Medium Term Planning & Scheduling under Uncertainty for BP Chemicals

Progress Report

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BP Chemicals

Outline

1) Products & Applications

- PX
- PTA
- By Products
- Applications

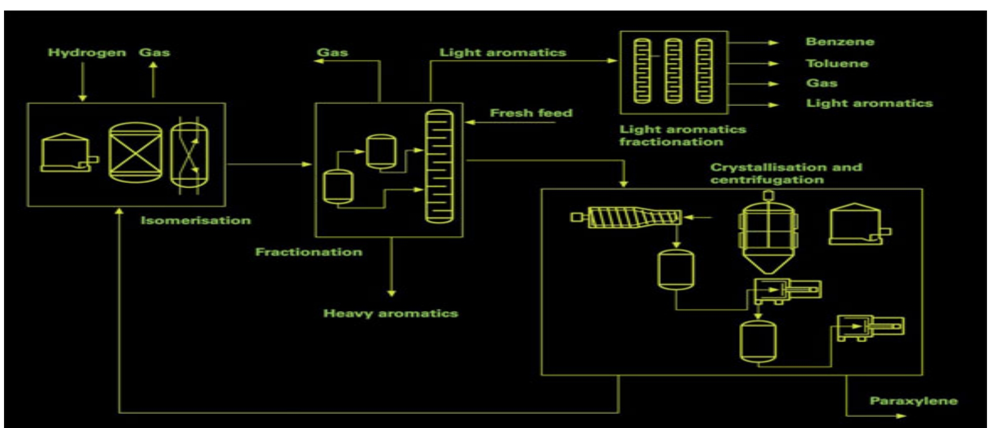
2) Models

- Background
- Overview of New Models
- Schematic Comparison
- Extension to Stochastic Models

3) Results & Future Research

Products - PX

- Paraxylene (PX) : Colorless, flammable liquid that has a sweet odor.
- Separated from a mixed xylene stream that results from the refining of petroleum.
- Areas of use:
  - Feedstock for the local manufacture of Purified Terephthalic Acid (PTA)
  - Sellable to the customers.



Mixed Xylenes

↓

PX

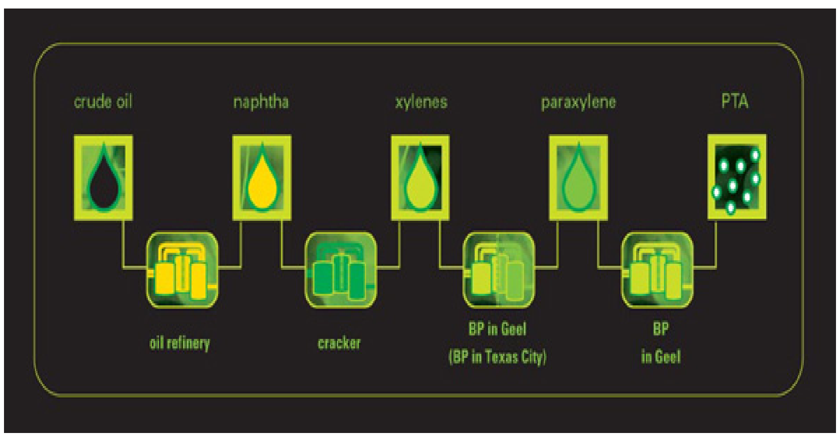
↓

fed into the PTA units / sold

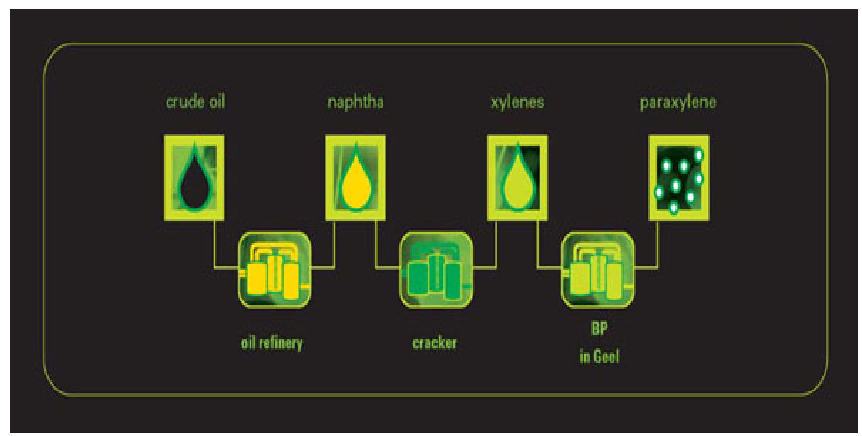
Products - PTA

- Purified Terephthalic Acid (PTA) : an aromatic acid.
- Primarily applied in the production of polyester
- The main raw material for PTA → PX.
- **Production Chains:**

From crude oil to PTA



From crude oil to PX



By Products

- Benzene:
  - Used elsewhere by other BP companies.
  - Used for production of styrene.
  - Styrene can be converted into polystyrene
  - Polystyrene can be used as an insulating material in the construction industry

- Fuel Additives:
  - Used as an additive for petrol production

Applications

- Users:
  - BP's business units
  - Customers of BP's business units

- Usage: Products as chemical intermediates are used in the manufacture of other downstream chemicals.

- Chemical intermediates
  - PX
  - PTA
  - Mixed xylenes
  - Benzene
  - Metaxylene
  - Toluene

- PX → PTA → Polyester fibers, Films, PET-bottles.

Background

- Existing deterministic model for planning medium term operations
  - Monthly production
  - Inventory targets
- What proportion of demand should be satisfied from which inventory location ?
- Types of businesses:
  - PX
  - PTA
- Deterministic model represents:
  - Global production assets & distribution system for these businesses

Background

- Deterministic model does not consider future uncertainties.

- Uncertainty lies behind the future forecasts, how can it be dealt with ?
  - Use Stochastic Programming

- Contributions:

Model extension to cover the probabilistic nature of future economies .

- **2 stage Stochastic Program**
- **Multistage Stochastic Program**

Overview of New Models

- Three models have been analyzed
- In all of the models:
  - 5 scenarios 5 different economic views
  - Operating policy for the first month as a whole constitutes the first-stage decision variables
- Model 1
  - 2 stage Stochastic Linear Program
  - No integrality restrictions
- Model 2
  - Extension of Model 1 with piecewise linear variables
- Model 3
  - Full model (All integrality restrictions)
  - 2 stage Stochastic MIP

Schematic Comparison of Models

LP	Deterministic	2 stage Stochastic
# of Constraints	2558	11086
# of Variables	4716	20444
# of Nonzeros	19354	84090
Time	0.61 sec.	2.24 sec.

PWL	Deterministic	2 stage Stochastic
# of Constraints	2948	12776
# of Variables	4950	21458
# of Nonzeros	21136	91812
Time	1.16 sec.	4.39 sec.

MIP	Deterministic	2 stage Stochastic
# of Constraints	3502	15342
# of Variables	5331	23119
# of Nonzeros	22802	99518
Time	2.52 sec.	~80 sec.

An Extension to Stochastic Models

	FIRST STAGE DECISIONS	SECOND STAGE DECISIONS
Initial	Decisions corresponding to the operating policy for the first time period *Time Periods: 1*	Decisions corresponding to the operating policy for the remaining time periods *Time Periods: 2,3,…*
Generic	Decisions corresponding to the operating policy for the first *i* time periods *Time Periods: 1,..,i*	Decisions corresponding to the operating policy for the remaining time periods *Time Periods: i+1,…*

Results

- Models were solved with data that was close to the actual data used by BP

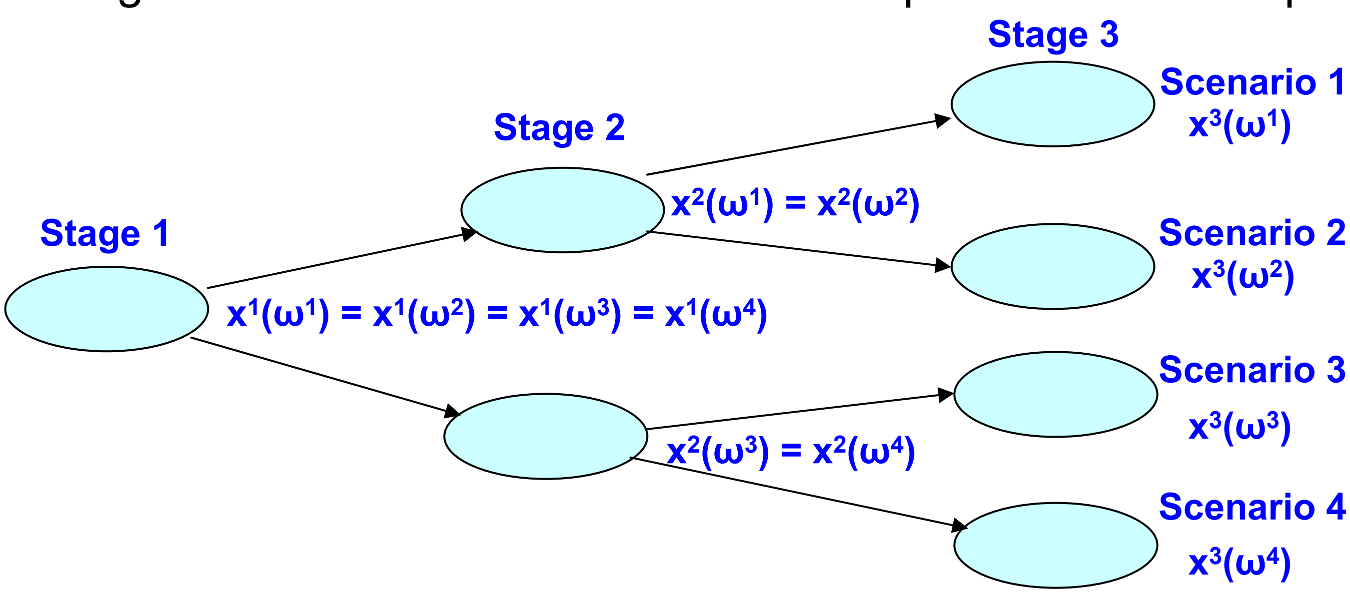
- Performance measures for the stochastic solution (Full 2-stage SMIP Model):
  - *Expected value of perfect information (EVPI)* : the maximum amount a decision maker would be ready to pay in return for complete information about the future
    - EVPI = $120,386
  - *Value of stochastic solution (VSS)*: the possible gain from solving the stochastic model
    - VSS = $1,450,842

Near-term Research

- Current Research: Building all the models as 3 stage stochastic program
- Limitation of solving the extensive form on
  - Number of scenarios
  - Number of stages
- Implementation of the L-shaped method for solving multi-stage stochastic programs
- Extension of the new model to Multistage SMIP using Lagrangian relaxation of nonanticipativity constraints
  - Many such constraints, even with few scenarios/stages
  - Duality gap
  - Finding an approximate solution

Mid-term Research

- Solving SP's with non-anticipativity
  - Decomposable by scenario
  - Non-anticipativity constraints are linking constraints
  - Lagrangian relaxation of linking constraints
  - Large scenario trees → enormous # of possible nonanticipativity constraints



Long-Term Research: Finding Bounds on Multistage SMIPs

- Birge (1982) provided a method for finding bounds on a 2-stage stochastic linear program
- Sandikci (2006) extended this to 2-stage SMIPs
- Gets much better bounds than SLP relaxation (which is how every stochastic branch-and-bound algorithm gets bounds
- We believe it will extend to multistage stochastic SMIPs (such as the BP problem)

Finding Bounds on Multistage SMIPs

- Computing the expected value of the optimal objective values of (Pξ) is called the wait-and-see (WS) solution, i.e.,

$$WS = \mathrm{E}_{\xi}[\ \min_x\ z(x,\xi)]$$
$$= \mathrm{E}_{\xi}[z(\bar{x}(\xi),\xi)].$$

- The optimal objective value to the (RP) problem is called the here-and-now solution, i.e.,

$$RP = min_x\ \ \mathrm{E}_{\xi}[z(x,\xi)].$$

- Proposition 1 (Madansky, 1960): WS $\leq$ RP.

Size-k Group Subproblem

Let ξ be a discrete random vector with finite support $\Xi = \{\xi^0, \xi^1, .., \xi^K\}$ and $P\{\xi = \xi^k\} = p_k$ for $k = 0, 1, .., K$. Suppose scenario ξ^0 is the reference scenario. Given a set Γ_k $\mathcal{P}_k(S)$ for any $k = 1, 2, \ldots, K$, then the *size-k group subproblem* is defined as:

$$(GR(\Gamma_k)) \quad \min \; z_{\{0,\Gamma_k\}} = c^T x + p_0 q^T_{\xi^0} y_{\xi^0} + (1 - p_0)\left(\sum_{i \in \Gamma_k} \frac{p_i}{\sum_{j \in \Gamma_k} p_j} q^T_{\xi^i} y_{\xi^i} \right)$$

$$\begin{aligned}
\text{s.t.} \quad & Ax = b, \\
& T_{\xi^0} x + W_{\xi^0} y_{\xi^0} = h_{\xi^0}, \\
& T_{\xi^i} x + W_{\xi^i} y_{\xi^i} = h_{\xi^i} \qquad i \in \Gamma_k, \\
& x \in R_+^{n_1 - k_1} \times Z_+^{k_1}, \\
& y_{\xi^0} \in R_+^{n_2 - k_2} \times Z_+^{k_2}, \\
& y_{\xi^i} \in R_+^{n_2 - k_2} \times Z_+^{k_2} \quad i \in \Gamma_k.
\end{aligned}$$

Sum of Group Expected Values

Proposition 2 (Birge, 1982): Let ξ be a discrete random variable with finite support whose cardinality is K. Then,

$$WS \leq SPEV \leq RP,$$

- Sum of group expected values with k scenarios, *SGEV*(*k*), is defined as:

$$SGEV(k) \equiv \frac{1}{C_k^K(1-p_0)}\left[\sum_{\Gamma_k \in \mathcal{P}_k(S)} \sum_{i \in \Gamma_k} p_i z^*_{\{0,\Gamma_k\}}\right],$$

- where $z^*_{\{0,\Gamma_k\}}$ is the optimal objective value of the problem *GR*(Γ*k*).

Bounds – Sandikci (2006)

Proposition 3:

$$
\begin{aligned}
WS \leq\ & SPEV = SGEV(1) \\
\leq\ & SGEV(2) \\
& \vdots \\
\leq\ & SGEV(K-1) \\
\leq\ & SGEV(K) = RP.
\end{aligned}
$$

- **Computations:**

- Three test problems from SIPLIB.
  - SIZES (Jorjani et al. 1995): 2 instance SIZES5 and SIZES10
  - DCAP (Ahmed and Garcia 2003): 9 instance dcap233_m, dcap243_m, dcap342_m, m = 200, 300, 500.
- LP relaxation comparison: relaxing 2nd-stage, relaxing both stages.

How Well does this Work for 2 stages?

- We compared the strength of the relaxation for SLP relaxations and SGEV
- For the SIZES and DCAP problems, the CPU times ranged from 0 seconds to 4-5 minutes
- The SLP gaps ranged from 37% to 83%.
- The SGEV gaps ranged from 1.3% to 5.0%
- If our experience in deterministic IP is any guide, this is a very significant improvement
- We believe that in multistage SMIP the relative improvement will be larger