Capacity Planning with Rational Markets and Demand Uncertainty

By: A. Kandiraju, P. Garcia-Herreros, E. Arslan, P. Misra, S. Mehta & I.E. Grossmann

Motivation

- Capacity planning : Anticipate demands and take investment, expansion and shut down decisions

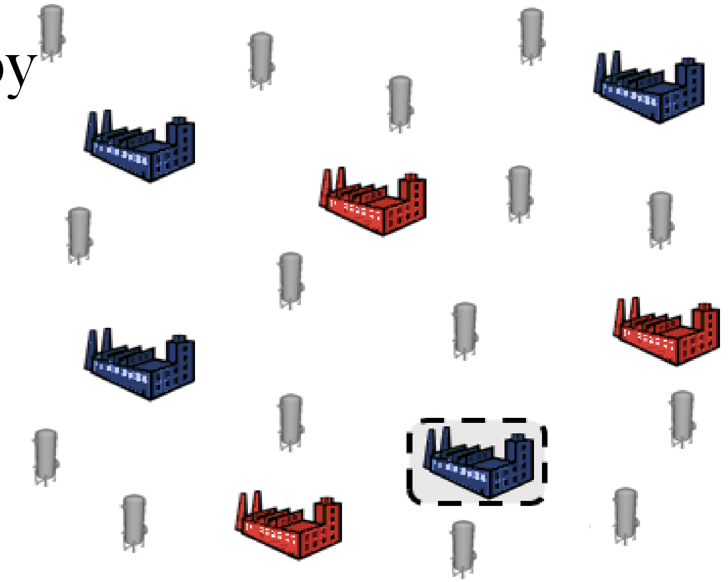
Market Preferences : Market chooses amongst the various existing suppliers to satisfy its demand

Demand Uncertainty : Demands are sensitive to customer needs and economic conditions

- Excess capacity - loss of capital investments
- Insufficient capacity - loss of market share

Problem statement

- Maximize expected Net Present Value (NPV) by determining optimal capacity planning strategy
- Set of capacitated plants and candidate locations for new plant from leading supplier
- Set of plants from independent suppliers with limited capacity



- Rational markets that select their suppliers according to their own objective function
- Different demand scenarios that can occur over the time horizon
- Probability of occurrence for each demand scenario

Solution Method

Bilevel optimization

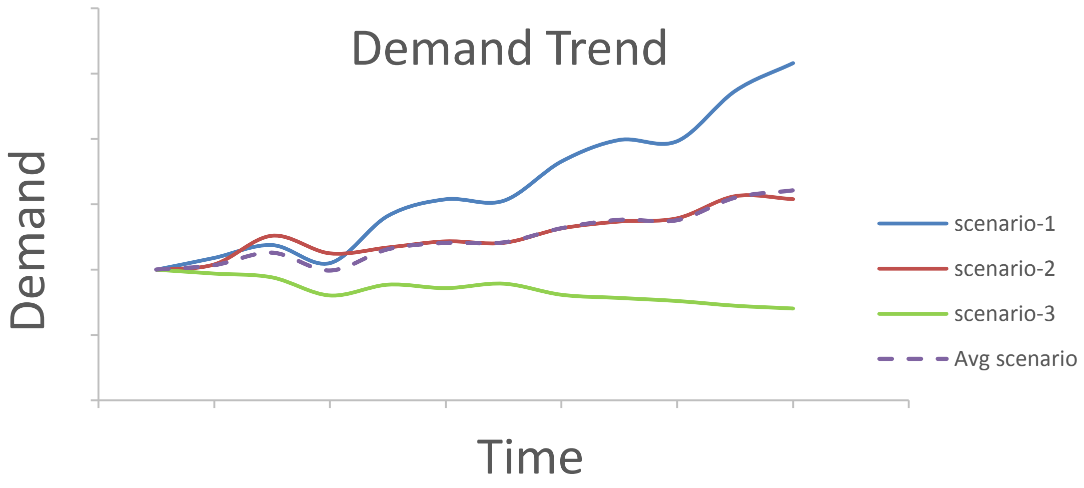
Upper Level :
Maximize expected NPV of the leader

"We want to sell as much as possible to maximize our profit "

Lower Level:
Minimize costs paid by the market

"We want to buy at a low price"

Two-stage Stochastic Programming



1^{st} stage decisions: **Capacity Planning decisions for the leader**

2^{nd} stage decisions: **Demand assignment decisions made by the market**

Stochastic Bilevel Model

$$max \sum_{t\epsilon T}\left(\frac{1}{(1+R)^t}\right)\left[\left\{\sum_{i\in I^1}\sum_{k\in K}\sum_{s\epsilon S}\left(\sum_{j\in J\backslash\{J'\}}(P_{t,i,j,k}-G_{t,i,j,k})-\sum_{j\in J}F_{t,i,k}\right)Pr_s y_{s,t,i,j,k}\right\}-\left\{\sum_{i\in I^1}(A_{t,i}v_{t,i}+B_{t,i}w_{t,i}+E_{t,i}x_{t,i})\right\}\right]$$

$w_{t,i} = V^o_{t,i} + \sum_{t'=1}^{t} v_{t,i} - \sum_{t'=1}^{t} z_{t,i}$ $(\forall t \in T, i \in I^1)$ Invest or divest in plants

$\sum_{t'=1}^{t} z_{t,i} = 1$ $(i \in I^1)$ Shutdown only once

$x_{t,i} \le w_{t,i}$ $(\forall t \in T, i \in I^1)$ Expand only open plants

$c_{t,i} = C^o_{t,i} + \sum_{t'=1}^{t} Hx_{t,i}$ $(\forall t \in T, i \in I^1)$ Capacity expansion

$\min \sum_{t\epsilon T}\sum_{i\in I^1\cup I^2\cup I^3}\sum_{j\in J\backslash\{J'\}}\sum_{k\in K}\frac{1}{(1+R)^t}P_{t,i,j,k}\, y_{s,t,i,j,k}$ $(\forall s \in S)$ Markets minimize cost paid

$\sum_{j\in J}\sum_{k\in K} y_{s,t,i,j,k} \le Mw_{t,i}$ $(\forall s \in S, t \in T, i \in I^1)$ No sales if plant is closed

$\sum_{j\in J}\sum_{k\epsilon K} y_{s,t,i,j,k} \le c_{t,i}$ $(\forall s \in S, t \in T, i \in I^1)$ Capacity of plants from leader

$\sum_{j\in J}\sum_{k\epsilon K} y_{s,t,i,j,k} \le C_{0,i}$ $(\forall s \in S, t \in T, i \in I^2)$ Capacity of independent plants

$-\sum_{j\in J} y_{s,t,i,j,k} \le -Q^{low}_{i,k}\sum_{j\in J}\sum_{k'\epsilon K} y_{s,t,i,j,k'}$ $(\forall s \in S, t \in T, i \in I, k' \in K)$ Lower proportion for product

$\sum_{j\in J} y_{s,t,i,j,k} \le Q^{up}_{i,k}\sum_{j\in J}\sum_{k'\epsilon K} y_{s,t,i,j,k'}$ $(\forall s \in S, t \in T, i \in I, k' \in K)$ Upper proportion for the product

$\sum_{i\in I^1\cup I^2\cup I^3} y_{s,t,i,j,k} = D_{s,t,j,k}$ $(\forall s \in S, t \in T, j \in J\backslash\{J'\}, k \in K)$ All markets are satisfied

$c_{t,i}, y_{s,t,j,i,k} \ge 0$; $x_{t,i}, v_{t,i}, z_{t,i}\ w_{t,i} \in \{0,1\}$

Pr(s) - Probability of scenario 's' to occur

Upper level maximizes expected NPV of leader

Lower level is LP minimizing costs paid by market for each scenario

Solution methods

Single level reformulation : The lower level optimization problem is equated to its dual to transform into single level problem.

Domain reduction strategy : Lower level LP with maximum capacity of leader is solved to determine variables that are never assigned to leader and are fixed to zero.

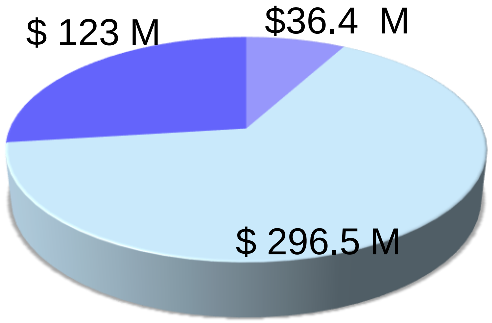
Lagrangean relaxation : Complicating constraint dualized and solved to obtain UB. Solution from UB fixed and solved with complicating constraint to obtain LB.

Benders decomposition: Iterates between master problem that gives expansion plans with UB and sub-problems give demand assignment decisions with the LB. Optimality cuts from sub-problems added to master problem till convergence.

Results and performance of solution methods

Example-1 (10 time periods)

Cost distribution for example-1



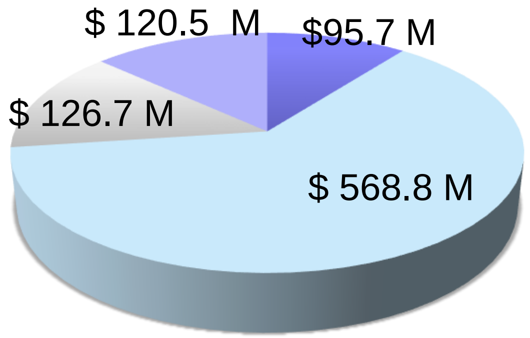
Maintenance costs　Production costs
Transportation costs

2 existing plants of leader
1 candidate plant
1 plant of other supplier
8 markets,
2 products,
10 time periods (quarters)
3 scenarios

NPV: $ 313 M
Sales: $ 769 M

Example-2 (30 time period)

Cost distribution for example-2



Expansion costs　Production costs
Maintenance costs　Transportation costs

2 existing plants of leader
1 candidate plant
3 plants of other suppliers
10 markets,
2 products,
30 time periods (quarters)
3 scenarios

NPV: $ 420 M
Sales: $ 1331 M

All methods could solve example-1 in 5 sec

Performance for example-2

	Single level	Domain reduction	Benders decomposition	Lagrangean relaxation
Solution time	50 sec	13 sec	2000 sec	6000 sec
Optimality gap	0%	0%	7%	0%

Deterministic model vs Stochastic model

Hybrid time: First 8 time periods are quarters, rest of 13 time periods are years

2 existing plants of leader, 1 candidate plant, 3 plants of other suppliers, 10 markets,
2 products, 21 hybrid time periods, 3 scenarios

Deterministic model solution: Expands in 1^{st} , 5^{th}, 9^{th} time periods

Stochastic model solution: Expands in 1^{st}, 9^{th} time periods

Model Results (NPV)	Deterministic model under uncertainty	Stochastic model under uncertainty
Scenario 1	$408 M ▼	$ 453 M ▲
Scenario 2	$574 M ▼	$ 599 M ▲
Scenario 3	$ 610 M ▲	$ 565 M ▼
Expected NPV	$ 530.5 M	**$ 539 M**

$ 8.5 M higher expected NPV

Novelty

- MILP two stage stochastic program with a bi-level optimization model for capacity expansion
- Considers both conflicting interest of producers and markets and also uncertainty in demands
- Allows to reduce capacity by shutdown of unprofitable plants
- Flexibility in production ratios of products

Conclusions

Stochastic model gives higher expected NPV than deterministic model

Takes decisions by considering rational markets and mitigates risks due to uncertainty in demands

Future work

Strategies to solve large problems within reasonable computational time

Multistage stochastic programming model for modeling uncertainties more accurately