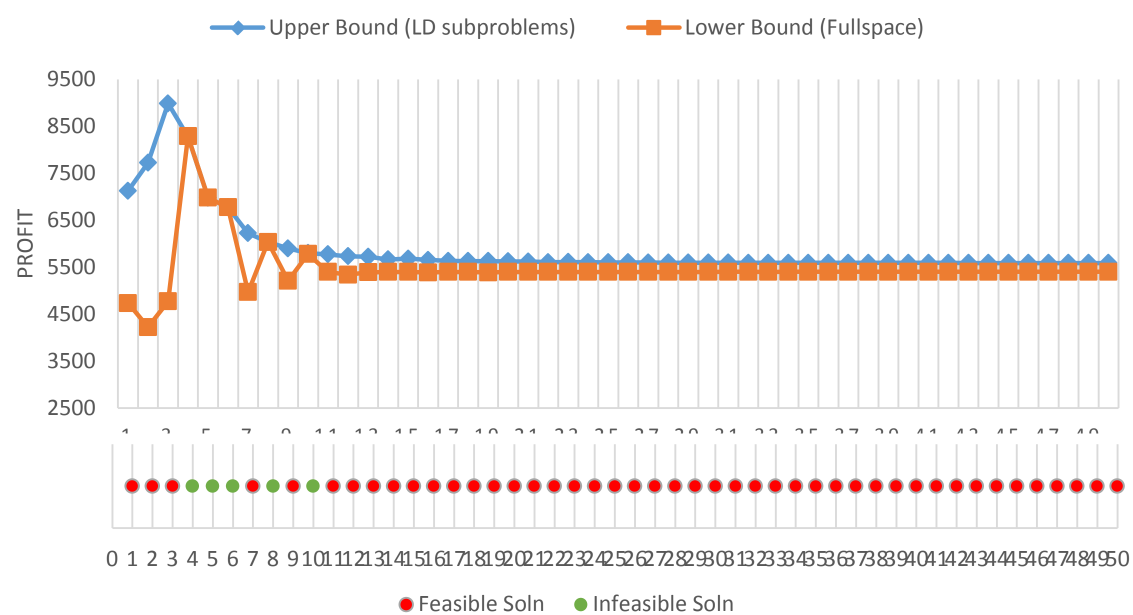
Temporal Lagrangean Decomposition

- Iter: 33/50
- Best solution: 5402.895 $
- Cpu time: 16.629
- GAP: 0.0014%



Hybrid approach

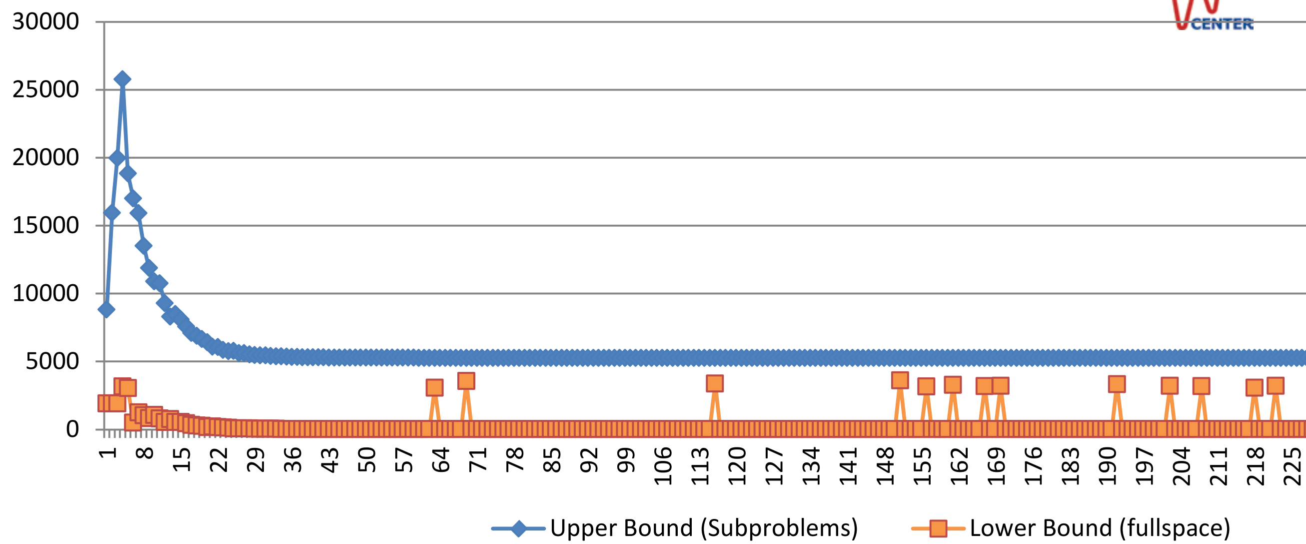
- Additional information is used at the initial stage to provide tighter bounds. Leading to feasible solutions in the fullspace problem
- Iter: 39/50
- Best solution: 5402.679 $
- Cpu time: 22.513
- GAP: 0.0054%



Small example: Hybrid vs Lagrangean Decomp

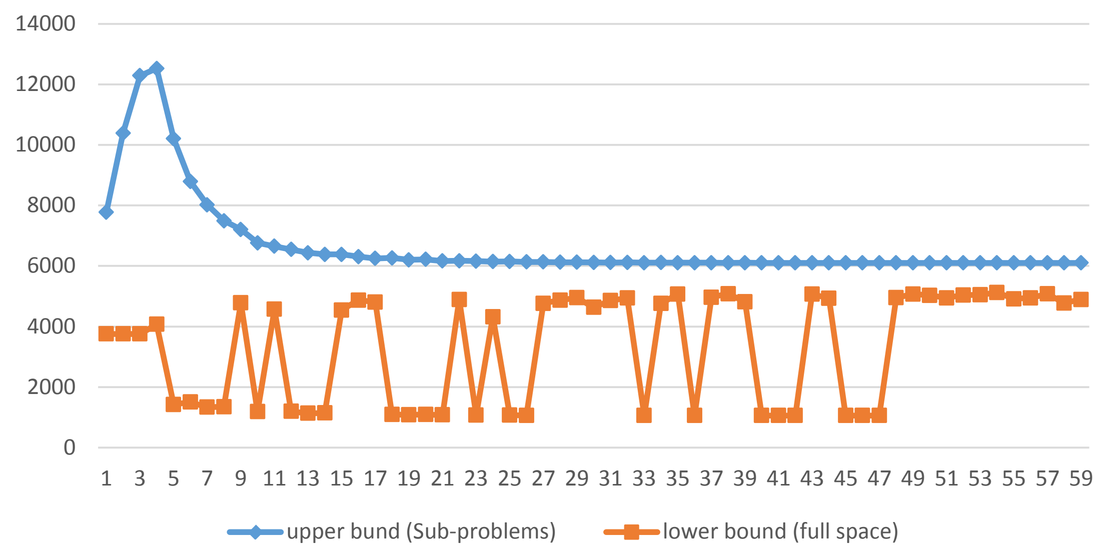
Spatial Lagrangean Decomposition

- Iter: 151/300
- Best solution: 3596 $
- Cpu time: 115.448
- GAP: 33%
- First feasible soln: iter 61

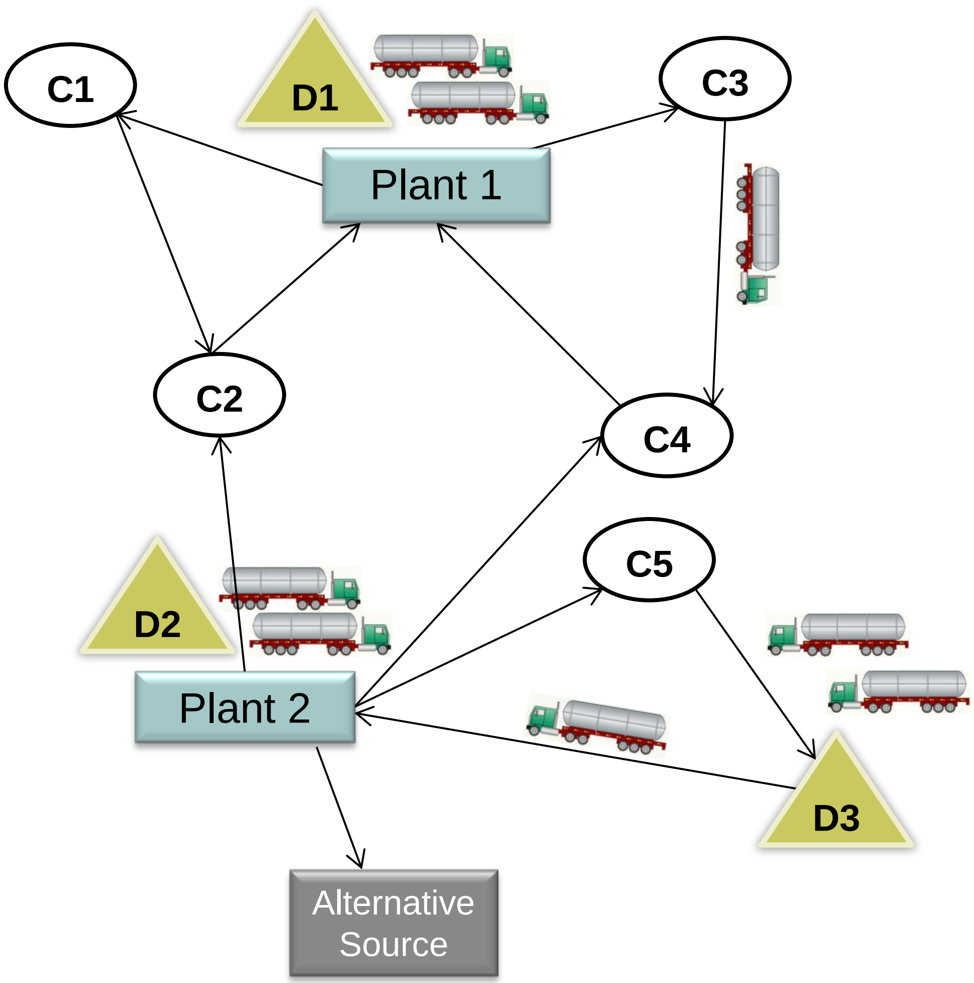


Hybrid approach

- Iter: 54/60
- Best solution: 5125.766$
- Cpu time: 34.2
- GAP: 5.13%



Industrial test case: Problem Statement



Given

- Plants, Products, Operating Modes and Production Limits
- Daily Electricity Prices (off-peak and peak)
- Customers and their demand/consumption profiles
- Max/Min inventory at production sites and customer locations
- Alternative sources and product availabilities
- Depots, Truck availabilities and capacities, Distances
- Fixed Planning Horizon (usually 1-2 weeks)

Decisions in each time period t

- **Modes and production rates at each plant**
- **Inventory level at customer location and plants**
- **How much product to be delivered to each customer through which route**

Objective Function

- **Minimize total production and distribution cost over planning horizon**

Main Assumptions – Distribution Side

- Two time periods per day (peak and off-peak) are considered
- Trucks do not visit more than 4 customers in a single delivery

Industrial test case

- 4 Plants / Depots
- 2 products (LIN, LOX)
- 2-3 production modes for each plant
- 15 alternative sources
- 282 customers → 105 customer demands
- 14 time periods (peak and off-peak)
- 46 trucks (25 for LIN, 12 for LOX)
- Min/max inventory, distances, electricity prices, truck deliveries, etc.

19 sources

3,900 possible routes

reduced to 890 by the route generation algorithm

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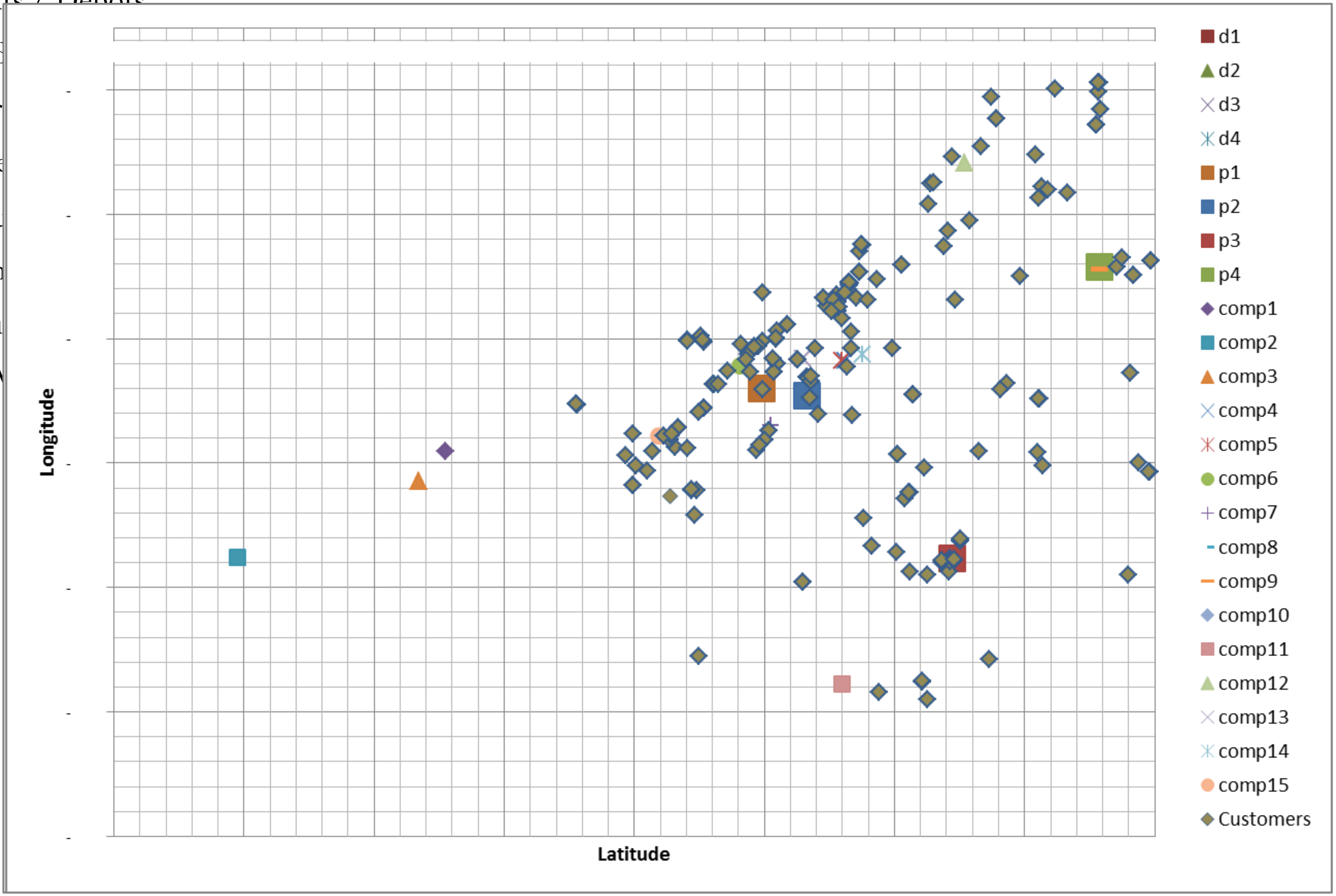
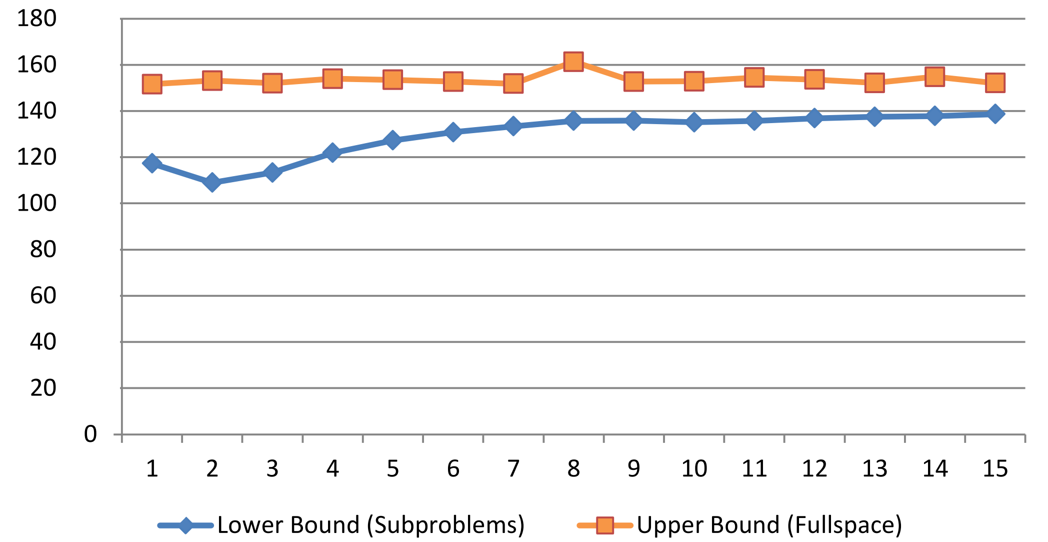


Industrial test case: Hybrid vs Lagrangean Decomp

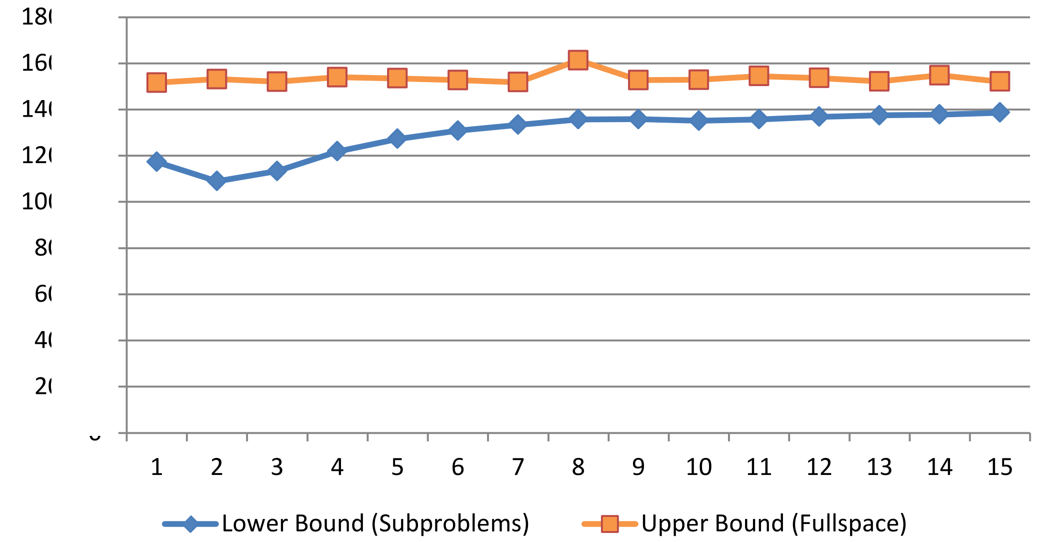
Temporal Lagrangean Decomposition

- Iter: 1 /15
- Best solution: 92 $
- Cpu time: 6000



Hybrid approach

- Iter: 13/15
- Best solution: 89 $
- Cpu time: 2713



Industrial Size Test Case – Results

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		Original	LD	Hybrid
Model Size	Binary variables	13,534	-	-
	Continuous variables	37,640	-	-
	Constraints	20,423	-	-
	Total cost	100	91	89,57

CPU results	Time	**15,000**	**6000**	**2700**
	Relative gap	**8%**	**12%**	**12%**

Concluding Remarks

- Hybrid decomposition approach has been tested under Temporal and Spatial decomposition techniques.
  - The computational expense and convergence properties have been improved
- Proposed framework provides optimal production and distribution coordination reducing the computational effort
- Production-Distribution large scale optimization problems are affected by symmetry. In this case mainly due to the combinatorial nature of the decisions, i.e:
  - produce/store/allocate the resources to be delivered to satisfy customer demands.
  - select the trucks to fulfill each route.
  - and select the customers visited trough each route.
- Hybrid decomposition approach exhibits slightly better cost estimation good convergence properties.
- The use of aggregate formulations could help providing tighter estimation of the distribution side constraints, improving the production-distribution coordination.