Closure for Production Planning under Power Uncertainty Project

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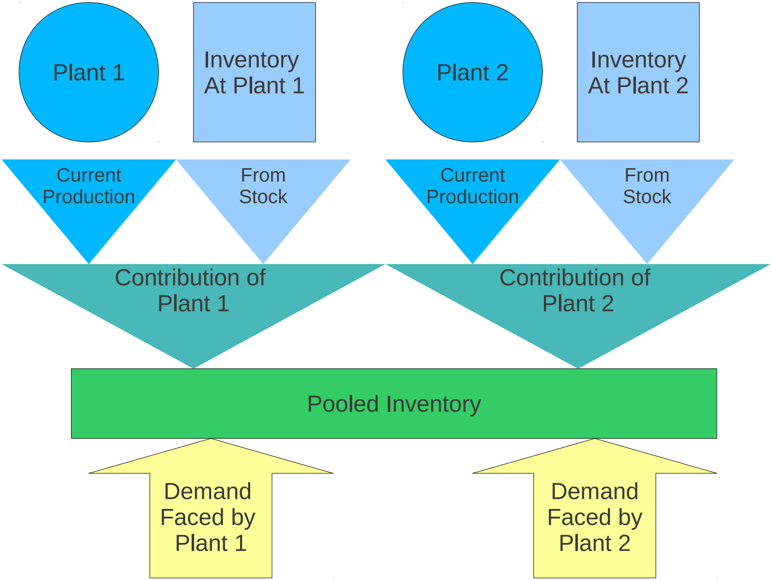
Setting

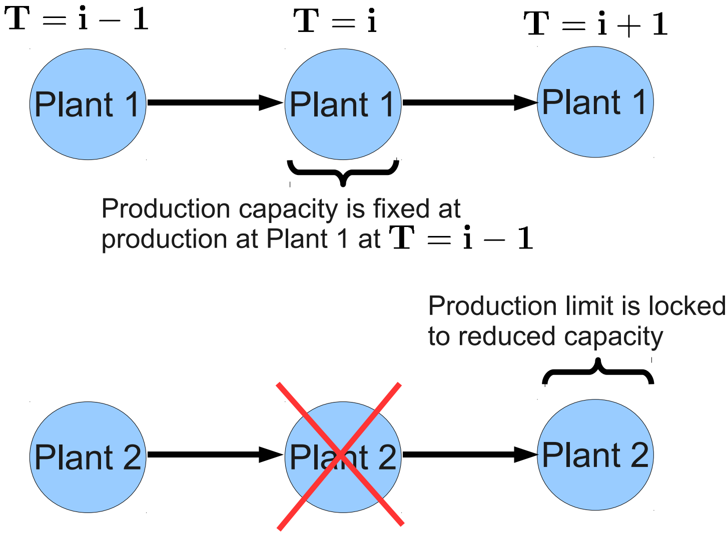
Overview

- Critical resource for production is electricity
- Limited production and storage capacity
- 2 plants in consideration
- There is one common inventory pool
- Demand is satisfied from this pool
- ILC participation:
  - maximum number of interruptions are defined.
  - times of interruption are not known.
- Interruptions affect production modes.

Setting


Plant 1
Inventory At Plant 1
Plant 2
Inventory At Plant 2
Current Production
From Stock
Current Production
From Stock
Contribution of Plant 1
Contribution of Plant 2
Pooled Inventory
Demand Faced by Plant 1
Demand Faced by Plant 2

Setting



$\mathbf{T = i - 1}$
$\mathbf{T = i}$
$\mathbf{T = i + 1}$
Plant 1
Plant 1
Plant 1
Production capacity is fixed at production at Plant 1 at $\mathbf{T = i - 1}$
Production limit is locked to reduced capacity
Plant 2
Plant 2
Plant 2

Polyhedral Uncertainty Set

Define

$$\xi_{t,p} = \begin{cases} 1 & \text{if an interruption occurs at time } t \text{ plant } p \\ 0 & \text{o.w.} \end{cases}$$

Then,

$$\mathcal{U} = \left\{ \xi_{t,p} \in \{0,1\} \ \forall (t,p) \in (T,P) \ \middle| \ \sum_{p \in P} \xi_{t,p} \leq 1 \ \forall t, \ \sum_{t \in T, p \in P} \xi_{t,p} \leq K \right\}$$

Scenarios

Each $\boldsymbol{\xi} \in \mathcal{U}$ corresponds to an interruption scenario.
There are exactly $\binom{|T|}{|K|} \times |P|^{|K|}$ interruption scenarios.

Soyster's Methodology

Approach based on Soyster ('73), "solve" the inner problem by taking its dual.

- Consider the columnwise uncertainty
- Construct inner optimization problems (uncertainty embedded)
- Take the duals and embed dual feasibility constraints to outer model
- Enforce non-negativity on dual objectives
- Dual feasibility, dual optimality and primal feasibility will be satisfied when outer problem solution is optimal.

Outer Model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t\in T}\sum_{p\in P}\sum_{g\in G} c_{t,p,g}x_{t,p,g} \\
\text{subject to} \quad & \\
& \text{[prod. constraints]} \\
\forall(t,g) \quad & \sum_{p\in P} w_{t,p,g} \geq d_{t,g} \\
\forall(t,g) \quad & \min_{\mathcal{U}}\{\sum_{p\in P} inv_{t,p,g}\} \geq 0 \\
\forall(t,p,g) \quad & x_{t,p,g}, w_{t,p,g} \geq 0
\end{aligned}
$$

Duality Gap

Need to use strong LP duality, therefore relax integrality

$$\mathcal{U}^R = \left\{ \xi_{t,p} \in [0,1] \ \forall (t,p) \in (T,P) \ \middle| \ \sum_{p \in P} \xi_{t,p} \leq 1 \ \forall t, \ \sum_{t \in T, p \in P} \xi_{t,p} \leq K \right\}$$

However in this case following holds since $\mathcal{U} \subset \mathcal{U}^R$

$$\min_{\mathcal{U}^R} \left\{ inv_{t,p,g} \right\} \leq \min_{\mathcal{U}} \left\{ inv_{t,p,g} \right\}$$

Implies over-conservatism in the inner problems, i.e. perceived severance of uncertainty is amplified!

Total Unimodularity

Consider the constraint matrix, $\mathbf{A}$, defined by $\mathcal{U}$. It has the following form:

$$\mathbf{A} = \begin{bmatrix} & \mathbf{e}^T & \\ \mathbf{I}_T & \ldots & \mathbf{I}_T \\ & \mathbf{I}_{T\times P} & \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{A}} \\ \mathbf{I}_{T\times P} \end{bmatrix}$$

where

$$\mathbf{e}^T = [1 \ldots 1]_{T\times P} \text{ and } \hat{\mathbf{A}} = \begin{bmatrix} & \mathbf{e}^T & \\ \mathbf{I}_T & \ldots & \mathbf{I}_T \end{bmatrix}$$

- $\hat{\mathbf{A}}$ is TUM by observation.
- Augmenting a matrix with $\mathbf{I}$ does retain TUM property.

A Numerical Example

Product 1

Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	0	63972	63972	185635	78337
2	0	63972	63972	136185	113422
3	0	63972	63972	27213	172944
4	63972	63972	127944	33108	122049
5	63972	63972	127944	13255	147796
6	63972	63972	127944	1154	140045
7	63972	63972	127944	0	129098

Product 2

Period	Plant 1	Plant 2	Total Production	Inventory Pool	Demand
0	-	-	-	200000	-
1	0	4544	4544	182458	22086
2	0	4544	4544	165034	21967
3	0	4544	4544	127328	42249
4	4544	4544	9087	80972	55444
5	4544	4544	9087	55594	34464
6	4544	4544	9087	26624	38057
7	4544	4544	9087	0	35711

Embedding Operational Rules

Inter-temporal impact of Interruption

An interruption in plant p at period t

- triggers a "safety-mode" at the other plant
  - Production is fixed at the previous day's production
- triggers a "recovery mode" for one period after the power is recovered
  - Production occurs at reduced rate

The New Uncertainty Set

- Requires more binary variables to be able to distinguish different modes
- The interaction between safety-mode rule and recovery-mode rule cause interesting circumstances, i.e. "interruption streaks"
- New uncertainty set is no longer TUM

Extended Model

$$
\begin{aligned}
\xi_{t,p} &= 1\text{, if plant p is interrupted at time t;}\\
\alpha_{t,p} &= 1\text{, if plant p is in safety mode at time t;}\\
\beta_{t,p} &= 1\text{, if plant p is in recovery mode at time t;}\\
\gamma_{t,p} &= 1\text{, if plant p is unaffected at time t;}\\
\delta_{t,p} &= 1\text{, if } \alpha_{t,p} \wedge \beta_{t,p} = 1\\
\eta_{t,p} &= 1\text{, if } \alpha_{t,p} \wedge \neg\beta_{t,p} = 1
\end{aligned}
$$

$$
\tilde{\mathcal{U}} = \left\{
\begin{array}{llll}
\sum_{t\in T}\sum_{p\in P}\xi_{t,p} = K & & \sum_{p\in P}\xi_{t,p} \le 1 & \forall p\\
\alpha_{1,p} = 0 & \forall p & \alpha_{t,p} - \alpha_{t-1,p} - \xi_{t,\bar{p}} \ge -1 & \forall p\ \forall t > 1\\
\xi_{1,p} - \beta_{1,\bar{p}} = 0 & \forall p & \alpha_{t,p} - \gamma_{t-1,p} - \xi_{t,\bar{p}} \ge -1 & \forall p\ \forall t > 1\\
\gamma_{1,p} - \gamma_{t,\bar{p}} = 0 & \forall p & \alpha_{t,p} + \xi_{t,p} \le 1 & \forall p\ \forall t > 1\\
\xi_{1,p} + \beta_{1,p} + \gamma_{1,p} = 1 & \forall p & \alpha_{t,p} - \xi_{t,\bar{p}} \le 0 & \forall p\ \forall t > 1\\
\alpha_{2,p} - \beta_{1,\bar{p}} - \xi_{2,p} \ge -1 & \forall p & \alpha_{t,p} + \gamma_{t,p} \le 1 & \forall p\ \forall t > 1\\
\beta_{t,p} - \xi_{t-1,p} + \xi_{t,p} \ge 0 & \forall p\ \forall t > 1 & \delta_{t,p} - \alpha_{t,p} - \beta_{t,p} \ge -1 & \forall p\ \forall t > 1\\
\beta_{t,p} - \beta_{t-1,p} - \xi_{t,\bar{p}} \ge -1 & \forall p\ \forall t > 1 & \delta_{t,p} - \alpha_{t,p} \le 0 & \forall p\ \forall t > 1\\
\beta_{t,p} + \xi_{t,p} \le 1 & \forall p\ \forall t > 1 & \delta_{t,p} - \beta_{t,p} \le 0 & \forall p\ \forall t > 1\\
\beta_{t,p} + \gamma_{t,p} \le 1 & \forall p\ \forall t > 1 & \eta_{t,p} - \alpha_{t,p} + \beta_{t,p} \ge 0 & \forall p\ \forall t > 1\\
\beta_{t,p} - \beta_{t-1,p} - \xi_{t-1,p} \le 0 & \forall p\ \forall t > 1 & \eta_{t,p} - \alpha_{t,p} \le 0 & \forall p\ \forall t > 1\\
\eta_{t,p} + \delta_{t,p} + \gamma_{t,p} + \xi_{t,p} = 1 & \forall p\ \forall t > 1 & \eta_{t,p} + \beta_{t,p} \le 1 & \forall p\ \forall t > 1\\
\\
\multicolumn{4}{c}{\xi_{t,p}, \alpha_{t,p}, \beta_{t,p}, \gamma_{t,p}, \delta_{t,p}, \eta_{t,p} \in \{0,1\} \quad \forall t, p}
\end{array}
\right\}
$$

New Duality Gap

$z^*(ROP^{new}) = 934776.3$. Now we know the optimal solution, we can observe the duality gap in individual subproblems.
Note that we can give $\mathbf{x}, \mathbf{w}$ as parameters to subproblems.
For instance consider IOP(7,1), which is

$$\min_{\tilde{\mathcal{U}}}\{\sum_{p=1}^{P} \widetilde{inv}_{7,p,1}\}$$

Solutions of the DRIOP(7,1), RIOP(7,1) and IOP(7,1) are given below

$$\{z^*(DRIOP(7,1)) = 0\} = \{z^*(RIOP(7,1)) = 0\} < \{z^*(IOP(7,1)) = 4000\}$$

If we had a TU Matrix:

$$z^*(DRIOP(t,g)) = z^*(RIOP(t,g)) = z^*(IOP(t,g)) \quad \forall t, g$$

Another Numerical Example

Consider the following scenario: plant 1 is interrupted at period 3, plant 2 is interrupted at period 4 and plant 1 is interrupted again at period 5. This would correspond to

$$\xi_{3,1} \wedge \xi_{4,2} \wedge \xi_{5,1} = 1 \implies x_{3,1,g} = x_{4,2,g} = x_{5,1,g} = 0 \;\; \forall g \in \mathcal{G}$$

Due to safety-mode rule

$$x_{3,2,g} = x_{2,2,g} \;\; \forall g \in \mathcal{G}$$

Due to recovery-mode rule

$$x_{4,1,g} = x_{1,g}^{rec} \;\; \forall g \in \mathcal{G}$$
$$x_{5,2,g} = x_{2,g}^{rec} \;\; \forall g \in \mathcal{G}$$

At optimality $z^* = 832931$.
If we had the complete interruption information at t=0 then there would be no uncertainty. Let's call this problem OP^{det}.
When we solve OP^{det}, we obtain the solution given below where $z^*(OP^{det}) = 753669$.

Folding Horizon Scheme (FHS)

- Re-solve at every time period
  - As time proceeds, newly acquired interruption information is used to refine the uncertainty set
  - The model tends to produce more in early periods so that future interruptions can be countered
- Helps us mitigate the overproduction caused by the duality gap between inner problem primal and dual

Now instead of using the robust solution obtained at t=0, we will use the FHS. The resulting production levels are given belowwhere $z^*(FHS) = 753669$ which is equal to $z^*(OP^{det})$ as expected.

New Project

- Our objective is to hedge the power against price and supply uncertainties.
- Currently studying how to construct such a power portfolio.

Questions?

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