Bi-level Optimization for Capacity Planning in Industrial Gas Markets

P. Garcia-Herreros, E. Arslan, P. Misra, & I.E. Grossmann

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Motivation

Industrial gas markets are dynamic:

- Suppliers must anticipate demand growth
- Most markets are served locally

Capacity expansion is a major strategic decision:

- Requires large investment cost
- Benefits are obtained over a long horizon

} Optimization

Benefits are sensitive to market behavior:

- Market preferences
- Economic environment

} Variability

Sensitivity can be reduced by assuming rational behavior:

- Producers try to maximize their profit
- Markets try to minimize their cost

} Bilevel optimization

Need to model the conflicting interests of producer and markets

Problem Statement

Given:

- Set of capacitated plants and candidate locations for new plants from leading supplier
- Set of plants from independent suppliers with limited capacity
- Rational markets that select their suppliers according to their own objective function
- Deterministic demands over the time horizon

Maximize net present value (NPV):

- Determine expansion plan
- Considering optimal distribution strategy in each time-period

Bilevel Approach (MILP)

Capacity expansion planning with rational market:

Plants are divided in two: plants from leading supplier (I^1) and plants from independent suppliers (I^2)

$$\max \; NPV = \sum_{t\in T} \frac{1}{(1+R)^t}\left\{\sum_{i\in I^1}\sum_{j\in J} P_{t,i,j}y_{t,i,j} - \sum_{i\in I^1}\left[A_{t,i}v_{t,i} + B_{t,i}w_{t,i} + E_{t,i}x_{t,i} + \sum_{j\in J}\left(E_{t,i}y_{t,i,j} + G_{t,i,j,k}y_{t,i,j}\right)\right]\right\}$$

s.t.

$$w_{t,i} = V^o_{t,i} + \sum_{t'=1}^{t} v_{t,i} \quad (\forall t\in T,\ i\in I^1) \quad \text{Invest in new plants}$$

$$x_{t,i} \le w_{t,i} \quad (\forall t\in T,\ i\in I^1) \quad \text{Expand only open plants}$$

$$c_{t,i} = C^o_{t,i} + \sum_{t'=1}^{t} H x_{t-1,i} \quad (\forall t\in T,\ i\in I^1) \quad \text{Capacity expansion}$$

$$\min \sum_{t\in T}\frac{1}{(1+r)^t}\left[\sum_{j\in J}\sum_{i\in I} P_{t,i,j}y_{t,i,j}\right]$$

s.t.

$$\sum_{j\in J} y_{t,i,j} \le c_{t,i} \quad (\forall t\in T,\ i\in I^1) \quad \text{Markets minimize cost paid}$$

$$\sum_{j\in J} y_{t,i,j} \le C_{t,i} \quad (\forall t\in T,\ i\in I^2) \quad \text{Capacity of plants from leader}$$

$$\sum_{i\in I} y_{t,i,j} = D_{t,j} \quad (\forall t\in T,\ j\in J) \quad \text{Capacity of independent plants}$$

$$c_{j,k},\ y_{s,j,i,k} \ \ge\ 0;\quad v_{t,i},\ w_{t,i},\ x_j \in \{0,1\} \quad (\forall t\in T,\ i\in I,\ j\in J) \quad \text{All markets are satisfied}$$

KKT Reformulation

Transform to single-level by using KKT conditions of lower-level problem

The optimal solution for LP:

$$\min \sum_{k=1}^{|K|} c_k x_k$$

$$\text{s.t.} \sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \qquad (\mu_i) \qquad i=1,\dots,|I|$$

$$\sum_{k=1}^{|K|} b_{k,j} x_k = b_0 \qquad (\lambda_j) \qquad j=1,\dots,|J|$$

$$x_k \in R \qquad k=1,\dots,|K|$$

Can be obtained by solving:

Stationarity:

$$c_k + \sum_{i=1}^{|I|} a_{k,i}\mu_i + \sum_{j=1}^{|J|} b_{k,j}\lambda_j = 0 \qquad k=1,\dots,|K|$$

Primal feasibility:

$$\sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \qquad i=1,\dots,|I|$$

$$\sum_{k=1}^{K} b_{k,j} x_k = b_0 \qquad j=1,\dots,|J|$$

Dual feasibility:

$$\mu_i \geq 0 \qquad i=1,\dots,|I|$$

Complementary slackness:

$$\mu_i \left(\sum_{k=1}^{K} a_{k,i} x_k - a_0 \right) = 0 \qquad i=1,\dots,|I|$$

$$x_k, \lambda_j \in R \qquad k=1,\dots,|K|;\ j=1,\dots,|J|$$

Complementary slackness can be transformed to disjunctive constraints:

$$\sum_{k=1}^{|K|} a_{k,i} x_k - a_0 - s_i = 0$$

$$[s_i = 0] \vee [\mu_i = 0]$$

→ **Use MILP reformulation**

Primal-Dual Reformulation

Transform to single-level optimization problem using strong duality:

Optimal objective value of the primal and dual formulations for LP lower-level problems is the same

Primal formulation:

$$\min \sum_{k=1}^{|K|} c_k x_k$$

$$\text{s.t.} \quad \sum_{k=1}^{|K|} a_{k,i} x_k \leq a_{0,i} \quad (\mu_i) \qquad i=1,\dots,|I|$$

$$x_k \in R \qquad k=1,\dots,|K|$$

Dual formulation:

$$\max \sum_{i=1}^{|I|} a_{0,i} \mu_i$$

$$\text{s.t.} \quad \sum_{i=1}^{|I|} a_{k,i} \mu_i = c_k \qquad k=1,\dots,|K|$$

$$\mu_i \in R^+ \qquad i=1,\dots,|I|$$

The optimal solution in the lower-level can be obtained by enforcing:

$$\sum_{k=1}^{|K|} c_k x_k = \sum_{i=1}^{|I|} a_{0,i} \mu_i$$

Strong duality

$$\sum_{k=1}^{|K|} a_{k,i} x_k \leq a_{0,i} \qquad i=1,\dots,|I|$$

Primal feasibility

$$\sum_{i=1}^{|I|} a_{k,i} \mu_i = c_k \qquad k=1,\dots,|K|$$

$$\mu_i \in R^+$$

Dual feasibility

Remark:
Upper-level variables are considered parameters for the lower-level problem. However, when transformed to single-level problem, bilinearities might appear. *Exact linearization* can be used for discrete capacities.

Garces et al, 2009

Illustrative Example

Problem structure:

- 3 existing plants which can be expanded
- 1 new candidate plant
- 3 existing plants which can not be expanded
- 15 markets with deterministic demand for 1 commodity
- 20 time-periods (quarters)

Formulations:

- Single-level (***SL***): leader selects the markets to satisfy
- Single-level evaluation (***SL-eval***): evaluation of single-level investment decisions in a market driven environment
- Bilevel KKT (***KKT***): KKT reformulation of the bilevel problem
- Bilevel Primal-Dual (***P-D***): Primal-dual reformulation of the bilevel problem

Results

Computational statistics:

Statistics	***SL***	***SL-eval***	***KKT***	***P-D***
No. of constraints:	680	520	8,460	4,380
No. of continuous variables:	2,240	2,240	6,060	4,220
No. of binary variables:	240	0	3,080	240
Solution time (CPLEX):	0.10 s	0.02 s	193 s	5.72 s
Optimality gap:	0.1%	0.1%	0.1%	0.1%

Results:

Items of objective function	***SL***	***SL-eval***	***KKT***	***P-D***
Income from sales [MM$]:	1,171	805	794	794
Investment in new plants [MM$]:	0	0	0	0
Capacity expansion cost [MM$]:	199	199	58	58
Maintenance cost[MM$]:	94	94	94	94
Production cost[MM$]:	424	292	279	279
Transportation cost[MM$]:	14	8	9	9
Total NPV [MM$]:	**440**	**212**	**354**	**354**
Market cost[MM$]:	**1,239**	**1,234**	**1,234**	**1,234**

Bilevel optimization yields **67% higher NPV (354 *vs* 212 million)** when compared to single-level expansion strategy

Conclusions

Novelty:

- MILP bi-level optimization model for capacity expansion
- Considers the conflicting interest of producers and markets
- Models market behavior according to their interests
- Alternative Primal-Dual reformulation with better computational performance

Industrial Impact:

- Allows developing capacity expansion plans that are less sensitive to changing business environments
- Determines expansion plan based on market preferences
- Reduces variability of business performance
- Avoids overestimating expansion

Future Work

- Decomposition strategy for large-scale problems
- Constraint the proportion in which different products can be produced
- Allow to reduce capacity and shut-down of plants
- Model expansion of plants from independent providers