Capacity Planning with uncertainty in Industrial Gas Markets

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Motivation

Industrial gas markets are dynamic

- Suppliers must anticipate demands
- Plan their capacity accordingly
- Improper planning may lead to new players

Benefits are sensitive to market behavior:

- Market preferences
- Economic environment

Misprediction may lead to :

- Loss of huge capital investment
- Loss of market share

Need a model to mitigate losses due to uncertainties

Problem Statement

Maximize net present value (NPV):

- Determine Optimal Capacity Planning strategy

Given:

- Set of capacitated plants and candidate locations for new plant from leading supplier
- Set of plants from independent suppliers with limited capacity
- Rational markets that select their suppliers according to their own objective function
- Different demands scenarios that can occur over the time horizon
- Probabilities for each demand scenario to occur

Bilevel capacity planning under uncertainty

$$max \sum_{t\in T}\left(\frac{1}{(1+R)^t}\right)\left[\left\{\sum_{i\in I^1}\sum_{k\in K}\sum_{s\in S}\left(\sum_{j\in J\setminus\{J'\}}(P_{t,i,j,k}-G_{t,i,j,k})-\sum_{j\in J}F_{t,i,k}\right)Pr_s y_{s,t,i,j,k}\right\}-\left\{\sum_{i\in i^1}(A_{t,i}v_{t,i}+B_{t,i}w_{t,i}+E_{t,i}x_{t,i})\right\}\right]$$

$w_{t,i} = V^o_{t,i} + \sum_{t'=1}^{t} v_{t,i} - \sum_{t'=1}^{t} z_{t,i}$ $(\forall t \in T, i \in I^1)$ Invest or divest in plants

$x_{t,i} \leq w_{t,i}$ $(\forall t \in T, i \in I^1)$ Expand only open plants

$c_{t,i} = C^o_{t,i} + \sum_{t'=1}^{t} H x_{t,i}$ $(\forall t \in T, i \in I^1)$ Capacity expansion

$\min \sum_{t\in T}\sum_{i\in I^1\cup I^2\cup I^3}\sum_{j\in J\setminus\{J'\}}\sum_{k\in K}\frac{1}{(1+R)^t}P_{t,i,j,k}\, y_{s,t,i,j,k}$ $(\forall s\in S)$ Markets minimize cost paid

$\sum_{j\in J}\sum_{k\in K} y_{s,t,i,j,k} \leq M w_{t,i}$ $(\forall s\in S, t\in T, i\in I^1)$ No sales if plant is closed

$\sum_{j\in J}\sum_{k\in K} y_{s,t,i,j,k} \leq c_{t,i}$ $(\forall s \in S, t \in T, i \in I^1)$ Capacity of plants from leader

$\sum_{j\in J}\sum_{k\in K} y_{s,t,i,j,k} \leq C_{0,i}$ $(\forall s \in S, t \in T, i \in I^2)$ Capacity of independent plants

$-\sum_{j\in J} y_{s,t,i,j,k} \leq -Q^{low}_{i,k}\sum_{j\in J}\sum_{k'\in K} y_{s,t,i,j,k'}$ $(\forall s \in S, t \in T, i \in I, k' \in K)$ Upper proportion for product

$\sum_{j\in J} y_{s,t,i,j,k} \leq Q^{up}_{i,k}\sum_{j\in J}\sum_{k'\in K} y_{s,t,i,j,k'}$ $(\forall s \in S, t \in T, i \in I, k' \in K)$ Lower proportion for the product

$\sum_{i\in I^1\cup I^2\cup I^3} y_{s,t,i,j,k} = D_{s,t,j,k}$ $(\forall s \in S, t \in T, j \in J\setminus\{J'\}, k \in K)$ All markets are satisfied

$c_{t,i}, y_{s,t,j,i,k} \geq 0$; $x_{t,i}, v_{t,i}, z_{t,i}\ w_{t,i} \in \{0,1\}$

Dual Reformulation

- Bi-level is transformed to single level by Dual reformulation
- Lower level LP is equated to its corresponding dual using strong duality for each scenario
- Primal and dual feasibility conditions are added

Primal formulation:

$$\min \sum_{k=1}^{K} c_k x_k$$

$$\text{s.t.} \sum_{k=1}^{K} a_{k,i} x_k \geq a_{0,i} \; (\mu_i) \qquad i=1\ldots I, \; k=1\ldots K$$

$$x_k \in R$$

=

Dual formulation:

$$\max \sum_{i=1}^{I} a_{0,i} \mu_i$$

$$\text{s.t.} \sum_{i=1}^{I} a_{k,i} \mu_i \leq c_k \qquad k=1\ldots K, \; i=1\ldots I$$

$$\mu_i \in R^+$$

- Bilinear terms arising from multiplication of upper level variables are linearized using Glovers linearization . These variables can be treated as parameters for lower level

Benders decomposition

- Single level problem can be decomposed by benders decomposition
- Facilitates computationally for large sized problem

Original problem :

$$\min_{x,y_s} c^T x + \sum_{s\in S} p_s q_s^T y_s$$

$$Ax \le b \quad x \ge 0$$

$$W y_s = h_s - T_s x \quad y_s \ge 0$$

Master Problem:

$$\min_{x,\theta} c^T x + \theta$$

$$\theta \ge d^{iter} x + e^{iter}$$
$$iter = 1,2 \dots N$$

$$Ax \le b \quad x \ge 0$$

Sub problem:

$$\min_{y_s} q_s^T y_s$$

$$W y_s = h_s - T_s x \qquad y_s \ge 0$$

$$d^{iter} = \sum_{s\in S} p_s \pi_{iter,s}^T T_s$$

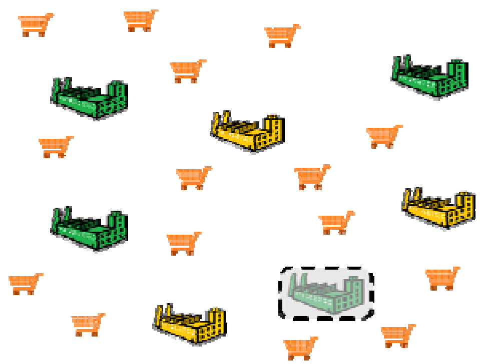
$$e^{iter} = \sum_{s\in S} p_s \pi_{iter,s}^T h_s$$

π_s *are optimal dual vectors of constraints*

- Planning decisions are made in master problem and sales decisions are made in the subproblem
- Master problem and subproblem are solved iteratively by adding optimality cuts to master problem till convergence

Illustrative example

Problem structure:



- 2 Existing plants from leading provider
- 1 New candidate plant
- 1 Existing plants from competitors
- 8 Markets
- 30 time periods (Quarters)
- Planning decisions allowed for every 4 periods
- Composition of product-1 is 15-25% & product-2 is 75-85%
- 3 Demand scenarios are considered
- Each scenario has equal probability of occurrence

Results

Model Statistics:	Deterministic model	Stochastic model
Number of continuous variables	4708	13638
Number of discrete variables	146	146
Number of constraints	4221	13633
Solution time (sec)	84	113

Model Results :	Deterministic model under uncertainty	Stochastic model under uncertainty
NPV (MM$)	349.22	358.24
Income from sales (MM$)	1101.83	1007.83
Investment in new facilities (MM$)	-	-
Expansion costs (MM$)	-	-
Production costs (MM$)	531.07	467.06
Transportation costs (MM$)	119.60	150.07
Maintenance costs (MM$)	31.61	71.26

Analysis: Stochastic model gives 9 MM $ higher expected Net present value compared to deterministic model

Novelty

- MILP two stage stochastic program with a bi-level optimization model for capacity expansion
- Considers both conflicting interest of producers and markets, and also uncertainty in demands
- Allows to reduce capacity by shutdown of unprofitable plants
- Flexibility in production ratios of products

Future Work

- Three stage stochastic programming model to model uncertainties more accurately
- Include different expansion levels to which capacity of existing facilities of leader can be increased
- Include expansion strategy of competitor in the formulation