Decomposition Approaches for Optimal Production – Distribution Coordination of Industrial Gases Supply Chains

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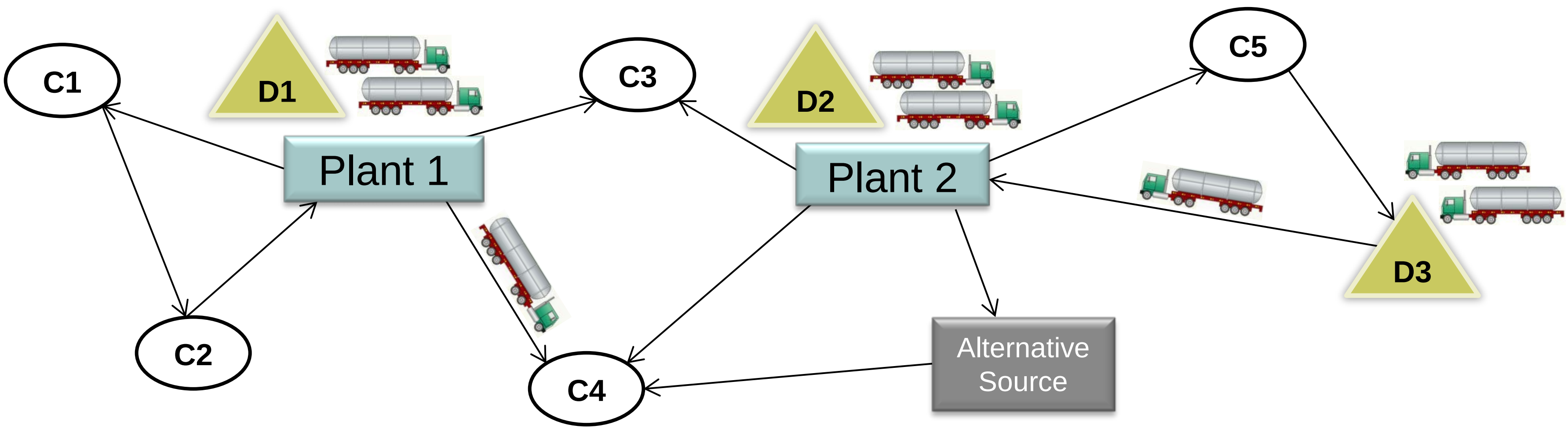
Background and Motivation

Industial Gases Supply-Chain

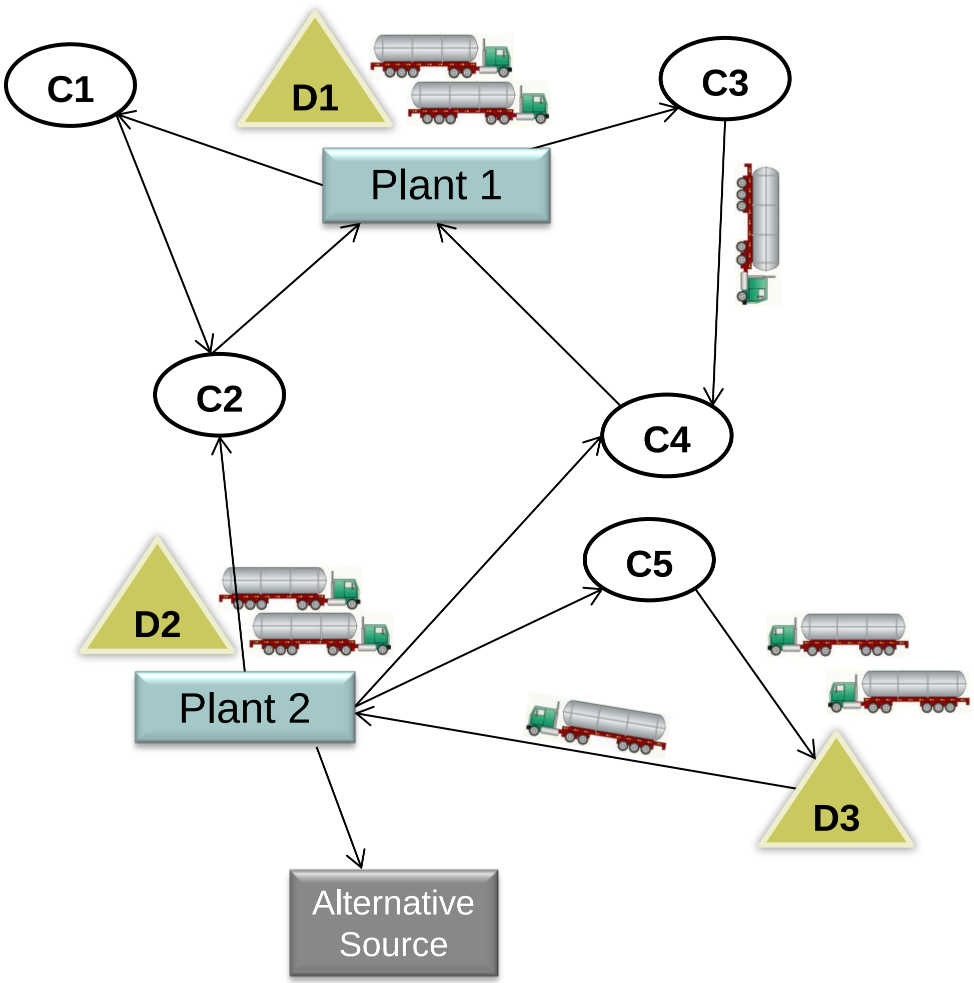
- Multiple Plants and Depots (located or not at plants)
- Multiple Products *(LIN, LOX etc.)* and Product Grades
- *Coordination among plants, distribution options and customers*
- Large computational effort (> 12 hrs. and > 4% gap)
- Large combinatorial problem



Goal

*Apply decomposition strategy in order to reduce the computational effort and improve the **optimal decision making** in complex production distribution problems*

Problem Statement and Main Assumptions



Given

- Plants, Products, Operating Modes and Production Limits
- Daily Electricity Prices (off-peak and peak)
- Customers and their demand/consumption profiles
- Max/Min inventory at production sites and customer locations
- Alternative sources and product availabilities
- Depots, Truck availabilities and capacities, Distances
- Fixed Planning Horizon (usually 1-2 weeks)

Decisions in each time period t

- Modes and production rates at each plant
- Inventory level at customer location and plants
- How much product to be delivered to each customer through which route

Objective Function

- Minimize total production and distribution cost over planning horizon

Main Assumptions – Distribution Side

- Two time periods per day (peak and off-peak) are considered
- Trucks do not visit more than 4 customers in a single delivery

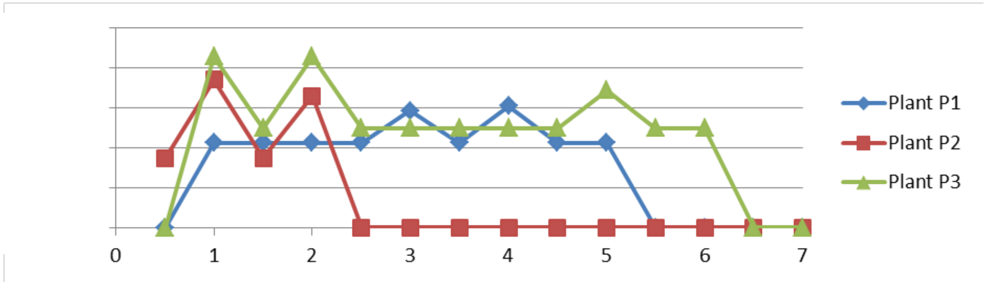
Mathematical Model (MILP)

Objective **Minimize total Production and Distribution Costs**

Constraints on Production Side

Production Cost = Fixed Start-up cost
+ Variable production cost

- **Min/Max Production Capacity Constraints in each mode of operation**
- **Logic Constraints for switching between various modes of operation**
- **Max/Min Inventory limits at the production sites**
- **Plant Inventory Balance Constraints**
- **Demand satisfaction for pick-up customers**
- **Ad Hoc Production Models**
  - To account for specific equipment configurations and production modes.



Main binary variables
mode selection
B_{pmt}

Mathematical Model (MILP)

Constraints on Distribution Side

Distribution Cost = Cost of deliveries by trucks + purchases from competitors

- **Max/Min Inventory limits at the customer locations**
- **Customer Inventory Balance Constraints**
- **Truck Capacity constraints**
- **Material balance constraints for product pick-up and delivery points**
- **Max product purchase limit from competitor sources**

Main binary variables

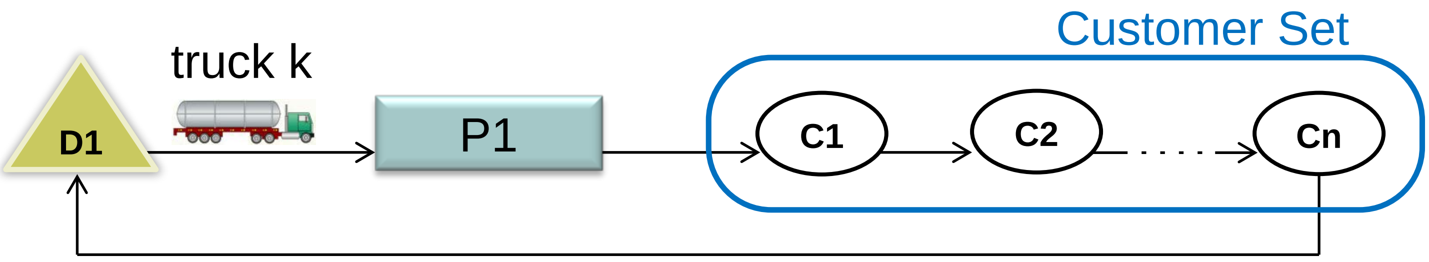
truck-customers association

y_{kst}

truck-source association

Y_{kpt}

Route = Depot + Plant + Customers



Route Generation Algorithm

- Parameters: maximum distance, number of customers visited, etc.
- Main ordering criteria: traveling distance

Industrial Size Test Case (previous work)

- 4 Plants / Depots
- 2 products (LIN, LOX)
- 2-4 production modes for each plant
- 15 alternative sources
- 229 customers
- 14 time periods (peak and off-peak)
- 46 trucks (25 for LIN, 12 for LOX)
- Min/max inventory, distances, electricity prices, truck deliveries, etc.

19 sources

10,032 possible routes reduced to 1,900 by the route generation algorithm

12 h gap > 4% !

Model Size	Binary variables	26,486
	Continuous variables	69,902
	Constraints	35,233

CPU results	Time	**7978 s**
	Nodes	**11,537**
	Relative gap	**10 %**

Models implemented with GAMS
CPU results obtained with solver CPLEX 12.3

Objective: propose a ***decomposition*** *strategy in order to reduce the computational effort*

Rolling Horizon approach

The detailed time block:

X $\in \{0, 1\}$ **(binary variable)**

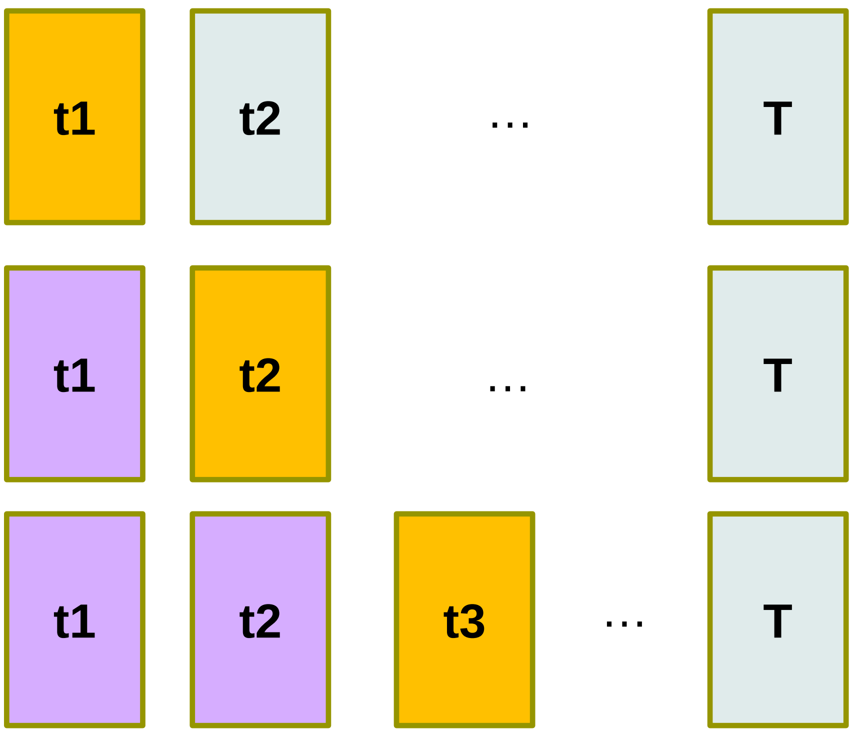
The aggregated time block:

$X' \in [0, 1]$ **(continuous variable)**

Full space

$$\min TotalCost(x, y)$$
$$s.t.$$
$$Ax_s^t = Bx_s^{t-1} \qquad \forall s \in \mathrm{S}, \mathrm{t} \in \mathrm{T}$$
$$Cx_s^t = Dy_s^t \qquad \forall s \in S, m \in M, \mathrm{t} \in \mathrm{T}$$

Algorithm:



Note:

- The planning horizon in detail (“the detailed time block”),
- Rest of the horizon is represented in an aggregated manner (“the aggregate time block”).

$\forall t \in T$ → **Sub-problems**

Industrial Size Test Case – Results

- 4 Plants / Depots
- 2 products (LIN, LOX)
- 2-3 production modes for each plant
- 15 alternative sources
- 229 customers
- 14 time periods (peak and off-peak)
- 46 trucks (25 for LIN, 12 for LOX)
- Min/max inventory, distances, electricity prices, truck deliveries, etc.

19 sources

10,032 possible routes reduced to 1,900 by the route generation algorithm

		Original	Rolling H
Model Size	Binary variables	26,486	-
	Continuous variables	69,902	-
	Constraints	35,233	-
	Normalized total cost	100	102
CPU results	Time	**7,978 s**	**1,147 s**
	Nodes	**11,537**	**1,278**
	Relative gap	**10 %**	**1.7 %**

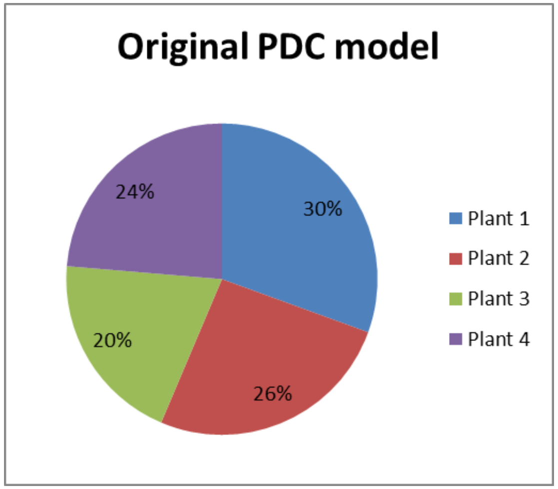
Models implemented with GAMS CPU results obtained with solver CPLEX 12.3

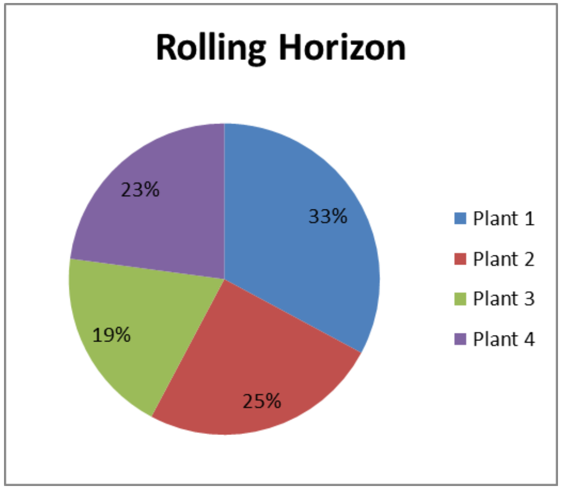
RH: 7 times faster!

1.8% worse

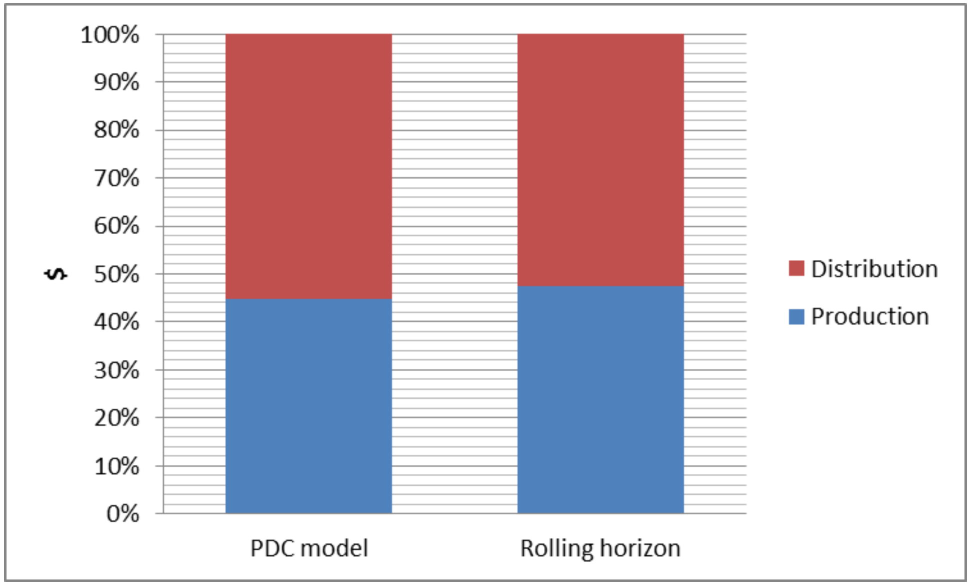
Industrial Size Test Case – Results

Production level





Total Cost (original model 1.83 % lower)



A ***Novel*** *Framework for* ***coordinated*** *production-distribution decision making has been developed*

Remarks and Conclusions

The proposed framework provides optimal production and distribution coordination reducing the computational effort

- The proposed rolling horizon is computationally faster than simultaneous approach with small penalty for suboptimality
- Lagrangean Decomposition might be also promising but for problems with strong linking no feasible solutions have been encountered.

Work under development

- Investigate extension of Lagrangean Decomposition to handle strong linking among the time periods (start up detection, high inventory levels, and high final inventory or safety stock considered).
- Comprehensive comparison between Rolling horizon vs Lagrangean Decomposition