Bi-level Optimization for Capacity Planning in Industrial Gas Markets

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Motivation

Industrial gas markets are dynamic:

- Suppliers must anticipate demand growth
- Most markets are served locally

Capacity expansion is a major strategic decision:

- Requires large investment cost
- Benefits are obtained during a long time-horizon

Optimization

Benefits are sensitive to market behavior:

- Market preferences
- Economic environment

Variability

Sensitivity can be reduced by assuming rational behavior:

- Producers try to maximize their profit
- Markets try to minimize their cost

Bilevel optimization

Need to model the conflicting interests of producer and markets

Problem Statement

Given:

- Set of capacitated plants from some of which can be expanded and some can not to be expanded
- Set of candidate locations for new plants
- Cost coefficients for investments, operation, and distribution
- Set of markets with deterministic demands during the time-horizon

Maximize net present value (NPV):

- Determine expansion plan
- While considering optimal distribution in each period

Standard Approach

Capacity expansion planning:

Maximize: Income – New plants – Maintenance – Expansion – Production – Transportation

$$NPV = \sum_{t\in T} \frac{1}{(1+r)^t} \left\{ \sum_{j\in J} \rho_{t,j} \sum_{i\in I} y_{t,i,j} - \sum_{i\in I} \left[\alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j\in J} \left(\varphi_{t,i,j} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j} \right) \right] \right\}$$

Subject to:

$$w_{t,i} = \sum_{t'=1}^{t} v_{t,i} \qquad (\forall t \in T,\ i \in I)$$ Open plants must be maintained

$$x_{t,i} \leq w_{t,i} \qquad (\forall t \in T,\ i \in I)$$ Expansion can only take place in open plants

$$c_{t,i} = c_{t-1,i} + \delta x_{t-1,i} \qquad (\forall t \in T,\ i \in I)$$ Capacity in plants is incremental

$$\sum_{j\in J} y_{t,i,j} \leq c_{t,i} \qquad (\forall t \in T,\ i \in I)$$ Demand satisfaction is constraint by capacities

$$\sum_{i\in I} y_{t,i,j} = D_{t,j} \qquad (\forall t \in T,\ j \in J)$$ All markets are satisfied

$$c_{j,k}, y_{s,j,i,k} \geq 0;\ v_{t,i}, w_{t,i}, x_j \in \{0,1\} \quad (\forall t \in T,\ i \in I,\ j \in J)$$ Bounds

Bilevel Optimization

Capacity expansion planning with rational market:

Two subsets: plants that can be expanded by leader(I^1) and that cannot be expanded by market (I^2)

$$\max \ NPV = \sum_{t\in T} \frac{1}{(1+r)^t} \left\{ \sum_{j\in J} \rho_{t,j} \sum_{i\in I^1} y_{t,i,j} - \sum_{i\in I^1} \left[\alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j\in J} \left(\varphi_{t,i,j} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j} \right) \right] \right\}$$

s.t.

$$w_{t,i} = \sum_{t'=1}^{t} v_{t,i} \qquad (\forall t \in T,\ i \in I^1)$$

$$x_{t,i} \le w_{t,i} \qquad (\forall t \in T,\ i \in I^1)$$

$$c_{t,i} = c_{t-1,i} + \delta x_{t-1,i} \qquad (\forall t \in T,\ i \in I^1)$$

$$\sum_{j\in J} y_{t,i,j} \le c_{t,i} \qquad (\forall t \in T,\ i \in I^1)$$

$$\min \sum_{t\in T} \frac{1}{(1+r)^t} \left[\sum_{j\in J} \sum_{i\in I} \left(\rho_{t,j} y_{t,i,j} + \tau_{t,j,i} y_{t,i,j} \right) \right]$$

Markets minimize their cost: price + transportation

s.t.

$$\sum_{j\in J} y_{t,i,j} \le c_{t,i} \qquad (\forall t \in T,\ i \in I^2)$$

Capacity of plants excluded from expansion plan

$$\sum_{i\in I} y_{t,i,j} = D_{t,j} \qquad (\forall t \in T,\ j \in J)$$

All markets are satisfied

$$c_{j,k},\ y_{s,j,i,k} \ \ge\ 0\ ;\ \ v_{t,i},\ w_{t,i},\ x_j \in \{0,1\} \qquad (\forall t \in T,\ i \in I,\ j \in J)$$

Solution Strategy

Transform to single-level by using KKT conditions of lower-level problem

The optimal solution for LP:

$$\min \sum_{k=1}^{|K|} c_k x_k$$

$$\text{s.t.} \sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \qquad (\mu_i) \qquad i=1,\ldots,|I|$$

$$\sum_{k=1}^{|K|} b_{k,j} x_k = b_0 \qquad (\lambda_j) \qquad j=1,\ldots,|J|$$

$$x_k \in R \qquad k=1,\ldots,|K|$$

Can be obtained by solving:

$$c_k + \sum_{i=1}^{|I|} a_{k,i}\mu_i + \sum_{j=1}^{|J|} b_{k,j}\lambda_j = 0 \qquad k=1,\ldots,|K|$$

Stationarity

$$\sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \qquad i=1,\ldots,|I|$$

$$\sum_{k=1}^{K} b_{k,j} x_k = b_0 \qquad j=1,\ldots,|J|$$

Primal feasibility

$$\mu_i \geq 0 \qquad i=1,\ldots,|I|$$

Dual feasibility

$$\mu_i \left(\sum_{k=1}^{K} a_{k,i} x_k - a_0 \right) = 0 \qquad i=1,\ldots,|I|$$

Complementary slackness

$$x_k, \lambda_j \in R \qquad k=1,\ldots,|K|; j=1,\ldots,|J|$$

Complementary slackness can be transformed to logic constraints:

$$\sum_{k=1}^{|K|} a_{k,i} x_k - a_0 - s_i = 0$$

$$[s_i = 0] \quad \vee \quad [\mu_i = 0]$$

$$s_i \geq 0$$

→ Use MILP reformulation

Illustrative Example

Problem structure:

- 3 existing plants which can be expanded
- 1 new candidate plant
- 3 existing plants which can not be expanded
- 15 markets with deterministic demand for 1 commodity
- 20 time-periods (quarters)

Formulations:

- Single-level (*SL*): leader selects the markets to satisfy
- Bi-level (*BL*): market minimizes cost in lower-level
- Single-level evaluation (*SL-eval*):evaluation of single-level investment decisions in the market environment

Results

Computational statistics:

Statistics	***SL***	***BL***	***SL-eval***
No. of constraints:	783	9,220	481
No. of continuous variables:	3,012	6,183	2701
No. of binary variables:	176	3,056	0
Solution time (CPLEX):	0.18 s	119 s	0.12 s
Optimality gap:	0.5%	0.5%	0.5%

Results:

Element of objective function	***SL***	***BL***	***SL-eval***
Income from sales [MM$]:	1,601	960	1,247
Investment in new plants [MM$]:	0	0	0
Capacity expansion cost [MM$]:	242	30	242
Maintenance cost[MM$]:	127	127	127
Production cost[MM$]:	580	357	454
Transportation cost[MM$]:	22	9	11
Total NPV [MM$]:	**630**	**438**	**413**
Market cost[MM$]:	**1,691**	**1,681**	**1,681**

Bilevel optimization **NPV higher by 25 million** ***(438 vs 413)*** when compared to single-level expansion strategy

Conclusions

Novelty:

- MILP bi-level optimization model for capacity expansion
- Considers the conflicting interest of producers and markets
- Models market behavior according to their interests
- Includes market preferences in capacity expansion planning

Impact for industrial applications:

- Allows developing capacity expansion plans that are less sensitive to market preferences and economic variability

- Avoids overestimating expansion

Future Steps

- Include uncertainty in market demands
- Model markets that can induce expansion and location of other plants
- Allow to reduce capacity and shut-down of plants