Medium Term Planning & Scheduling under Uncertainty for BP Chemicals

Progress Report

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BP Chemicals

Outline

1) Products & Applications

- PX
- PTA
- By Products
- Applications

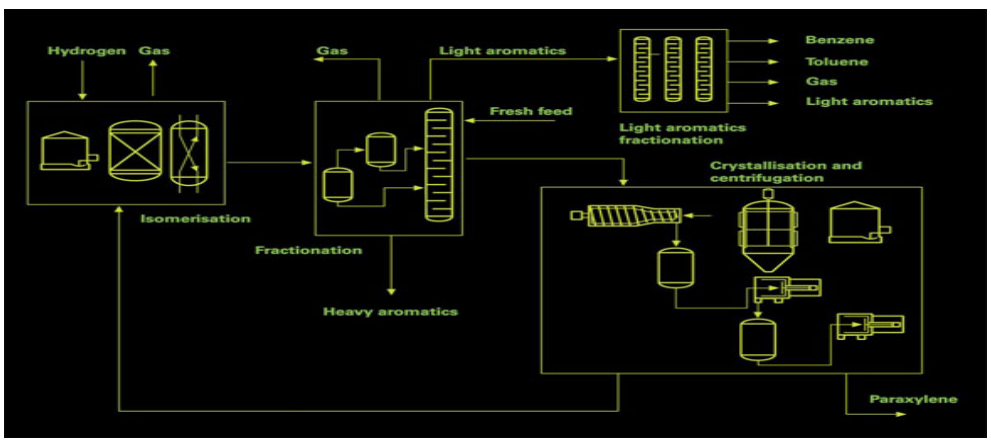
2) Models

- Background
- Overview of New Models
- Schematic Comparison
- Extension to Stochastic Models

3) Results

Products - PX

- Paraxylene (PX) : Colorless, flammable liquid that has a sweet odor.
- Separated from a mixed xylene stream that results from the refining of petroleum.
- Areas of use:
  - Feedstock for the local manufacture of Purified Terephthalic Acid (PTA)
  - Sellable to the customers.



Mixed Xylenes

↓

PX

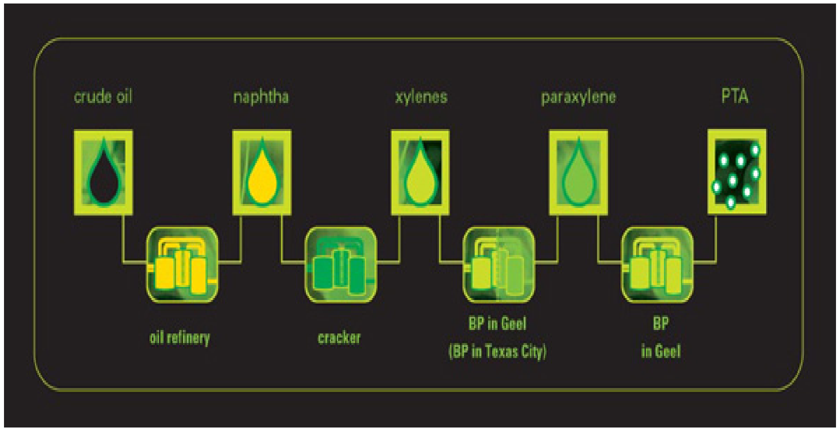
↓

fed into the PTA units / sold

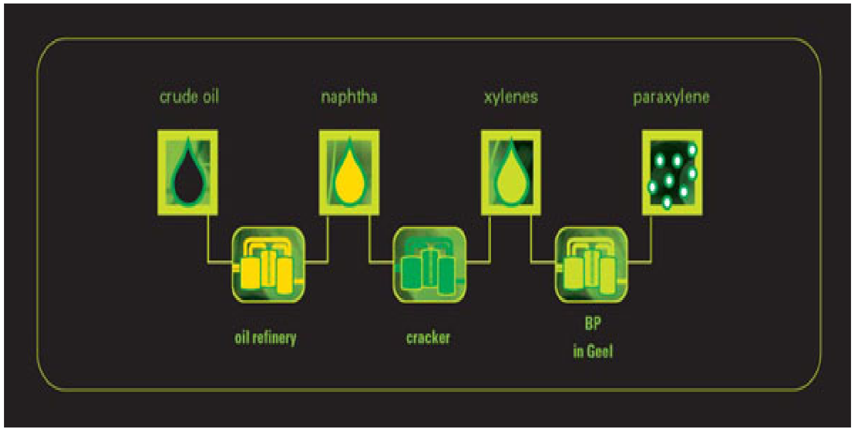
Products - PTA

- Purified Terephthalic Acid (PTA) : an aromatic acid.
- Primarily applied in the production of polyester
- The main raw material for PTA ⟶ PX.
- **Production Chains:**

From crude oil to PTA



From crude oil to PX



By Products

- Benzene:
  - Used elsewhere by other BP companies.
  - Used for production of styrene.
  - Styrene can be converted into polystyrene
  - Polystyrene can be used as an insulating material in the construction industry

- Fuel Additives:
  - Used as an additive for petrol production

Applications

- Users:
  - BP's business units
  - Customers of BP’s business units

- Usage: Products as chemical intermediates are used in the manufacture of other downstream chemicals.

- Chemical intermediates
  - PX
  - PTA
  - Mixed xylenes
  - Benzene
  - Metaxylene
  - Toluene

- PX → PTA → Polyester fibers
  - PTA → Films
  - PTA → PET-bottles.

Background

- Existing deterministic model for planning medium term operations
  - Monthly production
  - Inventory targets
- What proportion of demand should be satisfied from which inventory location ?
- Types of businesses:
  - PX
  - PTA
- Deterministic model represents:
  - Global production assets & distribution system for these businesses

Background

- Deterministic model does not consider future uncertainties.

- Uncertainty lies behind the future forecasts, how can it be dealt with ?

  - Use Stochastic Programming

- Contributions:

  Model extension to cover the probabilistic nature of future economies .

  - **2 stage Stochastic Program**
  - **Multistage Stochastic Program**

Overview of Models

- Initially, three models were analyzed
- In all of the models:
  - 5 scenarios 5 different economic views
  - Operating policy for the first month as a whole constitutes the first-stage decision variables
- Model 1
  - 2 stage Stochastic Linear Program
  - No integrality restrictions
- Model 2
  - Extension of Model 1 with piecewise linear variables
- Model 3
  - Full model (All integrality restrictions)
  - 2 stage Stochastic MIP

Schematic Comparison of Models

LP	Deterministic	2 stage Stochastic
# of Constraints	2558	11086
# of Variables	4716	20444
# of Nonzeros	19354	84090
Time	0.61 sec.	2.24 sec.

PWL	Deterministic	2 stage Stochastic
# of Constraints	2948	12776
# of Variables	4950	21458
# of Nonzeros	21136	91812
Time	1.16 sec.	4.39 sec.

MIP	Deterministic	2 stage Stochastic
# of Constraints	3502	15342
# of Variables	5331	23119
# of Nonzeros	22802	99518
Time	2.52 sec.	~80 sec.

An Extension to Stochastic Models

	FIRST STAGE DECISIONS	SECOND STAGE DECISIONS
Initial	Decisions corresponding to the operating policy for the first time period ***Time Periods: 1***	Decisions corresponding to the operating policy for the remaining time periods ***Time Periods: 2,3,…***
Generic	Decisions corresponding to the operating policy for the first *i* time periods ***Time Periods: 1,..,i***	Decisions corresponding to the operating policy for the remaining time periods ***Time Periods: i+1,…***

Results

- Models were solved with data that was close to the actual data used by BP

- Performance measures for the stochastic solution (Full 2-stage SMIP Model):
  - *Expected value of perfect information (EVPI)* : the maximum amount a decision maker would be ready to pay in return for complete information about the future
    - EVPI = $120,386
  - *Value of stochastic solution (VSS)*: the possible gain from solving the stochastic model
    - VSS = $1,450,842

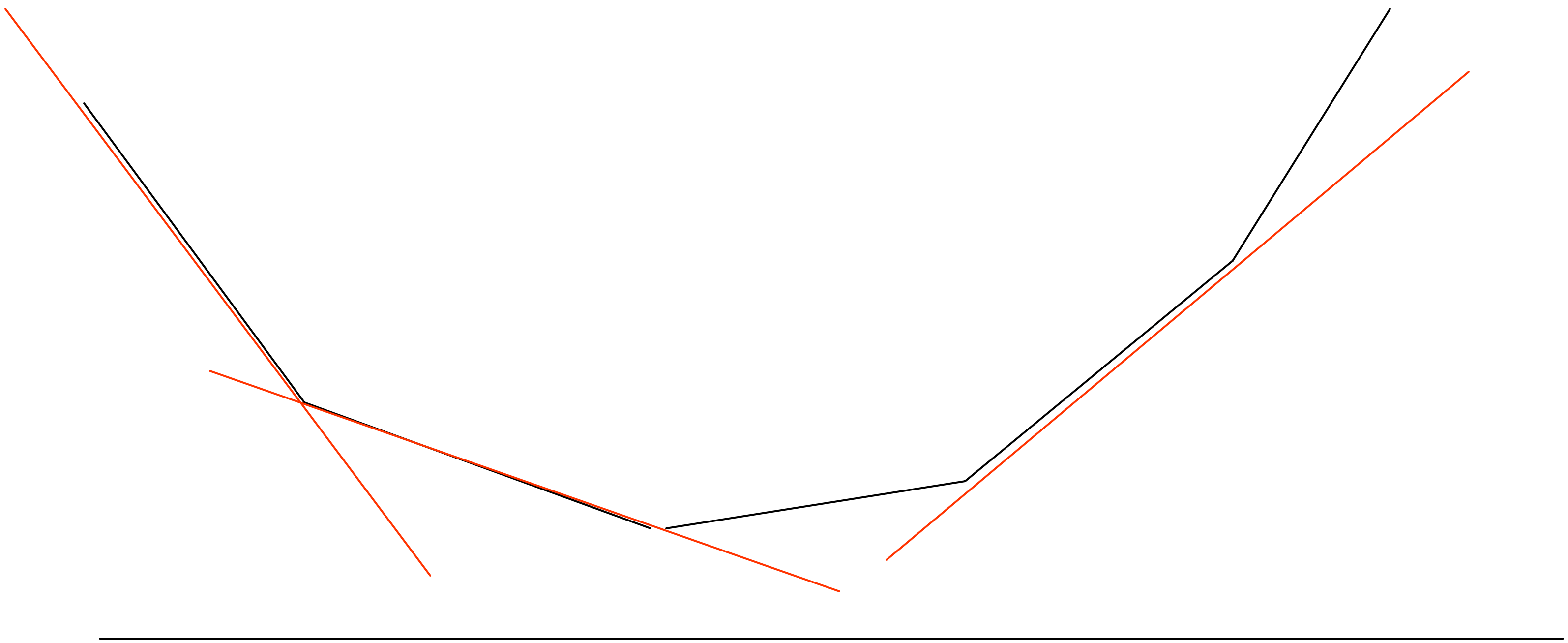
Second Phase

- After the November EWO meeting, the emphasis was shifted to implementing decomposition approaches in AIMMS

- Benders' Decomposition is the key to solving larger stochastic programs

- Before the project ended, we had implemented Benders' decomposition for a classical application (capacitated facility location) in AIMMS

Expected Recourse Function



The expected recourse function Q(x) is convex and, if there are a finite number of scenarios, is also piece-wise linear

- L-shaped optimality cuts support Q(x) from below. If there are a finite number of scenarios, there are a finite number of optimality cuts

Importance of SP in EWO

- Uncertainty is pervasive in EWO

- Many EWO problems are well suited for mathematical programming

- The resulting problems are extremely hard:
  - Stochastic MINLPs
  - Multistage stochastic programs
  - High-performance computing solutions (clusters, grid computing)
  - Approximate solution techniques
  - Exploiting special structure

Long-Term Research: Finding Bounds on Multistage SMIPs

- Birge (1982) provided a method for finding bounds on a 2-stage stochastic linear program
- Sandikci (2006) extended this to 2-stage SMIPs
- Gets much better bounds than SLP relaxation (which is how every stochastic branch-and-bound algorithm gets bounds
- We believe it will extend to multistage stochastic SMIPs
- Are currently extending this technique to finding upper bounds

How Well does this Work for 2 stages?

- We compared the strength of the relaxation for SLP relaxations and SGEV
- For the SIZES and DCAP problems, the CPU times ranged from 0 seconds to 4-5 minutes
- The SLP gaps ranged from 37% to 83%.
- The SGEV gaps ranged from 1.3% to 5.0%
- If our experience in deterministic IP is any guide, this is a very significant improvement
- We believe that in multistage SMIP the relative improvement will be larger