Capacity Expansion with Independent Decision Makers

Formulation and Solution Approaches

C. Florensa, P. Garcia-Herreros, P. Misra, E. Arslan & I.E. Grossmann

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Motivation

Industrial gas markets are dynamic:

- Suppliers must anticipate demand growth
- Most markets are served locally

Capacity expansion is a major strategic decision:

- Requires large investment cost
- Benefits are obtained during a long time-horizon

} **Optimization**

Benefits are sensitive to external agents behavior:

- Markets try to minimize their cost
- All producers try to maximize their profit

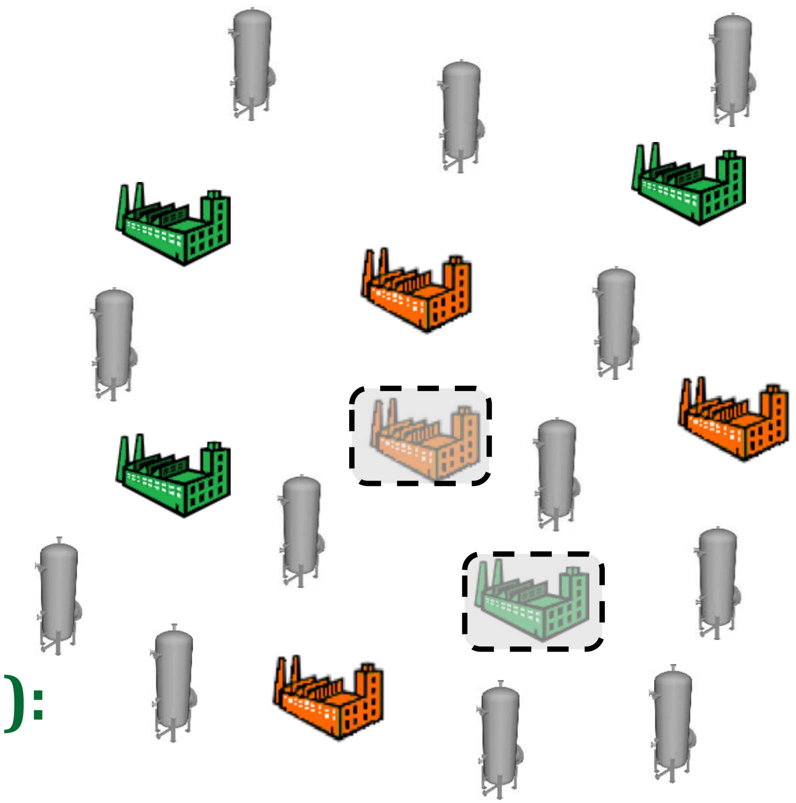
} **Multiple rational decision makers**

Need to model the conflicting interests of all producers and markets

Problem Statement

Given:

- Set of capacitated plants from two different companies
- Set of candidate locations from two different companies
- Set of markets with deterministic demands during the time-horizon
- Cost coefficients for investments, operation, and distribution



Maximize net present value (NPV_L):

- Determine expansion plan
- While considering optimal distribution in each time period
- Subject to the rational reaction of other decision makers

Previous Approaches

Capacity expansion planning: SINGLE LEVEL (Leader's profit)

Maximize: Income – New plants – Maintenance – Expansion – Production – Transportation

$$NPV_L = \sum_{t\in T} \frac{1}{(1+r)^t} \left\{ \sum_{j\in J} \rho_{t,j} \sum_{i\in I} y_{t,i,j} - \sum_{i\in I} \left[\alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j\in J} \left(\varphi_{t,i}\, y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j} \right) \right] \right\}$$

Subject to:

Open plants must be maintained $w_{t,i} = \sum_{t'=1}^{t} v_{t,i} \quad (\forall t \in T,\ i \in I^1) \quad (2)$

Expansion only in open plants $x_{t,i} \le w_{t,i} \quad (\forall t \in T,\ i \in I^1) \quad (3)$

Capacity is incremental $c_{t,i} = c_{t-1,i} + \delta x_{t-1,i} \quad (\forall t \in T,\ i \in I^1) \quad (4)$

Demand constrained by capacities $\sum_{j\in J} y_{t,i,j} \le c_{t,i} \quad (\forall t \in T,\ i \in I) \quad (5)$

All markets are satisfied $\sum_{i\in I} y_{t,i,j} = D_{t,j} \quad (\forall t \in T,\ j \in J) \quad (6)$

Bounds $c_{j,k}, y_{s,j,i,k} \ge 0;\ v_{t,i},\ w_{t,i},\ x_j \in \{0,1\} \quad (\forall t \in T,\ i \in I^1,\ j \in J)$

Impose Rational market: Replace feasibility (5)-(6) by the constraint:

⇩ BILEVEL

$$y_{t,i,j} = \operatorname{argmin} \sum_{\substack{j\in J \\ i\in I \\ t\in T}} \frac{\rho_{t,j} y_{t,i,j}}{(1+r)^t}$$

s.t. (5)-(6)

This still supposes competitor's plants $i \in I^2 = I \setminus I^1$ do not expand

Trilevel Optimization

Capacity expansion planning with rational market AND competitors

Competition's plants (I^2) expand maximizing the NPV_C subject to same constraints than (I^1)

$$\max \; NPV_L$$

s.t. (2)-(4) *(expansion constraints)*

$$\max \; NPV_C = \sum_{t \in T} \frac{1}{(1+r)^t} \left\{ \sum_{\substack{i \in I^2 \\ j \in J}} \rho_{t,j} y_{t,i,j} - \sum_{i \in I^2} \left[\alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j \in J} \left(\varphi_{t,i} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j} \right) \right] \right\}$$

s.t.

$$w_{t,i} = \sum_{t'=1} v_{t,i} \qquad (\forall t \in T,\; i \in I^2)$$

$$x_{t,i} \le w_{t,i} \qquad (\forall t \in T,\; i \in I^2)$$

$$c_{t,i} = c_{t-1,i} + \delta x_{t-1,i} \qquad (\forall t \in T,\; i \in I^2)$$

$$\min_{y_{t,j,i} \,\ge\, 0} \sum_{t \in T} \frac{1}{(1+r)^t} \left[\sum_{j \in J} \sum_{i \in I} \rho_{t,j} y_{t,i,j} \right]$$

s.t.

$$\sum_{j \in J} y_{t,i,j} \le C_{t,i} \qquad (\forall t \in T,\; i \in I)$$

$$\sum_{i \in I} y_{t,i,j} = D_{t,j} \qquad (\forall t \in T,\; j \in J)$$

$$c_j \ge 0\,; \;\; v_{t,i},\; w_{t,i},\; x_j \in \{0,1\} \qquad (\forall t \in T,\; i \in I,\; j \in J)$$

Solution Strategy

1) Optimality of the Market with duality-based reformulation

$$\sum_{t\in T}\sum_{j\in J}\sum_{i\in I}\frac{1}{(1+r)^t}\rho_{t,j}y_{t,i,j} = \sum_{t\in \mathrm{T}}\left(\sum_{j\in J} D_{t,j}\,\lambda_{t,j} - \sum_{i\in I} c_{t,i}\mu_{t,i}\right)$$

Primal = Dual

Also linearize this product of integer · continuous

$$y_{t,i,j} \geq 0,$$
$$\sum_{j\in J} y_{t,i,j} \leq C_{t,i}$$
$$\sum_{i\in I} y_{t,i,j} = D_{t,j}$$

Primal feasible

$$\lambda_{t,j} - \mu_{t,i} \leq \frac{1}{(1+r)^t}\rho_{t,i,j} \qquad (\forall t \in T,\ i \in I,\ j \in J)$$
$$\mu_{t,i} \geq 0, \qquad \lambda_{t,j} \in \mathbb{R}$$

Dual feasible

2) Integer variables in both levels: No reformulation

3125 MILPs to test the reaction of the follower for every decision of the leader… But:

A. Many leader’s decisions induce the same reaction of all followers

B. Many leader’s decisions don’t make sense: even if the competitors behave to the leader’s favor, the profit cannot beat previous studied strategies.

Generate cuts

Iteratively solve a Relaxation of the Bilevel MILP adding cuts

- We iteratively solve the High Point (ignoring the objective functions of the other players). This is a relaxation that gives an Upper Bound
- Then we add a cut eliminating all decisions of the leader inducing the same response from the other players. This response yields a Lower Bound

Extended No-good cut: **given an expansion decision *x* of the leader:**

Impose that at next iterations either:
- Expand less an open plant
- Expand more a plant at its limit

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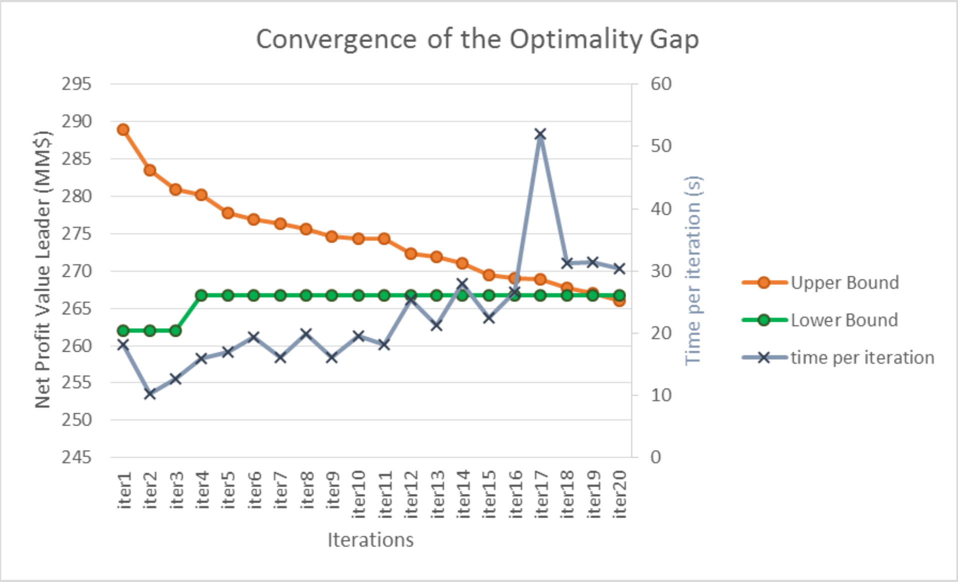
Modeled by a disjunction



Every iteration adds at most 2·|T| binary variables and 3·|T| constraints

Decomposition approach

Focus on the decisions of the competitor

- Save already observed reactions of the competitor x_C^k
- Introduce a constraint:

> *"The competitor has to do at least as good as it would with any of the already observed reactions x_C^k"*
>
> $$NPV_C(x_L, x_C, y) \geq NPV_C^k(x_L, x_C^k, y^k)$$

Impose duplicated variable y^k to be the market's decision when $x_C = x_C^k$ fixed:

$$\sum_{t \in T} \sum_{j \in J} \sum_{i \in I} \frac{1}{(1+r)^t} y_{t,i,j}^k = \sum_{t \in T} \left(\sum_{j \in J} D_{t,j} \lambda_{t,j}^k - \sum_{i \in I} c_{t,i} \mu^k(t,i) \right) \quad \textbf{P = D}$$

Primal feasible:

$$y_{t,i,j} \geq 0,$$

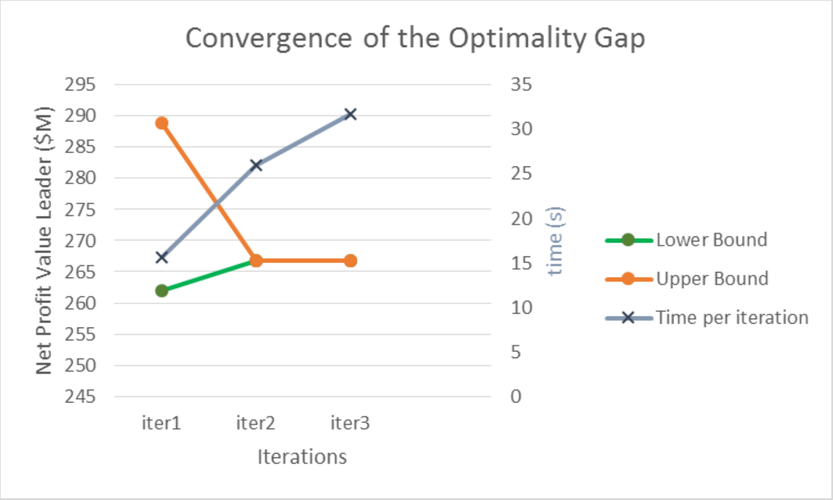
$$\sum_{j \in J} y_{t,i,j}^k \leq C_{t,i}$$

$$\sum_{i \in I} y_{t,i,j}^k = D_{t,j}$$

Dual feasible:

$$\lambda_{t,j}^k - \mu_{t,i}^k \leq \frac{1}{(1+r)^t} \rho_{t,i,j} \qquad (\forall t \in T,\ i \in I,\ j \in J)$$

$$\mu_{t,i}^k \geq 0, \qquad \lambda_{t,j} \in \mathbb{R}$$



Results

Formulations :

- **Bilevel (*BL*):** model not considering expansions of the competitor's plants
- **Trilevel (*TL*):** all players assumed to exhibit rational behavior
- **Regret (*BL-TR_eval*):** solve TL fixing decision obtained from BL model

Results:

Element of objective function	BL	BL-TR_eval	TL
Income from sales [MM$]:	772.325	706.108	692.819
Capacity expansion cost [MM$]:	55.901	55.901	29.272
Maintenance cost [MM$]:	127	94	94
Production cost [MM$]:	268	241	242
Transportation cost [MM$]:	65	53	61
Total NPV_C [MM$]:	**224**	**237**	**245**
Market cost [MM$]:	**1,336**	**1,334**	**1,334**
Total NPV_L [MM$]:	**289**	**262**	**267**

- Rational decision makers optimize own benefit → reduces leader's profit
- This framework anticipates their reactions and increases NPV_L by $5 million when compared to bilevel expansion strategy

Conclusions

Novelty:

- MILP tri-level optimization model for capacity expansion
- Considers the conflicting interest of all producers and markets
- Solution methods to quickly explore the feasible region of the leader

Future steps:

- Include uncertainty in market demands