Optimal Scheduling for Copper Concentrates Operations in Aurubis Production Process

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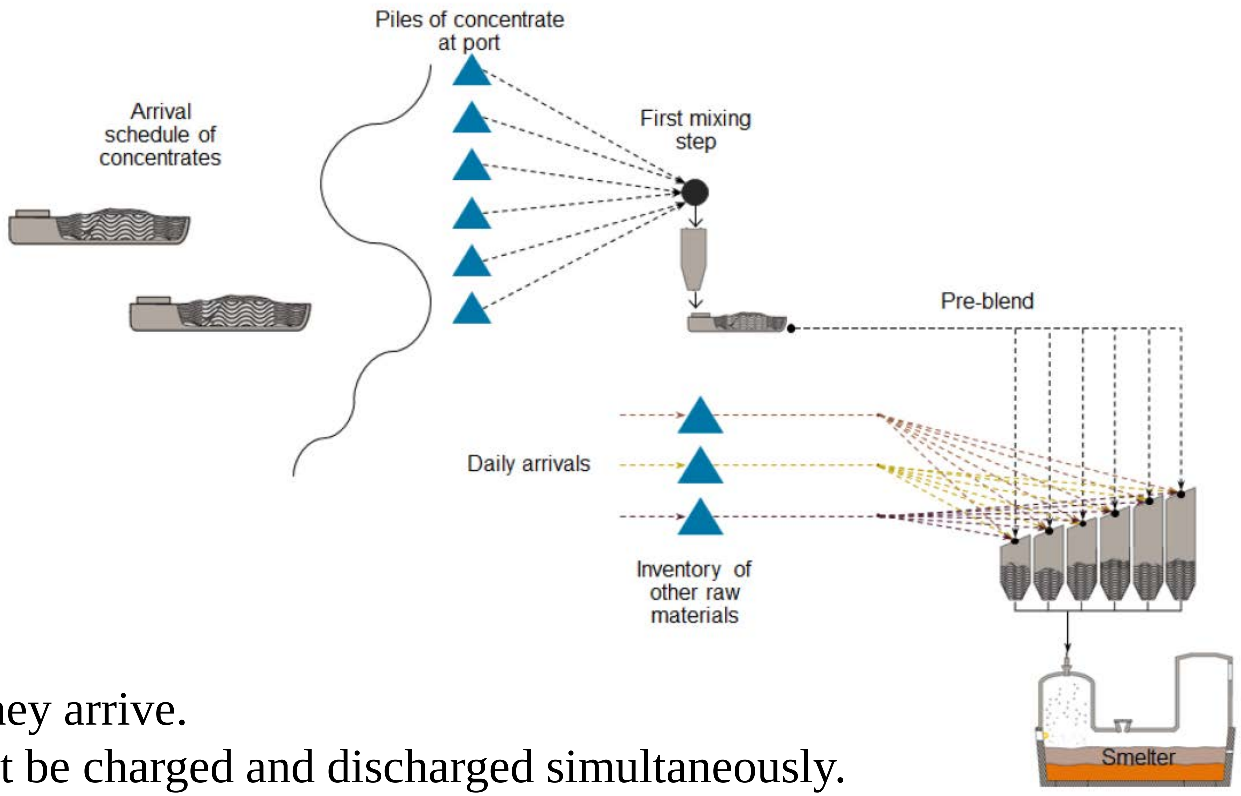
Aurubis

Outline

- **Background and motivation**
- **Model development**
  - Process flowsheet, variables and objective function
  - Mass balance equations
  - Logistic constraints
  - Special operation rules for bins
  - Quality constraint of material feeding the smelter
- **Solution strategy**
  - Quantity and logistics problem: MILP copper concentrates scheduling
  - Quality problem: NLP copper concentrates blend
- **Results**
- **Conclusion and future work**

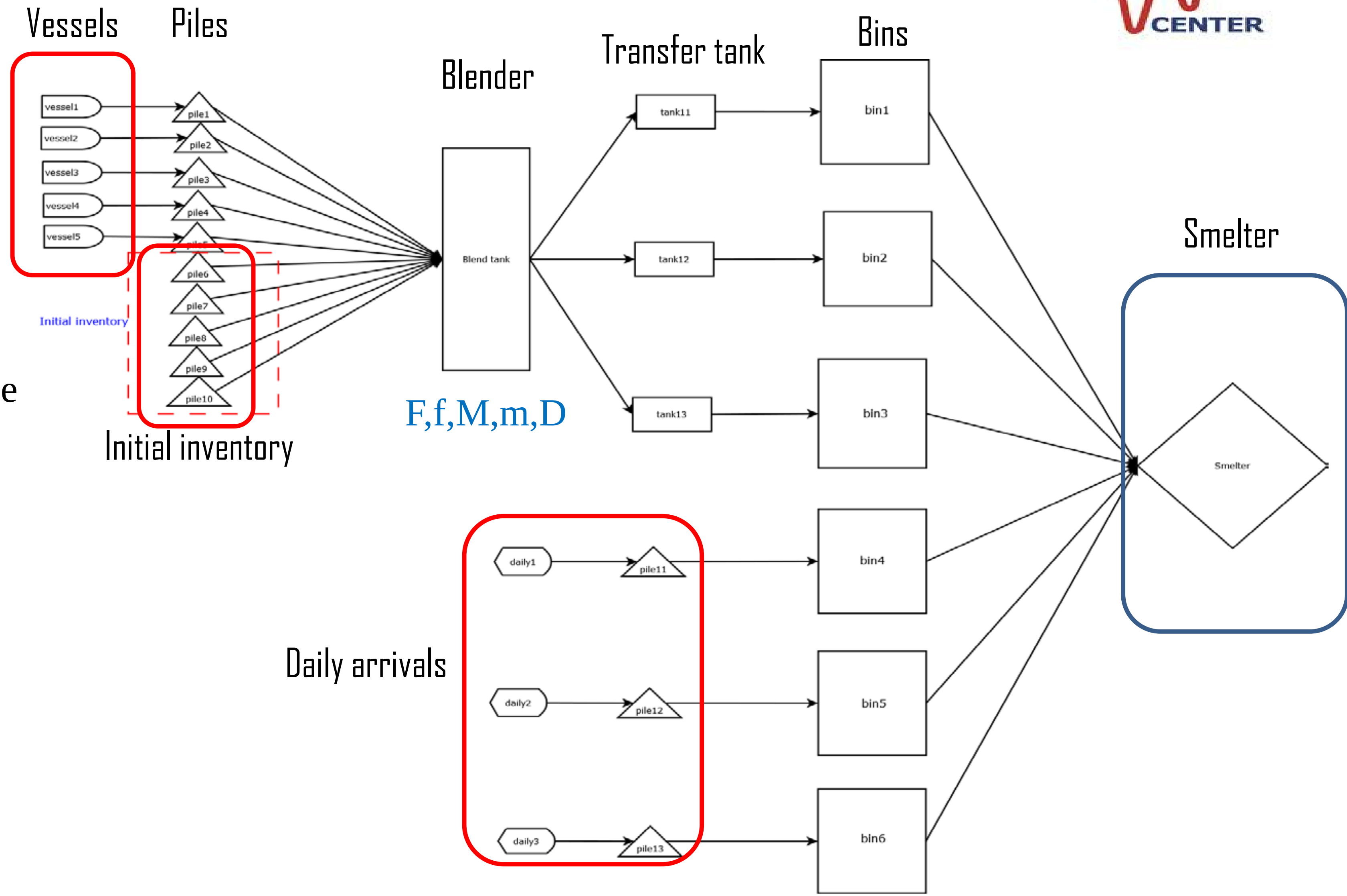
Background and Motivation

- **Given:**
  - Schedule of ship arrivals
  - Raw material from vessels, ports and daily arrivals and initial inventories of bins
    - 14 kinds of copper concentrates(C1…C14) with different composition(K1…K8)
  - Specific constraints
    - Intermediate units
    - Final blending
  - Economic data
    - Income
    - cost
  - Time horizon: 15
- **Assumption:**
  - The vessels unload all the raw material at the day they arrive.
  - The intermediate unit (blender, transfer tank) cannot be charged and discharged simultaneously.
  - The six bins work simultaneously during the time horizon.
- **Goal:**
  - Find schedule of transfer of concentrates to bins to max profit.



Model Development

- **Process flowsheet**
- **Continuous variables:**
  - Flowrate
  - Inventory
- **Binary variables:**
  - D - To denote each flowrate line in the flowsheet



Model Development

- **Model Summary:**

MINLP model	
Raw material	
Copper concentrates from vessels	C1-C5
Copper concentrates from port	C6-C10
Daily arrivals non-concentrate material	C11-C13
Initial inventory of bins	C14
Key components	K1-K8
Time horizon	15 days
Time period	1 day
Model size	
Discrete variables	**1365**
Continuous variables	**12567**
Constraints	**8043**

Model Development

- **Objective function:**

$$\max \sum_{t\in T}\sum_{c\in C} A_{t,c} f_{t,c} - \sum_{t\in T}\sum_{c\in C} B_{t,c} V_{t,c} - \sum_{t\in T}\sum_{k\in K} E_{t,k} Z_{t,k}$$

$A_{t,c}$ – The income for processing a unit of concentrate c at time t
$F_{t,c}$ – The mass flow of concentrate m being fed to the smelter at time t
$B_{t,c}$ – The cost of storing a unit of concentrate at time t
$V_{t,c}$ – The Inventory of concentrate m available at time t
$E_{t,k}$ – The linear cost of changing the concentration of key component k in the mixture at time t
$Z_{t,k}$ – The change in concentration of key component k at time t

Model Development

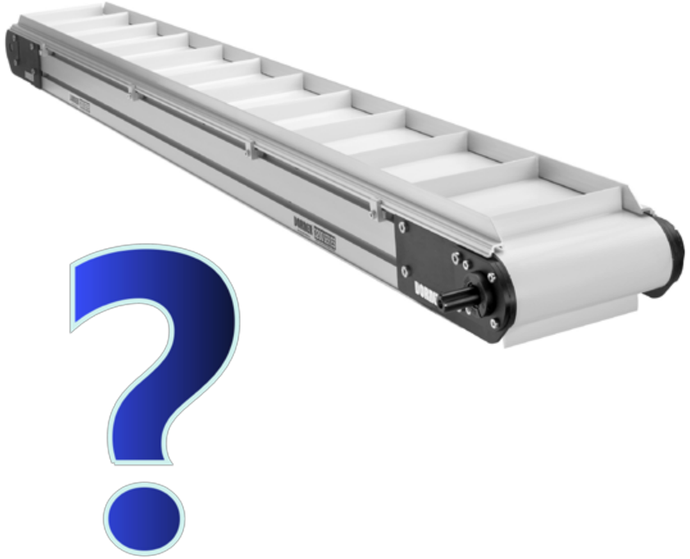
- **Mass balance equation:**
  - $\sum_{c=1}^{14} m_{B,j,c,t} = M_{B,j,t}$ $\quad j = 1,\ t = 1 \ldots 15$
  - $\sum_{c=1}^{14} f_{BBV,j,bv,c,t} = F_{BBV,j,bv,t}\ i = 1 \ldots 10$ $\quad j = 1,\ bv = 1, 2, 3,\ t = 1 \ldots 15$
  - $M_{B,j,t} = M_{B,j,t-1} + \sum_{i=1}^{10} F_{PB,i,j,t} - \sum_{bv=1}^{3} F_{BBV,j,bv,t}$ $\quad j = 1,\ t = 2 \ldots 15$
  - $m_{B,j,c,t} = m_{B,j,c,t-1} + \sum_{i=1}^{10} f_{PB,i,j,c,t} - \sum_{bv=1}^{3} f_{BBV,j,bv,c,t}$ $\quad j = 1,\ t = 2 \ldots 15,\ c = 1 \ldots 14$
  - $\frac{m_{B,j,c,t}}{M_{B,j,t}} = \frac{f_{BBV,j,bv,c,t}}{F_{BBV,j,bv,t}}$ $\quad j = 1,\ c = 1 \ldots 14,\ bv = 1,2,3,\ t = 1 \ldots 15$
- **Logistic constraints:**
  - $F_{BBV,j,bv,t} \leq F_{BBV,j,bv,max} \cdot D1_{j,bv,t}$ $\quad j = 1,\ bv = 1, 2,3,\ t = 1 \ldots 15$
  - $F_{BBV,j,bv,t} \geq F_{BBV,j,bv,min} \cdot D1_{j,bv,t}$ $\quad j = 1,\ bv = 1,2,3,\ t = 1 \ldots 15$
  - $\sum_{bv=1}^{3} D1_{j,bv,t} \leq 1$ $\quad j = 1,\ t = 1 \ldots 15$

Model Development

- **Quality and capacity constraint feeding the smelter:**
  - The maximum capacity of the smelter is 3000 tons/day.
  - Upper limits for mass fraction of key components:

$$compfrac_{k,t} \leq x_{k,max} \qquad k = 1 \dots 8, t = 1 \dots 15$$

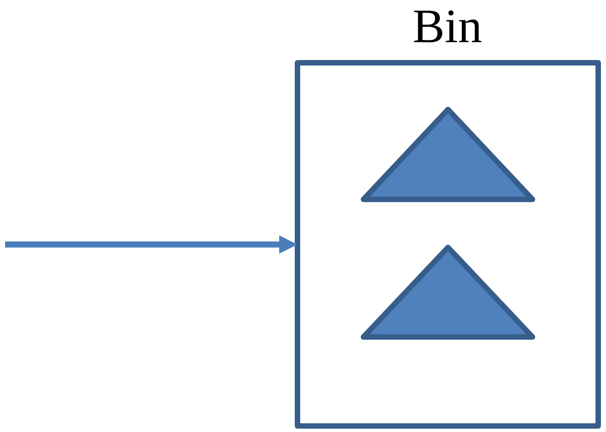
  - Independency constraints based on the individual key components flows
- **Special operation rules for bins:** Key part for modelling in GAMS
  - There are six bins feeding the smelter.
  - All bins feeding the smelter must be filled at same time.
  - The consumption from the bins feeding the smelter must be coordinated.
  - No blending in each bin.



Model Development

- **Special operation rules for bins:**
  - Create two piles for each bins.
  - The two pile work in turn to make the bin working continuously.
  - No material blending between two different recipe.
  - The first piles of 6 bins are coordinated to become empty.

    at the same time and then the second piles of 6 bins start working.



Solution Strategy

- **<u>Non-convex MINLP</u>**
- **Two step method**
  - ***Quantity and logistics problem: MILP copper concentrates scheduling***
    - Drop nonlinear constraints

Composition limits:

For blend tank:

$$\frac{m_{B,j,c,t}}{M_{B,j,t}} = \frac{f_{BBV,j,bv,c,t}}{F_{BBV,j,bv,t}} \qquad j = 1, \qquad c = 1 \dots 14, \qquad bv = 1,2,3, \qquad t = 1 \dots 15$$

For transfer tank:

$$\frac{m_{BV,bv,c,t}}{M_{BV,bv,t}} = \frac{f_{TBP,bv,bp,c,t}}{F_{TBP,bv,bp,t}} \qquad bv = 1,2,3, \qquad c = 1 \dots 14, \qquad bp = 1,2,\dots,12, \qquad t = 1 \dots 1$$

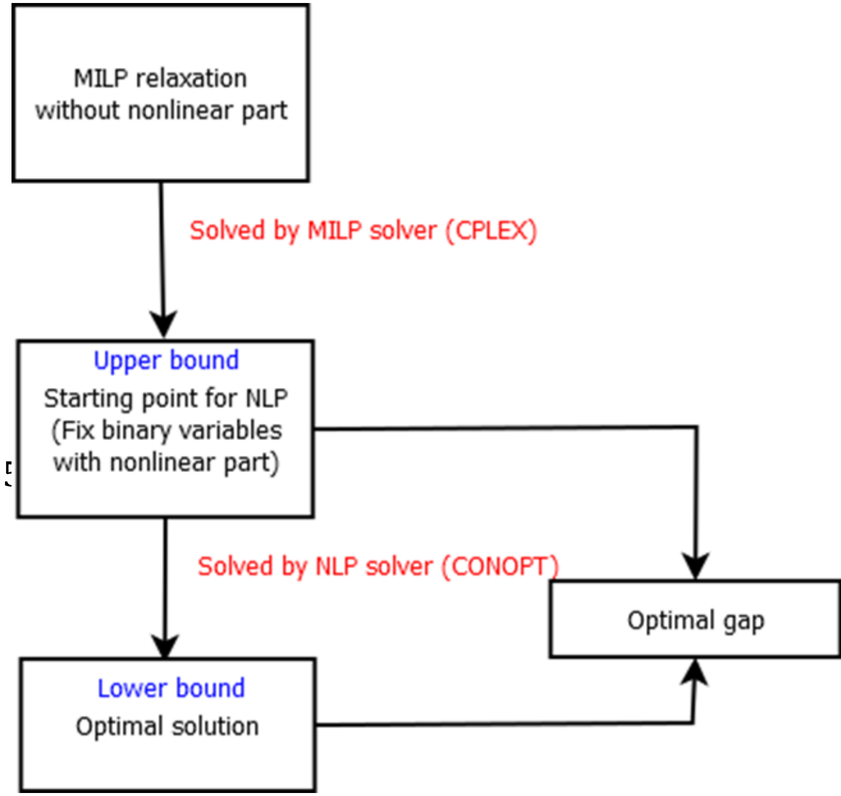
For bins:

$$\frac{m_{BP,bp,c,t}}{M_{BP,bp,t}} = \frac{f_{BPS,bp,s,c,t}}{F_{BPS,bp,s,t}} \quad bp = 1,2,\dots,12, \qquad c = 1 \dots 14, \qquad s = 1, \qquad t = 1 \dots 15$$

For smelter:

$$compfrac_{k,t} \cdot finalf_t = compf_{k,t}$$

  - ***Quality problem: NLP copper concentrates blend***



MILP relaxation without nonlinear part

Solved by MILP solver (CPLEX)

Upper bound
Starting point for NLP (Fix binary variables with nonlinear part)

Solved by NLP solver (CONOPT)

Lower bound
Optimal solution

Optimal gap

GAMS Results

- **Model Summary:**

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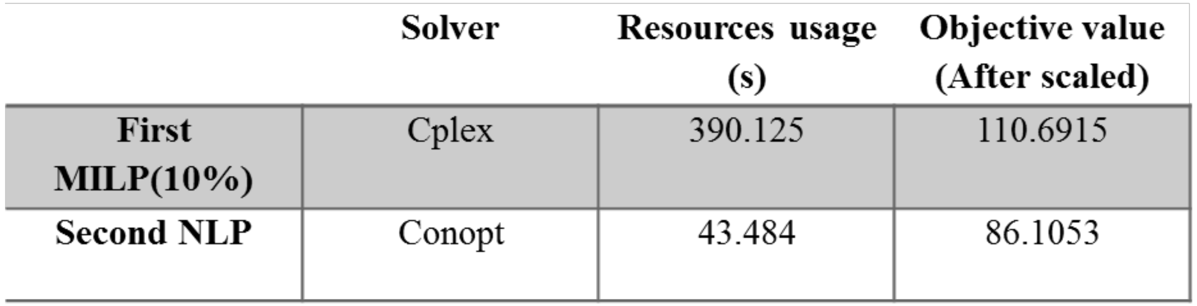
- **GAMS Performance:**



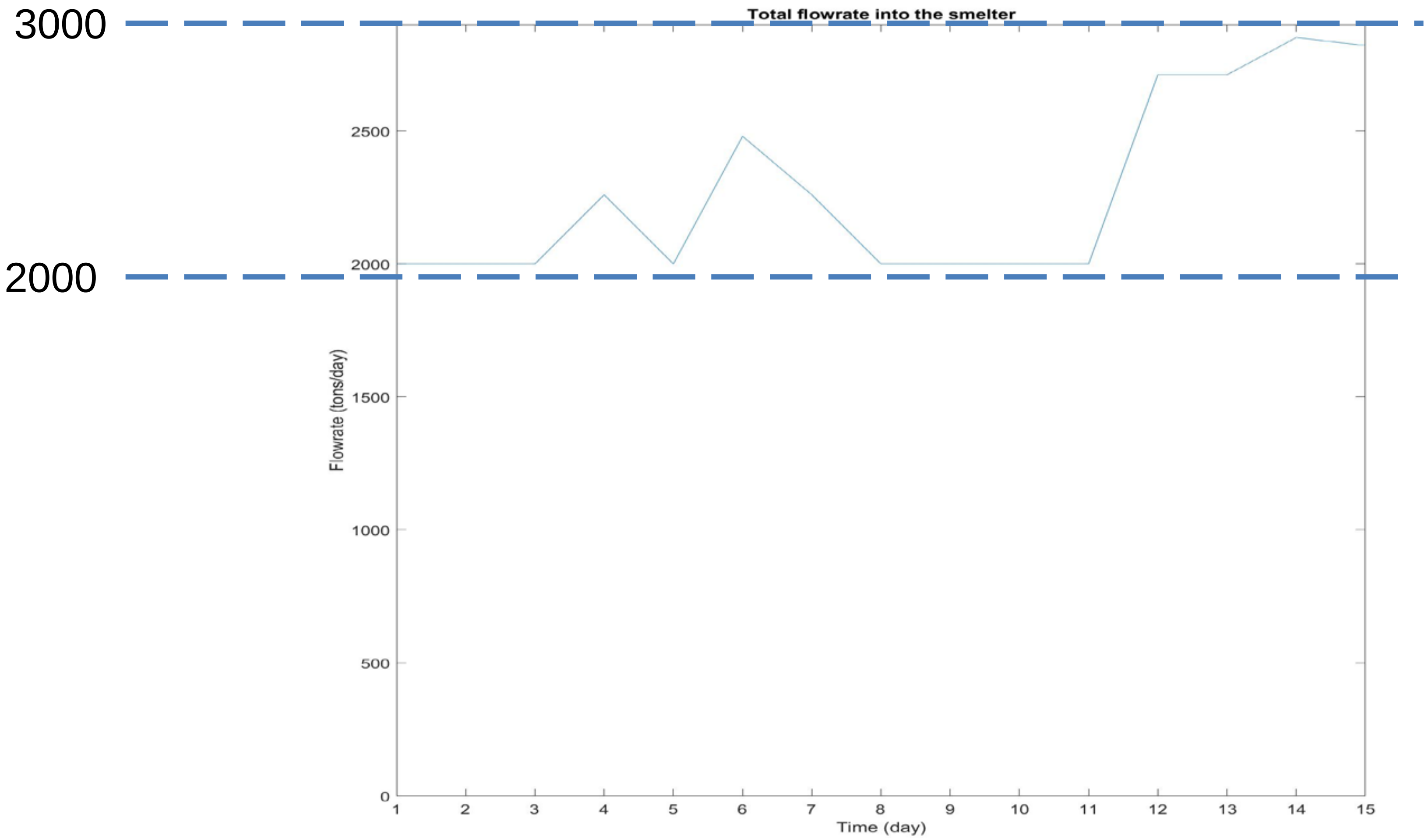
	Solver	Resources usage (s)	Objective value (After scaled)
First MILP(10%)	Cplex	390.125	110.6915
Second NLP	Conopt	43.484	86.1053

Gap = (110.6915 – 86.1053)/86.1053 = 28.55%

- The MILP-NLP gap shows that the quantity determined by the first MILP cannot meet the quality constraint into the smelter.

Results of Total Flowrates (tons/day)

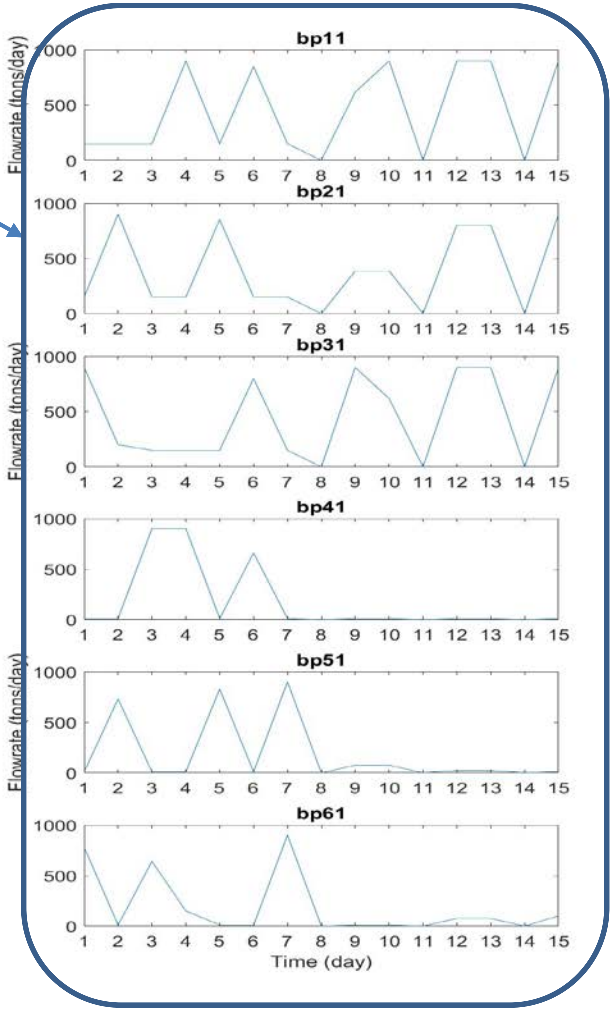
$2000 \leq final f_t \leq\ 3000$



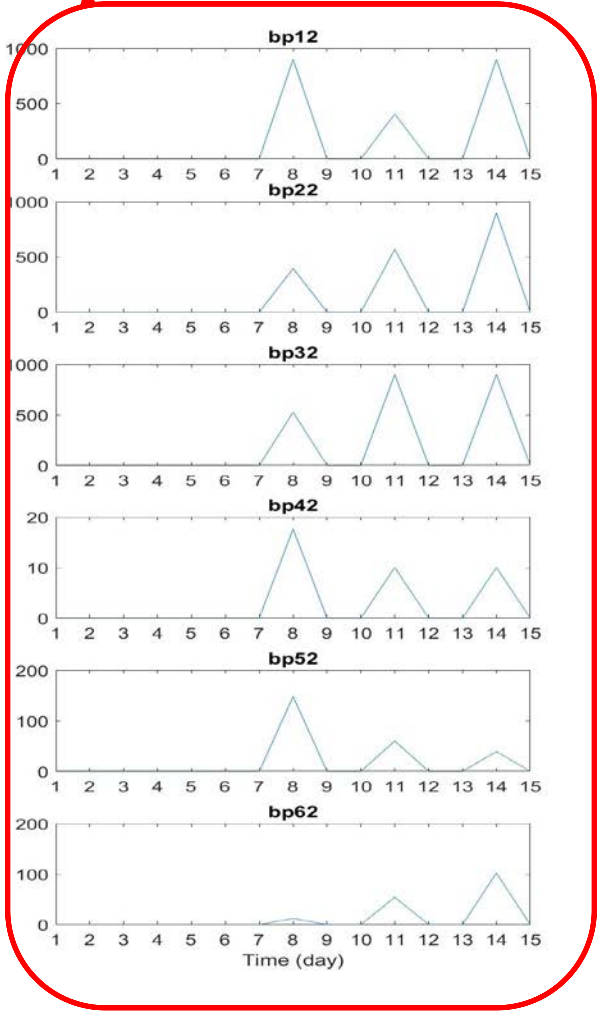
Results of Flowrates from Bins (tons/day)

- The first piles of bins work simultaneously at day 1,2,3,4,5,6,7,9,10,12,13.
- The second piles of bins work simultaneously at day 8,11,14.
- Two piles of each bin work in turn to keep the bin working continuously and no blending in the bin.

First pile of each bin

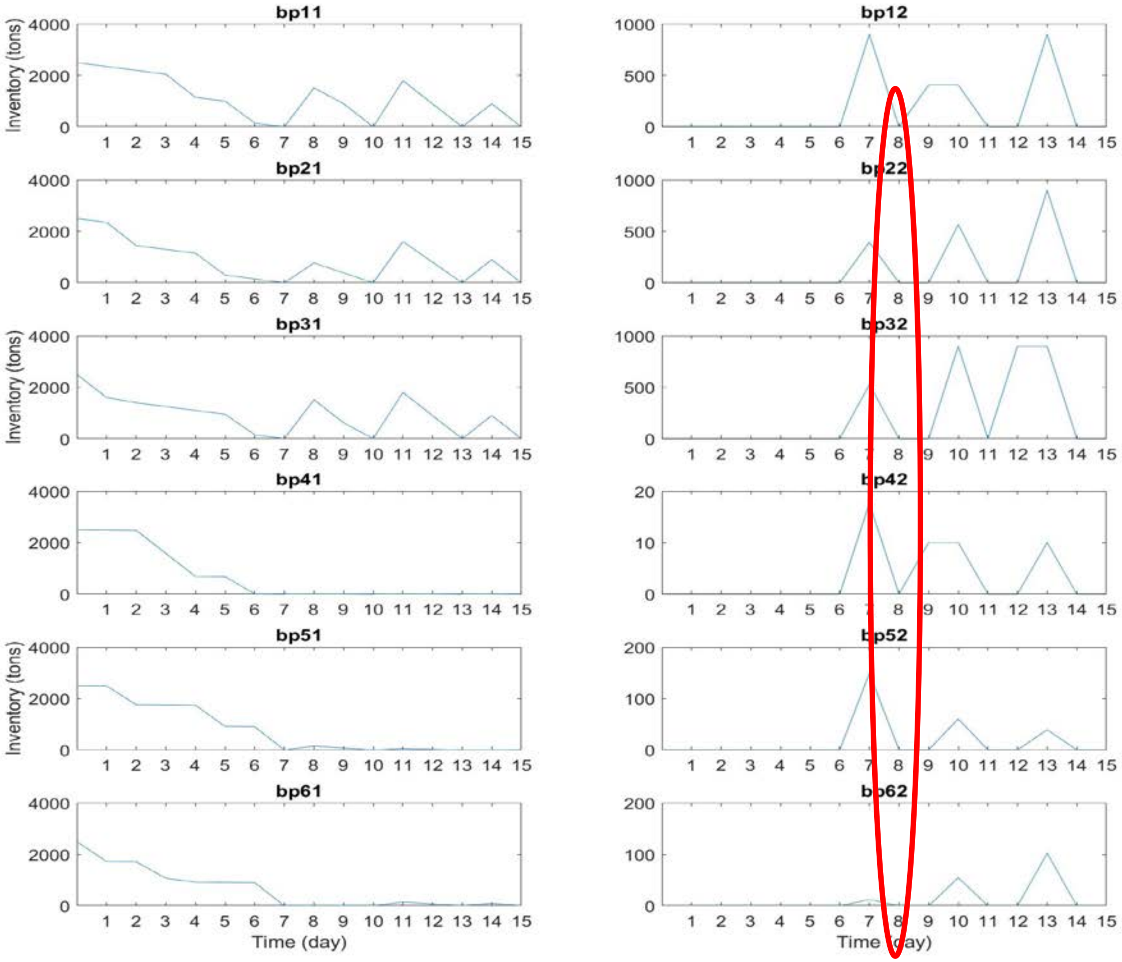


Second pile of each bin



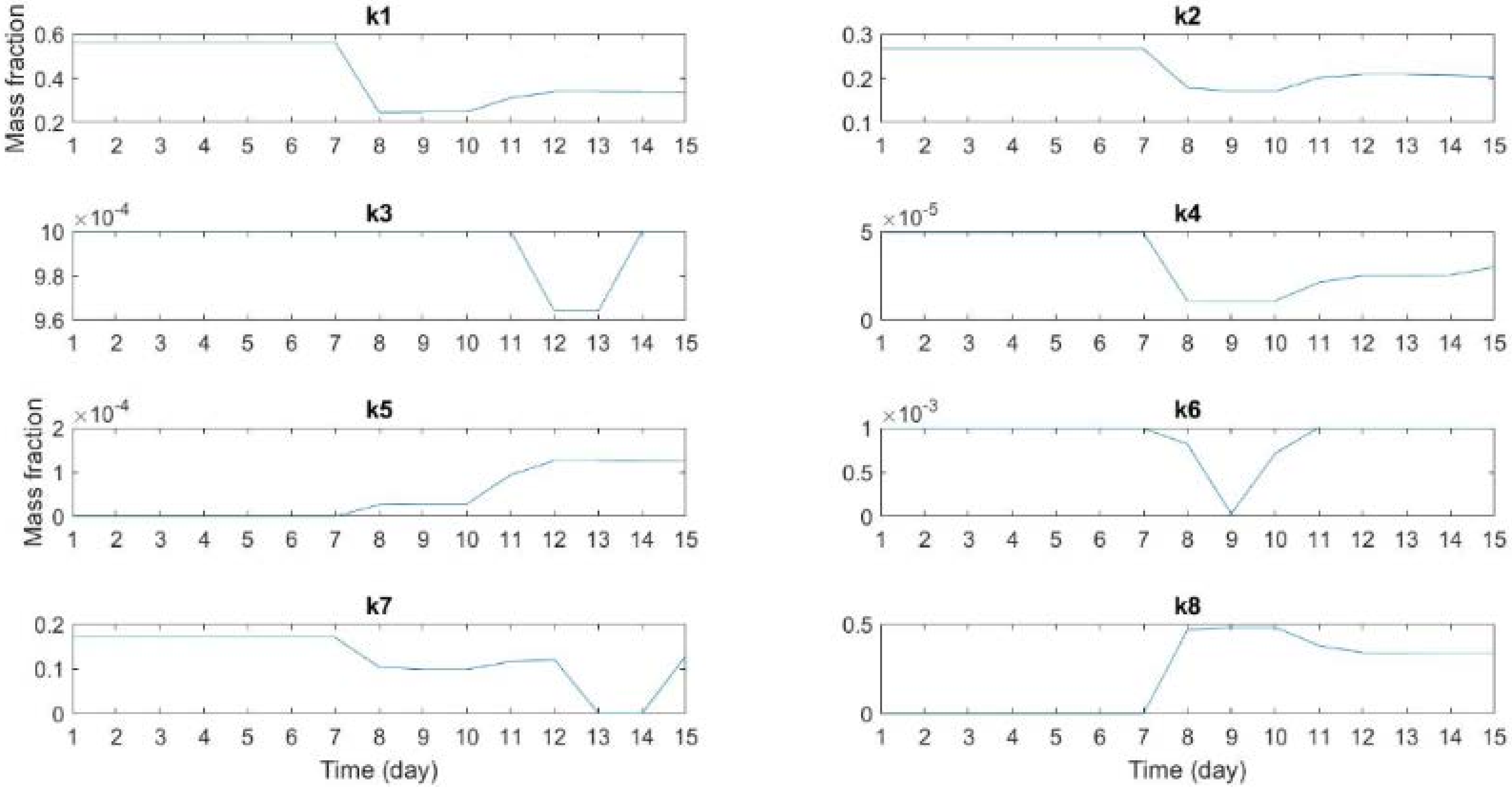
Results of Inventories of Bins (tons)

- The second piles of bins stop working at the end of day 8. Then they all become empty.
- The consumption from the bins feeding the smelter is coordinated.



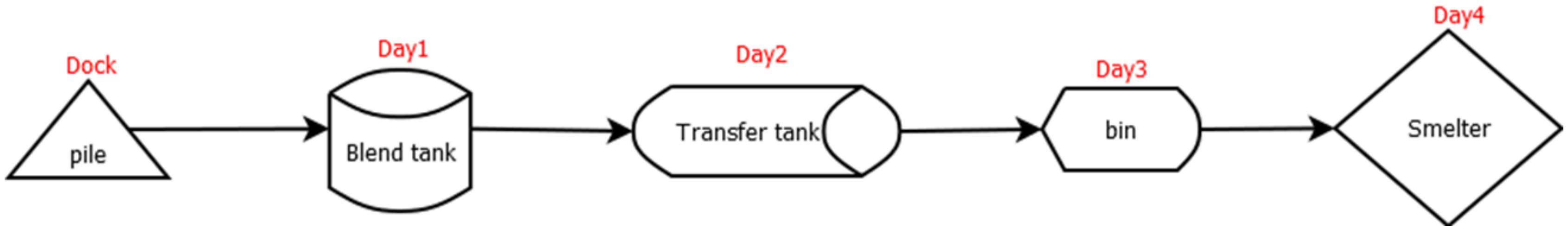
Results of Composition of Key Components

- The composition of key components (k1,…k8) in the mixture



Conclusion and Future Work

- **Conclusion:**
  - Non-convex MINLP is solved by MILP-NLP method.
  - The gap between NLP and MILP is relatively large which indicates that the quantity of concentrates determined by the first MILP model cannot meet the quality constraints feeding to smelter
- **Future work:**
  - Change the process flowsheet (delete the transfer tank to make the blender feeding the smelter immediately) to reduce the initial inventory of bins that can make the smelter work continuously.



  - Perform a sensitivity analysis to find the right weight on change-over penalties for key components.