Robust Optimization Approach for Industrial Gases Supply Chains under Demand Uncertainty

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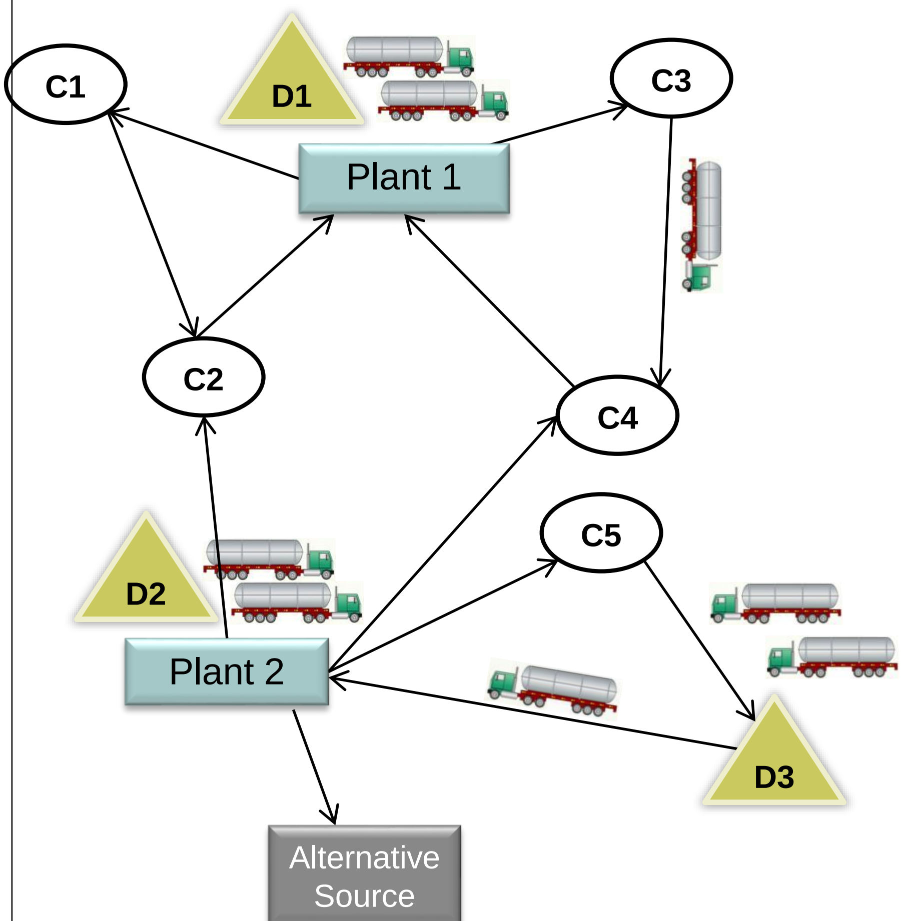
Problem Statement and Main Assumptions



Given

- Plants, Products, Operating Modes and Production Limits
- Daily Electricity Prices (off-peak and peak)
- Customers and their demand/consumption profiles
- Max/Min inventory at production sites
- Alternative sources and product availabilities
- Fixed Planning Horizon (usually 1-2 weeks)
- Flexible customer demands (initial inventory levels and tank capacities)

Decisions in each time period t

- **Modes and production rates at each plant**
- **Inventory level at plants**
- **Customer inventory management**
- **How much product to be delivered to each customer through which route**

Objective Function

- **Minimize total production and distribution cost over planning horizon**

Main Assumptions – Distribution Side

- Two time periods per day (peak and off-peak) are considered
- Trucks do not visit more than 4 customers in a single delivery
- Depots, Truck availabilities and capacities, Distances

Problem Statement

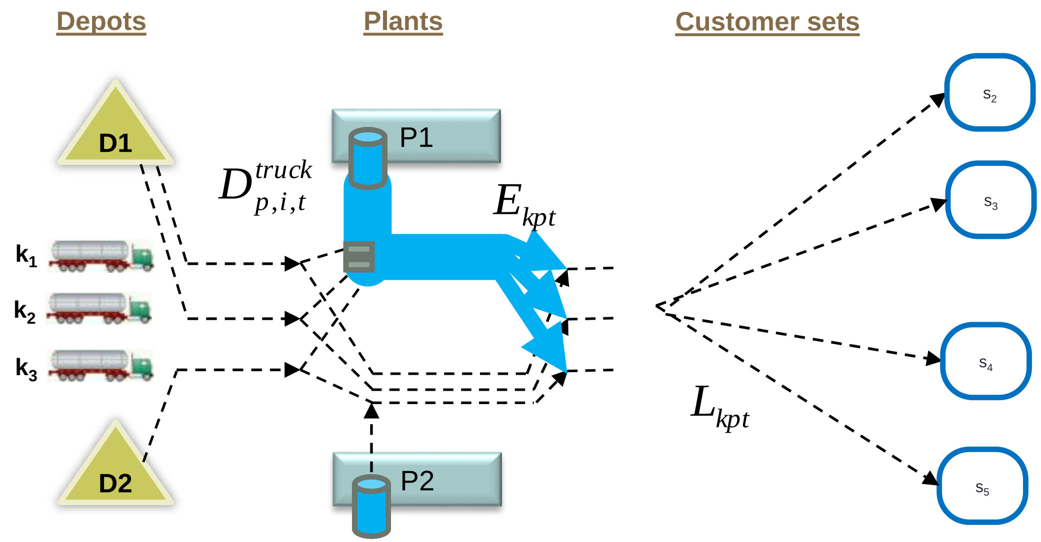
Detailed Distribution constraints

Detailed Production

Main binary variables

mode selection

$$B_{pmt}$$



Decision making

Production:

- Production mode
- Production load
- Inventory management

Link:

- Plant withdrawal
- Material Balances

Distribution:

- Truck assignment
- Truck load
- Route selection
- Customer replenishment
- Customer inventory levels

Sequential vs Simultaneous approach

Sequential approach

Distribution

$Dcust = Dem\ Forecast$

$E(p,i,t) = Dcust$

$TCost = Distr\ cost$

Fixed $\boldsymbol{E(p,i,t)}$

Production

$Inv = Prod - E(p,i,t)$

$TCost = Prod\ cost$

= Total Cost

$TCost = Prod\ cost + Distr\ cost$

Link:

- Plant withdrawal (E(p,i,t))
- Plant Inventory (Inv: Production - distribution)

Simultaneous approach *(Marchetti et al, 2013)*

- Coordinated production and distribution management

Distribution + Production = Total Cost

$Distribution = Demand\ Forecast$

$E(p,i,t) = Distribution$

$Inv = Production - E(p,i,t)$

$TCost = Prod\ cost + Distr\ cost$

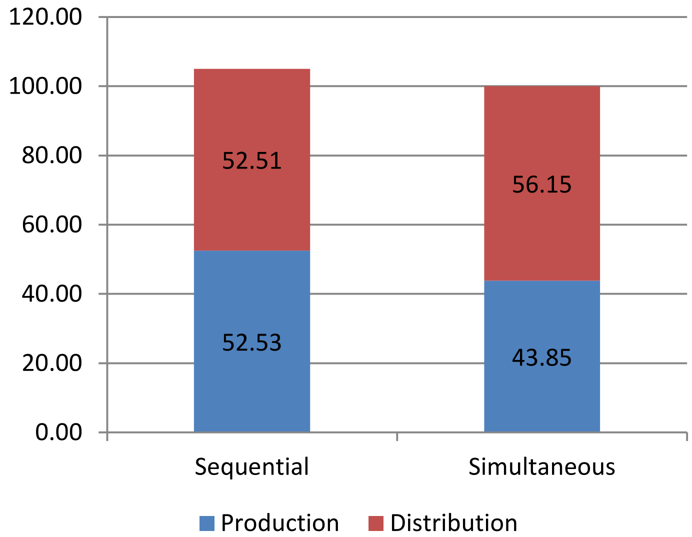
Production side constraints

- mode_selection
- same_production_mode
- start_up_detection_t0
- start_up_detection
- plant_inventory_t0
- plant_inventory
- Production plant adhoc models

Distribution side constraints

- one_route_per_truck
- truck_source
- product_grade_constraint
- plant_withdrawal
- plant_withdrawal_limit
- purchase_limit
- alternative_source_delivery
- alternative_source_bound
- default_source_bound
- truck_capacity
- added_distance_source
- distribution_balance_customers
- customer_accumulated_deliverie s
- force_deliveries_customer
- shutdown_mode_plant_delivery

Sequential vs Simultaneous approach



		Sequential Model		Simultaneous Model
		Production	Distribution	Prod-Dist
Model Size	Binary variables	168	26,374	26,486
	Continuous variables	1078	69,300	69,902
	Constraints	836	34,398	35,233
	Normalized cost	52,53	52,51	**100**
		105.03		
CPU results	Time	**0.172**	**952**	**41,511s**
	Nodes	**-**	**-**	**104,880**
	Relative gap	**2%**	**9.6%**	**4.7%**

Deterministic models

- Decisions for the future (ignoring uncertainty)
- In reality we are almost never certain about the parameters/data of the problem

Robust Optimization approach

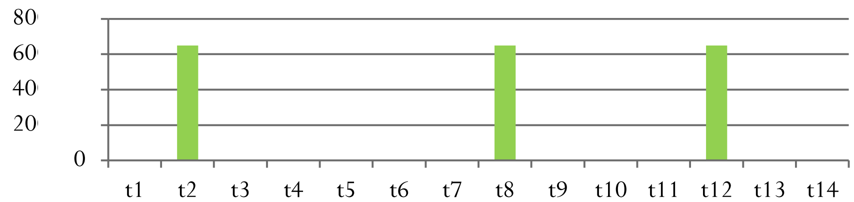
- Robust counterpart
- Robust solutions are feasible against all the uncertainty set
- Here and now decisions
  - Short time problems recursive actions are expensive or infeasible

Goal:
Develop a **realistic** customer inventory management approach able to capture customer demand uncertainties for the production distribution coordination of industrial gases supply chains.

Customer Inventory Management (CIM)

PDC model currently considers a simplified customer inventory management.

- Planned deliveries:
  - Fixed demand within a time window
  - i.e. customer 10: 650,000 (t1-t2, t7-t8, t11-t12)



Customer 10 (planned deliveries)

- Advantages:
  - Size & complexity of the model representation
  - Realistic approach for the "deterministic case"
- Limitations:
  - Simplified customer inventory management
  - Lack of reality when the customer demands are affected by the uncertainty

Detailed inventory management

- New variables
  - Inventory level ($\boldsymbol{Vo_{u,t}}$)
  - Replenishment ($p_{u,t}$)
- New parameters
  - Initial Inventory ($\boldsymbol{Vo_{u,t=0}}$)
  - Consumption rate ($r_{u,t}$)

Customer inventory management (CIM)

Cont. Vars.: Vo – volume at customer tanks, p – replenishment,
Parameter: r – consumption rate

- Inventory management (typical approach)

$$Vo_{u,t} = Iinv_{u,t=1} + Vo_{u,t-1} + p_{u,t} - r_{u,t} \quad \forall u,t$$

Mass Balances (inventory at the end of time t)

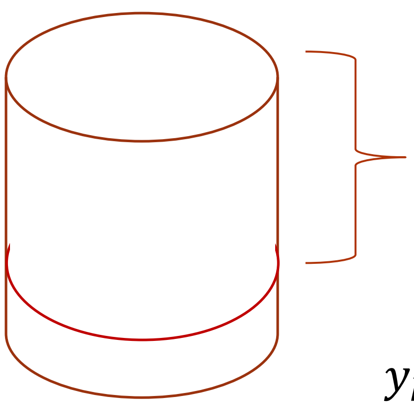
$$VI_u \leq Vo_{u,t} \quad \forall u,t$$

Minimum inventory level

$$Vo_{u,t} + p_{u,t} \leq Vu_{u,t} \quad \forall u,t$$

Maximum inventory level

$Vu_{u,t}$



$VI_{u,t}$

$p_{u,t}$ (continuous variable, scf)

$p_{u,t} \geq$ 80 % (V upper bound – V lower bound)

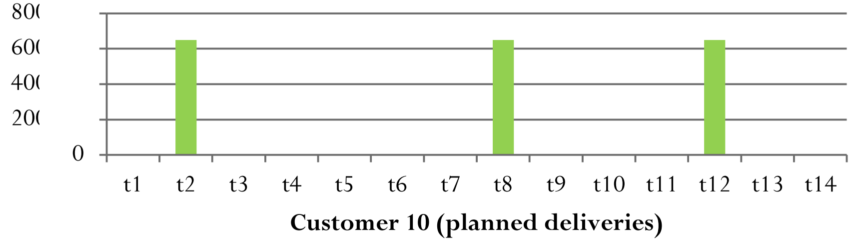
100 % (V upper bound – V lower bound) $\geq p_{u,t}$

$$y_{k,u,t} \cdot \left(Vu_{u,t} - Vl_{u,t}\right) \geq p_{u,t} \geq \beta \cdot \left(Vu_{u,t} - Vl_{u,t}\right) \cdot y_{k,u,t} \ \forall u,t,k$$

Fengqi You, Jose M. Pinto, Elisabet Capon, Ignacio E. Grossmann, Nikhil Arora, and Larry Megan 2011, Industrial & Engineering Chemistry Research

CIM formulation

- Demand (planned deliveries)
  - Dcust = 650,000 (t2, t8, t12)

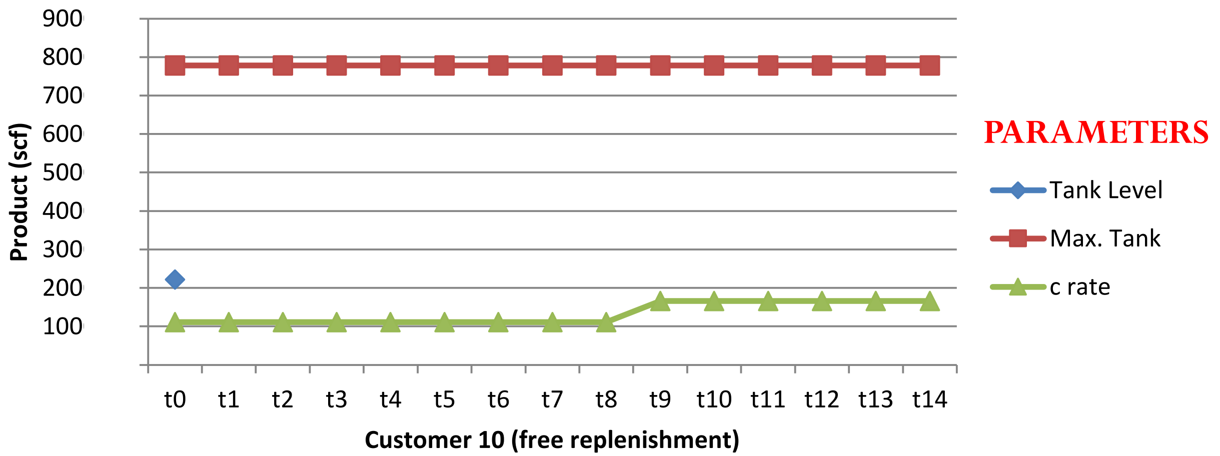


- Detailed inventory management
  - Initial Inventory ($Vo_{u,t=0}$)
  - Replenishment ($p_{u,t}$)
  - Consumption rate ($r_{u,t}$)

$$Vo_{u,t} = Vo_{u,t-1} + p_{u,t} - r_{u,t} \quad \forall u,t$$

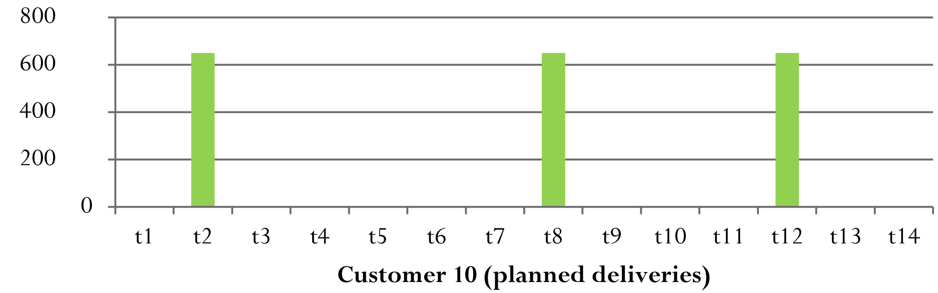
$$VI_u \leq Vo_{u,t} \quad \forall u,t$$ Minimum inventory level

$$Vo_{u,t} + p_{u,t} \leq Vu_{u,t} \quad \forall u,t$$ Maximum inventory level



Planned deliveries vs customer inventory management

- Demand (planned deliveries)
  - Dcust = 650,000 (t2, t8, t12)

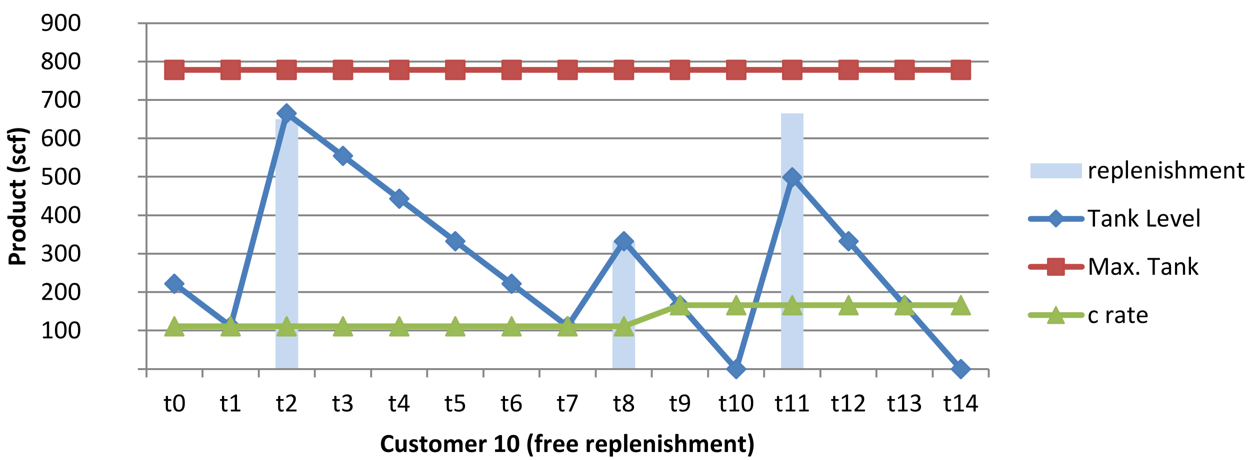


- Detailed inventory management
  - Initial Inventory
  - Replenishment ($p_{u,t}$)
  - Consumption rate ($r_{u,t}$)

$$Vo_{u,t} = Vo_{u,t-1} + p_{u,t} - r_{u,t} \quad \forall u, t$$

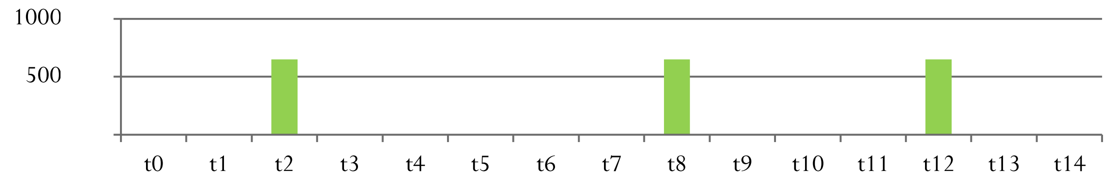
$$VI_u \leq Vo_{u,t} \quad \forall u, t$$ Minimum inventory level

$$Vo_{u,t} + p_{u,t} \leq Vu_{u,t} \quad \forall u, t$$ Maximum inventory level

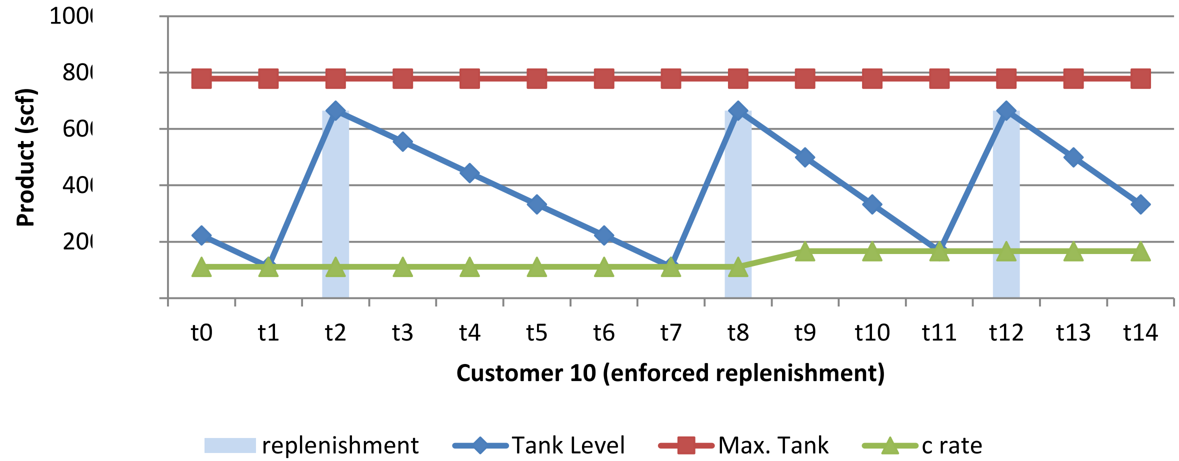


Planned deliveries vs inventory management

- Deterministic Demand
  - Dcust = 650,000 (t2, t8, t12)



- Enforced Replenishment
  - $p_{u,t} \geq 100\ \% \ (Vu - Vl)$



$$y_{k,u,t} \cdot \left(Vu_{u,t} - Vl_{u,t}\right) \geq p_{u,t} \geq \beta \cdot \left(Vu_{u,t} - Vl_{u,t}\right) \cdot y_{k,u,t} \ \forall u, t, k$$

Remarks

Planned deliveries

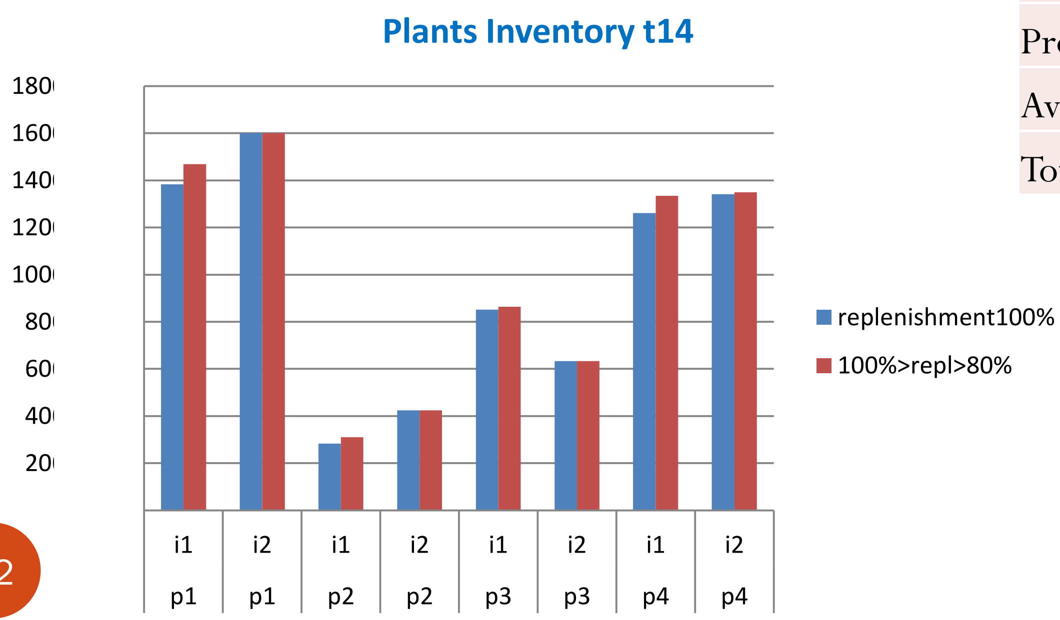
- Simplified inventory management problem

Detailed Inventory Management

- Flexible replenishment
- Enforced replenishment
  - More robust
  - 100 % ≥ p(u,t) ≥ 80 % max capacity of the tank

Replenishment	100 %	≥ 80%
Production cost	52	52
Distribution cost	21	19
Shutdown cost	17	17
Subcontracting cost		0
Total	91	89

Prate	360	360
Product Distributed	2110	**1902**
Average Filling ratio	**0.84**	0.78
Total trucks used	39	**36**



Robust Optimization Approach

- Applying RO tools developed by Bertsimas and Sim (2004).
- General framework of Linear RO by Bertsimas and Thiele (2006)
  - Derived closed form-expressions to consider budgets of uncertainty
    - Examples: single station inventory problem, and network case with demand uncertainty .
  - Zhang et al. (2014) applied budgets of uncertainty for reserve market of electricity production

Consider the following problem:

- Subject to data uncertainty

$$minimize\ c'x$$

$$\text{s.t.} \quad Ax \leq b$$

$$l \leq x \leq u$$

Interval data uncertainty

- Each uncertain coefficient

$$a_{i,j} \in [\bar{a}_{i,j} - \hat{a}_{i,j}, \bar{a}_{i,j} + \hat{a}_{i,j}]$$

- $\bar{a}_{i,j}$ Nominal value
- $\hat{a}_{i,j}$ Deviation from its nominal value
- Scaled deviation from its nominal value $w_{i,j} = (a_{i,j} - \bar{a}_{i,j})/\hat{a}_{i,j}$ with values in [-1,1]
- Budget uncertainty, why budget? R: the total variation (scaled) cannot exceed some threshold $\sum_{(i,j)\in J} |w_{ij}| \leq \Gamma$

Why to apply budget uncertainty?

- Coefficients equal to their nominal value (**unlikely**)
- Coefficients at their worst-case value (**unlikely**)

There is a reasonable **tradeoff** between robustness and performance

$\Gamma = 0$, nominal case

$\Gamma = |J|$ worst case

$0 < \Gamma < |J|$ provides flexibility

Robust Approach

- The robust problem is then formulated as:

$$\text{minimize } c'x$$
$$s.t. \; Ax \leq b \qquad \forall A \in U$$
$$l \leq x \leq u$$

Uncertainty set:

$$U = \left\{ U \in R^{mxn}, \; a_{ij} \in [\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}] \; \forall \, i, j, \sum_{j \in J} |w_{ij}| \leq \Gamma \right\}$$

$$w_{i,j} = (a_{i,j} - \bar{a}_{i,j}) / \hat{a}_{i,j}$$

To ensure robustness requires for ith constraint to

$$max_{A \in \mathcal{A}} \sum_j a_{ij} x_j \leq b_j$$

Robust Counterpart (RC) for all i constraint:

- Inner problem (or auxiliary problem)

$$\text{maximize } \sum_j (\bar{a}_{ij} - w_{i,j} \hat{a}_{ij}) \, x_j$$

$$\text{s.t.} \quad \textstyle\sum_{j \in J} w_{ij} \leq \Gamma$$

$$0 \leq w_{ij} \leq 1$$

- Applying duality theory (strong duality)

$$\text{minimize } \; q_i \Gamma + \textstyle\sum_{i \in J_i} s_{ij}$$

$$\text{s.t. } q_i + s_{ij} \geq \hat{a}_{ij} \; \forall j \in J$$

$$q_i \geq 0, s_{ij} \geq 0 \; \forall j \in J$$

$$\text{minimize } c'x$$

$$\text{s.t. } \textstyle\sum_j \bar{a}_{ij} X_J + q_i \Gamma + \sum_{i \in J_i} s_{ij} \leq b_i \qquad \forall i$$

$$q_i + s_{ij} \geq \hat{a}_{ij} y_i \qquad \forall j \in J$$

$$l \leq x \leq u \qquad -y \leq x \leq y$$

$$q \geq 0, s \geq 0, y \geq 0$$

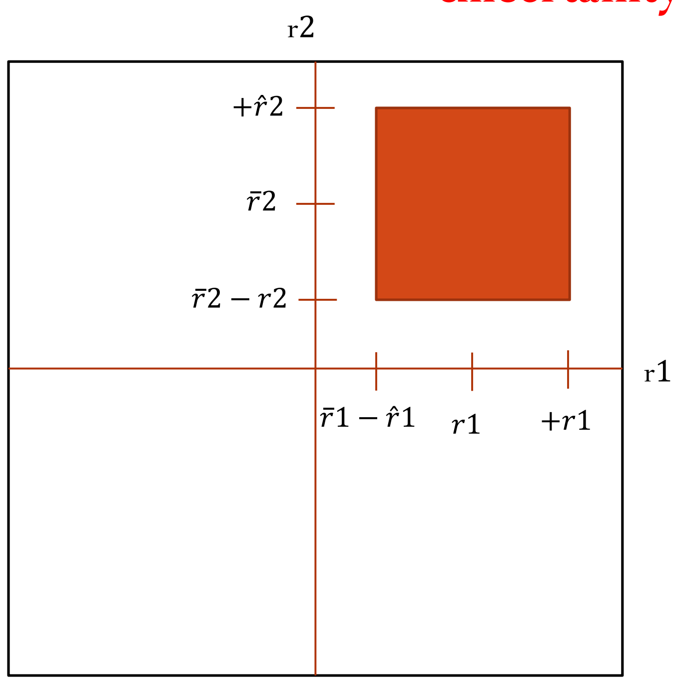
*If b and c are uncertain we can rewrite the problem to represent the uncertainty as part of A

Uncertainty Set (A)

- Inv. Management approach considers fixed **customer demand** forecast
  - Customers with fixed demands over the time horizon
  - i.e. customer one (u1) has 2 demands: (r1 and r2, in time period 1 and 2)
  - Lets consider closed box uncertainty (or a deviation from its nominal value)

$$r_{i,j} \in \left[\bar{r}_{i,j} - \hat{r}_{i,j}, \bar{r}_{i,j} + \hat{r}_{i,j}\right]$$

Uncertainty set U



$\bar{D}$ nominal value $\hat{D}$ deviation

- RO – Robustifies the solution
  - Optimal solution provides a feasible solution for all the uncertainty set

Why to apply budget uncertainty?

- Coefficients equal to their nominal value (unlikely)

Uncertain Set

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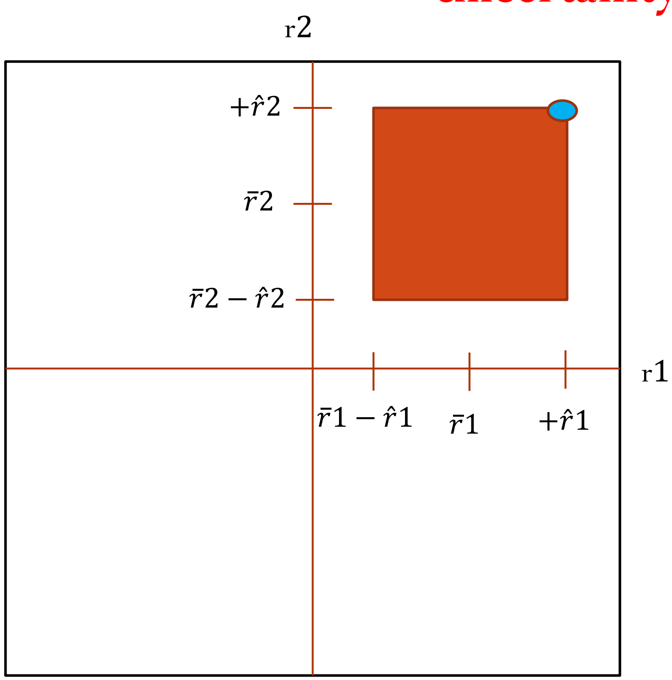
Uncertainty set U



$\overline{D}$ nominal value $\widehat{D}$ deviation

- RO – Robustifies the solution
  - Optimal solution provides a feasible solution for all the uncertainty set
  - **Budgets of uncertainty**

w1=1, w2=1

$\Gamma = 2$

Worst case

$$maximize \sum_{k=1}^{t} \hat{r}_k \ w_k$$

s.t.

$$\sum_{k}^{t} w_k \leq \Gamma \qquad \forall k \leq t$$

$$0 \leq w_k \leq 1$$

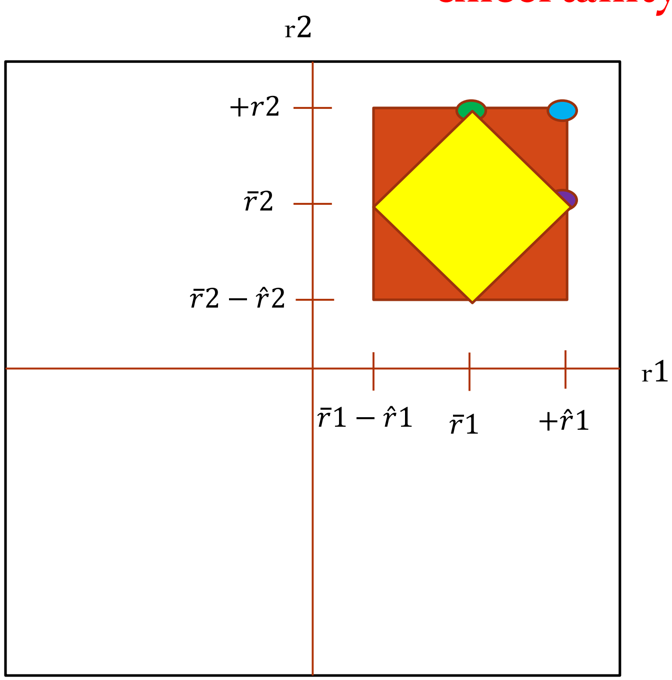
Why to apply budget uncertainty?

- Coefficients equal to their nominal value (unlikely)
- Coefficients at their worst-case value (unlikely)

Uncertain Set

- Inv. Management approach considers fixed **customer demand** forecast
  - Customers with fixed demands over the time horizon
  - i.e. customer one (u1) has 2 demands: (r1 and r2, in time period 1 and 2 respectively)
  - Lets consider closed box uncertainty (or a deviation from its nominal value)

Uncertainty set *U*



$\overline{D}$ nominal value $\widehat{D}$ deviation

- RO – Robustifies the solution
  - Optimal solution provides a feasible solution for all the uncertainty set
  - **Budgets of uncertainty**

Γ= 2: **w1=1, w2=1**

Γ= 1: **w1=1, w2=0**

Γ= 1: **w1=0, w2=1**

This budgets of uncertainty rule out some demand deviations, and can be seen as **"reasonable worst-case approach "**

Robust approach

- Inventory management problem:
  - Inventory at customer tanks over the time is described:

$$Vo_{u,t} = Iinv_{u,t=1} + Vo_{u,t-1} + p_{u,t} - r_{u,t} \quad \forall u, t$$

- Derive Closed form expression (cumulative representation)

$$Vo_{u,t} = Iinv_u + \sum_{k=0}^{t} (p_{u,k} - r_{u,k}) \quad \forall u, t$$

	Original	Closed Form
Production	52	52
Distribution	17	17
Shutdown	17	17
Sub	0	0
Total	87	87

Robust approach

- Closed form expression

$$Vo_{u,t} = Iinv_u + \sum_{k=0}^{t} (p_{u,k} - r_{u,k}) \quad \forall u, t$$

Robust Counterpart

- Closed box uncertainty

$$r_{u,k} = [\bar{r}_{uk} - \hat{r}_{uk}, \bar{r}_{uk} + \hat{r}_{uk}]$$

$$Vo_{u,t} = Iinv_u + \sum_{k}^{t} (p_{u,k} - \bar{r}_{u,k} + \hat{r}_{u,k} w_{u,k}) \quad \forall u, t$$

$$\text{maximize} \quad \sum_{k=1}^{t} \hat{r}_{u,k} \; w_{u,k}$$

$$\text{s.t.} \quad \sum_{k}^{t} w_{u,k} \leq \Gamma_{u,k} \quad \forall k \leq t$$

$$0 \leq w_{u,k} \leq 1$$

$$\text{minimize} \;\; q_t \Gamma + \textstyle\sum_{k=1} s_{tk}$$

$$\text{s.t. } q_t + s_{tk} \geq \hat{D}_t \; \forall k \leq t$$

$$q_t \geq 0, s_{tk} \quad \geq 0 \; \forall k \leq t$$

$$Vo_{u,t} = Iinv_u + \sum_{k=0}^{t} (p_{u,k} - \bar{r}_{u,k}) + q_t \Gamma_{u,t} + \sum_{k=1}^{t} S_{k,t} \quad \forall u, t$$

$$\text{s.t. } q_t + s_{tk} \geq \hat{D}_t \; \forall k \leq t \qquad q_t \geq 0, s_{tk} \quad \geq 0 \; \forall k \leq t$$

Robust approach

- Equality constraints: at optimality are tight, avoid equality constraints.

$$Vo_{u,t} = Iinv_u + \sum_{k=0}^{t} (p_{u,k} - \bar{r}_{u,k}) + q_t \Gamma_{u,t} + \sum_{k=1}^{t} S_{k,t} \quad \forall u, t$$

s.t. $q_t + s_{tk} \geq \widehat{D}_t \ \forall t, k \leq t$ $\qquad q_t \geq 0, s_{tk} \geq 0 \ \forall t, k \leq t$

Inequality constraints

- Two sided inequalities

$$Vo_{u,t} \leq Iinv_u + \sum_{k=0}^{t} (p_{u,k} - \bar{r}_{u,k}) + q_t \Gamma_{u,t} + \sum_{k=1}^{t} S_{k,t} \quad \forall u, t$$

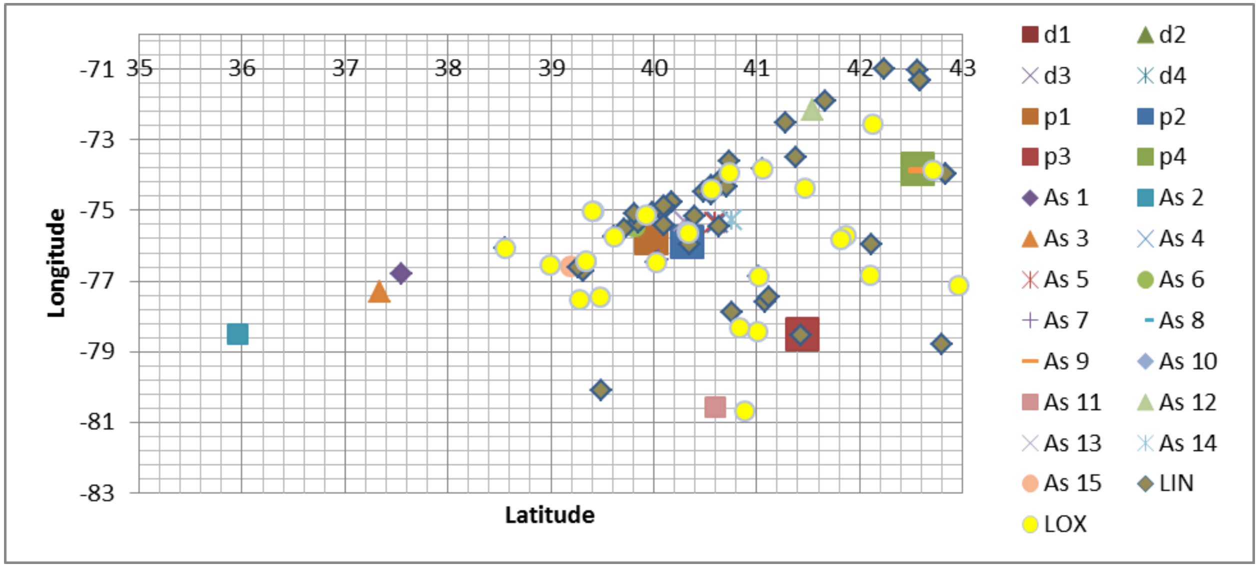
$$Vo_{u,t} \geq Iinv_u + \sum_{k=0}^{t} (p_{u,k} - \bar{r}_{u,k}) + q_t \Gamma_{u,t} + \sum_{k=1}^{t} S_{k,t} \quad \forall u, t$$

s.t. $q_t + s_{tk} \geq \widehat{D}_t \ \forall k \leq t$

$$q_t \geq 0, s_{tk} \quad \geq 0 \ \forall k \leq t$$

Case study 1

- Small industrial case study
  - provide optimality
  - compare the quality of the solutions by the rolling horizon approach
  - Industrial example 205 customer demands → 40 customer demands
  - 4 plants, 11 alternative sources



- Uncertainty set $r_{u,t}$ = customer demand (**consumption rate**)
  - Deviation 5% →

$$r_{u,k} = [\bar{r}_{uk} - \hat{r}_{uk}, \bar{r}_{uk} + \hat{r}_{uk}]$$

Results

- PDC Original (Nominal value)
- Closed box
  - Uncertain customer consumption rate (± 5 %)

$$r_{u,k} = [\bar{r}_{uk} - \hat{r}_{uk}, \bar{r}_{uk} + \hat{r}_{uk}]$$

Customer demand		Robust optimization approach (Budgets of uncertainty)				
	Original	Γ=0	Γ=1	Γ=3	Γ=5	Γ=14
Production cost	52,2	52,6	52,6	52,6	52,6	52,603
Distribution cost	19,1	19,3	19,3	19,3	19,3	19,398
Shutdown cost	17,8	17,8	17,8	17,8	17,8	17,893
Subcontracting cost	0	0	0	0	0	0
Total Cost	89.30	89.80	89.80	89.80	89.83	89.89
Prate	36	36	36	36	36	36
Product Distributed	19,133	19,274	19,726	19,724	19,734	20,025
Average Filling ratio	0.82	0.81	0.76	0.79	0.80	0.80

Normalized values

Conclusions

Proposed framework provides **optimal** production and distribution coordination under demand uncertainty

- Robust optimization framework provides here and now decisions for problems with limited recursive actions.
- Robust optimal production and distribution plans over the planning horizon
- Budgets of uncertainty rule out some demand deviations
  - "reasonable worst-case approach"
  - A reasonable tradeoff between robustness and performance is considered
- Customer inventory management provides flexibility to the production planning.
  - Enforced and flexible replenishment has been considered