Optimal Scheduling for Copper Concentrates Operations in Aurubis Production Process

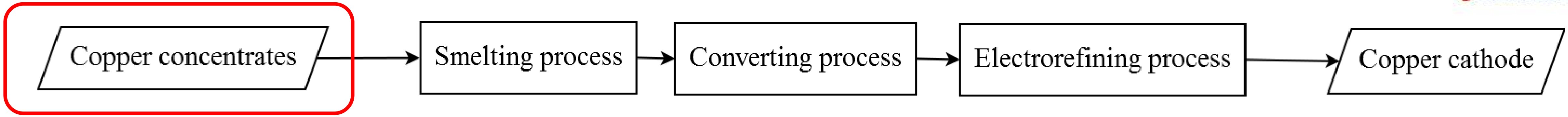
Yingkai Song, Brenno C. Menezes, Mangalam Lalpuria, Ignacio E. Grossmann

Center of Advanced Process Decision-making

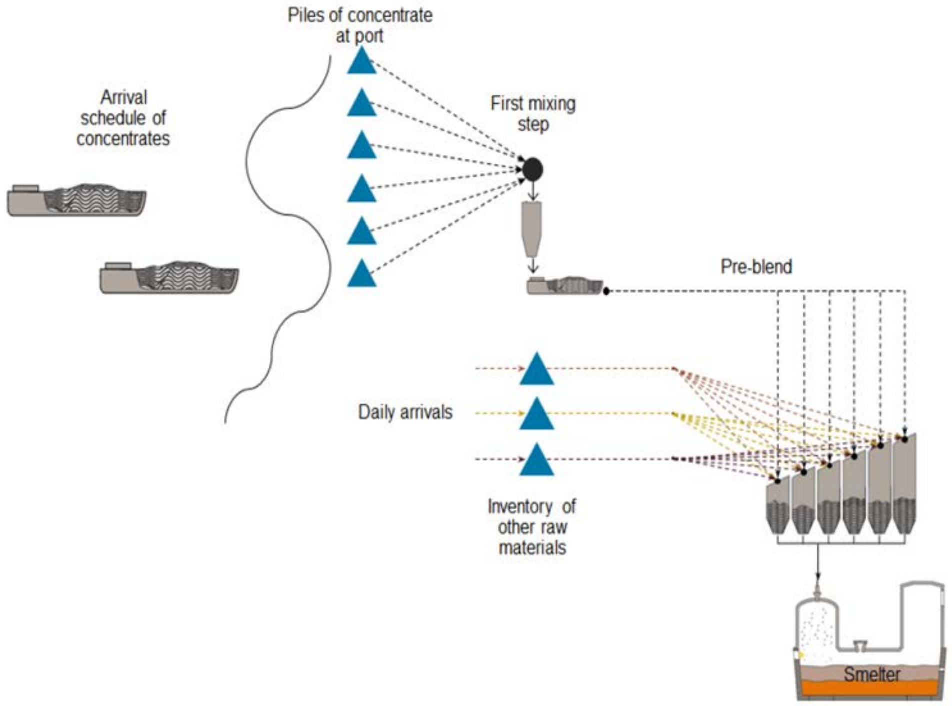
Carnegie Mellon University

Aurubis

Motivation

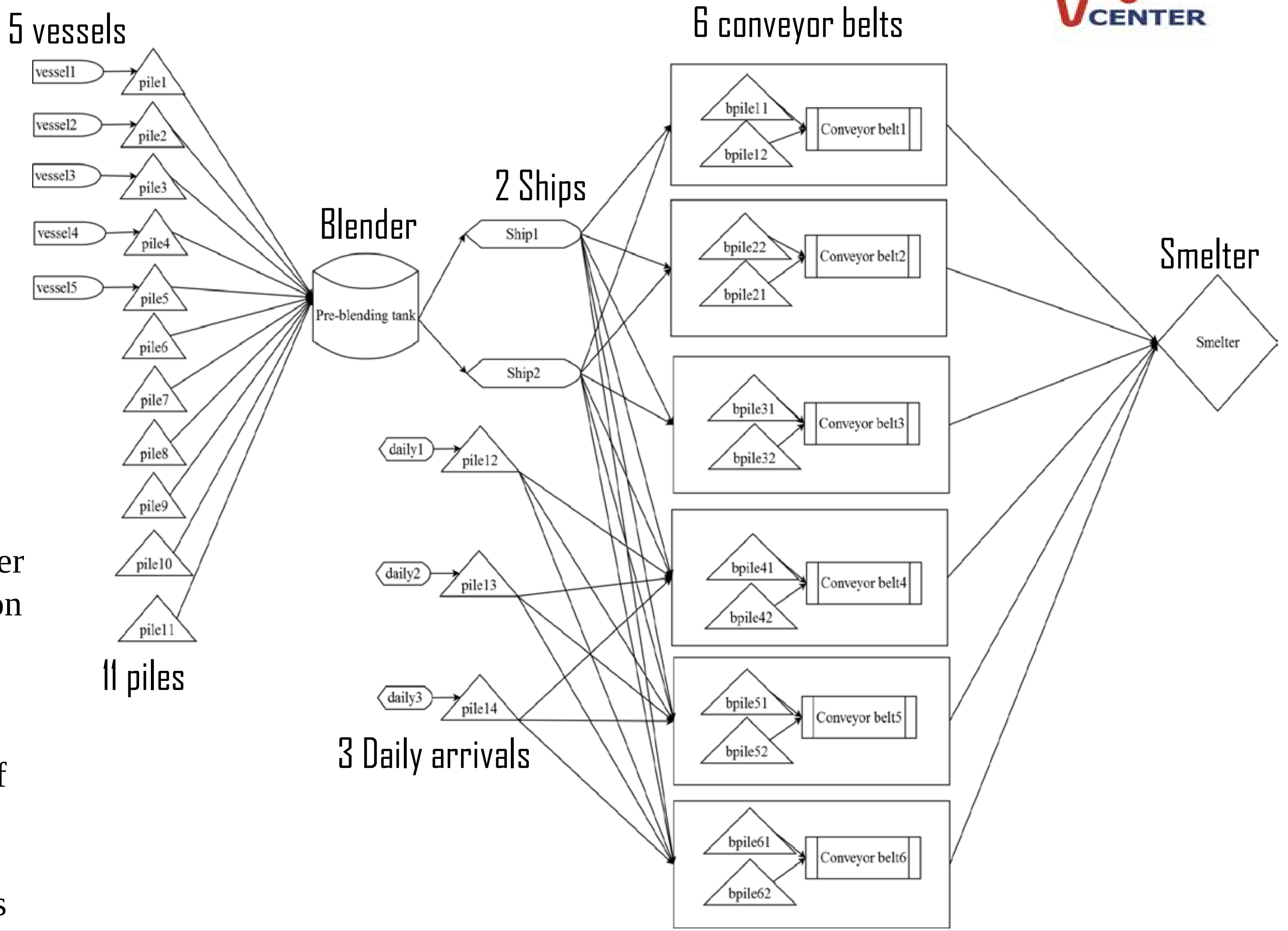


- **Scheduling of copper concentrates operations** is the head of copper refining process.
- **Qualified copper concentrates** for smelting process is important.
    - Restrictions on the element concentration in the smelter.
    - Quality restrictions for subsequent refining process.
- Achieve **maximum profit** of smelting process within the constraints.



Problem statement

- Given:
  - **Time horizon: 15 days**
  - Schedule of vessels
  - **Raw material** from vessels, ports and daily arrivals：
    - 14 kinds of copper concentrates(C1…C14) with different composition(K1…K8)
  - Main constraints:
    - **Logistic constraints** for **operation rules:**
      - The transfer ships
      - The conveyor belts
    - Final blending:
      - **Quality constraints** to limit the concentration of elements in the smelter
      - **Interdependency constraints** based on the individual element flows
      - **Final flowrate limits**
  - Economy data:
    - Income: Gross margin for processing unit of copper concentrates
    - Cost: Cost of changing concentration of key components



Problem statement

- Operation rules:
    - The vessels unload all the raw material at the day they arrive.
    - The intermediate units **cannot be charged and discharged simultaneously in one time period.**
  - **Rules for transfer ships:**
    - There are two ships (maximum capacity: 3000 tons) transporting material from the port to plant.
    - The ships must unload all the material in the plant.
    - The loading time, transporting time and unloading time are all the same, **0.5 day**.
    - The cycle time for transfer operation is **2 days** **(Load → Go to the plant → Unload → return to the port)**
  - **Rules for conveyor belts:**
    - The six conveyor belts work **simultaneously and continuously** during the time horizon.
    - There are two piles working in turn for each conveyor belt.
    - The piles for conveyor belts can only be created by **one resource (transfer ships or daily arrivals)** in one time period.
    - The flowrate of each conveyor belt should be **as constant as possible**.
- Goal:
    - Find optimal schedule to **maximize profit** (income – cost) under **specific constraints** within the time horizon.

Model development

- **Objective function:**

$$\max \sum_{t\in T}\sum_{c\in C} A_{t,c}F_{t,c} - \sum_{t\in T}\sum_{k\in K} E_{t,k}Z_{t,k} - \sum_{t\in T}\sum_{conv\in CONV} X_{t,conv} - \sum_{t\in T}\sum_{k\in K} Slack_{k,t} - \sum_{t\in T}\sum_{i\in I} Slackf_{i,t}$$

$A_{t,c}$ – The income for processing a unit of concentrate ***c*** at time ***t***
$F_{t,c}$ – The mass flow of concentrate **c** being fed to the smelter at time **t**
$E_{t,k}$ – The linear cost of changing the concentration of key component ***k*** in the mixture at time ***t***
$Z_{t,k}$ – The change in concentration of key component ***k*** at time ***t***
$X_{t,conv}$ - The change in flowrate of conveyor belt *conv* at time ***t***
$Slack_{k,t}$ - The slack variable for key component ***k*** at time ***t***
$Slackf_{i,t}$ - The slack variable for interdependency constraint ***i*** at time ***t***

Add **penalties of conveyor belts flowrate changing** to objective function!

$$compfrac_{k,t} \le x_{k,max} + Slack_{k,t}$$
$$f_{25} \le 0.64 f_2 + Slackf_{\prime 3\prime,t}$$
$$\ldots\ldots\ldots\ldots$$

Add slack variables to final blending constraints
To **avoid infeasible issue!**

Model development

- **Equality & Inequality:**
  - **Mass balance** equation with continuous flowrate
  - **Nonlinear constraints** for intermediate units (blender, transfer ships, piles for conveyor belts)
    - **The composition of outflow should be identical to the original tank.**

$$\frac{m_{B,j,c,t}}{M_{B,j,t}} = \frac{f_{BTV,j,bp,c,t+1}}{F_{BTV,j,bp,t+1}} \qquad j = 1,\ c = 1 \ldots 14,\ tv = 1{,}2,\ t = 1 \ldots 29$$

  - **Nonlinear constraints** for smelter:

$$compfrac_{k,t} \cdot finalf_t = compf_{k,t} \qquad \text{k}= 1 \ldots 8, t = 1 \ldots 30$$

  - **Operation rules** modeled by 0-1 constraints with propositional logic
  - **Final blending constraints**
- **Model Summary:**

MINLP model	
Copper concentrates from vessels	C1-C5
Copper concentrates from ports	C6-C11
Daily arrival non-concentrate material	C12-C14
Key conponents	K1-K8
Time horizon	15 days
Time interval	0.5 day
Number of time periods	30
Model statistics	
Discrete variables	3390
Continuous variables	38932
Constraints	39199
Non zero elements	240169
Non linear N-Z	64320

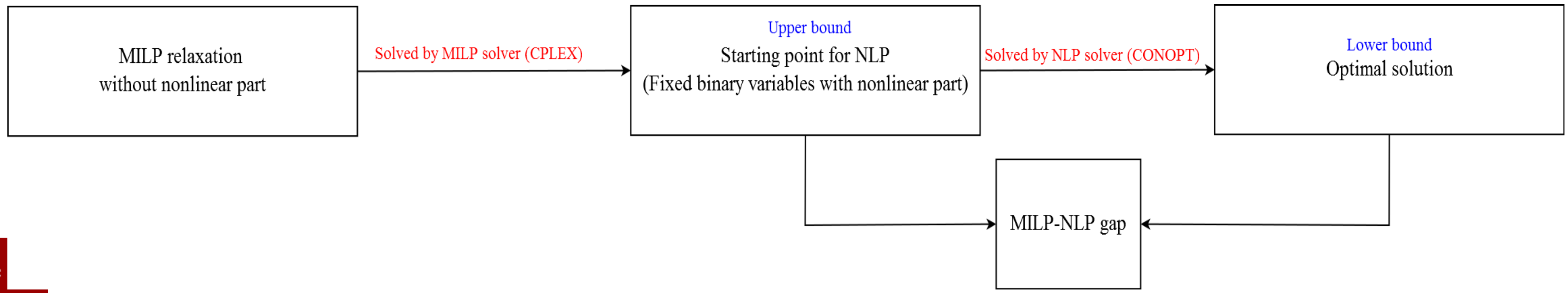
Solution strategy, Rolling horizon and MILP-NLP decomposition

- **GAMS**
- Rolling horizon
  - Solve 8 sub-problems in sequence:

	Fixed time block	Detailed time block	Aggregate time block
Subproblem1	None	1-6	7-30
Subproblem2	1-4	5-10	11-30
Subproblem3	1-8	9-14	15-30
Subproblem4	1-11	12-17	18-30
Subproblem5	1-14	15-20	21-30
Subproblem6	1-18	19-24	25-30
Subproblem7	1-21	22-26	27-30
Subproblem8	1-24	25-30	None

- Fixed time block: **binary variables fixed**.
- Detailed time block: **nonlinear constraints and binary variables considered (MINLP)**.
- Aggregate time block: **nonlinear dropped and binary variables**
- **relaxed from 0 to 1 (LP).**

  - The result of sub-problem 8 is a feasible solution of the MINLP model in a full space.
- MILP-NLP decomposition for each sub-problem



Result

- GAMS performance for 8 sub-problems:

	MILP: CPLEX MILP Solving time (s)	MILP Objective value	NLP: CONOPT NLP Solving time (s)	NLP Objective value	Objective value (Subtract effect of slacks)
Subproblem 1	162	188.58	425	188.58	188.78
Subproblem 2	262.73	176.21	602	173.04	173.48
Subproblem 3	241	156.78	491	156.77	157.07
Subproblem 4	321	142.49	452	139.79	140.15
Subproblem 5	143	136.35	469	121.72	122.12
Subproblem 6	19	125.36	389	107.83	108.20
Subproblem 7	8.5	123.82	Infeasible		
Subproblem 8	9.5	109.24	418	48.91	83.05

The objective value is after scaled.

- **Iteration to find solution with less non-zero slack variables**:
  - Better solution found with **two non-zero slack variables**:

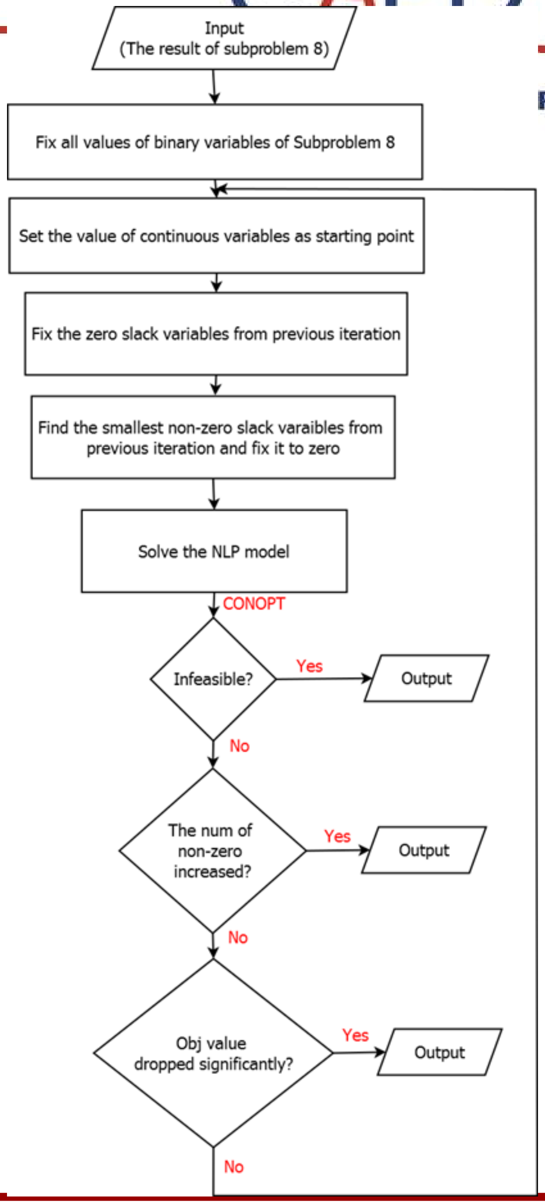
Objective value	Gross margin	Change-over cost of key components	Penalties of flowrate change	The sum of slack variables	**Profit**
48.56	102.871	14.005	6.403	33.852	**88.87**

$$f_{25} \leq 0.64 f_2 + Slack_{,3',t}$$

Slack3	
Time period	Value
27	14.073
28	19.724

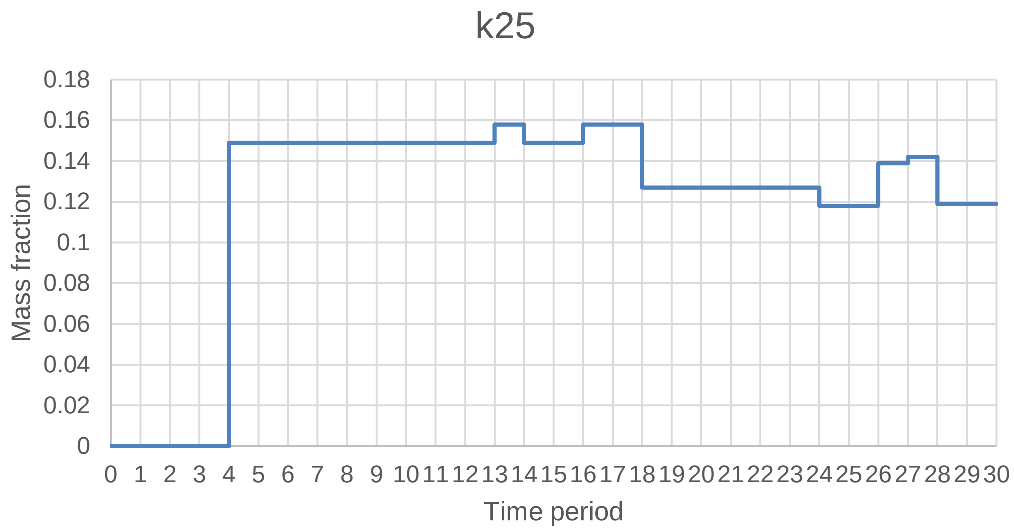
	Time period 27	**Time period 28**
F2 (tons/tp)	303.101	302.317
F25 (tons/tp)	208.098	213.222

Only interdependency constraint 3 is violated in time period 27 and 28.



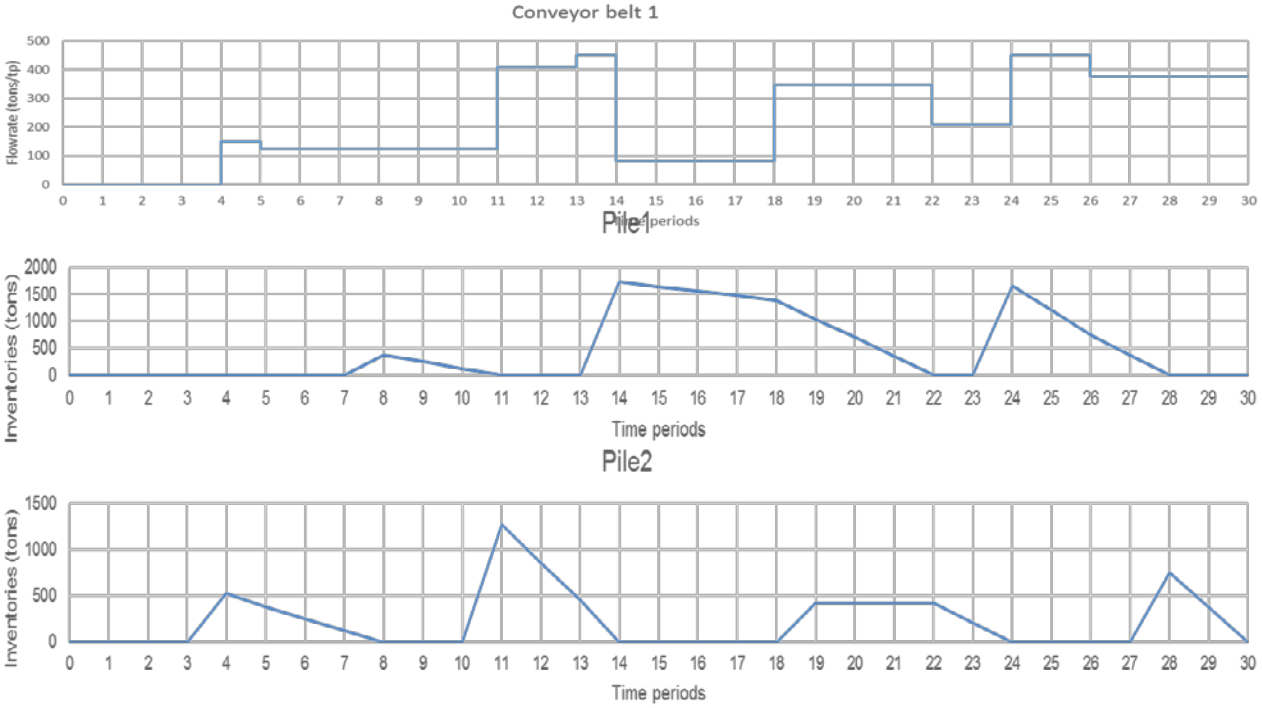
Conclusion & Uniqueness

- Conclusion:
  - The problem in a discrete time formulation is **too large to be solved in a full space**.
  - By applying rolling horizon and MILP-NLP decomposition, A **feasible solution** is available with two non-zero slack variables.
- Uniqueness:
  - The change in concentration of key elements is available:



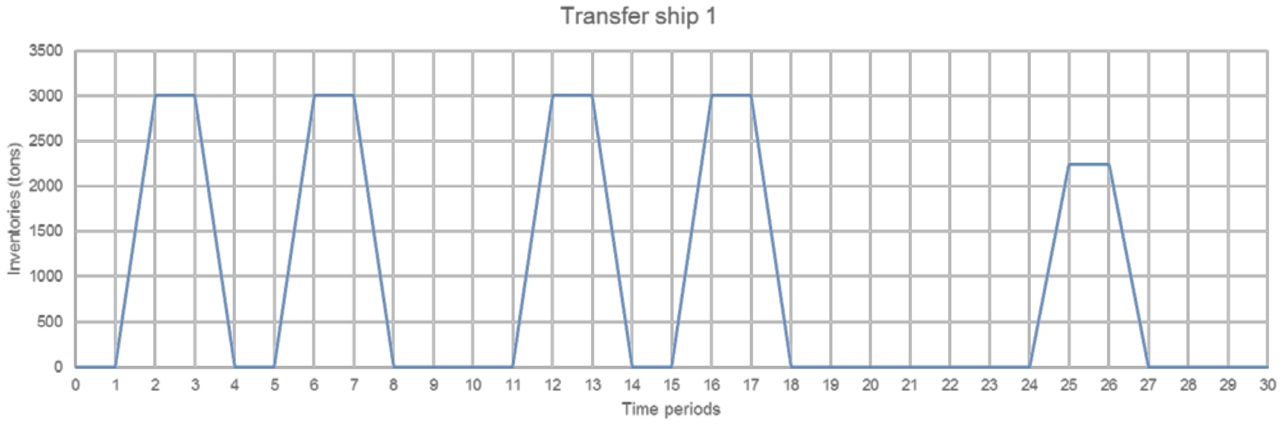
- Pile 1 is set up 3 times.
- Pile 2 is set up 4 times.
- The pile should be empty before switching to another one.

- The behavior of conveyor belts is ruled by specific operation rules **(shut down when empty).**



Conclusion & Uniqueness

- The behavior of transfer ships is ruled by specific operation rules **(transport delay).**



"Transit cycle" means "loading-transporting-unloading-returning"

Transit cycle: 2-5
6-9
12-15
16-19
25-28

- **Future work:**
  - Try to find better solution (**less non-zero slack variables and higher profit**) in GAMS.
  - Try the scheduling problem in **IMPL** (Industrial Modeling and Programming Language).
  - Try the scheduling problem based on **continuous time formulation** in GAMS (mainly by Mangalam Lalpuria).